

## Fuzzy $D'$ -Baire spaces

S. Anjalmose

Received 22 September 2014; Accepted 31 March 2015

---

**ABSTRACT.** In this paper the concepts of fuzzy  $D'$ -Baire spaces are introduced. Several characterizations of fuzzy  $D'$ -Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

**2010 AMS Classification:** 54A40, 03E72

**Keywords:** Fuzzy dense, Fuzzy nowhere dense, Fuzzy first category space, Fuzzy Baire space, Fuzzy  $D'$ -Baire space.

**Corresponding Author:** S. Anjalmose ([ansalmose@gmail.com](mailto:ansalmose@gmail.com))

---

### 1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper[15] in the year 1965. Thereafter the paper of C.L.Chang[3] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. Tang[8] has used a slightly changed version of Chang's fuzzy topological spaces to model spatial objects for GIS databases and Structured Query Language (SQL) for GIS. Fuzzy set theory finds its applications for Modeling[4], uncertainty and vagueness in various fields of Science and Engineering such as Nonlinear Dynamical Systems, Control of Chaos, Quantum Physics.

The concepts of Baire spaces have been studied extensively in classical topology in [6],[7],[16] and [17]. The concept of Baire space in fuzzy setting was introduced and studied by G. Thangaraj and S. Anjalmose in [9]. In this paper we introduce the concepts of fuzzy  $D'$ -Baire spaces. Also we discuss several characterizations of fuzzy  $D'$ -Baire spaces. In section 4, inter-relations between fuzzy  $D'$ -Baire spaces and some other fuzzy topological spaces. In section 5, the relations of fuzzy  $D'$ -Baire

spaces, fuzzy D-Baire spaces and fuzzy Baire spaces are also investigated. Several examples are given to illustrate the concepts introduced in this paper.

## 2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by  $(X, T)$  or simply by  $X$ , we will denote a fuzzy topological space due to Chang[3].

**Definition 2.1.** Let  $\lambda$  and  $\mu$  be any two fuzzy sets in  $(X, T)$ . Then we define  $\lambda \vee \mu : X \rightarrow [0, 1]$  as follows :  $(\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\}$ . Also we define  $\lambda \wedge \mu : X \rightarrow [0, 1]$  as follows :  $(\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\}$ .

**Definition 2.2** ([1]). Let  $(X, T)$  be a fuzzy topological space and  $\lambda$  be any fuzzy set in  $(X, T)$ . we define  $\text{int}(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$  and  $\text{cl}(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$ . For any fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$ , it is easy to see that  $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$  and  $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$ .

**Definition 2.3** ([13]). A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy dense if there exists no fuzzy closed set  $\mu$  in  $(X, T)$  such that  $\lambda < \mu < 1$ . That is,  $\text{cl}(\lambda) = 1$ .

**Definition 2.4** ([13]). A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy nowhere dense if there exists no non-zero fuzzy open set  $\mu$  in  $(X, T)$  such that  $\mu < \text{cl}(\lambda)$ . That is,  $\text{int} \text{cl}(\lambda) = 0$ .

**Definition 2.5** ([13]). A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy first category if  $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$  where  $\lambda_i$ 's are fuzzy nowhere dense sets in  $(X, T)$ . Any other fuzzy set in  $(X, T)$  is said to be of **fuzzy second category**.

**Definition 2.6** ([9]). Let  $\lambda$  be a fuzzy first category set in a fuzzy topological space  $(X, T)$ . Then  $1 - \lambda$  is called a **fuzzy residual set** in  $(X, T)$ .

**Definition 2.7** ([13]). A fuzzy topological space  $(X, T)$  is called fuzzy first category if  $1 = \vee_{i=1}^{\infty}(\lambda_i)$  where  $\lambda_i$ 's are fuzzy nowhere dense sets in  $(X, T)$ . A topological space which is not of fuzzy first category, is said to be of **fuzzy second category**.

