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Soft compactification of soft topological spaces: soft star topological spaces

Ridvan ŞAHİN

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Abstract. Soft set theory is a novel mathematical tool to overcome uncertainties which classic mathematical tools can not handle. Recently, the concept of soft topological spaces are defined on an initial universe with a fixed set of parameters. In this paper, we extent the soft topological spaces to soft star topological spaces called soft star compactification under certain conditions, and discuss their basic properties as well as examples. Contrary to general topological spaces, in the soft star compactification, we not only extent the discussion space X but also the parameter set E. Also we give a necessary and sufficient condition for soft star compactification of X.

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Corresponding Author: Rıdvan Şahin (mat.ridone@gmail.com)

1. Introduction

To solve complicated problems in fields such as economics, medical science, engineering, management, environmental science and social science, classical methods may not be successfully used because of various uncertainties arising in these problems. However, there are some theories: the theory of probability, fuzzy sets [44], vague sets [16], rough sets [37] and interval mathematics [18] which were established by researchers to modelling uncertainties appearing in the above fields. The intuitionistic fuzzy sets that has highly useful dealing with uncertainties, have been introduced as a generalization of Zadeh's fuzzy sets by Atanassov [6]. Atanassov and Gargov [7] then extended the notion of the intuitionistic fuzzy set to the interval-valued intuitionistic fuzzy set whose membership function and non-membership function are intervals. However, all of these theories have their own difficulties which are pointed out in [34]. Soft set theory [34], orginally initiated by Molodtsov, is a new mathematical tool to overcome uncertainties that can not be handled by tradional mathematical tools, and it is free from the difficulties of the above mentioned theories and also the inadequacy of the parameterization tools. In soft set theory, a soft set is a mapping from parameter set to crisp subset of universe, and it is easily applied to many different fields. Recently, studies on soft sets and its applications have become a rapidly progressing reserch area. Maji et al. [27, 28] presented an application of soft sets in a decision making problem and made a detailed theoretical study on soft sets. Ali et al. [5] introduced several new operations on soft sets by revising some errors in [34]. Qin and Hong [39] defined the notion of soft equality and investigated some related properties as well as some conditions of equal for soft sets. Kharal and Ahmad [25] proposed the concept of a soft mapping on soft classes and discussed some properties of images and inverse images of soft sets and discussed them by examples and counterexamples. Chen et al [11] proposed a new definition of parameter reduction for soft sets and used it to improve the application of soft sets to the decision making problem. Also they discussed the basic difference between parameterization reduction of soft sets and attributes reduction in rough sets. Aktas and Cagman [4] compared soft sets to the notions of rough sets and fuzzy sets, introduced the concept of soft groups and studied their basic properties. Feng et al. [15] proposed the notions of soft subsemiring and soft ideal over a semiring. Later, Jun et al. [23, 24] introduced the soft BCK/BLC-algebras and presented their applications in ideal theory.

Studies on hybrid structure involving soft set theory have made by many researshers. By integrating fuzzy sets and soft sets, Maji et al. [30] introduced the concept of fuzzy soft sets. Feng et.al. [13] investigated to combining the theory of soft set with the theories of fuzzy set and rough set. The concept of generalised fuzzy soft sets which is further generalized of fuzzy sets have studied by Majumdar and Samanta [31]. Yang et al. [43] extended the fuzzy soft sets to interval-valued fuzzy soft sets. Maji et al. [29] proposed the concept of intuitionistic fuzzy soft set by combining the intuitionistic fuzzy set with soft set. Interval-valued intuitionistic fuzzy soft set obtained by the interval-valued intuitionistic fuzzy set and soft set were proposed by Jiang et al. [22].

As well as the algebraic nature of soft sets, studies on topological structure of their are widely maintained by researchers. Shabir and Naz [42], and Cagman et al. [9] independently initiated the studies on soft topological spaces and soft topology, respectively. Based on a fixed set of parameters, Shabir and Naz [42] introduced the concept of soft topological spaces on an initial universe and showed the relationship between a soft topological space and a topological space which is a parameterized family. Also they studied the concepts of soft open set, soft interior point, soft closed set, soft closure, soft neighborhood of a point, soft subspace and soft separation axioms in a soft topological space. Hussain and Ahmad [21] then obtained some important results related soft boundary, soft interior, soft closure and soft exterior in soft topological spaces. Sahin and Kucuk [40] introduced concepts of soft filter and soft ideal, and presented their related properties. They also investigated the convergence theory of the soft filter in a soft topological space. Aygunoglu and Aygun [8] introduced the soft continuity of a soft mapping, soft product topology and studied soft compactness and generalized Tychonoff theorem to soft topological spaces. Zorlutuna et al. [45] showed that the soft topological space is more general than a fuzzy topological space, and that an ordinary topological space can be considered a soft topological space. Recently, some hybrid structures of soft sets and their applications are discussed by some researchers [1, 3, 14, 17, 19, 20, 26, 33, 36, 38, 41].

