Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 10, No. 4, (October 2015), pp. 629–637
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

On the parameter reduction of soft binary relations

Guangji Yu

Received 5 February 2015; Revised 14 March 2015; Accepted 1 April 2015

Abstract. Soft set theory is a mathematical tool for dealing with uncertain problems. In this paper, the parameter reduction of soft binary relations is investigated and its algorithm is given. Moreover, parameters in a soft binary relation are divided into four categories (i.e., necessary parameters, relatively necessary parameters, absolutely dispensable parameters, dispensable parameters) according to the importance.

2010 AMS Classification: 49J53.

Keywords: Soft set; Soft binary relation; Core; Parameter reduction; Algorithm.

Corresponding Author: Guangji Yu (guangjiyu100@126.com)

1. Introduction

To solve complicated problems in economics, engineering, environmental science and social science, methods in classical mathematics are not always successful because of various types of uncertainties present in these problems. There are several theories: theory of probability, theory of fuzzy sets [14], rough set theory [13] and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties (see [8]). To overcome these kinds of difficulties, Molodtsov [7] proposed a completely new approach, which is called soft set theory, for modeling uncertainty.

Presently, works on theory of soft sets are progressing rapidly. Maji et al. [9, 10] further studied the theory of soft sets, used this theory to solve some decision making problems. Jiang et al. [4] extended soft sets with description logics. Ge et al. [3] discussed relationships between soft sets and topological spaces. Li et al. [6] investigated relationships among soft sets, soft rough sets and topologies. Majumdar et al. [11] studied softness of a soft set.

Soft set itself has classification ability. Parameter reductions of soft sets mean reducing the number of parameters for a soft set to the minimum without distorting its original classification ability. Parameter reductions of soft sets play a vital role in decision-making problems and can save expensive tests and time. Thus, parameter reductions of soft sets are very important. Many authors studied parameter reductions of soft sets (see [2, 5, 12]).

Soft binary relations are introduced by Ali [1]. The purpose of this paper is to investigate the parameter reduction of soft binary relations.

2. **Soft binary relations**

Definition 2.1. Let U be an initial universe and let E be a set of parameters. A pair (f, E) is called a soft set over U, if f is a mapping given by $f : E \to 2^U$ where 2^U is the power set of U. We denote (f, E) by f_E.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in E$, $f(e)$ may be considered as the set of e-approximate elements of the soft set f_E.

Example 2.2. Let $U = \{h_1, h_2, h_3, h_4, h_5\}$ be a universe consisting of five stores. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ be is a set of status of stores where e_1, e_2, e_3, e_4, e_5, e_6 and e_7 represent respectively the parameters "high empowerment of sales personnel", "medium empowerment of sales personnel", "low empowerment of sales personnel", "good perceived quality of merchandise", "average perceived quality of merchandise", "high traffic location" and "low traffic location", respectively. We define f_A as follows

$$f(e_1) = \{h_1\},\ \ f(e_2) = \{h_2, h_3, h_5\},\ \ f(e_3) = \{h_4\},\ \ f(e_4) = \{h_1, h_2, h_3\},$$
$$f(e_5) = \{h_4, h_5\},\ \ f(e_6) = \{h_1, h_2, h_3\},\ \ f(e_7) = \{h_4, h_5\}.$$

Soft sets f_A can be described as the following Table 1. If $h_i \in f(e_j)$, then $h_{ij} = 1$; otherwise $h_{ij} = 0$, where h_{ij} are the entries in Table 1.

TABLE 1. Tabular representation of the soft set f_E

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
h_1	1	0	0	1	0	1	0
h_2	0	1	0	1	0	1	0
h_3	0	1	0	1	0	1	0
h_4	0	0	1	0	1	0	1
h_5	0	1	0	0	1	0	1

Definition 2.3 ([1])**.** A pair (σ, E) is called a soft binary relation over U, if (σ, E) is a soft set over $U \times U$. We denote (σ, E) by σ_E.

In other words, a soft binary relation over U is a parameterized family of binary relations on U.