**Lemma 2.8** ([1]). For a family of  $\{\lambda_\alpha\}$  of fuzzy sets of a fuzzy space  $X$ ,  $\vee \text{cl}(\lambda_\alpha) \leq \text{cl}(\vee \lambda_\alpha)$ . In case  $A$  is a finite set,  $\vee \text{cl}(\lambda_\alpha) = \text{cl}(\vee \lambda_\alpha)$ . Also  $\vee \text{int}(\lambda_\alpha) \leq \text{int}(\vee \lambda_\alpha)$ .

**Definition 2.9** ([12]). A fuzzy topological space  $(X, T)$  is called a fuzzy resolvable space if there exists a fuzzy dense set  $\lambda$  in  $(X, T)$  such that  $\text{cl}(1 - \lambda) = 1$ . Otherwise  $(X, T)$  is called a **fuzzy irresolvable space**.

**Definition 2.10** ([2]). A fuzzy topological space  $(X, T)$  is called a fuzzy submaximal space if for each fuzzy set  $\lambda$  in  $(X, T)$  such that  $\text{cl}(\lambda) = 1$ , then  $\lambda \in T$  in  $(X, T)$ .

**Definition 2.11** ([14]). A fuzzy topological space  $(X, T)$  is called a fuzzy almost resolvable space if  $\vee_{i=1}^{\infty}(\lambda_i) = 1$ , where the fuzzy sets  $(\lambda_i)$ 's in  $(X, T)$  are such that  $\text{int}(\lambda_i) = 0$ . Otherwise  $(X, T)$  is called a **fuzzy almost irresolvable space**.

**Definition 2.12** ([11]). A fuzzy topological space  $(X, T)$  is called a fuzzy nodec space if every non-zero fuzzy nowhere dense set  $\lambda$  is fuzzy closed in  $(X, T)$ . That is, if  $\lambda$  is a fuzzy nowhere dense set in  $(X, T)$ , then  $1 - \lambda \in T$ .

### 3. Fuzzy $D'$ -Baire spaces

Motivated by the classical concept introduced in [5] we shall now define:

**Definition 3.1.** A fuzzy topological space  $(X, T)$  is fuzzy Baire space. Then  $(X, T)$  is called a fuzzy  $D'$ -Baire space if every fuzzy set with empty interior is fuzzy nowhere dense in  $(X, T)$ .

**Example 3.2.** Let  $X = \{a, b, c\}$  and  $\lambda$  and  $\mu$  be the fuzzy sets defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7$ .

Then  $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$  is a fuzzy topology on  $X$ .

Now consider the following fuzzy sets defined on  $X$  as follows:

$\alpha : X \rightarrow [0, 1]$  defined as  $\alpha(a) = 0.4; \alpha(b) = 0.5; \alpha(c) = 0.2$ .

$\beta : X \rightarrow [0, 1]$  defined as  $\beta(a) = 0.3; \beta(b) = 0.4; \beta(c) = 0.2$ .

$\delta : X \rightarrow [0, 1]$  defined as  $\delta(a) = 0.5; \delta(b) = 0.5; \delta(c) = 0.3$ .

The fuzzy nowhere dense sets in  $(X, T)$  are  $(1 - \mu), 1 - (\lambda \vee \mu), \alpha, \beta, \delta$ . Therefore  $(1 - \mu) \vee (1 - \lambda \vee \mu) \vee \alpha \vee \beta \vee \delta = 1 - \mu$  implies that  $\text{int}(1 - \mu) = 0$ . Therefore  $(X, T)$  is fuzzy Baire space. Now the fuzzy sets  $\alpha, \beta, \delta, 1 - \mu$  and  $1 - \lambda \vee \mu$  are empty interior in  $(X, T)$ . Then the fuzzy sets  $\alpha, \beta, \delta, 1 - \mu$  and  $1 - \lambda \vee \mu$  are fuzzy nowhere dense sets in  $(X, T)$ . Therefore  $(X, T)$  is a fuzzy Baire space and the fuzzy sets with empty interior in  $(X, T)$  is fuzzy nowhere dense. Hence  $(X, T)$  is fuzzy  $D'$ -Baire space.

