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On T_0 and T_1 fuzzy soft topological spaces

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Abstract. In this paper, we have introduced and studied T_0 and T_1 separation axioms in a fuzzy soft topological space. Several basic desirable results have been proved which establish the appropriateness of these definitions.

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1. Introduction

Molodtsov [11] introduced the concept of soft sets as a new approach for modelling uncertainties. Soft set theory has been applied in many directions e.g., game theory and operations research [11], soft analysis [11], group theory [1], topology theory [4, 12] etc. Later on Maji et al.[8] introduced fuzzy soft sets and using it, Tanay and Kandemir [17] introduced and studied the topological structure of fuzzy soft sets. Since then, many researchers have been working in this direction e.g., Varol and Aygün [18], Cetkin and Aygün [5], Mahanta and Das [7].

Hausdorffness in a fuzzy soft topological space has been studied in detail by Mishra and Srivastava [10]. In this paper we have introduced and studied T_0 and T_1 separation axioms in a fuzzy soft topological space. Several basic desirable results have been obtained which establish the appropriateness of these notions.

2. Preliminaries

Throughout this paper, X denotes a non empty set, called the universe, E the set of parameters for the universe X and $A \subseteq E$.

Definition 2.1 ([19])**.** A fuzzy set in X is a function $f : X \to [0, 1]$. Now we define some basic fuzzy set operations as follows:
Let f and g be fuzzy sets in X. Then

(1) $f = g$ if $f(x) = g(x)$, for each $x \in X$.
(2) $f \subseteq g$ if $f(x) \leqslant g(x)$, for each $x \in X$.
(3) $(f \cup g)(x)$ =max$\{f(x), g(x)\}$, for each $x \in X$.
(4) $(f \cap g)(x)$ =min$\{f(x), g(x)\}$, for each $x \in X$.
(5) $f^c(x) = 1 - f(x)$, for each $x \in X$ (here f^c denotes the complement of f).

The constant fuzzy set in X, taking value $\alpha \in [0, 1]$, will be denoted by α_X.

Definition 2.2 ([9]). Let Ω be an index set and $\{f_i : i \in \Omega\}$ be a family of fuzzy sets in X. Then their union $\bigcup_{i\in\Omega} f_i$ and intersection $\bigcap_{i\in\Omega} f_i$ are defined, respectively as follows:

(1) $(\bigcup_{i\in\Omega} f_i)(x)$=sup$\{f_i(x) : i \in \Omega\}$, for each $x \in X$.
(2) $(\bigcap_{i\in\Omega} f_i)(x)$=inf$\{f_i(x) : i \in \Omega\}$, for each $x \in X$.

Definition 2.3 ([13]). A fuzzy point x_λ $(0 < \lambda < 1)$ in X is a fuzzy set in X given by

$$x_\lambda(x') = \begin{cases} \lambda, & \text{if } x' = x \\ 0, & \text{otherwise.} \end{cases}$$

Here x and λ are respectively called the support and value of x_λ.

Definition 2.4 ([8]). A pair (f, E) is called a fuzzy soft set over X if f is a mapping from E to I^X i.e., $f : E \to I^X$, where I^X is the collection of all fuzzy sets in X.

Definition 2.5 ([18]). A fuzzy soft set f_A over X is a mapping from E to I^X i.e., $f_A : E \to I^X$ such that $f_A(e) \neq 0_X$, if $e \in A \subseteq E$ and $f_A(e) = 0_X$, otherwise, where 0_X denotes the constant fuzzy set in X taking value 0.

Definition 2.6 ([18]). The universal fuzzy soft set 1_E over X is given by $1_E(e) = 1_X$, for each $e \in E$ and the null fuzzy soft set 0_E over X is given by $0_E(e) = 0_X$, for each $e \in E$, where 1_X denotes the constant fuzzy set in X taking value 1.

The constant fuzzy soft set over X, taking value α_X, $\alpha \in [0, 1]$, will be denoted by α_E.

From here onwards, we will denote by $\mathcal{F}(X, E)$, the set of all fuzzy soft sets over X.

Definition 2.7 ([18]). Let $f_A, g_B \in \mathcal{F}(X, E)$. Then

(1) f_A is said to be a fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $f_A(e) \subseteq g_B(e)$, for each $e \in E$.
(2) f_A and g_B are said to be equal, denoted by $f_A = g_B$, if $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$.
(3) The union of f_A and g_B, denoted by $f_A \sqcup g_B$, is the fuzzy soft set over X defined by $(f_A \sqcup g_B)(e) = f_A(e) \cup g_B(e)$, for each $e \in E$.
(4) The intersection of f_A and g_B, denoted by $f_A \sqcap g_B$, is the fuzzy soft set over X defined by $(f_A \sqcap g_B)(e) = f_A(e) \cap g_B(e)$, for each $e \in E$. Two fuzzy soft sets f_A and g_B over X are said to be disjoint if $f_A \sqcap g_B = 0_E$.

(5) Let Ω be an index set and $\{f_{A_i} : i \in \Omega\}$ be a family of fuzzy soft sets over X. Then their union $\bigsqcup_{i\in\Omega} f_{A_i}$ and intersection $\sqcap_{i\in\Omega} f_{A_i}$ are defined, respectively as follows:
 (a) $(\bigsqcup_{i\in\Omega} f_{A_i})(e) = \bigcup_{i\in\Omega} f_{A_i}(e)$, for each $e \in E$.
 (b) $(\sqcap_{i\in\Omega} f_{A_i})(e) = \bigcap_{i\in\Omega} f_{A_i}(e)$, for each $e \in E$.

(6) The complement of f_A, denoted by f_A^c, is the fuzzy soft set over X, defined by $f_A^c(e) = 1_X - f_A(e)$, for each $e \in E$.

Definition 2.8 ([3])**.** Let $\mathcal{F}(X, E)$ and $\mathcal{F}(Y, K)$ be the collections of all the fuzzy soft sets over X and Y respectively and E, K be the parameters sets for the universe X and Y respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two maps. Then a fuzzy soft mapping from X to Y is a pair (φ, ψ) and is denoted by

$$(\varphi, \psi) : \mathcal{F}(X, E) \to \mathcal{F}(Y, K).$$

(1) Let $f_A \in \mathcal{F}(X, E)$. Then the image of f_A under the fuzzy soft mapping (φ, ψ) is the fuzzy soft set over Y, denoted by $(\varphi, \psi)f_A$ and is defined as

$$(\varphi, \psi)f_A(k)(y) = \begin{cases} \sup\limits_{\varphi(x)=y} \sup\limits_{\psi(e)=k} f_A(e)(x), & \text{if } \varphi^{-1}(y) \neq \phi \text{ and } \psi^{-1}(k) \neq \phi \\ 0, & \text{otherwise,} \end{cases}$$

for each $k \in K$, for each $y \in Y$.