3. **The parameter reduction of soft binary relations**

Parameter reductions of soft binary relations mean reducing the number of parameters for a soft binary relation to the minimum without distorting its original classification ability. Specific approach is first classifying the parameter according to the importance of parameters and then finding the minimum set of parameters (ie., the core for a soft binary relation) without distorting the original classification ability of soft sets.

Let σ_E be a soft binary relation over U. For $A \subseteq E$, denote

$$ind(A) = \bigcap_{e \in A} \sigma(e).$$

Definition 3.1. Let σ_E be a soft binary relation over U and $A \subseteq E$.

(1) A is called coordinate in σ_E, if $ind(E) = ind(A)$.

(2) $e \in A$ is called independent in A, if $ind(A - \{e\}) \neq ind(A)$; A is called independent in σ_E, if $\forall\ e \in A$, e is independent in A.

(3) A is called a parameter reduction in σ_E, if A is both coordinate and independent in σ_E.

In this paper, the family of all coordinate sets (resp., all parameter reductions) in σ_E is denoted by $co(\sigma_E)$ (resp., $red(\sigma_E)$).

Obviously,

$$A \in red(\sigma_E) \iff A \in co(\sigma_E)\ and\ \forall\ B \subset A, B \notin co(\sigma_E).$$

Proposition 3.2. *Let σ_E be a soft binary relation over U. Then there always exist a parameter reduction in σ_E.*

Proof. Suppose $\forall\ e \in E$, $E - \{e\} \notin co(\sigma_E)$. Then $E \in red(\sigma_E)$.

Suppose $\exists\ e_1 \in E$, $E - \{e_1\} \in co(\sigma_E)$. Then, we consider $E - \{e_1\}$. Again suppose $\forall\ e \in E - \{e_1\}$, $(E - \{e_1\}) - \{e\} \notin co(\sigma_E)$. Then $E - \{e_1\} \in red(\sigma_E)$. Again suppose $\exists\ e_2 \in E - \{e_1\}$, $(E - \{e_1\}) - \{e_2\} \in co(\sigma_E)$. Then, we consider $E - \{e_1, e_2\}$. Repeat this process. Since E is finite, we can find a parameter reduction in σ_E. Thus, there always exist a parameter reduction in σ_E. □

Definition 3.3. Let σ_E be a soft binary relation over U. Put

$$\mathcal{D}(x, y) = \{e \in E | (x, y) \notin \sigma(e)\}\ (x, y \in U).$$

Then

(1) $\mathcal{D}(x, y)$ is called the discernibility set in σ_E on x and y.

(2) $\mathfrak{D}(\sigma_E) = (d_{ij})_{n \times n}$ is called the discernibility matrix in σ_E where $U = \{x_1, x_2, \cdots, x_n\}$ and $d_{ij} = \mathcal{D}(x_i, x_j)$ $(1 \leq i, j \leq n)$.

Example 3.4. Let σ_E be a soft binary relation over U where $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $E = \{e_1, e_2, e_3, e_4\}$.

$U/\sigma(e_1) = \{\{x_1, x_2, x_5\}, \{x_3, x_4, x_6\}\}$,
$U/\sigma(e_2) = \{\{x_1, x_6\}, \{x_2, x_3, x_4, x_5\}\}$,
$U/\sigma(e_3) = \{\{x_1, x_2, x_5, x_6\}, \{x_3, x_4\}\}$,
$U/\sigma(e_4) = \{\{x_1, x_2, x_5\}, \{x_3, x_4, x_6\}\}$.

