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On fixed points in complete fuzzy normed linear spaces

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Abstract. In this paper , we state and prove some fixed point theorems in a complete fuzzy normed linear space introduced in a slightly different way. Our results contain extensions and generalizations of multitudes of fixed point theorems in complete fuzzy normed linear spaces which include the classical fixed point theorems mainly due to Banach , Caccioppoli and Kannan. We also prove a common fixed point theorem which generalizes the fuzzy analogues of Jungck's fixed point theorem in classical metric spaces and Vasuki's common fixed point theorem in fuzzy metric spaces. We deduce a corollary and support our results with suitable examples.

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1. Introduction

The concept of fuzzy norm on a linear space was introduced by several authors in various ways. In particular , A. K. Katsaras [15] , while studying fuzzy topological vector spaces , introduced the notion of a fuzzy norm in the year 1984. In fact , he was the first to introduce the notion of fuzzy normed linear spaces. In 1992 , C. Felbin [9] introduced an idea of fuzzy norm on a linear space so that the resulting fuzzy metric is of O. Kaleva and S. Seikkala [12] type. In 1994 , S. C. Cheng and J. N. Mordeson [5] introduced another idea of a fuzzy norm in such a way that the induced fuzzy metric is of I. Kramosil and J. Michalek [16] type. In 2003 , T. Bag and S. K. Samanta [4] introduced the concept of fuzzy norm in a slightly different way and proved some fixed point theorems along with some other results in some of his papers [1, 2, 3, 4]. Further , the authors introduced α - norms on a linear space and proved the Decomposition Theorem. For further references on fixed point theory , we cite [6, 7, 8, 10] . In our present attempt , we have proved some fixed point theorems in fuzzy normed linear spaces by extending and generalizing some classical fixed point theorems. Also we deduce a corollary and illustrate our results with suitable examples.

2. Preliminaries

In this section , we recall some definitions and known results which are already in the literature. Here we also introduce a new definition of a fuzzy normed linear space defined in a slightly different way other than [4]. The fuzzy metric induced by the newly defined fuzzy norm is of Kramosil and Michalek [16] type.

Definition 2.1 ([19]). A fuzzy set A in X is a mapping $A : X \to [0, 1]$. For $x\epsilon X, A(x)$ is called the grade of membership of x .

Definition 2.2 ([17]). A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm , if $([0, 1], *)$ is an abelian topological monoid with unity 1 such that $a * b \leq c * d$, whenever $a \leq c, b \leq d$, for all $a, b, c, d\epsilon[0, 1]$.The followings are some continuous t - norms: (i) $a * b = min\ \{a, b\}$, (ii) $a * b = ab$, (iii) $a * b = max$ $\{0, a + b - 1\}$, $a, b\ \epsilon$[0,1] .

We now introduce a new definition of a fuzzy norm N on a linear space X in a slightly different way as follows. Here N is a fuzzy set in $X \times [0, \infty[$.

Definition 2.3. The 3-tuple $(X, N, *)$ is called a fuzzy normed linear space (in short,F-nls) and N, a fuzzy norm, if X is a real linear space, $*$ is a continuous t-norm and N is a fuzzy set in $X \times [0, \infty[$, satisfying the following conditions:

For all $x, y\epsilon X$ and $s, t > 0$,

(2.3.1)$N(x, 0) = 0$,

(2.3.2)$N(x, t) = 1$, for all $t > 0$ if and only if $x = 0$,

(2.3.3) $N(cx, t) = N\left(x, \frac{t}{|c|}\right)$,if $c \neq 0$,

(2.3.4)$N(x + y, s + t) \geq N(x, s) * N(y, t)$,

(2.3.5) $N(x, .) : [0, \infty[\to [0, 1]$ is left continuous , and $lim_{t\to\infty} N(x, t) = 1$.

Example 2.4 ([1]). Let $(X, \| \ . \ \|)$ be a real normed linear space and $a * b = min\{a, b\}$,

$a, b\epsilon[0,1]$. Let N be a fuzzy set in $X \times [0,\infty[$, given by $N(x,t) = \frac{t}{t+\|x\|}$, and $N(x,0) = 0$. Then N is a fuzzy norm on X and $(X, N, *)$ is a F-nls. N is called the standard fuzzy norm on X induced by $\| \, . \, \|$. Thus every norm on a real linear space induces a fuzzy norm.