(2) Let $g_B \in \mathcal{F}(Y, K)$. Then the inverse image of g_B under the fuzzy soft mapping (φ, ψ) is the fuzzy soft set over X, denoted by $(\varphi, \psi)^{-1}g_B$ and is defined as $(\varphi, \psi)^{-1}g_B(e)(x) = g_B(\psi(e))(\varphi(x))$, for each $e \in E$, for each $x \in X$.

A fuzzy soft mapping (φ, ψ) is constant if both φ and ψ are constant[18].

Definition 2.9 ([18])**.** Let $f_A \in \mathcal{F}(X, E)$ and $g_B \in \mathcal{F}(Y, K)$. Then the fuzzy soft product of f_A and g_B, denoted by $f_A \times g_B$, is the fuzzy soft set over $X \times Y$ and is defined by $(f_A \times g_B)(e, k) = f_A(e) \times g_B(k)$, for each $(e, k) \in E \times K$ and for $(x, y) \in X \times Y$,

$$(f_A(e) \times g_B(k))(x, y) = \min\{f_A(e)(x), g_B(k)(y)\}.$$

Definition 2.10 ([17, 18])**.** A fuzzy soft topological space relative to the parameters set E is a pair (X, τ) consisting of a non empty set X and a family τ of fuzzy soft sets over X satisfying the following conditions :

(1) 0_E, $1_E \in \tau$;
(2) If $f_A, g_B \in \tau$, then $f_A \sqcap g_B \in \tau$;
(3) If $f_{A_j} \in \tau, \forall j \in \Omega$, where Ω is some index set, then $\bigsqcup_{j\in\Omega} f_{A_j} \in \tau$.

Then τ is called a fuzzy soft topology over X and members of τ are called fuzzy soft open sets. A fuzzy soft set g_B over X is called fuzzy soft closed if $(g_B)^c \in \tau$.

In particular, $\tau^o = \{0_E, 1_E\}$ and $\tau^1 = \mathcal{F}(X, E)$ are called the indiscrete and the discrete fuzzy soft topologies over X, respectively.

Definition 2.11 ([18])**.** A fuzzy soft topology τ is called enriched if $\alpha_E \in \tau$, for each $\alpha \in [0,1]$.

A fuzzy soft topology τ is called indiscrete enriched if $\tau = \{\alpha_E \mid \alpha \in [0,1]\}$.

Definition 2.12 ([18])**.** Let (X,τ) be a fuzzy soft topological space. Then a subfamily $\mathcal{B}$ of τ is called a base for τ if every member of τ can be written as a union of members of $\mathcal{B}$.

Definition 2.13 ([18])**.** Let (X,τ) be a fuzzy soft topological space. Then a subfamily $\mathcal{S}$ of τ is called a subbase for τ if the family of finite intersections of its members forms a base for τ.

Definition 2.14 ([18])**.** A fuzzy soft topology τ over X is said to be generated by a subfamily $\mathcal{S}$ of fuzzy soft sets over X if every member of τ is a union of finite intersections of members of $\mathcal{S}$.

Definition 2.15 ([18])**.** Let $\{(X_i,\tau_i)\}_{i\in\Omega}$ be a family of fuzzy soft topological spaces relative to the parameters sets E_i respectively, X be a set with parameters set E and for each $i \in \Omega$, $(\varphi,\psi)_i : X \to (X_i,\tau_i)$ be a fuzzy soft mapping. Then the fuzzy soft topology τ over X is said to be initial with respect to the family $\{(\varphi,\psi)_i\}_{i\in\Omega}$ if τ has as subbase the set

$$\mathcal{S} = \{(\varphi,\psi)_i^{-1}(f_{A_i}) : i \in \Omega,\ f_{A_i} \in \tau_i\}$$

i.e., the fuzzy soft topology τ over X is generated by $\mathcal{S}$.

Definition 2.16 ([18])**.** Let $\{(X_i,\tau_i)\}_{i\in\Omega}$ be a family of fuzzy soft topological spaces relative to the parameters sets E_i respectively. Then their product is defined as the fuzzy soft topological space (X,τ) relative to the parameters set E, where $X = \prod\limits_i X_i$, $E = \prod\limits_i E_i$ and τ is the fuzzy soft topology over X which is initial with respect to the family $\{(p_{X_i}, q_{E_i})\}_{i\in\Omega}$, where $p_{X_i} : \prod\limits_i X_i \to X_i$ and $q_{E_i} : \prod\limits_i E_i \to E_i$ are the projection maps i.e., τ is generated by

$$\{(p_{X_i}, q_{E_i})^{-1}(f_{A_i}) : i \in \Omega, f_{A_i} \in \tau_i\}.$$

Definition 2.17 ([10])**.** A fuzzy soft point e_{x_λ} over X is a fuzzy soft set over X defined as follows:

$$e_{x_\lambda}(e') = \begin{cases} x_\lambda, & \text{if } e' = e \\ 0_X, & \text{if } e' \in E - \{e\}, \end{cases}$$

where x_λ is the fuzzy point in X with support x and value λ, $\lambda \in (0,1)$. A fuzzy soft point e_{x_λ} is said to belong to a fuzzy soft set f_A, denoted by $e_{x_\lambda} \in f_A$ if $\lambda < f_A(e)(x)$ and two fuzzy soft points e_{x_λ} and e'_{y_s} are said to be distinct if $x \neq y$ or $e \neq e'$.

Proposition 2.18 ([10])**.** *Let $\{f_{A_i} : i \in \Omega\}$ be a family of fuzzy soft sets over X, then $e_{x_\lambda} \in \bigsqcup\limits_{i\in\Omega} f_{A_i}$ iff $e_{x_\lambda} \in f_{A_i}$ for some $i \in \Omega$.*

Proposition 2.19 ([10])**.** *A fuzzy soft set f_A over X is the union of all fuzzy soft points belonging to it i.e.,*

$$f_A = \bigsqcup\{e_{x_\lambda} : e_{x_\lambda} \in f_A\}.$$

Proposition 2.20 ([10]). *Let (X, τ) be a fuzzy soft topological space relative to the parameters set E. Then a fuzzy soft set f_A is fuzzy soft open iff for each $e_{x_r} \in f_A$, there exists a basic fuzzy soft open set g_B such that $e_{x_r} \in g_B \sqsubseteq f_A$.*

Definition 2.21 ([10]). Let (X, τ) be a fuzzy soft topological space relative to the parameters set E and $G \subseteq E$. Then (X, τ_G) is also a fuzzy soft topological space relative to the parameters set G where

$$\tau_G = \{f_A \mid_G : f_A \in \tau\},$$

and is called a fuzzy soft subspace of (X, τ).