We can obtain the discernibility matrix $\mathfrak{D}(\sigma_E)$ as follows:

$$\begin{pmatrix} \varnothing & \{e_2\} & E & E & \{e_2\} & \{e_1,e_4\} \\ \{e_2\} & \varnothing & \{e_1,e_3,e_4\} & \{e_1,e_3,e_4\} & \varnothing & \{e_1,e_2,e_4\} \\ E & \{e_1,e_3,e_4\} & \varnothing & \varnothing & \{e_1,e_3,e_4\} & \{e_2,e_3\} \\ E & \{e_1,e_3,e_4\} & \varnothing & \varnothing & \{e_1,e_3,e_4\} & \{e_2,e_3\} \\ \{e_2\} & \varnothing & \{e_1,e_3,e_4\} & \{e_1,e_3,e_4\} & \varnothing & \{e_1,e_2,e_4\} \\ \{e_1,e_4\} & \{e_1,e_2,e_4\} & \{e_2,e_3\} & \{e_2,e_3\} & \{e_1,e_2,e_4\} & \varnothing \end{pmatrix}$$

Proposition 3.5. *Let σ_E be a soft binary relation over U. Then*

$$A \in co(\sigma_E) \iff If\ (x,y) \notin ind(E),\ then\ A \cap \mathcal{D}(x,y) \neq \varnothing.$$

Proof. (1) "$\Longrightarrow$". Since $A \in co(\sigma_E)$, we have $ind(A) = ind(E)$. Note that $(x,y) \notin ind(E)$. Then $(x,y) \notin ind(A)$. So $(x,y) \notin \sigma(e)\ for\ some\ e \in A$.

$(x,y) \notin \sigma(e)$ implies $e \in \mathcal{D}(x,y)$. Then $e \in A \cap \mathcal{D}(x,y)$.

Thus $A \cap \mathcal{D}(x,y) \neq \varnothing$.

"$\Longleftarrow$". Suppose $A \notin co(\sigma_E)$. Then $ind(A) \neq ind(E)$. This implies $ind(A) - ind(E) \neq \varnothing$. Pick

$$(x,y) \in ind(A) - ind(E).$$

Since $(x,y) \notin ind(E)$, we have $A \cap \mathcal{D}(x,y) \neq \varnothing$.

Note that $(x,y) \in ind(A)$. Then $\forall\ e \in A$, $(x,y) \in \sigma(e)$. So $e \notin \mathcal{D}(x,y)$. Thus $A \cap \mathcal{D}(x,y) = \varnothing$. This is a contradiction. Thus $A \in co(\sigma_E)$. □

The discernibility set can easily determine parameter reductions.

Theorem 3.6. *Let σ_E be a soft binary relation over U. Then $A \in red(\sigma_E) \iff$ (1) If $(x,y) \notin ind(E)$, then $A \cap \mathcal{D}(x,y) \neq \varnothing$;*

(2) $\forall\ e \in A$, $\exists\ (x_e,y_e) \in ind(E)$, $(A - \{e\}) \cap \mathcal{D}(x_e,y_e) = \varnothing$.

Proof. This holds by Proposition 3.5. □

Definition 3.7. Let σ_E be a soft binary relation over U. Put

$$core(\sigma_E) = \bigcap_{A \in red(\sigma_E)} B.$$

Then $core(\sigma_E)$ is called the core of E. Moreover,

(1) $e \in E$ is called a necessary parameter, if $e \in core(\sigma_E)$.

(2) $e \in E$ is called a relatively necessary parameter, if $e \in \bigcup_{A \in red(\sigma_E)} A - core(\sigma_E)$.

(3) $e \in E$ is called a absolutely dispensable parameter, if $e \in E - \bigcup_{A \in red(\sigma_E)} A$.

(4) $e \in E$ is called a dispensable parameter, if $e \in E - core(\sigma_E)$.

Obviously, e is dispensable $\iff$ e is relatively necessary or absolutely dispensable.

Proposition 3.8. *Let σ_E be a soft binary relation over U. Then*

$$|red(\sigma_E)| = 1 \iff core(\sigma_E) \in red(\sigma_E).$$

Proof. Necessity. This is obvious.

Sufficiency. Denote $red(\sigma_E) = \{A_k : 1 \leq k \leq n\}$. We only need to prove $n = 1$.