In what follows, the symbol X will denote a F - nls $(X, N, *)$.

Definition 2.5 ([1])**.** A sequence $\{x_n\}$ in a F - nls X is called

(a) a Cauchy sequence , if $lim_{n\to\infty}N(x_{n+p} - x_n, t) = 1$, for all $t > 0, p = 1, 2, 3, \ldots$

(b) convergent to $x\epsilon X$(in symbols , $\lim_{n\to\infty} x_n = x$ or $x_n \to x$) , if $lim_{n\to\infty}N(x_n - x, t)$ $= 1$, for all $t > 0$.

Definition 2.6 ([1])**.** A F - nls X is said to be complete,if every Cauchy sequence in X is convergent.

Definition 2.7. Let $(X, N, *)$ be a F- nls and $T : X \to X$ be a mapping. T is said to be continuous, if for every $x\epsilon X, x_n \to x$ implies $Tx_n \to Tx$.

Definition 2.8 ([18])**.** A pair $[A, B]$ self maps of a F - nls X is said to be be R-weakly commuting, if there exists a positive real nuber R such that $N(ABx - BAx, t) \geq N\left(Ax - Bx, \frac{t}{R}\right)$, for every $x\epsilon X, t > 0$.

Lemma 2.9. *In a F - nls X, the followings hold:*

(i)$N(x, .) : [0,\infty[\to [0, 1]$ *is non - decreasing.*

(ii) $N(-x, t) = N(x, t)$, *for every* $x\epsilon X, t \geq 0$.

Proof. These follow immediately from (2.3.4) and (2.3.3) respectively. □

Lemma 2.10. *If* $\{x_n\}$ *is a sequence in a F - nls X then* $N(x_n - x_{n+p}, t) \geq$ $N(x_n - x_{n+1}, \frac{t}{p}) * N(x_{n+1} - x_{n+2}, \frac{t}{p}) * \ldots * N(x_{n+p-1} - x_{n+p}, \frac{t}{p})$, *for every* $t > 0, p > 0$.

Proof. It is easy to see the result follow by induction from (2.3.4). □

Lemma 2.11 ([11])**.** *In a F - nls X, if for* $x\epsilon X, N(x, kt) \geq N(x, t)$, *for every* $t > 0$ *and some* $0 < k < 1$, *then* $x = 0$.

Lemma 2.12 ([11])**.** *A sequence* $\{x_n\}$ *in a F - nls X satisfying* $N(x_{n+1} - x_n, kt) \geq$ $N(x_n - x_{n-1}, t)$, *for every* $t > 0$ *and some* $0 < k < 1$, *is a Cauchy sequence.*

3. Main Results

Grabiec [11] extended Banach Contraction Principle to fuzzy metric spaces in the year 1988.The present authors [6] extended the Banach Contraction Principle to fuzzy 2- metric spaces. In the following Theorem, we extend the Principle to the newly introduced F - nls.

Definition 3.1. Let $(X, N, *)$ be a fuzzy normed linear space.A mapping $T : X \to X$ is called a contraction if $N(Tx - Ty, kt) \geq N(x - y, t)$, for every $x, y \epsilon X$ and some $0 < k < 1$.

Theorem 3.2. *If T is a self map of a complete fuzzy normed linear space $(X, N, *)$ satisfying*

$$N(Tx - Ty, kt) \geq N(x - y, t), \tag{3.2.1}$$

for every $x, y \epsilon X$ and some $0 < k < 1$, then T has a unique fixed point.