Definition 2.22 ([10]). Let (X, τ) be a fuzzy soft topological space relative to the parameters set E. Then (X, τ) is said to be Hausdorff if for each pair of distinct fuzzy soft points e_{x_λ}, e'_{y_s} over X, there exist fuzzy soft open sets f_A and g_B such that $e_{x_\lambda} \in f_A$, $e'_{y_s} \in g_B$ and $f_A \sqcap g_B = 0_E$.

Definition 2.23 ([18]). Let (X_1, τ_1) and (X_2, τ_2) be two fuzzy soft topological spaces relative to the parameters sets E_1 and E_2 respectively. Then a fuzzy soft mapping

$$(\varphi, \psi) : (X_1, \tau_1) \to (X_2, \tau_2)$$

is said to be fuzzy soft continuous if $(\phi, \psi)^{-1} f_B \in \tau_1$, for each $f_B \in \tau_2$.

3. Fuzzy soft T_0 topological spaces

In the case of fuzzy topology, there exist many definitions of fuzzy T_0 separation axiom(cf.[2, 6, 14]) but the following definition given in [6] turns out to be the 'categorically right' definition.

Definition 3.1 ([6]). A fuzzy topological space (X, τ) is said to be fuzzy T_0 if for every pair of distinct $x, y \in X$, there exists a fuzzy open set U in X such that $U(x) \neq U(y)$.

Motivated by this, we give the following:

Definition 3.2. Let (X, τ) be a fuzzy soft topological space relative to the parameters set E. Then (X, τ) is said to be fuzzy soft T_0 if for each pair $(x_1, e_1), (x_2, e_2) \in X \times E, (x_1, e_1) \neq (x_2, e_2)$, there exists $f_A \in \tau$ such that $f_A(e_1)(x_1) \neq f_A(e_2)(x_2)$.

We mention here that earlier Mahanta and Das[7] have introduced fuzzy soft T_0 topological spaces. Their definition in a fuzzy soft topological space (X, τ) is as follows:

Definition 3.3 ([7]). A fuzzy soft topological space (X, τ) is said to be fuzzy soft T_0 if for every pair of disjoint fuzzy soft sets e_{h_A}, e_{g_B}(where e_{h_A} is a fuzzy soft set over X such that $e_{h_A}(e) \neq 0_X$ and $e_{h_A}(e') = 0_X$, for $e' \neq e$, similarly e_{g_B} is defined), there exists a fuzzy soft open set p_A over X containing one but not the other.

The above two definitions of a fuzzy soft T_0 space are independent as exhibited through the following examples.

Example 3.4. Definition 3.2 $\not\Rightarrow$ Definition 3.3

Let $X = \{a, b\}$ and E be the parameter set which consists of only one element, say $E = \{e\}$ and $\tau = \{p_A : E \to I^X \mid p_A(e)(a) = 1/2, p_A(e)(b) = 0\} \cup \{0_E, 1_E\}$. Then (X, τ) is fuzzy soft T_0 in the sense of Definition 3.2, since for $(a, e), (b, e) \in X \times E$, there exists $p_A \in \tau$ such that $p_A(e)(a) \neq p_A(e)(b)$. But it fails to be a fuzzy soft T_0 space in the sense of Definition 3.3, since for two disjoint fuzzy soft sets e_{h_A} and e_{g_B} such that

$$h_A(e)(x) = \begin{cases} 3/4, & \text{if } x = a \\ 0, & \text{if } x = b \end{cases}$$

and

$$g_B(e)(x) = \begin{cases} 0, & \text{if } x = a \\ 1, & \text{if } x = b, \end{cases}$$

there does not exist any non trivial fuzzy soft open set over X containing any of them.

Example 3.5. Definition 3.3 $\not\Rightarrow$ Definition 3.2

Let $X = \{a, b\}$ and E be the parameters set which consists of two elements, say $E = \{e_1, e_2\}$ and $\tau = \{p_A : E \to I^X \mid p_A(e_1) = p_A(e_2) = \chi_{\{a\}}\} \cup \{0_E, 1_E\}$. Then (X, τ) is fuzzy soft T_0 in the sense of Definition 3.3. To show this, consider the disjoint fuzzy soft sets $(e_1)_{h_A}$ and $(e_1)_{g_B}$. Since $h_A(e_1) \cap g_B(e_1) = 0_X$, we may assume that $h_A(e_1) = a_\lambda$ and $g_B(e_1) = b_s$, where $\lambda,\ s \in (0, 1]$. Then there exists $p_A \in \tau$ such that p_A contains $(e_1)_{h_A}$ but does not contain $(e_1)_{g_B}$. Similarly, we can deal the case of disjoint fuzzy soft open sets of the form $(e_2)_{h_A}$ and $(e_2)_{g_B}$. But it fails to be a fuzzy soft T_0 space in the sense of Definition 3.2, since for $(a, e_1) \neq (a, e_2)$, there does not exist any fuzzy soft open set f_A such that $f_A(e_1)(a) \neq f_A(e_2)(a)$.

From now onwards we mean fuzzy soft T_0 topological space in the sense of Definition 3.2.

Theorem 3.6. *Let (Y, σ) be the indiscrete enriched fuzzy soft topological space relative to the parameters set K such that $Y = \{y_1, y_2\}$. Then (X, τ) is fuzzy soft T_0 implies that every fuzzy soft continuous mapping $(\varphi, \psi) : (Y, \sigma) \to (X, \tau)$ is constant.*

Proof. First suppose that (X, τ) is fuzzy soft T_0. Then for each pair $(x_1, e_1), (x_2, e_2) \in X \times E, (x_1, e_1) \neq (x_2, e_2)$, there exists $f_A \in \tau$ such that $f_A(e_1)(x_1) \neq f_A(e_2)(x_2)$. We have to show that every fuzzy soft continuous mapping $(\varphi, \psi) : (Y, \sigma) \to (X, \tau)$ is constant. For this, suppose on the contrary that there exists a fuzzy soft continuous mapping $(\varphi, \psi) : (Y, \sigma) \to (X, \tau)$ which is not constant. Then,

Case 1: If φ is not constant and ψ is constant.