One of the most elementary and functional notions of finiteness in analysis, algebra and topology is the notion of compactness. In fact, it is easier to work on compact spaces than a non-compact space, because they don't include the concept of infinity. Compactification is the process of obtaining a compact space by expanding a non-compact space. It has a great importance due to the fundamental role of compact spaces in two basic branches such as topology and functional analysis of mathematics analysis. In this paper, based on the soft set theory, we attempt to consruct a soft compact topological space from non-soft compact space. Contrary to general topological spaces, in the soft compactification, we not only extent the discussion space X but also the parameter set E.

This paper is organized as follows. In Section 2, we briefly review some basic notions and framework on soft sets and soft topological spaces, In Section 3, we propose the concept of soft base via soft point and present some basic properties related with the soft base. In Section 4, we introduce the soft star topological spaces which are a extension of soft topological spaces. Also we investigate that soft topological spaces would be a soft star compactification under what conditions. Then we give a necessary and sufficient condition for soft star compactification of a soft non-compact space. Finally, in Section 6, we present the conclusion and give some related topics for future research.

2. Preliminaries

In this section, we give some preliminaries about soft set. We make some small modifications to some of them in order to make theoretical study in detail.

Throughout this paper, X refers to an initial universal set, E is a set of all possible parameters, $P(X)$ is the power set of X, and $A \subseteq E$. Moreover, $\mathcal{S}(X,E)$ denotes the collection of all soft sets over X.

Definition 2.1 ([34])**.** A pair (F,A) is called a soft set on the universe X, where F is a mapping given by $F : E \to P(X)$.

In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be expressed as the set of e-approximate elements of the soft set (F,A). Clearly, a soft set is not a set.

Definition 2.2 ([10])**.** The soft set $(F,A) \in \mathcal{S}(X,E)$ is called the null soft set, denoted by Φ if $F(e) = \varnothing$ for all $e \in A$.

Definition 2.3 ([10])**.** Let $(F,A) \in \mathcal{S}(X,E)$. If $F(e) = X$, $\forall e \in A$, then (F,A) is called $A-$absolute soft set, denoted by $\tilde{A}$. If $A = E$, then the $A-$absolute soft set is called absolute soft set and denoted by $\widetilde{X}$.

Definition 2.4 ([10])**.** Let $(F,A),(G,B) \in \mathcal{S}(X,E)$. (F,A) is a soft subset of (G,B), denoted $(F,A) \sqsubseteq (G,B)$ if $F(e) \subseteq G(e)$, for all $e \in E$.

Definition 2.5 ([10])**.** Let $(F,A),(G,B) \in \mathcal{S}(X,E)$. (F,A) and (G,B) are soft equal, denote by $(F,A) = (G,B)$ if $(F,A) \sqsubseteq (G,B)$ and $(G,B) \sqsubseteq (F,A)$.

Definition 2.6 ([10]). Let $(F,A),(G,B)\in\mathcal{S}(X,E)$. Union of (F,A) and (G,B) is a soft set (H,C) defined by $H(e)=F(e)\cup G(e)$, $\forall e\in E$, where $C=A\cup B$. That is, $(H,C)=(F,A)\sqcup(G,B)$.

Definition 2.7 ([10]). Let $(F,A),(G,B)\in\mathcal{S}(X,E)$. Intersection of (F,A) and (G,B) is a soft set (H,C) defined by $H(e)=F(e)\cap G(e)$, $\forall e\in E$, where $C=A\cap B$. That is, $(H,C)=(F,A)\sqcap(G,B)$.

Definition 2.8 ([5]). Let $(F,A)\in\mathcal{S}(X,E)$. Complement of (F,A), denoted by (F^c,A) is defined by $F^c(e)=X-F(e)$ for all $e\in A$.

Definition 2.9 ([21]). Difference (H,C) of two soft sets (F,A) and (G,B) over X, denoted by $(H,C)=(F,A)-(G,B)$, is defined as $H(e)=F(e)-G(e)$ for all $e\in E$.

Definition 2.10 ([25]). Let $\mathcal{S}(X,E)$ and $\mathcal{S}(Y,M)$ be the collections of all soft sets over X and Y, respectively. The mapping φ_ψ is called a soft mapping from X to Y, denoted by $\varphi_\psi: S(X,E)\longrightarrow S(Y,M)$ where $\varphi: X\longrightarrow Y$ and $\psi: E\longrightarrow M$ are two mappings.

(1) Let $(F,A)\in S(X,E)$, then the image of (F,A) under the soft mapping φ_ψ is a soft set on Y denoted by $\varphi_\psi((F,A))$ and it is defined by
$$\varphi_\psi((F,A))(k)=\begin{cases}\bigcup\limits_{e\in\psi^{-1}(k)\cap A}\varphi(F(e)), & \text{if } \psi^{-1}(k)\cap A\neq\varnothing;\\ \varnothing, & \text{otherwise.}\end{cases}$$
(2) Let $(G,B)\in S(Y,M)$, then the pre-image of (G,B) under the soft mapping φ_ψ is a soft set on X denoted by $\varphi_\psi^{-1}((G,B))$, where
$$\varphi_\psi^{-1}((G,B))(e)=\begin{cases}\varphi^{-1}(G(\psi(e))), & \text{if } \psi(e)\in B;\\ \varnothing, & \text{otherwise.}\end{cases}$$

The soft mapping φ_ψ is called injective mapping, if φ and ψ are injective. The soft mapping φ_ψ is called surjective mapping, if φ and ψ are surjective.