Suppose $n \geq 2$. Since $core(\sigma_E) \in red(\sigma_E)$, we have $core(\sigma_E) = A_i$ for some i. Pick $j \neq i$. Then $A_i = \bigcap_{k=1}^{n} A_k \subseteq A_j$. But $A_i \neq A_j$. Thus $A_i \subset A_j$. Since $A_j \in red(\sigma_E)$, we have $A_i \notin co(\sigma_E)$. Then $A_i \notin red(\sigma_E)$. This is a contradiction. Thus $n = 1$. □

The discernibility set can easily determine the core.

Proposition 3.9. *Let σ_E be a soft binary relation over U. The following are equivalent:*
(1) *e is a necessary parameter;*
(2) *e is independent in E;*
(3) *$\exists\, x, y \in U$, $\mathcal{D}(x, y) = \{e\}$.*

Proof. (1) $\Longrightarrow$ (2). Suppose that e is not independent in E. Then

$$ind(E - \{e\}) = ind(E).$$

This implies $E - \{e\} \in co(\sigma_E)$. Consider $E - \{e\}$. By Proposition 3.2, $\exists\, A \subseteq E - \{e\}$, $A \in red(\sigma_E)$.

$A \subseteq E - \{e\}$ implies $e \notin A$. Then e is not necessary. This is a contradiction.

(2) $\Longrightarrow$ (1). Suppose that e is not necessary. Then $\exists\, A \in red(\sigma_E)$, $e \notin A$. So $A \subseteq E - \{e\} \subseteq E$. This implies

$$ind(A) \supseteq ind(E - \{e\}) \supseteq ind(E).$$

By $A \in red(\sigma_E)$, $ind(A) = ind(E)$. Then $ind(E - \{e\}) = ind(E)$. So e is not independent in E. This is a contradiction.

(2) $\Longrightarrow$ (3). Since e is independent in E, we have $ind(E - \{e\}) \neq ind(E)$. Then $ind(E - \{e\}) - ind(E) \neq \varnothing$. Pick

$$(x, y) \in ind(E - \{e\}) - ind(E).$$

Denote $E = \{e_1, e_2, \ldots, e_n\}$. Then $e = e_j$ for some $j \leq n$. So

$$(x, y) \in \bigcap_{1 \leq i \leq n, i \neq j} e_i - \bigcap_{1 \leq i \leq n} e_i.$$

This implies $(x, y) \notin e_j$ and $(x, y) \in e_i$ $(i \neq j)$.

Thus $\mathcal{D}(x, y) = \{e\}$.

(3) $\Longrightarrow$ (2). Since $\exists\, x, y \in U$, $\mathcal{D}(x, y) = \{e\}$, we have

$$(x, y) \notin \sigma(e),\ (x, y) \in \sigma(e')\ (e' \neq e).$$

Then $(x, y) \in ind(E - \{e\})$. But $(x, y) \notin ind(E)$. This implies $ind(E - \{e\}) \neq ind(E)$. Thus e is independent in E. □

Proposition 3.10. *Let σ_E be a soft binary relation over U. Denote*

$$R^\star = \bigcup_{A \in co(\sigma_E)} ind(A - \{e\}).$$

Then the following are equivalent:
(1) *e is a absolutely dispensable parameter;*
(2) *$\forall\, A \in co(\sigma_E)$, $ind(A - \{e\}) = ind(E)$;*
(3) *$R^\star = ind(E)$;*

(4) $R^\star \subseteq \sigma(e)$.

Proof. (1) $\Longrightarrow$ (2). By Proposition 3.2, $\exists\ B \subseteq A$, $B \in red(\sigma_E)$. Since e is not necessary, we have $e \notin B$, which implies $B \subseteq E - \{e\}$. Then

$$B \subseteq A \cap (E - \{e\}) = A - \{e\} \subseteq A.$$

We have

$$ind(B) \supseteq ind(A - \{e\})) \supseteq ind(A).$$

Note that $A \in co(\sigma_E)$ and $B \in red(\sigma_E)$. Then $ind(A) = ind(E) = ind(B)$. Thus

$$ind(A - \{e\}) = ind(E).$$

(2) $\Longrightarrow$ (3) $\Longrightarrow$ (4) are obvious.