Let $\varphi(y_1) \neq \varphi(y_2)$ and $\psi(k) = e$, for each $k \in K$. Then, for $(\varphi(y_1), e)$, $(\varphi(y_2), e) \in X \times E$, there exists $f_A \in \tau$ such that $f_A(e)(\varphi(y_1)) \neq f_A(e)(\varphi(y_2))$.

Now, for $k_1, k_2 \in K$, we have

$$\begin{array}{rcl}(\varphi,\psi)^{-1}f_A(k_1)(y_1) & = & f_A(\psi(k_1))(\varphi(y_1)) \\ & = & f_A(e)(\varphi(y_1)) \\ & \neq & f_A(e)(\varphi(y_2)) \\ & = & f_A(\psi(k_2))(\varphi(y_2)) \\ & = & (\varphi,\psi)^{-1}f_A(k_2)(y_2),\end{array}$$

which implies that $(\varphi,\psi)^{-1}f_A$ is not a constant fuzzy soft set over Y, therefore (φ,ψ) is not fuzzy soft continuous, a contradiction.
Case 2: If φ is constant and ψ is not constant.
Let $\varphi(y_1)=\varphi(y_2)$ and $\psi(k_1)\neq\psi(k_2)$, for some $k_1,k_2\in K$. Then, for $(\varphi(y_1),\psi(k_1))$, $(\varphi(y_1),\psi(k_2))\in X\times E$, there exists $f_A\in\tau$ such that $f_A(\psi(k_1))(\varphi(y_1))\neq f_A(\psi(k_2))(\varphi(y_1))$.
Now,

$$\begin{array}{rcl}(\varphi,\psi)^{-1}f_A(k_1)(y_1) & = & f_A(\psi(k_1))(\varphi(y_1)) \\ & \neq & f_A(\psi(k_2))(\varphi(y_1)) \\ & = & (\varphi,\psi)^{-1}f_A(k_2)(y_1),\end{array}$$

which implies that $(\varphi,\psi)^{-1}f_A$ is not a constant fuzzy soft set over Y, therefore (φ,ψ) is not fuzzy soft continuous, a contradiction.
Case 3: If φ and ψ both are not constant.
Let $\varphi(y_1)\neq\varphi(y_2)$ and $\psi(k_1)\neq\psi(k_2)$, for some $k_1,k_2\in K$. Then, for $(\varphi(y_1),\psi(k_1))$, $(\varphi(y_2),\psi(k_2))\in X\times E$, there exists $f_A\in\tau$ such that $f_A(\psi(k_1))(\varphi(y_1))\neq f_A(\psi(k_2))(\varphi(y_2))$.
Now,

$$\begin{array}{rcl}(\varphi,\psi)^{-1}f_A(k_1)(y_1) & = & f_A(\psi(k_1))(\varphi(y_1)) \\ & \neq & f_A(\psi(k_2))(\varphi(y_2)) \\ & = & (\varphi,\psi)^{-1}f_A(k_2)(y_2),\end{array}$$

which implies that $(\varphi,\psi)^{-1}f_A$ is not a constant fuzzy soft set over Y, therefore (φ,ψ) is not fuzzy soft continuous, a contradiction.

□

T_0-ness in fuzzy soft topological spaces satisfies the hereditary property as shown in the following proposition.

Proposition 3.7. *Fuzzy soft subspace of a fuzzy soft T_0 space is fuzzy soft T_0.*

Proof. Let (X,τ) be a fuzzy soft T_0 space relative to the parameters set E and $G\subseteq E$. Then, for each pair $(x_1,e_1),(x_2,e_2)\in X\times E$, $(x_1,e_1)\neq(x_2,e_2)$, there exists $f_A\in\tau$ such that $f_A(e_1)(x_1)\neq f_A(e_2)(x_2)$. In particular, for $(x_1,g),(x_2,g')\in X\times G$, $(x_1,g)\neq(x_2,g')$, there exists $f_A\in\tau$ such that $f_A(g)(x_1)\neq f_A(g')(x_2)$. So, $f_A\mid_G(g)(x_1)\neq f_A\mid_G(g')(x_2)$. This implies that (X,τ_G) is fuzzy soft T_0. □

In the following theorem it is proved that T_0-ness in fuzzy soft topological spaces satisfies the productive and projective properties.

Theorem 3.8. *If $\{(X_i,\tau_i): i\in\Omega\}$ is a family of fuzzy soft topological spaces relative to the parameters sets E_i respectively. Then the product fuzzy soft topological space*

$(X,\tau)=\prod_{i\in\Omega}(X_i,\tau_i)$ is fuzzy soft T_0 iff each coordinate fuzzy soft topological space (X_i,τ_i) is fuzzy soft T_0.

Proof. First, let us assume that (X_i,τ_i) is fuzzy soft T_0, for each $i\in\Omega$. To show that (X,τ) is fuzzy soft T_0, choose (x,e), $(y,e')\in X\times E$, $(x,e)\neq(y,e')$, where $x=\prod\limits_{j\in\Omega}x_j$, $y=\prod\limits_{j\in\Omega}y_j$, $e=\prod\limits_{j\in\Omega}e_j$ and $e'=\prod\limits_{j\in\Omega}e'_j$. Then $x_i\neq y_i$ or $e_k\neq e'_k$ for some $i,\,k\in\Omega$. Let $x_i\neq y_i$. Since (X_i,τ_i) is fuzzy soft T_0, for (x_i,e_i), $(y_i,e'_i)\in X_i\times E_i$, there exists $f_{A_i}\in\tau_i$ such that $f_{A_i}(e_i)(x_i)\neq f_{A_i}(e'_i)(y_i)$. Set $f_A=\prod\limits_{j\in\Omega}f^1_{A_j}$ such that $f^1_{A_j}=1_{E_j}$, for $j\neq i$ and $f^1_{A_i}=f_{A_i}$. Then f_A is a fuzzy soft open set such that $f_A(e)(x)\neq f_A(e')(y)$ implying that (X,τ) is fuzzy soft T_0. The other cases can be handled similarly.