**Example 3.3.** Let  $X = \{a, b, c\}$  and  $\lambda$  and  $\mu$  be the fuzzy sets defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7$ .

Then  $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$  is a fuzzy topology on  $X$ .

Now consider the following fuzzy sets defined on  $X$  as follows:

$\alpha : X \rightarrow [0, 1]$  defined as  $\alpha(a) = 0.4; \alpha(b) = 0.5; \alpha(c) = 0.2$ .

$\beta : X \rightarrow [0, 1]$  defined as  $\beta(a) = 0.3; \beta(b) = 0.4; \beta(c) = 0.2$ .

$\delta : X \rightarrow [0, 1]$  defined as  $\delta(a) = 0.5; \delta(b) = 0.6; \delta(c) = 0.4$ .

The fuzzy nowhere dense sets in  $(X, T)$  are  $(1 - \mu), 1 - (\lambda \vee \mu), \alpha, \beta$ . Therefore  $(1 - \mu) \vee (1 - \lambda \vee \mu) \vee \alpha \vee \beta = 1 - \mu$  implies that  $\text{int}(1 - \mu) = 0$ . Therefore  $(X, T)$  is fuzzy Baire space. Now the fuzzy sets  $\alpha, \beta, \delta, 1 - \mu$  and  $1 - \lambda \vee \mu$  are empty interior in  $(X, T)$ . Then the fuzzy sets  $\alpha, \beta, 1 - \mu$  and  $1 - \lambda \vee \mu$  are fuzzy nowhere dense sets in  $(X, T)$ . Now the fuzzy set  $\delta$  is empty interior but not a fuzzy nowhere dense set in  $(X, T)$ . Therefore  $(X, T)$  is a fuzzy Baire space but not fuzzy  $D'$ -Baire space.

**Theorem 3.4 ([9]).** If  $\lambda$  is a fuzzy closed set in a fuzzy topological space  $(X, T)$  with  $\text{int}(\lambda) = 0$ , then  $\lambda$  is a fuzzy nowhere dense set in  $(X, T)$ .

**Proposition 3.5.** If every fuzzy closed set  $\lambda$  with empty interior in a fuzzy Baire space  $(X, T)$  is fuzzy  $D'$ -Baire space.

*Proof.* Let  $\text{int}(\lambda_i) = 0$ , where  $\lambda_i$ 's are fuzzy closed set in  $(X, T)$ . By Theorem 3.4,  $(\lambda_i)$ 's are fuzzy nowhere dense sets in  $(X, T)$ . Therefore  $(X, T)$  is fuzzy Baire space and each fuzzy set with empty interior is fuzzy nowhere dense in  $(X, T)$ . Hence  $(X, T)$  is fuzzy  $D'$ -Baire space.  $\square$

**Proposition 3.6.** *If  $cl(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 1$ , where  $\lambda_i$ 's are fuzzy open and fuzzy dense in  $(X, T)$ , then  $(X, T)$  is fuzzy  $D'$ -Baire space.*