Proposition 2.11 ([25]). *Let φ_ψ be a soft mapping from $\mathcal{S}(X,E)$ to $\mathcal{S}(Y,M)$, where $\varphi: X\longrightarrow Y$ and $\psi: E\longrightarrow M$ are two mappings. Then for soft sets $(F,A)_1$, $(F,A)_2$ over X and $(G,B)_1$, $(G,B)_2$ over Y we have that*

(1) $\varphi_\psi^{-1}(\Phi_Y)=\Phi_X$ *and* $\varphi_\psi(\Phi_X)=\Phi_Y$,
(2) $\varphi_\psi^{-1}\left(\widetilde{Y}\right)=\widetilde{X}$ *and* $\varphi_\psi\left(\widetilde{X}\right)\sqsubseteq\widetilde{Y}$,
(3) $\varphi_\psi^{-1}((G,B)_1\sqcup(G,B)_2)=\varphi_\psi^{-1}((G,B)_1)\sqcup\varphi_\psi^{-1}((G,B)_2)$ *and* $\varphi_\psi((F,A)_1\sqcup(F,A)_2)=\varphi_\psi((F,A)_1)\sqcup\varphi_\psi((F,A)_2)$,
(4) $\varphi_\psi^{-1}((G,B)_1\sqcap(G,B)_2)=\varphi_\psi^{-1}((G,B)_1)\sqcap\varphi_\psi^{-1}((G,B)_2)$ *and* $\varphi_\psi((F,A)_1\sqcap(F,A)_2)\sqsubseteq\varphi_\psi((F,A)_1)\sqcap\varphi_\psi((F,A)_2)$,
(5) *if* $(F,A)_1\sqsubseteq(F,A)_2$, *then* $\varphi_\psi((F,A)_1)\sqsubseteq\varphi_\psi((F,A)_2)$,
(6) *if* $(G,B)_1\sqsubseteq(G,B)_2$, *then* $\varphi_\psi^{-1}((G,B)_1)\sqsubseteq\varphi_\psi^{-1}((G,B)_2)$.

2.1. **Soft topology.** In this section, we give some basic properties of soft topological spaces.

Definition 2.12 ([42]). A soft topology $\mathcal{T}$ is a collection of soft sets over X which satisfy the following properties;

(1) $\Phi,\widetilde{X}\in\mathcal{T}$

(2) the union of any number of soft sets in $\mathcal{T}$ belongs to $\mathcal{T}$
(3) the intersection of any two of soft sets in $\mathcal{T}$ belongs to $\mathcal{T}$.

The pair $(X, \mathcal{T})$ is called a soft topological space.

Definition 2.13 ([42])**.** Let $(X, \mathcal{T})$ be a soft topological space and E be a set of all parameters. Then the collection $\mathcal{T}^e = \{F(e) : (F, E) \in \mathcal{T}\}$ for each $e \in E$, defines a topology on X. It is called $e-$parameter topology on X.

Above definition shows that corresponding to each parameter $e \in E$, we have a topology $\mathcal{T}^e$ on X. Thus a soft topology on X gives a parameterized family of topologies on X.

Definition 2.14 ([42])**.** Let $(X, \mathcal{T})$ be a soft topological space. Then every element of $\mathcal{T}$ is called a soft open set. Clearly, Φ and $\widetilde{X}$ are soft open sets.

Definition 2.15 ([42])**.** Let $(X, \mathcal{T})$ be a soft topological space and $(F, E) \in \mathcal{S}(X, E)$. Then (F, E) is said to be soft closed if the soft set (F^c, E) is soft open.

Definition 2.16 ([42])**.** Let $(X, \mathcal{T})$ be a soft topological space and (F, E) be a soft set over X. Then the soft closure of (F, E) is a soft set defined by

$$\overline{(F, E)} = \sqcap \{(K, E) : (K, E) \text{ is a soft closed and } (F, E) \sqsubseteq (K, E)\}.$$

Definition 2.17 ([42])**.** Let X be an initial universal set, E a set of parameters and Y be a non-empty classical subset of X. Then the sub-soft set of (F, E) over Y denoted by ${}^Y(F, E)$, is defined as follows

$${}^Y F(e) = Y \cap F(e), \text{ for all } e \in E$$

In other words, ${}^Y(F, E) = \tilde{Y} \sqcap (F, E)$.

Definition 2.18 ([42])**.** Let $(X, \mathcal{T})$ be a soft topological space and Y be a non-empty subset of X. Then

$$\mathcal{T}_Y = \{{}^Y(F, E) : (F, E) \in \mathcal{T}\}$$

is said to be the soft relative topology on Y and $(Y, \mathcal{T}_Y)$ is called a soft subspace of $(X, \mathcal{T})$.