Proof. Let $x \epsilon X, x_n = T^n x, n = 1, 2, 3, \ldots$. Now $\{x_n\}$ is a sequence in X such that $x_1 = Tx, x_2 = Tx_1, \ldots, x_{n+1} = Tx_n, n = 1, 2, 3, \ldots$. We get ,$N(x_{n+1} - x_n, kt) = N(Tx_n - Tx_{n-1}, kt) \geq N(x_n - x_{n-1}, t)$, by (3.2.1) , for every $t > 0$.
Therefore , by Lemma (2.12) , we get $\{x_n\}$ is a Cauchy sequence in X. Since X is complete ,$x_n \to x$, for some $x \epsilon X$, we get ,

$$\begin{aligned} N(Tx - x, t) &\geq N(Tx - Tx_n, \tfrac{t}{2}) * N(Tx_n - x, \tfrac{t}{2}) \\ &\geq N(x - x_n, \tfrac{t}{2k}) * N(x_{n+1} - x, \tfrac{t}{2}) \to 1 * 1 = 1 , \end{aligned}$$

as $n \to \infty$, for every $t > 0$. Therefore ,$N(Tx - x, t) = 1$, for every $t > 0$ and so $Tx = x$, a fixed point of T. For uniqueness , let $Ty = y$, for some $y \epsilon X$. Now $N(x - y, kt) = N(Tx - Ty, kt) \geq N(x - y, t)$, for every $t > 0$.
Therefore , by Lemma 2.11 , $x = y$ and so the fixed point is unique. □

The present authors [7] established the fuzzy analogue of Caccioppoli's fixed point Theorem. In our next Theorem , we extend Caccioppoli's fixed point Theorem to fuzzy normed linear spaces.

Theorem 3.3. *Let T be a self map of a complete fuzzy normed linear space $(X, N, *)$ satisfying*

$$N(T^n x - T^n y, k_n t) \geq N(x - y, t), \tag{3.3.1}$$

for every $x, y \epsilon X, t > 0$ and $k_n > 0$, is independent of x, y . If $k_n \to 0$, then T has a unique fixed point.

Proof. Let $x \epsilon X$. We consider the sequence $\{x_n\}$ such that $x_n = T^n x, n = 1, 2, 3, \ldots$ Then

$$\begin{aligned} N(x_n - x_{n+p}, t) &\geq N(x_n - x_{n+1}, \tfrac{t}{p}) * N(x_{n+1} - x_{n+2}, \tfrac{t}{p}) * \ldots * N(x_{n+p-1} - x_{n+p}, \tfrac{t}{p}) \\ &\geq N(x - x_1, \tfrac{t}{pk_n}) * N(x - x_1, \tfrac{t}{pk_{n+1}}) * \ldots * N(x - x_1, \tfrac{t}{pk_{n+p-1}}), \text{by} \\ (3.3.1) \quad &\to 1 * 1 * \ldots * 1 = 1, \text{as}\, n \to \infty, \text{ for every}\, t > 0, p > 0. \end{aligned}$$

Therefore, $lim_{n\to\infty} N(x_n - x_{n+p}, t) = 1$, for every $t > 0, p > 0$ and so $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, $x_n \to x$, for some $x \epsilon X$. We now get ,

$$\begin{aligned} N(x - Tx, t) &\geq N(x - x_{n+1}, \tfrac{t}{2}) * N(x_{n+1} - Tx, \tfrac{t}{2}) \\ &\geq N(x_{n+1} - x, \tfrac{t}{2}) * N(x_n - x, \tfrac{t}{2k_1}), \text{by } (3.3.1). \\ &\to 1 * 1 = 1, as\, n \to \infty, \text{for every} t > 0. \end{aligned}$$

Hence $N(x - Tx, t) = 1$ for every $t > 0$ and so $Tx = x$, a fixed point of T. For uniqueness , let $Ty = y$, for some $y \epsilon X$. We get $T^n x = x$ and $T^n y = y$, for every $n = 1, 2, 3, \ldots$. Now $N(x - y, t) = N(T^n x - T^n y, t) \geq N(x - y, \frac{t}{k_n}) \to 1$, as $n \to \infty$, for every $t > 0$. Therefore , $x = y$ and so the fixed point is unique. □

Remark 3.4. It may be noted that for $n = 1$, the condition (3.3.1) does not reduce to the condition (3.2.1) as $k_1 < 1$ is not assured. But we may deduce the Theorem 3.2 as a Corollary as follows.