*Proof.* Let  $\lambda_i$  be fuzzy dense and fuzzy open sets in a fuzzy topological space  $(X, T)$ . Now  $cl(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 1$  implies that  $1 - cl(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 0$  implies that  $int(\bigvee_{i=1}^{\infty}(1 - \lambda_i)) = 0$ . Let  $\mu_i = 1 - \lambda_i$ , then we have  $int(\bigvee_{i=1}^{\infty}(\mu_i)) = 0$ . Now  $\lambda_i \in T$  implies that  $1 - \lambda_i$  is a fuzzy closed set in  $(X, T)$  and hence  $\mu_i$  is fuzzy closed and  $int(\mu_i) = int(1 - \lambda_i) = 1 - cl(\lambda_i) = 1 - 1 = 0$ . Hence by Theorem 3.4,  $\mu_i$  is a fuzzy nowhere dense sets in  $(X, T)$ . Hence  $int(\bigvee_{i=1}^{\infty}(\mu_i)) = 0$ , where  $\mu_i$ 's are fuzzy nowhere dense sets, implies that  $(X, T)$  is fuzzy Baire space. Now we claim every fuzzy set with empty interior is fuzzy nowhere dense. Suppose  $int(\mu_j) = 0$  and  $\mu_j$  is not a fuzzy nowhere dense set in  $(X, T)$  implies that  $intcl(\mu_j) \neq 0$  implies that  $int(\mu_j) \neq 0$  since  $\mu_j$  is fuzzy closed, which is a contradiction to  $int(\mu_j) = 0$ . Therefore  $\mu_j$  is fuzzy nowhere dense set in  $(X, T)$ . Therefore every fuzzy set with empty interior is nowhere dense in  $(X, T)$ . Hence  $(X, T)$  is a fuzzy  $D'$ -Baire space.  $\square$

#### 4. Fuzzy $D'$ -Baire spaces and some other fuzzy topological spaces

**Theorem 4.1** ([9]). *A fuzzy second category space need not be a fuzzy Baire space.*

**Proposition 4.2.** *If the fuzzy topological space  $(X, T)$  is a fuzzy  $D'$ -Baire space, then  $(X, T)$  is a fuzzy second category space.*

*Proof.* Let  $(X, T)$  be a fuzzy  $D'$ -Baire space, therefore  $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ , where  $\lambda_i$ 's are fuzzy nowhere dense set and each  $int(\lambda_i) = 0$ . Suppose  $(X, T)$  is fuzzy first category space, therefore  $\bigvee_{i=1}^{\infty}(\lambda_i) = 1$  implies that  $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = int(1) = 1$  implies that  $0 = 1$  which is a contradiction. Hence  $(X, T)$  is a fuzzy second category space.  $\square$

**Remark 4.3.** The converse of the above proposition need not be true.

*Proof.* By Theorem 4.1, a fuzzy second category space need not be fuzzy Baire space, then the fuzzy second category space need not be fuzzy  $D'$ -Baire space.  $\square$

**Proposition 4.4.** *If the fuzzy topological space  $(X, T)$  is fuzzy first category, then  $(X, T)$  is not of fuzzy  $D'$ -Baire space.*

*Proof.* Since  $(X, T)$  is fuzzy first category space, then  $\bigvee_{i=1}^{\infty}(\lambda_i) = 1$ , where  $\lambda_i$ 's are fuzzy nowhere dense sets. Therefore  $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = int(1) = 1 \neq 0$ . Hence  $int(\bigvee_{i=1}^{\infty}(\lambda_i)) \neq 0$ , where  $\lambda_i$ 's are fuzzy nowhere dense sets in  $(X, T)$ . Therefore  $(X, T)$  is not a fuzzy Baire space. Hence  $(X, T)$  is not a fuzzy  $D'$ -Baire space.  $\square$

**Proposition 4.5.** *A fuzzy  $D'$ -Baire space need not be fuzzy submaximal space. Consider the following example*

**Example 4.6.** Let  $X = \{a, b, c\}$  and  $\lambda$  and  $\mu$  be the fuzzy sets defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7$ .

Then  $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$  is a fuzzy topology on  $X$ .

Now consider the following fuzzy sets defined on  $X$  as follows:

$\alpha : X \rightarrow [0, 1]$  defined as  $\alpha(a) = 0.4; \alpha(b) = 0.5; \alpha(c) = 0.2$ .

$\beta : X \rightarrow [0, 1]$  defined as  $\beta(a) = 0.3; \beta(b) = 0.4; \beta(c) = 0.2$ .