Definition 2.19 ([8])**.** Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two soft topological spaces. A soft mapping $\varphi_\psi : (X, \mathcal{T}_1) \longrightarrow (Y, \mathcal{T}_2)$ is called soft continuous if $\varphi_\psi^{-1}(G, E) \in \mathcal{T}_1$, $\forall (G, E) \in \mathcal{T}_2$.

2.2. **Star topology.** Based on generalization of topological spaces, star topological spaces and star compactifications are firtsly defined by Nagasis and Papastavridis [35]. Then Acosta and Rubio [2] discussed a theoretical study of star compactifications in more detail.

Let $(X, \mathcal{T})$ be a non-compact topological space and $X_n = X \sqcup \{y_1, y_2, ..., y_n\}$. We present the concept of star topology as follows:

Definition 2.20 ([35])**.** Let (X, τ) be a topological space and U_i be open subset contained in X for $i = 1, 2, ..., n$. Then

$$B = \tau \cup \{(U_i - K) \cup \{y_i\} : K \text{ closed and compact}, \ i = 1, 2, ..., n\}$$

is a base for a topology τ_n on X. This topology is called the star topology associated to $U_1, U_2, ..., U_n$.

3. Soft point and soft base

In soft theory, the concept of soft point has been defined in different ways. But throughout this paper, we will adopt the concepts of soft point in [8, 12].

In the section, we give the concept of the soft base [32] and study some of its properties.

Definition 3.1 ([12])**.** A soft set (P, A) over X is said to be a soft point if there is exactly one $\lambda \in A$, such that $P(\lambda) = \{x\}$ for some $x \in X$ and $P(\mu) = \varnothing$, $\forall \mu \in A \backslash \{\lambda\}$. It will be denoted by P_x^λ.

Definition 3.2 ([8])**.** A soft set (P, A) over X is said to be a absolute soft point if there exists some $x \in X$ such that $P(\lambda) = \{x\}$ for every $\lambda \in E$.

Definition 3.3 ([12])**.** A soft set P_x^λ is said to belongs to a soft set (F, A) if $\lambda \in A$ and $P(\lambda) = \{x\} \subset F(\lambda)$. It will be denoted by $P_x^\lambda \tilde{\in} (F, A)$.

Definition 3.4 ([12])**.** Two soft sets P_x^λ and P_y^μ are said to be equal if $\lambda = \mu$ and $P(\lambda) = P(\mu)$ i.e., $x = y$. Then $P_x^\lambda \neq P_y^\mu \Leftrightarrow \lambda \neq \mu$ or $\lambda \neq \mu$.

Proposition 3.5. *Let X be a set and $\mathcal{C}$ a collection of soft subsets of X. Suppose that $(F, A)_{P_x^\lambda}$ means any soft set containin P_x^λ. If for every $P_x^\lambda \breve{\in} \tilde{X}$, there exists a soft set $(F, A)_{P_x^\lambda}$ in $\mathcal{C}$ such that $P_x^\lambda \breve{\in} (F, A)_{P_x^\lambda}$, then $\sqcup_{P_x^\lambda \tilde{\in} \tilde{X}} (F, A)_{P_x^\lambda} = \tilde{X}$.*

Proof. We show that $\sqcup (F, A)_{P_x^\lambda} \sqsubseteq \tilde{X}$ and $\tilde{X} \sqsubseteq \sqcup (F, A)_{P_x^\lambda}$. Since every $(F, A)_{P_x^\lambda} \sqsubseteq \tilde{X}$, it implies that $\sqcup (F, A)_{P_x^\lambda} \sqsubseteq \tilde{X}$. On the other hand, let us take $P_y^\mu \tilde{\in} \tilde{X}$. Then there is a soft set $(F, A)_{P_y^\mu}$ in $\mathcal{C}$ such that $P_y^\mu \tilde{\in} (F, A)_{P_y^\mu}$. Hence $P_y^\mu \tilde{\in} (F, A)_{P_y^\mu} \sqsubseteq \sqcup (F, A)_{P_x^\lambda}$ and so $P_y^\mu \tilde{\in} \sqcup (F, A)_{P_x^\lambda}$. Then it follows that $\tilde{X} \sqsubseteq \sqcup (F, A)_{P_x^\lambda}$. □

Definition 3.6 ([45])**.** Let $(X, \mathcal{T})$ be a soft topological space and $P_x^\lambda \tilde{\in} \tilde{X}$. A soft open set (F, A) containing P_x^λ is said to be a soft neighborhood of P_x^λ.