Conversely, assume that (X,τ) is fuzzy soft T_0. To show (X_i,τ_i) is fuzzy soft T_0, choose $(x_i,e_i),(y_i,e'_i)\in X_i\times E_i$, $(x_i,e_i)\neq(y_i,e'_i)$. Then $x_i\neq y_i$ or $e_i\neq e'_i$. Let $x_i\neq y_i$. Now construct two points $x=\prod\limits_{j\in\Omega}x'_j$ and $y=\prod\limits_{j\in\Omega}y'_j$ in X, where $x'_j=y'_j$, for $j\neq i$ and $x'_i=x_i$, $y'_i=y_i$ and two points $e^1=\prod\limits_{j\in\Omega}e^1_j$ and $e^2=\prod\limits_{j\in\Omega}e^2_j$ in E, where $e^1_j=e^2_j$, for $j\neq i$ and $e^1_i=e_i$, $e^2_i=e'_i$. Then, since (X,τ) is fuzzy soft T_0, for $(x,e^1),(y,e^2)\in X\times E$, there exists $f_A\in\tau$ such that $f_A(e^1)(x)\neq f_A(e^2)(y)$. Also, since each fuzzy soft open set can be written as a union of basic fuzzy soft open sets, so we can write f_A in the following form:

$$f_A=\bigcup_{k\in T}\prod_{j\in\Omega}(f_{A_j})_k$$

Now, if we assume that

$$\begin{aligned}
&\prod_{j\in\Omega}(f_{A_j})_k(e^1)(x)=\prod_{j\in\Omega}(f_{A_j})_k(e^2)(y),\quad \text{for each}\, k\in T\\
&\Rightarrow \inf_j(f_{A_j})_k(e^1_j)(x'_j)=\inf_j(f_{A_j})_k(e^2_j)(y'_j),\quad \text{for each}\, k\in T\\
&\Rightarrow \sup_k\inf_j(f_{A_j})_k(e^1_j)(x'_j)=\sup_k\inf_j(f_{A_j})_k(e^2_j)(y'_j)\\
&\Rightarrow (\bigcup_{k\in T}\prod_{j\in\Omega}(f_{A_j})_k)(e^1)(x)=(\bigcup_{k\in T}\prod_{j\in\Omega}(f_{A_j})_k)(e^2)(y)\\
&\Rightarrow f_A(e^1)(x)=f_A(e^2)(y),\ \text{a contradiction.}
\end{aligned}$$

Therefore, there exists some $k\in T$ such that

$$\begin{aligned}
\prod_{j\in\Omega}(f_{A_j})_k(e^1)(x) &\neq \prod_{j\in\Omega}(f_{A_j})_k(e^2)(y)\\
\Rightarrow \inf_j(f_{A_j})_k(e^1_j)(x'_j) &\neq \inf_j(f_{A_j})_k(e^2_j)(y'_j).
\end{aligned}$$

Since, for $j\neq i$, $x'_j=y'_j$, $e^1_j=e^2_j$, so we have

$$(f_{A_i})_k(e_i)(x_i)\neq(f_{A_i})_k(e'_i)(y_i),$$

which implies that (X_i,τ_i) is fuzzy soft T_0.
The other cases can be handled similarly. □

4. Fuzzy soft T_1 topological spaces

In this section, we introduce and study fuzzy soft T_1 topological spaces. The following definition of fuzzy soft T_1 topological spaces is motivated by the definition 5.1 of fuzzy T_1-topological spaces given by Srivastava et al.[16].

Definition 4.1. Let (X, τ) be a fuzzy soft topological space relative to the parameters set E. Then (X, τ) is said to be fuzzy soft T_1 if for each pair $(x_1, e_1), (x_2, e_2) \in X \times E, (x_1, e_1) \neq (x_2, e_2)$, there exist $f_A,\ g_B \in \tau$ such that $f_A(e_1)(x_1) = 1,\ f_A(e_2)(x_2) = 0,\ g_B(e_1)(x_1) = 0,\ g_B(e_2)(y_2) = 1$.

Earlier Mahanta and Das[7] have introduced fuzzy soft T_1 topological spaces as follows:

Definition 4.2 ([7])**.** A fuzzy soft topological space (X, τ) is said to be fuzzy soft T_1 if for every pair of disjoint fuzzy soft sets $e_{h_A},\ e_{g_B}$(where e_{h_A} is a fuzzy soft set over X such that $e_{h_A}(e) \neq 0_X$ and $e_{h_A}(e') = 0_X$, for $e' \neq e$, similarly e_{g_B} is defined), there exist fuzzy soft open sets p_A and q_A over X such that $e_{h_A} \subseteq p_A$ and $e_{g_B} \nsubseteq p_A$; $e_{h_A} \nsubseteq q_A$ and $e_{g_B} \subseteq q_A$.

We observe that the Definition 4.1 $\Rightarrow$ Definition 4.2, the proof of which is as follows:

Let (X, τ) be a fuzzy soft T_1 space in the sense of Definition 4.1.

$\Rightarrow e_{\{x\}}$ is fuzzy soft closed, for each $e \in E$ and $x \in X$ (in view of Proposition 4.4).

$\Rightarrow (e_{\{x\}})^c$ is fuzzy soft open, for each $e \in E$ and $x \in X$.

Now choose any two disjoint fuzzy soft sets e_{h_A} and e_{g_B}. Since $e_{h_A}(e) \neq 0_X$ and $e_{g_B}(e) \neq 0_X$, there exist $a,\ b \in X$ such that $e_{h_A}(e)(a) \neq 0$ and $e_{g_B}(e)(b) \neq 0$. Since $h_A(e) \cap g_B(e) = 0_X$, so $a \neq b$. Now consider the fuzzy soft open sets $(e_{\{b\}})^c$, $(e_{\{a\}})^c$. Then $e_{h_A} \subseteq (e_{\{b\}})^c$ and $e_{g_B} \nsubseteq (e_{\{b\}})^c$; $e_{h_A} \nsubseteq (e_{\{a\}})^c$ and $e_{g_B} \subseteq (e_{\{a\}})^c$, showing that (X, τ) is fuzzy soft T_0 in the sense of Definition 4.2.

But the converse is not true which is exhibited from the following example.