Corollary 3.5. *If T is a self map of a complete fuzzy normed linear space $(X, N, *)$ satisfying*

$$N(Tx - Ty, kt) \geq N(x - y, t), \tag{3.5.1}$$

for every $x, y \epsilon X, t > 0$ and some $0 < k < 1$, then T has a unique fixed point.

Proof. For any positive integer n , we have

$$\begin{aligned} N(T^n x - T^n y, k^n t) &\geq N(T^{n-1} x - T^{n-1} y, k^{n-1} t), \text{ by } (3.5.1). \\ &\geq N(T^{n-2} x - T^{n-2} y, k^{n-2} t) \geq \ldots \geq N(x - y, t), \end{aligned}$$

for every $x, y\epsilon X, t > 0$ and some $0 < k < 1$. As $k^n \to 0$, we have by the preceding Theorem ,T has a unique fixed point. □

The following Example illustrates the Theorem 3.3.

Example 3.6. Let $X = \mathbb{R}$, which is a complete normed linear space with the absolute value norm. Now $(\mathbb{R}, N, *)$ is the induced complete F - nls. Let T be a self map of $\mathbb{R}$ given by $Tx = \frac{x}{5}$.

$$\begin{aligned} \text{Now } N(T^n x - T^n y, k^n t) &= \frac{k^n t}{k^n t + |T^n x - T^n y|} \\ &= \frac{\frac{t}{2^n}}{\frac{t}{2^n} + |T^n x - T^n y|}, \text{with } k = \frac{1}{2}. \\ &= \frac{t}{t + (\frac{2}{5})^n |x-y|} \geq \frac{t}{t+|x-y|} = N(x-y,t), \end{aligned}$$

for every $x, y\epsilon X, t > 0, n > 0$. Also $k^n = \frac{1}{2^n} \to 0$. Therefore , the conditions of the Theorem 3.3 are satisfied and $0\epsilon\mathbb{R}$ is the unique fixed point of T.

We know that the Drastic t - norm is the point wise smallest t - norm and the minimum t - norm is the point wise largest t - norm , i.e. ,$T_D(a,b) \leq T(a,b) \leq T_{min}(a,b)$, for every $a, b\epsilon[0,1]$, where T is an arbitrary t - norm. R. Kannan , an Indian Mathematician , proved a fixed point theorem in the year 1968 by removing the continuity requirements and with a different contraction modulus $0 < \beta < \frac{1}{2}$. In the following Theorem , we have extended R. Kannan's fixed point Theorem [14] to fuzzy normed linear spaces with the minimum t - norm.

Theorem 3.7. *If T is a self map of a complete fuzzy normed linear space $(X, N, *)$ satisfying*

$$N(Tx - Ty, \beta t) \geq N(x - Tx, t) * N(y - Ty, t), \tag{3.7.1}$$

for every $x, y\epsilon X, t > 0$ and some $0 < \beta < \frac{1}{2}$, where $*$ is the minimum t- norm , then T has a unique fixed point.

Proof. Let $x_0\epsilon X$ and $\{x_n\}$ be a sequence in X where $x_n = T^n x_0, n = 1, 2, 3, \ldots$. We now get ,

$$\begin{aligned} N(x_{n+1} - x_n, \beta t) &\geq N(x_n - x_{n+1}, t) * N(x_{n-1} - x_n, t), \text{ by (3.7.1)}. \\ \text{If } N(x_{n+1} - x_n, \beta t) &\geq N(x_n - x_{n+1}, t) = N(x_{n+1} - x_n, t), \end{aligned}$$

for every $t > 0$, then by Lemma 2.11, $x_{n+1} = x_n$, for every $n > 0$. On the other hand , if $N(x_{n+1} - x_n, \beta t) \geq N(x_{n-1} - x_n, t) = N(x_n - x_{n-1}, t)$, for every $t > 0$, then by Lemma 2.12 ,$\{x_n\}$ is a Cauchy sequence in X. Thus in any case , $\{x_n\}$ is a