Proposition 3.7 ([32])**.** *Let $(X, \mathcal{T})$ be a soft topological space and (F, A) be a soft subset of X. Then (F, A) is soft open in X iff if for every $P_x^\lambda \tilde{\in} (F, A)$, there is a soft neighborhood (G, B) of P_x^λ such that $P_x^\lambda \tilde{\in} (G, B) \sqsubseteq (F, A)$.*

Definition 3.8. Let $(X, \mathcal{T})$ be a soft topological space and $P_x^\lambda, P_y^\mu \tilde{\in} \tilde{X}$ such that $P_x^\lambda \neq P_y^\mu$. $(X, \mathcal{T})$ is called soft Hausdorff space or soft T_2-space if there exist soft open sets (G, B) and (F, A) such that $P_x^\lambda \tilde{\in} (F, A)$, $P_y^\mu \tilde{\in} (G, B)$ and $(F, A) \sqcap (G, B) = \Phi$.

Definition 3.9. A soft topological space X is called soft locally compact if every $P_x^\lambda \tilde{\in} \tilde{X}$ has a soft neighborhood that is contained in a soft compact subset of X.

Definition 3.10. A soft subset (F, A) of a soft topological space X is said to be soft dense (or everywhere soft dense) in X if the soft closure of (F, A) is equal to $\tilde{X}$, i.e., $\overline{(F, A)} = \tilde{X}$.

Example 3.11. Let $X = \{x_1, x_2, x_3\}$ and $E = \{\lambda_1, \lambda_2\}$. We consider the soft sets (F_1, E), (F_2, E) and (F_3, E) over X defined as follows:

$$\begin{aligned}(F_1, E) &= \{(\lambda_1, \{x_1\}, (\lambda_2, \{x_2\}\},\\ (F_2, E) &= \{(\lambda_1, \{x_3\}, (\lambda_2, \{x_1, x_3\}\},\\ (F_3, E) &= \{(\lambda_1, \{x_1, x_3\}, (\lambda_2, X\}.\end{aligned}$$

Then, the $(X, \mathcal{T})$ with $\mathcal{T} = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ is a soft topological space. Thus, we have the soft closed sets,

$$\begin{aligned}(F_1^c, E) &= \{(\lambda_1, \{x_2, x_3\}, (\lambda_2, \{x_1, x_3\}\},\\ (F_2^c, E) &= \{(\lambda_1, \{x_1, x_2\}, (\lambda_2, \{x_2\}\},\\ (F_3^c, E) &= \{(\lambda_1, \{x_2\}, (\lambda_2, \varnothing\}.\end{aligned}$$

Then since $\overline{(F_1, E)} = (F_2^c, E)$ and $\overline{(F_2, E)} = (F_1^c, E)$, the soft sets (F_1, E) and (F_2, E) are not soft dense on X. But since $\overline{(F_3, E)} = \tilde{X}$, the soft set (F_3, E) is a soft dense set over X.

Proposition 3.12. *Let $(X, \mathcal{T})$ is a soft topological space and (F, E) be a soft dense set over X. Then for $\lambda \in E$, $F(\lambda)$ is a dense set according to the topology $\mathcal{T}^\lambda$ on X.*

Proof. It is clear. □

Proposition 3.13. *Let $(X, \mathcal{T})$ be a soft Hausdorff and $P_x^\lambda \tilde{\in} \tilde{X}$. If for every soft open set (F, E) with $P_x^\lambda \tilde{\in} (F, E)$, there is a soft open (G, E) which $\overline{(G, E)}$ is soft compact, such that $P_x^\lambda \tilde{\in} (G, E) \sqsubseteq \overline{(G, E)} \sqsubseteq (F, E)$, then $(X, \mathcal{T})$ is soft locally.*

Proof. It is clear. □

Proposition 3.14. *Let $(X, \mathcal{T})$ be a soft Hausdorff and Y be a non-empty subset of X. Then*

(1) *$(Y, \mathcal{T}_Y)$ is soft Hausdorff the according to soft relative topology on Y.*
(2) *Every single-soft point subset of X is soft closed.*
(3) *If $(X, \mathcal{T})$ is soft locally and Y is soft open in X, then $(Y, \mathcal{T}_Y)$ is soft locally.*

Proof. (1) Suppose that $(Y, \mathcal{T}_Y)$ is soft subspace of X and $P_x^\lambda, P_y^\mu \tilde{\in} \tilde{Y}$ $(P_x^\lambda \neq P_y^\mu)$. Then $P_x^\lambda, P_y^\mu \tilde{\in} \tilde{X}$. Since $(X, \mathcal{T})$ is soft Hausdorff, there exist two soft open sets $P_x^\lambda \tilde{\in} (F, E)$ and $P_y^\mu \tilde{\in} (G, E)$ in X such that $(F, E) \sqcap (G, E) = \Phi$. On the other hand, we have that

$$P_x^\lambda \tilde{\in} (F, E) \sqcap (Y, \mathcal{T}_Y),\ P_y^\mu \tilde{\in} (G, E) \sqcap (Y, \mathcal{T}_Y)$$

such that

$$((F, E) \sqcap (Y, \mathcal{T}_Y)) \sqcap ((G, E) \sqcap (Y, \mathcal{T}_Y)) = \Phi.$$