Example 4.3. Let $X = \{a, b\}$, E be the parameters set which consists of two elements, say $E = \{e_1, e_2\}$ and $\tau = \{0_E, 1_E\} \cup \{p_A : E \to I^X, q_A : E \to I^X \mid p_A(e_1) = p_A(e_2) = \chi_{\{a\}}, q_A(e_1) = q_A(e_2) = \chi_{\{b\}}\}$. Then (X, τ) is a fuzzy soft T_1 space in the sense of Definition 4.2. For this, let $(e_1)_{h_A}$ and $(e_1)_{g_B}$ be disjoint fuzzy soft sets. Since $h_A(e_1) \cap g_B(e_1) = 0_X$, we may assume that $h_A(e_1) = a_\lambda$ and $g_B(e_1) = b_s$, where $\lambda,\ s \in (0, 1]$. Then for $(e_1)_{h_A}$ and $(e_1)_{g_B}$, there exist $p_A, q_A \in \tau$ such that p_A contains $(e_1)_{h_A}$ but does not contain $(e_1)_{g_B}$ and q_A contains $(e_1)_{g_B}$ but does not contain $(e_1)_{h_A}$. Similarly, we can deal the case of disjoint fuzzy soft open sets of the form $(e_2)_{h_A}$ and $(e_2)_{g_B}$. But it fails to be a fuzzy soft T_1 space in the sense of Definition 4.1, since for $(a, e_1) \neq (a, e_2)$, there does not exist any fuzzy soft open set f_A such that $f_A(e_1)(a) = 1$ and $f_A(e_2)(a) = 0$.

From now onwards we mean fuzzy soft T_1 topological space in the sense of Definition 4.1.

Proposition 4.4. *The following statements are equivalent in a fuzzy soft topological space (X, τ) relative to the parameters set E:*

(1) (X, τ) *is fuzzy soft* T_1.
(2) $e_{\{x\}}$ *is fuzzy soft closed, for each* $e \in E$ *and* $x \in X$*, where* $e_{\{x\}}$ *denotes the fuzzy soft set over* X *such that*

$$e_{\{x\}}(e')(x') = \begin{cases} 1, & \textit{if } e' = e,\, x' = x \\ 0, & \textit{otherwise.} \end{cases}$$

Proof. (1) $\Rightarrow$ (2) To show that $e_{\{x\}}$ is fuzzy soft closed, equivalently, $e^c_{\{x\}}$ is fuzzy soft open, choose any fuzzy soft point $e'_{y_\lambda} \in e^c_{\{x\}}$. Then,
Case 1: If $y = x$ and $e' \neq e$.
Then, for $(x, e), (x, e') \in X \times E$, there exist $f_A, g_B \in \tau$ such that $f_A(e)(x) = 1$, $f_A(e')(x) = 0$, $g_B(e)(x) = 0$, $g_B(e')(x) = 1$. Now $e'_{x_\lambda} \in g_B \sqsubseteq e^c_{\{x\}}$.
Case 2: If $y \neq x$ and $e' = e$.
Then, for (x, e), $(y, e) \in X \times E$, there exist $f_A, g_B \in \tau$ such that $f_A(e)(x) = 1$, $f_A(e)(y) = 0$, $g_B(e)(x) = 0$, $g_B(e)(y) = 1$. Now $e_{y_\lambda} \in g_B \sqsubseteq e^c_{\{x\}}$.
Case 3: If $y \neq x$ and $e' \neq e$.
Then, for (x, e), $(y, e') \in X \times E$, there exist $f_A, g_B \in \tau$ such that $f_A(e)(x) = 1$, $f_A(e')(y) = 0$, $g_B(e)(x) = 0$, $g_B(e')(y) = 1$. Now $e'_{y_\lambda} \in g_B \sqsubseteq e^c_{\{x\}}$.
Therefore, $e^c_{\{x\}}$ is fuzzy soft open.
(2) $\Rightarrow$ (1) Suppose that $e_{\{x\}}$ is fuzzy soft closed for each $e \in X$, for each $x \in X$. To show that (X, τ) is fuzzy soft T_1, choose (x', e'), $(x'', e'') \in X \times E$, $(x', e') \neq (x'', e'')$. Now consider the fuzzy soft open sets $(e'_{\{x'\}})^c$ and $(e''_{\{x''\}})^c$. Then, $(e'_{\{x'\}})^c(e')(x') = 0$, $(e'_{\{x'\}})^c(e'')(x'') = 1$, $(e''_{\{x''\}})^c(e')(x') = 1$ and $(e''_{\{x''\}})^c(e'')(x'') = 0$. $\square$

In the following proposition, we show that T_1-ness in fuzzy soft topological spaces satisfies the hereditary property.

Proposition 4.5. *Fuzzy soft subspace of a fuzzy soft* T_1 *space is fuzzy soft* T_1.

Proof. Let (X, τ) be a fuzzy soft T_1 space relative to the parameters set E and $G \subseteq E$. Then, for each pair (x_1, e_1), $(x_2, e_2) \in X \times E, (x_1, e_1) \neq (x_2, e_2)$, there exist $f_A, g_B \in \tau$ such that

$$f_A(e_1)(x_1) = 1,\ f_A(e_2)(x_2) = 0,\ g_B(e_1)(x_1) = 0,\ g_B(e_2)(x_2) = 1$$

In particular, for (x_1, g), $(x_2, g') \in X \times G$, $(x_1, g) \neq (x_2, g')$, there exist $f_A, g_B \in \tau$ such that

$$f_A(g)(x_1) = 1,\ f_A(g')(x_2) = 0,\ g_B(g)(x_1) = 0,\ g_B(g')(x_2) = 1$$
$$\Rightarrow f_A \mid_G (g)(x_1) = 1,\ f_A \mid_G (g')(x_2) = 0,\ g_B \mid_G (g)(x_1) = 0,\ g_B \mid_G (g')(x_2) = 1$$

This implies that (X, τ_G) is fuzzy soft T_1. $\square$

In the following theorem we prove that T_1-ness in fuzzy soft topological spaces, satisfies the productive and projective properties. The proof is based on the proof of the corresponding result given in [15].

Theorem 4.6. *If* $\{(X_i, \tau_i); i \in \Omega\}$ *is a family of fuzzy soft topological spaces relative to the parameters sets* E_i *respectively. Then the product fuzzy soft topological space* $(X, \tau) = \prod_{i \in \Omega}(X_i, \tau_i)$ *is fuzzy soft* T_1 *iff each coordinate fuzzy soft topological space* (X_i, τ_i) *is fuzzy soft* T_1.