Cauchy sequence in X and so $x_n \to x$ for some $x\epsilon X$, as X is complete. We now get , $N(x - Tx, 2\beta t) \geq N(x - x_n, \beta t) * N(x_n - Tx, \beta t)$
$\geq N(x_n - x, \beta t) * N(x_{n-1} - x_n, t) * N(x - Tx, t)$, for every $t > 0$.
Letting $n \to \infty$, we get $N(x - Tx, 2\beta t) \geq N(x - Tx, t)$, for every $t > 0$. As $0 < 2\beta < 1$, therefore by Lemma 2.11, $Tx = x$, a fixed point of T. For uniqueness , let $Ty = y$, for some $y\epsilon X$. We get ,

$$N(x - y, t) = N(Tx - Ty, t) \quad \geq N(x - x, \tfrac{\beta}{t}) * N(y - y, \tfrac{\beta}{t}) \text{ , by (3.7.1).}$$
$$= 1 * 1 = 1, \text{for every}\, t > 0.$$

Therefore , $x = y$ and so the fixed point is unique. □

The following example shows that a function which is neither a contraction nor continuous may have a unique fixed point and also illustrates the Kannan's fixed point Theorem in fuzzy normed linear spaces.

Example 3.8. Let us consider the complete fuzzy normed linear space $(\mathbb{R}, N, *)$ induced by the complete normed linear space $(\mathbb{R}, \| \ . \ \|)$ with the absolute value norm. Let T be a self map of $\mathbb{R}$ given by $Tx = \frac{x}{5}$, if $0 \leq x \leq 1; Tx = 0$, otherwise. We see that $x_n = 1 + \frac{1}{n} \to 1$ but $Tx_n \to 0 \neq T1$ and so T is discontinuous at 1 and hence T is discontinuous. Therefore ,T is not a contraction. For $x, y\epsilon[0, 1]$, we get $N(Tx - Ty, \beta t) \geq N(x - Tx, t)$, with $\beta = \frac{1}{3}$.
For $x, y \not\epsilon[0, 1]$, we get, $N(Tx - Ty, \beta t) = 1$.
For $x\epsilon[0, 1], y \not\epsilon[0, 1]$, we get, $N(Tx - Ty, \beta t) \geq N(x - Tx, t)$, with $\beta = \frac{1}{3}$.
For $x \not\epsilon[0, 1], y\epsilon[0, 1]$, we get ,$N(Tx - Ty, \beta t) \geq N(y - Ty, t)$, with $\beta = \frac{1}{3}$.
Therefore , the condition (3.7.1) is satisfied as $* = t_{min}$. We now note that $0\epsilon\mathbb{R}$ is the unique fixed point of T .

In the following Theorem , we have established a common fixed point Theorem which extends and generalizes a result due to Vasuki [18] and also generalizes a result of Jungck [13]. In fact , this is also a generalization of the Theorem 3.2 as the Theorem 3.2 can be deduced from it by taking $B = I$.

Theorem 3.9. *If A and B are self maps of a complete fuzzy normed linear space $(X, N, *)$ satisfying*
(3.9.1) $AX \subseteq BX$,
(3.9.2) $N(Ax - Ay, kt) \geq N(Bx - By, t)$, *for every* $x, y\epsilon X, t > 0$ *and some* $0 < k < 1$,
(3.9.3) A or B is continuous ,

(3.9.4) $[A, B]$ *is R - weakly commuting , and*

(3.9.5) $x_n \to x, y_n \to y$ *in* X *implies* $N(x_n - y_n, t) \to N(x - y, t)$, *for every* $t > 0$, *then* A *and* B *have a unique common fixed point.*

Proof. Let $x_0 \epsilon X$ be arbitrary. By (3.9.1) , we get some $x_1 \epsilon X$ such that $Ax_0 = Bx_1$ and likewise $Ax_1 = Bx_2$, for some $x_2 \epsilon X$, and so on. Thus we get a sequence $\{x_n\}$ in X such that $Ax_{n-1} = Bx_n$, for $n = 1, 2, 3, \ldots$. Now we get ,

$$\begin{aligned} N(Ax_{n+1} - Ax_n, kt) &\geq N(Bx_{n+1} - Bx_n, t), by (3.9.2). \\ &= N(Ax_n - Ax_{n-1}, t), for\, every\, t > 0, n > 0. \end{aligned}$$