Then $(Y, \mathcal{T}_Y)$ is soft Hausdorff. (2) Let $P_x^\lambda \tilde{\in} \tilde{X}$. We show that $\tilde{X} - \left\{P_x^\lambda\right\}$ is soft open. Suppose that $P_y^\mu \tilde{\in} \tilde{X} - \left\{P_x^\lambda\right\}$ is an arbitrary soft point. There exist two soft open sets $P_x^\lambda \tilde{\in} (F, E)$ and $P_y^\mu \tilde{\in} (G, E)$ such that $(F, E) \sqcap (G, E) = \Phi$, since $(X, \mathcal{T})$ is soft Hausdorff. This implies that $P_x^\lambda \tilde{\notin} (G, E)$ and so $P_y^\mu \tilde{\in} (G, E) \sqsubseteq \tilde{X} - \left\{P_x^\lambda\right\}$. Then $\tilde{X} - \left\{P_x^\lambda\right\}$ is soft open. (3) Let $P_x^\lambda \tilde{\in} \tilde{Y}$. Then we have $P_x^\lambda \tilde{\in} \tilde{X}$. Since $(X, \mathcal{T})$ is soft locally, there exists a soft open set (F,E) which $\overline{(F,E)}$ is soft compact, such that $P_x^\lambda \tilde{\in} (F,E) \sqsubseteq \overline{(F,E)} \sqsubseteq \tilde{Y}$. By Proposition 3.13, $(Y, \mathcal{T}_Y)$ is soft locally. □

Definition 3.15 ([32])**.** Let $(X,\mathcal{T})$ be a soft topological space and $\mathfrak{B}$ be a collection of soft open subsets of X.Then $\mathfrak{B}$ is a soft base for a soft topology on X, if it satisfies the following conditions for every $(F,E) \in \mathcal{T}$;

(1) $\mathfrak{B} \subseteq \mathcal{T}$,
(2) There exists a index sets I such that $(F,E) = \sqcup_{i\in I} (S_i,E)$, where $(S_i,E) \in \mathfrak{B}$ for $i \in I$.

Example 3.16. Let $\mathbb{R}$ be the real numbers, $E = \mathbb{R}^+$ be the positive real numbers. Let us consider

$$\mathfrak{B}=\{(S,E)_\lambda : (S,E)_\lambda = (x,[x,x+\lambda)) : \ \lambda \in E \text{ and } \lambda < |x|\}$$

Then the collection $\mathfrak{B}$ is a soft base for a soft topology defined by

$$\mathcal{T} = \left\{ (F,E)_y : y \in E \right\} \cup \left\{ \tilde{\mathbb{R}}, \Phi \right\}$$

on $\mathbb{R}$, where $(F,E)_y = \{(x,[x,y)) : \ x,y \in E \text{ and } x < y\}$.

Example 3.17. Let X be a initial universe set and let $\mathfrak{B}=\left\{\left\{P_x^\lambda\right\} : x \in X\right\}$. Then the collection $\mathfrak{B}$ is a soft base for the discrete soft topology on X.

Definition 3.18. Let $\mathfrak{B}$ be a soft base on X. The soft topology $\mathcal{T}$ generated by $\mathfrak{B}$ is obtained by defining the soft open sets to be the empty soft set and every soft set that is equel to a union of soft base elements.

Proposition 3.19 ([32])**.** *Let $(X,\mathcal{T})$ be a soft topological space and $\mathfrak{B} \subseteq \mathcal{S}(X,E)$. Then $\mathfrak{B}$ is a soft base on X if and only if $\mathfrak{B} \subseteq \mathcal{T}$ and for every $(F,E) \in \mathcal{T}$ and $P_x^\lambda \tilde{\in} (F,E)$, there exists a soft base element $(S,E)_{P_x^\lambda} \in \mathfrak{B}$ such that $P_x^\lambda \tilde{\in} (S,E)_{P_x^\lambda} \sqsubseteq (F,E)$.*

Proposition 3.20. *Let X be a initial universe set, $\mathcal{T}_1$ and $\mathcal{T}_2$ be two soft topological spaces on X. Suppose that $\mathfrak{B}_1$ and $\mathfrak{B}_2$ are two soft bases for soft topologies $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively. Then $\mathcal{T}_1 \subseteq \mathcal{T}_2$ iff for every $(S_1,E) \in \mathfrak{B}_1$ and $P_x^\lambda \tilde{\in} (S_1,E)$, there exists a soft base element $(S_2,E) \in \mathfrak{B}_2$ such that $P_x^\lambda \tilde{\in} (S_2,E) \sqsubseteq (S_1,E)$.*

Proof. ($\Rightarrow$) Let $\mathcal{T}_1 \subseteq \mathcal{T}_2$. Suppose that (S_1,E) is a soft base element in $\mathfrak{B}_1$ and P_x^λ is a soft point in (S_1,E). Since $\mathfrak{B}_1 \subseteq \mathcal{T}_1$, we have that $(S_1,E) \in \mathcal{T}_1$ and so $(S_1,E) \in \mathcal{T}_2$. By Proposition 3.19, there exists a soft base element $(S_2,E) \in \mathfrak{B}_2$ such that $P_x^\lambda \tilde{\in} (S_2,E) \sqsubseteq (S_1,E)$.