$\delta : X \rightarrow [0, 1]$  defined as  $\delta(a) = 0.5; \delta(b) = 0.6; \delta(c) = 0.7$ .

The fuzzy nowhere dense sets in  $(X, T)$  are  $(1 - \mu), 1 - (\lambda \vee \mu), \alpha, \beta$ . Therefore  $(1 - \mu) \vee (1 - \lambda \vee \mu) \vee \alpha \vee \beta = 1 - \mu$  implies that  $\text{int}(1 - \mu) = 0$ . Therefore  $(X, T)$  is fuzzy Baire space. Now the fuzzy sets  $\alpha, \beta, 1 - \mu$  and  $1 - \lambda \vee \mu$  are empty interior in  $(X, T)$ . Then the fuzzy sets  $\alpha, \beta, 1 - \mu$  and  $1 - \lambda \vee \mu$  are fuzzy nowhere dense sets in  $(X, T)$ . Therefore  $(X, T)$  is fuzzy  $D'$ -Baire space. Hence  $(X, T)$  is a fuzzy  $D'$ -Baire space but not of fuzzy submaximal space. Since the fuzzy set  $\delta$  is fuzzy dense set but not a fuzzy open set in  $(X, T)$ .

**Proposition 4.7.** *If the fuzzy topological space  $(X, T)$  is fuzzy  $D'$ -Baire space, then  $(X, T)$  is not of fuzzy almost resolvable space.*

*Proof.* Since  $(X, T)$  is fuzzy  $D'$ -Baire space. By Proposition 4.2, every fuzzy  $D'$ -Baire space is fuzzy second category space, therefore  $(X, T)$  is not a fuzzy first category space. Then  $\bigvee_{i=1}^{\infty} (\lambda_i) \neq 1$ , where  $\lambda_i$ 's are fuzzy nowhere dense sets in  $(X, T)$ . Now  $\lambda_i$ 's are fuzzy nowhere dense sets implies that  $\text{intcl}(\lambda_i) = 0$ . Since  $\text{int}(\lambda_i) \leq \text{intcl}(\lambda_i)$  we have  $\text{int}(\lambda_i) = 0$ . Hence  $\bigvee_{i=1}^{\infty} (\lambda_i) \neq 1$ , where  $\text{int}(\lambda_i) = 0$ . Hence  $(X, T)$  is not a fuzzy almost resolvable space.  $\square$

**Proposition 4.8.** *If the fuzzy topological space  $(X, T)$  is fuzzy nodec space, then  $(X, T)$  is not of fuzzy  $D'$ -Baire space.*

*Proof.* Let  $(X, T)$  be a fuzzy nodec space. Now  $(\lambda_i)$  be fuzzy nowhere dense sets in  $(X, T)$ . Then  $\lambda_i$ 's are fuzzy closed sets in  $(X, T)$ , that is  $\text{cl}(\lambda_i) = \lambda_i$  for each  $i \in I$ . Now  $\bigvee_{i=1}^{\infty} \text{cl}(\lambda_i) = \bigvee_{i=1}^{\infty} (\lambda_i)$  and  $\bigvee_{i=1}^{\infty} (\lambda_i)$  is a fuzzy first category set in  $(X, T)$ . Hence  $\bigvee_{i=1}^{\infty} \text{cl}(\lambda_i)$  is a fuzzy first category set in  $(X, T)$ . Now  $\text{int}(\bigvee_{i=1}^{\infty} \text{cl}(\lambda_i)) \geq \bigvee_{i=1}^{\infty} \text{intcl}(\lambda_i) = 0$ . Hence  $\text{int}(\bigvee_{i=1}^{\infty} \text{cl}(\lambda_i)) \neq 0$ . Therefore  $(X, T)$  is not a fuzzy Baire space. Hence  $(X, T)$  is not of fuzzy  $D'$ -Baire space.  $\square$

## 5. Fuzzy $D'$ -Baire space, fuzzy D-Baire space and fuzzy Baire spaces

If the fuzzy topological space  $(X, T)$  is fuzzy  $D'$ -Baire space then  $(X, T)$  is fuzzy D-Baire space. Consider the following example:

**Example 5.1.** Let  $X = \{a, b, c\}$ . Consider the fuzzy sets  $\lambda$  and  $\mu$  defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7$ .