Proof. Let (X_i, τ_i) be fuzzy soft T_1, for each $i \in \Omega$. Let (x, e), $(y, e') \in X \times E$, $(x, e) \neq (y, e')$, where $x = \prod_{j\in\Omega} x_j$, $y = \prod_{j\in\Omega} y_j$, $e = \prod_{j\in\Omega} e_j$ and $e' = \prod_{j\in\Omega} e'_j$. Then there exist some $i, k \in \Omega$ such that $x_i \neq y_i$ or $e_k \neq e'_k$. Let $x_i \neq y_i$. Now since (X_i, τ_i) is fuzzy soft T_1, for (x_i, e_i), (y_i, e'_i), there exist $f_{A_i}, g_{B_i} \in \tau_i$ such that $f_{A_i}(e_i)(x_i) = 1$, $f_{A_i}(e'_i)(y_i) = 0$, $g_{B_i}(e_i)(x_i) = 0$, $g_{B_i}(e'_i)(y_i) = 1$. Now consider the fuzzy soft open sets $f_A = (p_{X_i}, q_{E_i})^{-1} f_{A_i}$ and $g_B = (p_{X_i}, q_{E_i})^{-1} g_{B_i}$. Then, $f_A(e)(x) = ((p_{X_i}, q_{E_i})^{-1} f_{A_i})(e)(x) = f_{A_i}(q_{E_i}(e))(p_{X_i}(x)) = f_{A_i}(e_i)(x_i) = 1$ and $f_A(e')(y) = f_{A_i}(e'_i)(y_i) = 0$. Similarly, we get $g_B(e')(y) = 1, g_B(e)(x) = 0$. Hence (X, τ) is fuzzy soft T_1. The other cases can be handled similarly.

Conversely, let us assume that (X, τ) is fuzzy soft T_1. To show that (X_i, τ_i) is fuzzy soft T_1, choose (x_i, e_i), $(y_i, e'_i) \in X_i \times E_i$ such that $(x_i, e_i) \neq (y_i, e'_i)$. Then $x_i \neq y_i$ or $e_i \neq e'_i$. Let $x_i \neq y_i$. Now consider two points $x = \prod_{j\in\Omega} x'_j$ and $y = \prod_{j\in\Omega} y'_j$ in X, where $x'_j = y'_j$, for $j \neq i$ and $x'_i = x_i$, $y'_i = y_i$ and two points $e^1 = \prod_{j\in\Omega} e^1_j$ and $e^2 = \prod_{j\in\Omega} e^2_j$ in E, where $e^1_j = e^2_j$, for $j \neq i$ and $e^1_i = e_i$, $e^2_i = e'_i$. Since the product (X, τ) is fuzzy soft T_1, for (x, e^1), $(y, e^2) \in X \times E$, there exist $f_A, g_B \in \tau$ such that $f_A(e^1)(x) = 1$, $f_A(e^2)(y) = 0$, $g_B(e^1)(x) = 0$, $g_B(e^2)(y) = 1$. Now since f_A and g_B are fuzzy soft open, so for $e^1_{x_r} \in f_A$ and $e^2_{y_s} \in g_B$, we can find basic fuzzy soft open sets $\prod_{j\in\Omega} f^r_{A_j}$ and $\prod_{j\in\Omega} g^s_{B_j}$ such that

$$e^1_{x_r} \in \prod_{j\in\Omega} f^r_{A_j} \sqsubseteq f_A,$$

$$e^2_{y_s} \in \prod_{j\in\Omega} g^s_{B_j} \sqsubseteq g_B.$$

Since $f_A(e^1)(x) = 1$, so for each $r \in (0, 1)$, $e^1_{x_r} \in f_A$ and

$$\begin{aligned}
&& r < (\prod_{j\in\Omega} f^r_{A_j})(e^1)(x) \leqslant f_A(e^1)(x), \quad \text{for each}\, r \in (0,1) && (4.1)\\
&\Rightarrow & r < \inf_{j\in\Omega} f^r_{A_j}(e^1_j)(x'_j) \leqslant 1, \quad \text{for each}\, r \in (0,1) &&\\
&\Rightarrow & 1 \leqslant \sup_{0<r<1} \inf_{j\in\Omega} f^r_{A_j}(e^1_j)(x'_j) \leqslant 1 &&\\
&\Rightarrow & \sup_{0<r<1} \inf_{j\in\Omega} f^r_{A_j}(e^1_j)(x'_j) = 1 &&\\
&\Rightarrow & (\bigcup_{0<r<1} \prod_{j} f^r_{A_j})(e^1)(x) = 1 && (4.2)
\end{aligned}$$

Next, since

$$\begin{aligned}
& f^r_{A_j}(e^1_j)(x'_j) \leqslant \sup_r f^r_{A_j}(e^1_j)(x'_j), \quad \text{for each}\, r \in (0,1)\\
\Rightarrow\ & \inf_{j\in\Omega} f^r_{A_j}(e^1_j)(x'_j) \leqslant \inf_{j\in\Omega} \sup_r f^r_{A_j}(e^1_j)(x'_j), \quad \text{for each}\, r \in (0,1)\\
\Rightarrow\ & \sup_r \inf_{j\in\Omega} f^r_{A_j}(e^1_j)(x'_j) \leqslant \inf_{j\in\Omega} \sup_r f^r_{A_j}(e^1_j)(x'_j)
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \quad (\bigcup_r \prod_{j\in\Omega} f^r_{A_j})(e^1)(x) \leqslant (\prod_{j\in\Omega}\bigcup_r f^r_{A_j})(e^1)(x),\\
&\Rightarrow \quad (\prod_{j\in\Omega}\bigcup_r f^r_{A_j})(e^1)(x) = 1, \quad \text{(using (4.2))}\\
&\Rightarrow \quad \inf_j \sup_r f^r_{A_j}(e^1_j)(x'_j) = 1\\
&\Rightarrow \quad \sup_r f^r_{A_j}(e^1_j)(x'_j) = 1, \quad \text{for each } j \in \Omega\\
&\Rightarrow \quad (\bigcup_r f^r_{A_i})(e_i)(x_i) = 1.
\end{aligned}$$

Further, since

$$\begin{aligned}
& && f_A(e^2)(y) = 0\\
&\Rightarrow && (\prod_{j\in\Omega} f^r_{A_j})(e^2)(y) = 0, \quad \text{for each } r \in (0,1)\\
&\Rightarrow && \inf_{j\in\Omega} f^r_{A_j}(e^2_j)(y'_j) = 0, \quad \text{for each } r \in (0,1)
\end{aligned}$$