($\Leftarrow$) Let (F,E) be a soft open set in $\mathcal{T}_1$. Then by Proposition 3.19, there exists a soft base element $(S,E)_{P_x^\lambda} \in \mathfrak{B}$ such that $P_x^\lambda \tilde{\in} (S_1,E)_{P_x^\lambda} \sqsubseteq (F,E)$ for every $P_x^\lambda \tilde{\in} (F,E)$. This implies that (S_2,E) is a soft base element in $\mathfrak{B}_2$ with $P_x^\lambda \tilde{\in} (S_2,E) \sqsubseteq (S_1,E)$ and so that $P_x^\lambda \tilde{\in} (S_2,E) \sqsubseteq (F,E)$. Then $(F,E) \in \mathcal{T}_2$. □

Corollary 3.21. *Let X be an initial universe set and $\mathcal{T}_1, \mathcal{T}_2$ be two soft topological spaces on X. Suppose that $\mathfrak{B}_1$ and $\mathfrak{B}_2$ are two soft bases for soft topologies $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively. Then $\mathcal{T}_1 = \mathcal{T}_2$ iff*

(1) *For every $(S_1,E) \in \mathfrak{B}_1$ and $P_x^\lambda \tilde{\in} (S_1,E)$ there exists a soft base element $(S_2,E) \in \mathfrak{B}_2$ such that $P_x^\lambda \tilde{\in} (S_2,E) \sqsubseteq (S_1,E)$.*

(2) *For every* $(S_2, E) \in \mathfrak{B}_2$ *and* $P_x^\lambda \tilde{\in} (S_2, E)$ *there exists a soft base element* $(S_1, E) \in \mathfrak{B}_1$ *such that* $P_x^\lambda \tilde{\in} (S_1, E) \sqsubseteq (S_2, E)$.

Proposition 3.22. *Let* $\mathfrak{B}$ *be a soft base on* X. *Suppose that* $(S_1, E), (S_2, E), ..., (S_k, E)$ *are soft base elements in* $\mathfrak{B}$ *and* $P_x^\lambda \tilde{\in} (S_1, E) \sqcap (S_2, E) \sqcap ... \sqcap (S_k, E)$. *Then there exists a soft base element* (S, E) *in* $\mathfrak{B}$ *such that* $P_x^\lambda \tilde{\in} (S, E) \sqcap_{i=1}^{k} (S_i, E)$ *for* $i \in I$.

Proof. Let $\mathfrak{B}$ be a soft base on X. We show by induction that fact. By the definition of soft base, it is provided for $k = 1$. Suppose that the result is true for $k - 1$ and we show that it holds for k. Let us consider $(S_1, E), (S_2, E), ..., (S_k, E)$ and $P_x^\lambda \tilde{\in} \sqcap_{i=1}^{k} (S_i, E)$. Then $P_x^\lambda \tilde{\in} \sqcap_{i=1}^{k-1} (S_i, E)$ and so there exists a soft base element $(T, E) \tilde{\in} \mathfrak{B}$ such that $P_x^\lambda \tilde{\in} (T, E) \sqsubseteq \sqcap_{i=1}^{k-1} (S_i, E)$. Since $P_x^\lambda \tilde{\in} (S_k, E)$, we have a soft set (S, E) such that $P_x^\lambda \tilde{\in} (S, E) \sqsubseteq (T, E) \sqcap (S_k, E)$. Then $P_x^\lambda \tilde{\in} (S, E) \sqcap_{i=1}^{k} (S_i, E)$. □

Theorem 3.23. *Let* $(X, \mathcal{T})$ *and* $(Y, \mathcal{V})$ *be two soft topological spaces and* $\mathfrak{B}$ *be a soft base for the soft topology on* Y. *Then*

(1) $\varphi_\psi : (X, \mathcal{T}) \to (Y, \mathcal{V})$ *is a soft continuous iff* $\varphi_\psi^{-1}((S, E))$ *is soft open in* X *for every* $(S, E) \in \mathfrak{B}$.

(2) *If* $\varphi_\psi : (X, \mathcal{T}) \to (Y, \mathcal{V})$ *is a soft continuous and soft open, then* $\varphi_\psi^{-1}(\mathfrak{B})$ *is a soft base for the soft topology on* X.

Proof. (1) $(\Rightarrow)$ Assume that $\varphi_\psi : (X, \mathcal{T}) \to (Y, \mathcal{V})$ is a soft continuous mapping. Then $\varphi_\psi^{-1}((F, E))$ is soft open in X for every $(F, E) \in \mathcal{V}$. Since every soft base element (S, E) is soft open in Y, this implies that $\varphi_\psi^{-1}((S, E))$ is soft open in X for all $(S, E) \in \mathfrak{B}$.