Then  $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$  is a fuzzy topology on  $X$ . The fuzzy nowhere dense sets in  $(X, T)$  are  $(1 - \mu), 1 - (\lambda \vee \mu)$ . Therefore  $(1 - \mu) \vee (1 - \lambda \vee \mu) = 1 - \mu$  implies that  $\text{int}(1 - \mu) = 0$ . Therefore  $(X, T)$  is fuzzy Baire space. Now the fuzzy sets  $1 - \mu$  and  $1 - \lambda \vee \mu$  are empty interior in  $(X, T)$ . Then the fuzzy sets  $1 - \mu$  and  $1 - \lambda \vee \mu$  are fuzzy nowhere dense sets in  $(X, T)$ . Therefore the fuzzy sets with empty interior in  $(X, T)$  is fuzzy nowhere dense. Hence  $(X, T)$  is fuzzy  $D'$ -Baire space. Now  $1 - \mu$  is fuzzy first category in  $(X, T)$ , then  $\text{intcl}(1 - \mu) = 0$ . Therefore the fuzzy first category in  $(X, T)$  is fuzzy nowhere dense in  $(X, T)$ . Hence a fuzzy  $D'$ -Baire space is fuzzy D-Baire space.

The converse need not be true, consider the following example:

**Example 5.2.** Let  $X = \{a, b, c\}$ . Consider the fuzzy sets  $\lambda, \mu$  and  $\gamma$  defined on  $X$  as follows:

$\lambda : X \rightarrow [0, 1]$  defined as  $\lambda(a) = 0.7; \lambda(b) = 0.6; \lambda(c) = 0.2$ .

$\mu : X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.8$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.5; \gamma(b) = 0.3; \gamma(c) = 0.6$ .

Then  $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\lambda \vee \gamma), (\lambda \wedge \gamma), \mu \wedge (\lambda \vee \gamma), \gamma \vee (\lambda \wedge \mu), 1\}$  is a fuzzy topology on  $X$ . Now consider the following fuzzy sets defined on  $X$  as follows:

$\alpha : X \rightarrow [0, 1]$  defined as  $\alpha(a) = 0.2; \alpha(b) = 0.5; \alpha(c) = 0.7$ .

$\beta : X \rightarrow [0, 1]$  defined as  $\beta(a) = 0.3; \beta(b) = 0.5; \beta(c) = 0.6$ .

The fuzzy nowhere dense sets in  $(X, T)$  are  $(1 - \mu), 1 - (\lambda \vee \mu)$  and  $1 - (\lambda \vee \gamma)$ . Now the fuzzy first category sets  $(1 - \lambda)$  and  $1 - (\lambda \vee \gamma)$  are fuzzy nowhere dense sets in  $(X, T)$ . Therefore  $(X, T)$  is fuzzy D-Baire space but not of fuzzy  $D'$ -Baire space. Since the fuzzy sets  $\alpha$  and  $\beta$  are empty interior but not a fuzzy nowhere dense set in  $(X, T)$ . Hence a fuzzy D-Baire space need not be fuzzy  $D'$ -Baire space.