Therefore , by Lemma 2.12 , $\{Ax_n\}$ is a Cauchy sequence in X. As X is complete , $Ax_{n-1} = Bx_n \to z$, for some $z \epsilon X$. Let us assume that A is continuous. Then $AAx_n \to Az, ABx_n \to Az$. [If B is continuous , we get $BAx_n \to Bz$ and $N(AAx_n - Az, t) \geq N(BAx_n - Bz, \frac{t}{k}) \to 1$, as $n \to \infty$. Therefore ,$AAx_n \to Az$. Again , $ABx_n = AAx_{n-1} \to Az$. Thus we get ,$AAx_n \to Az, ABx_n \to Az$]. As $[A, B]$ is R - weakly commuting , we have a positive real number R satisfying $N(ABx_n - BAx_n, t) \geq N(Ax_n - Bx_n, \frac{t}{R})$. Taking limit as $n \to \infty$, we get , $lim_{n\to\infty} N(ABx_n - BAx_n, t) = 1$, for every $t > 0$. As $ABx_n \to Az$, we get $BAx_n \to Az$. Therefore ,$N(Ax_n - AAx_n, kt) \geq N(Bx_n - BAx_n, t)$ and so in the limit as $n \to \infty$, we get $N(z - Az, kt) \geq N(z - Az, t)$, for every $t > 0$. Therefore , we get $Az = z$. By (3.9.1) , we get $z = Az = Bz_1$, for some $z_1 \epsilon X$. Now , $N(AAx_n - Az_1, kt) \geq N(BAx_n - Bz_1, t)$, for every $t > 0$. Letting $n \to \infty$, we get $N(Az - Az_1, kt) \geq N(Az - Bz_1, t) = 1$, for every $t > 0$. Therefore , $Az = Az_1$ and so $z = Az = Az_1 = Bz_1$. Again , $N(Az - Bz, t) = N(ABz_1 - BAz_1, t) \geq N(Az_1 - Bz_1, \frac{t}{R}) = 1$, for every $t > 0$. This gives , $Az = Bz = z$, a common fixed point of A and B. For uniqueness , let $w \epsilon X$ be such that $Aw = Bw = w$. Now $N(z - w, kt) = N(Az - Aw, kt) \geq N(Bz - Bw, t) = N(z - w, t)$, for every $t > 0$. Therefore , $z = w$ and so the common fixed point is unique. □

We now present an Example below to illustrate the Theorem 3.9

Example 3.10. Let us consider the complete fuzzy normed linear space $(\mathbb{R}, N, *)$ induced by the complete normed linear space $(\mathbb{R}, ||.||)$, with the absolute value norm.

Let A and B be

$$\text{self maps of}\,\mathbb{R}\,\text{given by}\,Ax = 1\,and\,Bx = \begin{cases} 1 & \text{if}\ x\epsilon\,\mathbb{Q} \\ 0 & \text{if}\ x\ \not\epsilon\mathbb{Q} \end{cases}$$

Then (i) $AX \subseteq BX$,

(ii) $N(Ax - Ay, kt) = 1$, for every $x, y\epsilon X, t > 0$ and $0 < k < 1$ and so the condition (3.9.2) is satisfied.

(iii) A is continuous and B is discontinuous.

(iv) $AB = BA$ and so the condition (3.9.4) is satisfied.

(v) $x_n \to x, y_n \to y\,\text{in}X$ implies $N(x_n - y_n, t) \to N(x - y, t)$, for every $t > 0$.

So all the conditions of the Theorem 3.9 are satisfied and $1\epsilon\mathbb{R}$ is the unique common fixed point of A and B.

Remarks/Conclusions 3.11. We have proved the fuzzy analogue of Kannan's fixed point theorem in a complete fuzzy normed linear space $(X, N, *)$ but with the minimum t - norm. So a query naturally arises: what happens to the result for the general t - norm , for it creates great annoyance as the result holds only for the minimum t - norm. So this remains an open problem which we shall try to resolve in our subsequent investigations.

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