(4) $\Longrightarrow$ (1). Suppose $\exists\ A \in red(\sigma_E)$, $e \in A$. Then $A - \{e\} \subset A$. Since $A \in red(\sigma_E)$, we have $A - \{e\} \notin co(\sigma_E)$. Then $ind(A - \{e\}) - ind(E) \neq \varnothing$. $A \in red(\sigma_E)$ implies $ind(A) = ind(E)$. Then

$$ind(A - \{e\}) - ind(A) \neq \varnothing.$$

Pick $(x, y) \in ind(A - \{e\}) - ind(A)$. Note that $ind(A) = ind(A - \{e\}) \cap \sigma(e)$. Then $(x, y) \notin \sigma(e)$.

Since $A \in co(\sigma_E)$ and $R^\star \subseteq \sigma(e)$, we have $ind(A - \{e\}) \subseteq \sigma(e)$. Then $(x, y) \in \sigma(e)$. This is a contradiction. □

Theorem 3.11. *Let σ_E be a soft binary relation over U. Then*
(1) *e is necessary $\Leftrightarrow E - \{e\} \notin co(\sigma_E)$.*
(2) *e is relatively necessary $\Leftrightarrow E - \{e\} \in co(\sigma_E)$ and $R^\star \nsubseteq \sigma(e)$.*
(3) *e is absolutely dispensable $\Leftrightarrow R^\star \subseteq \sigma(e)$.*
(4) *e is dispensable $\Leftrightarrow E - \{e\} \in co(\sigma_E)$.*

Proof. This holds by Propositions 3.9 and 3.10. □

Example 3.12. In Example 3.4, we have
(1) e_2 is necessary.
(2) e_1 and e_4 are relatively necessary.
(3) e_3 is absolutely dispensable.
(4) e_1, e_3 and e_4 are dispensable.

4. **An algorithm**

It is more convenient to calculate parameter reductions and the core in a soft binary relation by using the following discernibility function when there are many binary relations in a soft binary relation.

Below, we give an algorithm on parameter reductions of a soft binary relation with the help of mathematical logic.

"$\bigvee$"(disjunction), "$\bigwedge$"(conjunction), "$\longrightarrow$"(implication), "$\longleftrightarrow$"(biimplication) are propositional connectives in mathematical logic. They are read as "or", "and", "if-then", "if and only if", respectively.

Let σ_E be a soft binary relation over U. $\forall\ e \in E$, we specify a Boolean variable "e". If $\mathcal{D}(x, y) = \{e_1, e_2, \cdots, e_k\}$ $(x, y \in U)$, then we specify a Boolean function $e_1 \vee e_2 \vee \cdots \vee e_k$.

Denote

$$\bigvee\{e_1, e_2, \cdots, e_k\} \text{ or } \bigvee_{i=1}^{k} e_i = e_1 \vee e_2 \vee \cdots \vee e_k,$$

$$\bigwedge\{e_1, e_2, \cdots, e_k\} \text{ or } \bigwedge_{i=1}^{k} e_i = e_1 \wedge e_2 \wedge \cdots \wedge e_k.$$

We stipulate that $\vee \varnothing = 1$ and $\wedge \varnothing = 0$ where 0 and 1 are two Boolean constants.

Definition 4.1. Let σ_E be a soft binary relation over U where $U = \{x_1, x_2, \cdots, x_n\}$ and $\mathfrak{D}(\sigma_E) = (d_{ij})_{n\times n}$ the discernibility matrix in σ_E. We define the discernibility function $\Delta(\sigma_E)$ in σ_E as follows:

$$\Delta(\sigma_E) = \bigwedge(\bigvee d_{ij}).$$

Example 4.2. In Example 3.4, we have

$$\triangle(\sigma_E) = e_2 \wedge (e_1 \vee e_2 \vee e_3 \vee e_4) \wedge (e_1 \vee e_4) \wedge (e_1 \vee e_3 \vee e_4) \wedge (e_1 \vee e_2 \vee e_4) \wedge (e_2 \vee e_3).$$