Next, since $x'_j = y'_j$ and $e^1_j = e^2_j$ for $j \neq i$, so

$$\begin{aligned}
f^r_{A_j}(e^2_j)(y'_j) &= f^r_{A_j}(e^1_j)(x'_j), \quad j \neq i, \quad \text{for each } r \in (0,1)\\
&> 0 \quad (\text{ using (4.1)})
\end{aligned}$$

Therefore, $f^r_{A_i}(e'_i)(y_i) = 0,$ for each $r \in (0,1)$. Put $f_{A_i} = \bigcup_{0<r<1} f^r_{A_i}$. Then $f_{A_i} \in \tau_i$ such that $f_{A_i}(e_i)(x_i) = 1$ and $f_{A_i}(e'_i)(y_i) = 0$. Similarly, we can obtain another fuzzy soft open set g_B such that $g_{B_i}(e_i)(x_i) = 0$ and $g_{B_i}(e'_i)(y_i) = 1$. The other cases can be handled similarly. □

Theorem 4.7. *Let (X, τ) be a fuzzy soft topological space. Then (X, τ) is fuzzy soft Hausdorff $\Rightarrow (X, \tau)$ is fuzzy soft $T_1 \Rightarrow (X, \tau)$ is fuzzy soft T_0.*

Proof. First, assume that (X, τ) is fuzzy soft Hausdorff. Then to show that (X, τ) is fuzzy soft T_1, choose $(x, e), (y, e') \in X \times E, (x, e) \neq (y, e')$. Consider e_{x_λ} and e'_{y_s} which are distinct fuzzy soft points over X. Next, since (X, τ) is fuzzy soft Hausdorff, there exist f^λ_A and $g^s_B \in \tau$ such that

$$e_{x_\lambda} \in f^\lambda_A,\ e'_{y_s} \in g^s_B \text{ and } f^\lambda_A \sqcap g^s_B = 0_E \tag{4.3}$$
$$\Rightarrow \lambda < f^\lambda_A(e)(x) \text{ and } s < g^s_B(e')(y)$$
$$\Rightarrow 1 \leq \sup_{0<\lambda<1} f^\lambda_A(e)(x) \text{ and } 1 \leq \sup_{0<s<1} g^s_B(e')(y)$$
$$\Rightarrow \sup_{0<\lambda<1} f^\lambda_A(e)(x) = 1 \text{ and } \sup_{0<s<1} g^s_B(e')(y) = 1. \tag{4.4}$$

Now, set $f_A = \bigcup_{0<\lambda<1} f^\lambda_A$. Then,

$$\begin{aligned}
f_A(e)(x) &= \sup_{0<\lambda<1} f^\lambda_A(e)(x)\\
&= 1, \qquad \text{(using (4.4))}.
\end{aligned}$$

Next, since

$$
\begin{aligned}
& f_A^{\lambda} \sqcap g_B^{s} = 0_E, \quad \text{for each } \lambda \in (0,1), \text{ for each } s \in (0,1) \\
& \Rightarrow \min\{f_A^{\lambda}(e_1)(y), g_B^{s}(e_1)(y)\} = 0, \quad \text{for each } \lambda \in (0,1), s \in (0,1) \text{ and } e_1 \in E \\
(4.5) \quad & \Rightarrow f_A^{\lambda}(e')(y) = 0, \quad \text{for each } \lambda \in (0,1) \text{ (using (4.3))}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
f_A(e')(y) &= \sup_{0<\lambda<1} f_A^{\lambda}(e')(y) \\
&= 0, \qquad \text{(using (4.5))}
\end{aligned}
$$

Similarly, we can construct a fuzzy soft open set g_B such that $g_B(e)(x) = 0$, $g_B(e')(y) = 1$.

Next, assume that (X, τ) is fuzzy soft T_1. So, for $(x_1, e_1), (x_2, e_2) \in X \times E$, $(x_1, e_1) \neq (x_2, e_2)$, there exist f_A, $g_B \in \tau$ such that $f_A(e_1)(x_1) = 1$, $f_A(e_2)(x_2) = 0$, $g_B(e_1)(x_1) = 0$, $g_B(e_2)(y_2) = 1$. Then, clearly (X, τ) is fuzzy soft T_0. □

The converse of the above theorem 4.7 does not hold good as can be seen in the following counter examples.

Example 4.8. Let X be an infinite set and E be a parameters set. Suppose

$$\tau = \{f_A : E \to I^X\} \cup \{0_E\}.$$

where f_A is defined as follows:

$$f_A(e)(x) = \begin{cases} 1, & \text{except for finitely many pairs } (e,x) \\ 0, & \text{otherwise.} \end{cases}$$

Then τ is a fuzzy soft topology over X as shown below:

(1) 0_E, $1_E \in \tau$.
(2) If $f_A, g_B \in \tau$, then

$$
\begin{aligned}
(f_A \sqcap g_B)(e)(x) &= \min\{f_A(e)(x), g_B(e)(x)\} \\
&= \begin{cases} 1, & \text{except for finitely many pairs } (e,x) \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
$$

Hence $f_A \sqcap g_B \in \tau$.

(3) If $(f_A)_i \in \tau$, then

$$
\begin{aligned}
(\bigsqcup_{i\in\Omega} (f_A)_i)(e)(x) &= \sup_{i\in\Omega} (f_A)_i(e)(x) \\
&= \begin{cases} 1, & \text{except for finitely many pairs } (e,x) \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
$$

Hence $\bigsqcup_{i\in\Omega} f_{A_i} \in \tau$.

Now this fuzzy soft topological space is fuzzy soft T_1 as $e_{\{x\}}$ is fuzzy soft closed, for each $e \in E$ and $x \in X$. But (X, τ) is not fuzzy soft T_2, since there does not exist any pair of non trivial disjoint fuzzy soft open sets.

Example 4.9. Let $X = \{a, b\}$ and E be a parameter set which consists of only one element say, $E = \{e\}$ and $\tau = \{f_A : E \to I^X \mid f_A(e) = \chi_{\{a\}}\} \cup \{0_E, 1_E\}$. Then (X, τ) is fuzzy soft topological space which is fuzzy soft T_0, since for $(a, e), (b, e) \in X \times E$, there exists $f_A \in \tau$ such that $f_A(e)(a) \neq f_A(e)(b)$ but it is not fuzzy soft T_1 since there does not exist any fuzzy soft open set g_B such that $g_B(e)(b) = 1$ and $g_B(e)(a) = 0$.

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