**Proposition 5.3.** *If the fuzzy topological space  $(X, T)$  is fuzzy  $D'$ -Baire space, then  $(X, T)$  is fuzzy Baire space.*

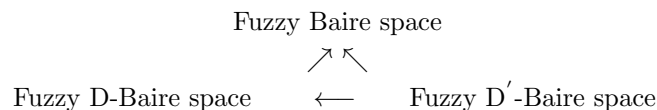
*Proof.* Proof is straight forward. □

**Remark 5.4.** The converse of the above proposition need not be true.

*Proof.* In example 3.3, the fuzzy topological space  $(X, T)$  is fuzzy Baire space, but not of fuzzy  $D'$ -Baire space. □

**Theorem 5.5** ([10]). *If the fuzzy topological space  $(X, T)$  is fuzzy D-Baire space, then  $(X, T)$  is fuzzy Baire space. Converse need not be true.*

The following implications are hold



## 6. Conclusions

In this paper we first defined fuzzy  $D'$ -Baire spaces. Inter relations between fuzzy  $D'$ -Baire spaces and some other fuzzy topological spaces and relations of fuzzy  $D'$ -Baire spaces, fuzzy D-Baire spaces and fuzzy Baire spaces.

## REFERENCES

- [1] K. K. Azad., On fuzzy semi continuity, Fuzzy almost continuity and Fuzzy weakly continuity, J. Math. Anal. Appl. 82 (1981) 14–32.
- [2] G. Balasubramanian, Maximal fuzzy topologies, Kybernetika 31 (5) (1995) 459–464.
- [3] C. L. Chang, Fuzzy Topological Spaces, J.M ath. Anal. Appl. 24 (1968) 182–190.
- [4] S. DU, Q. Qin, Q. Wang and B. Li, Fuzzy description of topological relations I. a Unified fuzzy 9-intersection model. Lecture Notes in Comput. Sci. 3612 : 1261 : 1273, 2005.
- [5] S. Garcia-Ferreira, A. Garcia-Maynez and M. Hrusak, Spaces in which every dense subset is Baire, Houston J. Math. 39 (1) (2013) 247–263.

- [6] G. Gruenhage and D. Lutzer, Baire and Volterra Spaces, Proc. Amer. Soc. 128 (2000) 3115–3124.
- [7] T. Neubrunn, A Note on Mappings of Baire spaces, Math. Slovaca 27 (2) (1977) 173–176.
- [8] X. Tang, Spatial Object modeling in fuzzy topological spaces with applications to land cover change in China, Ph.D. Dissertation, University of Twente, Enschede, The Netherlands, 2004 ITC Dissertation No. 108.
- [9] G. Thangaraj and S. Anjalmose, On Fuzzy Baire Spaces, J. Fuzzy Math. 21 (3) (2013) 667–676.
- [10] G. Thangaraj and S. Anjalmose, On fuzzy D-Baire spaces, Ann. fuzzy Math. Inform. 7 (1) (2014) 99–108.
- [11] G. Thangaraj and S. Anjalmose, Some remarks on Fuzzy Baire Spaces, Scientia Magna 9 (1) (2013) 1–6.
- [12] G. Thangaraj and G. Balasubramanian, On Fuzzy Resolvable and Fuzzy Irresolvable spaces, Fuzzy sets, Rough sets and Multi-valued operations and Applications 1 (2) (2009) 173–180.
- [13] G. Thangaraj and G. Balasubramanian, On Somewhat Fuzzy Continuous functions, J. Fuzzy Math. 11 (3) (2003) 725–736.
- [14] G. Thangaraj and G. Vijayann, On Fuzzyalmost resolvable and fuzzy almost irresolvable spaces, Inter. J. Statistika and Mathmatika 9 (2) (2014) 61–65.
- [15] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.
- [16] Frolik Zdenek , Baire Spaces and some Generalizations of Complete Metric spaces. Czech. Math. J. 11 (2) (1961) 237–247.
- [17] Frolik Zdenek , Remarks concerning the invariance of Baire Spaces under mappings, Czech, Math. J. 11 (3) (1961) 381–385.

S. ANJALMOSE ([ansalmose@gmail.com](mailto:ansalmose@gmail.com))

Research and Development Centre, Bharathiar University, Coimbatore - 641 046,  
Tamilnadu, India