Definition 4.3. Let σ_E be a soft binary relation over U. If $\Delta(\sigma_E) = \bigvee\limits_{k=1}^{q}(\bigwedge\limits_{l=1}^{p_k} e_{kl})$, where every $A_k = \{e_{kl} : l \leq p_k\} \subseteq E$ has not repetitive elements, then $\bigvee\limits_{k=1}^{q}(\bigwedge\limits_{l=1}^{p_k} e_{kl})$ is called the standard minimum formula of $\Delta(\sigma_E)$. We denote it by $\Delta^*(\sigma_E)$. That is,

$$\Delta^*(\sigma_E) = \bigvee_{k=1}^{q}(\bigwedge_{l=1}^{p_k} e_{kl}).$$

Example 4.4. In Example 3.4, we have

$$e_2 \leq (e_1 \vee e_2 \vee e_3 \vee e_4), \quad e_2 \leq (e_1 \vee e_2 \vee e_4), \quad e_2 \leq (e_2 \vee e_3), \quad (e_1 \vee e_4) \leq (e_1 \vee e_3 \vee e_4).$$

Obviously,

$$e_2 \wedge (e_1 \vee e_2 \vee e_3 \vee e_4) = e_2, \quad e_2 \wedge (e_1 \vee e_2 \vee e_4) = e_2,$$

$$e_2 \wedge (e_2 \vee e_3) = e_2, \quad (e_1 \vee e_4) \wedge (e_1 \vee e_3 \vee e_4) = e_1 \vee e_4.$$

$$\begin{aligned} \text{Then } \Delta(\sigma_E) &= e_2 \wedge (e_1 \vee e_2 \vee e_3 \vee e_4) \wedge (e_1 \vee e_4) \wedge (e_1 \vee e_3 \vee e_4) \wedge (e_1 \vee e_2 \vee e_4) \wedge (e_2 \vee e_3) \\ &= e_2 \wedge (e_1 \vee e_4) \\ &= (e_1 \wedge e_2) \vee (e_2 \wedge e_4). \end{aligned}$$

Thus $\Delta^*(\sigma_E) = (e_1 \wedge e_2) \vee (e_2 \wedge e_4)$.

Theorem 4.5. *Let σ_E be a soft binary relation over U. If $\Delta^*(\sigma_E) = \bigvee\limits_{k=1}^{q}(\bigwedge\limits_{l=1}^{p_k} e_{kl})$ is the standard minimum formula of $\Delta(\sigma_E)$. Then $red(\sigma_E) = \{A_k : k \leq q\}$ where $A_k = \{e_{kl} : l \leq p_k\}$.*

Proof. (1) Let $A_{k_0} \in \{A_k : k \leq q\}$.

(*i*) Obviously, $\Delta^*(\sigma_E) = \bigvee\limits_{k=1}^{q}(\bigwedge\limits_{l=1}^{p_k} e_{kl}) = \bigvee\limits_{k=1}^{q}(\bigwedge A_k)$. Then $\bigwedge A_{k_0} \longrightarrow \Delta^*(\sigma_E)$.

Let σ_E be a soft binary relation over U. The algorithm of parameter reductions of σ_E is shown as follows:

Input: the soft binary relation σ_E over U;
Output: $red(\sigma_E)$ and $core(\sigma_E)$.

Step 1. Input the soft binary relation σ_E over U;
Step 2. Calculate the discernibility matrix $\mathfrak{D}(\sigma_E)$ in σ_E;
Step 3. Give discernibility function $\Delta(\sigma_E)$ in σ_E;
Step 4. Calculate standard minimum formula $\Delta^*(\sigma_E)$ of $\Delta(\sigma_E)$;
Step 5. Output all parameter reductions and the core in σ_E.

Since $\Delta^*(\sigma_E) = \Delta(\sigma_E) = \bigwedge(\bigvee d_{ij})$, we have

$$\Delta^*(\sigma_E) \Longleftrightarrow \bigvee d_{ij} \ for \ any \ 1 \leq i, j \leq n.$$

Then $\forall\ x, y \in U$, $\bigwedge A_{k_0} \longrightarrow \bigvee \mathcal{D}(x, y)$.

So $\forall\ (x, y) \notin ind(E)$, $\bigwedge A_{k_0} \longrightarrow \bigvee \mathcal{D}(x, y)$.

Now $\bigwedge A_{k_0} \Longleftrightarrow e_{k_0 l}$ for any $l \leq p_{k_0}$ and $\bigvee \mathcal{D}(x, y) \longleftrightarrow e$ for some $e \in \mathcal{D}(x, y)$. Then $\forall\ (x, y) \notin ind(E)$, $e_{k_0 l}$ for any $l \leq p_{k_0} \longrightarrow e$ for some $e \in \mathcal{D}(x, y)$. So $\forall\ (x, y) \notin ind(E)$, there exists $l_0 \leq p_{k_0}$ such that $e = e_{k_0 l_0}$, i.e., $e \in A_{k_0} \cap \mathcal{D}(x, y)$. Thus $\forall\ (x, y) \notin ind(E)$, $A_{k_0} \cap \mathcal{D}(x, y) \neq \varnothing$.

By Proposition 3.5, $A_{k_0} \in co(\sigma_E)$.

(ii) To prove $A_{k_0} \in red(\sigma_E)$, by Theorem 3.6, we only need to show that

$$\forall\ e \in E_{k_0},\ \exists\ (x_e, y_e) \in ind(E),\ (A_{k_0} - \{e\}) \cap \mathcal{D}(x_e, y_e) = \varnothing.$$

Suppose that $\exists\ e_0 \in A_{k_0}$ such that $(A_{k_0} - \{e_0\}) \cap \mathcal{D}(x, y) \neq \varnothing$ for any $(x, y) \notin ind(E)$. Pick $e_{xy} \in (A_{k_0} - \{e_0\}) \cap \mathcal{D}(x, y)$. Then $\bigwedge(A_{k_0} - \{e_0\}) \longrightarrow e_{xy}$ and $e_{xy} \longrightarrow \bigvee \mathcal{D}(x, y)$. Thus $\forall\ (x, y) \notin ind(E)$, $\bigwedge(A_{k_0} - \{e_0\}) \longrightarrow \bigvee \mathcal{D}(x, y)$.

$\forall\ (x, y) \in ind(E)$, we have $\mathcal{D}(x, y) = \varnothing$. Then $\bigwedge(A_{k_0} - \{e_0\}) \longrightarrow \bigvee \mathcal{D}(x, y)$.

It follows that $\forall\ x, y \in U$,

$$\bigwedge(A_{k_0} - \{e_0\}) \longrightarrow \bigvee \mathcal{D}(x, y).$$

Since $\Delta^*(\sigma_E)$ contains all true explanations of $\Delta(\sigma_E)$, we have $A_{k_0} - \{e_0\} \in \{A_k : k \leq q\}$. Then

$$\begin{aligned}
&(\bigwedge A_{k_0}) \bigvee (\bigwedge(A_{k_0} - \{e_0\})) \\
&= ((\bigwedge(A_{k_0} - \{e_0\})) \bigwedge \{e_0\}) \bigvee ((\bigwedge(A_{k_0} - \{e_0\})) \bigwedge 1) \\
&= (\bigwedge(A_{k_0} - \{e_0\})) \bigwedge(\{e_0\} \bigvee 1) \\
&= (\bigwedge(A_{k_0} - \{e_0\})) \bigwedge 1 \\
&= \bigwedge(A_{k_0} - \{e_0\}).
\end{aligned}$$

This implies $A_{k_0} \notin \{A_k : k \leq q\}$. This is a contradiction.

Thus $A_{k_0} \in red(\sigma_E)$. This show $red(\sigma_E) \supseteq \{A_k : k \leq q\}$.

(2) Let $A \in red(\sigma_E)$. Then $A \in co(\sigma_E)$. By Proposition 3.5, $A \cap \mathcal{D}(x, y) \neq \varnothing$ ($(x, y) \notin ind(E)$). Similar to the proof of (1) (ii), we have $A \in \{A_k : k \leq q\}$.

Thus $red(\sigma_E) \subseteq \{A_k : k \leq q\}$. Hence $red(\sigma_E) = \{A_k : k \leq q\}$. □

Example 4.6. We consider Example 3.4.

In Step 1, we input the soft binary relation σ_E over U.

In Step 2, we obtain the discernibility matrix $\mathfrak{D}(\sigma_E)$.

In Step 3, we obtain

$$\Delta(\sigma_E) = e_2 \wedge (e_1 \vee e_2 \vee e_3 \vee e_4) \wedge (e_1 \vee e_4) \wedge (e_1 \vee e_3 \vee e_4) \wedge (e_1 \vee e_2 \vee e_4) \wedge (e_2 \vee e_3).$$

In Step 4, we obtain $\Delta^*(\sigma_E) = (e_1 \wedge e_2) \vee (e_2 \wedge e_4)$.

In Step 5, we obtain all parameter reductions of σ_E: $\{e_1, e_2\}$, $\{e_2, e_4\}$ and $core(\sigma_E) = \{e_2\}$.

5. Conclusions

In this paper, we have investigated the parameter reduction of soft binary relations by using discernibility matrix and discernibility functions and given its algorithm. In the future work, we will consider concrete applications of the parameter reduction of soft binary relations.

Acknowledgements. This work is supported by Quantitative Economics Key Laboratory Program of Guangxi University of Finance and Economics (2014SYS11), Guangxi University Science and Technology Research Project (KY2015YB266) and Guangxi Province Universities and Colleges Excellence Scholar and Innovation Team Funded Scheme.

References

[1] M. Irfan Ali, A note on soft sets, rough soft sets and fuzzy soft sets, Appl. Soft Comput. 13 (2011) 3329–3332.
[2] D. Chen, E. C. C. Tsang, D. S. Yeung and X. Wang, The parameterization reduction of soft sets and its applications, Comput. Math. Appl. 49 (2005) 757–763.
[3] X. Ge, Z. Li and Y. Ge, Topological spaces and soft sets, J. Comput. Anal. Appl. 13 (2011) 881–885.
[4] Y. Jiang, Y. Tang, Q. Chen, J. Wang and S. Tang, Extending soft sets with description logics, Comput. Math. Appl. 59 (2010) 2087–2096.
[5] Z. Kong, L. Gao, L. Wang and S. Li, The normal parameter reduction of soft sets and its algorithm, Comput. Math. Appl. 56 (2008) 3029–3037.
[6] Z. Li and T. Xie, The relationship among soft sets, soft rough sets and topologies, Soft Comput. 18(2014) 717-728.
[7] D. Molodtsov, Soft set theory-first result, Comput. Math. Appl. 37 (1999) 19–31.
[8] D. Molodtsov, The theory of soft sets, URSS Publishers, Moscow 2004.
[9] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[10] P. K. Maji and A. R. Roy, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002) 1077–1083.
[11] P. Majumdar and S. K. Samanta, Softness of a soft set: Soft set entropy, Ann. Fuzzy Math. Inform. 6 (1) (2013) 59–68.
[12] X. Ma, N. Sulaiman, H. Qin, T. Herawan and J. M. Zain, A new efficient normal parameter reduction algorithm of soft sets, Comput. Math. Appl. 62 (2011) 588–598.
[13] Z. Pawlak, Rough sets, Int. J. Comput. Inform. Sci. 11 (1982) 341–356.
[14] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Guangji Yu (guangjiyu100@126.com)
School of Information and Statistics, Guangxi Univresity of Finance and Economics, Nanning, Guangxi 530003, P. R. China