$(\Leftarrow)$ Now, assume that $\varphi_\psi^{-1}((S, E))$ is soft open in X for every $(S, E) \in \mathfrak{B}$ and $(G, E) \in \mathcal{V}$. Then (G, E) is a union of soft base elements such that $(G, E) = \sqcup (S_i, E)$ for $i \in I$. So,

$$\varphi_\psi^{-1}((G, E)) = \varphi_\psi^{-1}(\sqcup_{i \in I} (S_i, E)) = \sqcup_{i \in I} \varphi_\psi^{-1}((S_i, E)).$$

Since every $\varphi_\psi^{-1}((S_i, E))$ is soft open subset of X for every $i \in I$, we have that $\sqcup_{i \in I} \varphi_\psi^{-1}((S_i, E)) \in \mathcal{T}$ and so $\varphi_\psi^{-1}((G, E)) \in \mathcal{T}$. Then φ_ψ is a soft continuous mapping.

(2) Let $\varphi_\psi : (X, \mathcal{T}) \to (Y, \mathcal{V})$ be a soft continuous and soft open mapping. Suppose that (F, E) is an arbitrary soft open in X and $P_x^\lambda \tilde{\in} (F, E)$. Then we have that $\varphi_\psi(P_x^\lambda) \tilde{\in} \varphi_\psi((F, E))$. Since φ_ψ is soft open and $\mathfrak{B}$ is soft base for the soft topology on Y, the set $\varphi_\psi((F, E))$ is soft open in Y and so there exists a soft base element (S, E) such that $\varphi_\psi(P_x^\lambda) \tilde{\in} (S, E) \sqsubseteq \varphi_\psi((F, E))$. Then $P_x^\lambda \tilde{\in} \varphi_\psi^{-1}((S, E)) \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi((F, E))) \sqsubseteq (F, E)$ and therefore, $P_x^\lambda \tilde{\in} \varphi_\psi^{-1}((S, E)) \sqsubseteq (F, E)$. By first option of the theorem, $\varphi_\psi^{-1}((S, E))$ is soft open in X and thus, $\varphi_\psi^{-1}(\mathfrak{B}) = \left\{ \varphi_\psi^{-1}((S, E)) : (S, E) \in \mathfrak{B} \right\}$ is a soft base for the soft topology on X. □

4. Soft star topological spaces

In the section, we extend the star topological spaces to soft star topological spaces.

Let $(X, \mathcal{T})$ be a soft non-compact topological space and $X_n = X \sqcup \{y_1, y_2, ..., y_n\}$, $E_n = E \sqcup \{\mu_1, \mu_2, ..., \mu_n\}$. Assume that $P_{y_1}^{\mu_1}, P_{y_2}^{\mu_2}, ...P_{y_n}^{\mu_n}$ are n distinct soft points not belonging to $\tilde{X}$ and $\tilde{X}_n = \tilde{X} \sqcup \left\{P_{y_1}^{\mu_1}, P_{y_2}^{\mu_2}, ...P_{y_n}^{\mu_n}\right\}$. Here, we use a fixed parameter set of soft sets to construct the soft star topology and give the definition as following.

Definition 4.1. Let $(X, \mathcal{T})$ be a soft topological space and $(F_1, E), (F_2, E), ..., (F_n, E)$ be n soft open subsets of X for $i = 1, 2, ..., n$. Then

$$\mathfrak{B} = \mathcal{T} \cup \left\{((F_i, E) - (K, E)) \sqcup \left\{P_y^{\mu}\right\} : (K, E) \text{ soft closed and soft compact}\right\}$$

is a soft base for a soft topology $\mathcal{T}_n$, denoted by $\mathcal{T}_n = \langle (F_1, E), (F_2, E), ..., (F_n, E)\rangle$ on X_n.

The soft topological space $(X_n, \mathcal{T}_n)$ generated by $\mathfrak{B}$ is called a soft star topological space on X_n, where $\mathcal{T}_n = \langle (F_1, E), (F_2, E), ..., (F_n, E)\rangle$.

Theorem 4.2. *The $\mathcal{T}_n$ generated by $\mathfrak{B}$ given above is a soft topological space.*

Proof. By Definition 4.1, it is clear that Φ is in $\mathcal{T}_n$. Since every soft point in X_n is contained in soft base elements and X is the union of all of the elements, this implies that $\tilde{X}_n \in \mathcal{T}_n$. Now, we show that a finite intersection of soft sets in $\mathcal{T}_n$ belongs to $\mathcal{T}_n$. Assume that for $(F_i, E) \in \mathcal{T}_n$, $(F, E) = (F_1, E) \sqcap (F_2, E) \sqcap ... \sqcap (F_k, E)$. If for some $i \in I$, $(F_i, E) = \Phi$, then $(F, E) \in \mathcal{T}_n$. Suppose that P_x^{λ} is an arbitrary soft point of $\tilde{X}_n$. Then for $i = 1, 2, ..., k$, $P_x^{\lambda} \tilde{\in} (F_i, E)$. Since every (F_i, E) is a union of soft base elements, there exists a soft base element (S_i, E) such that $P_x^{\lambda} \tilde{\in} (S_i, E) \sqsubseteq (F_i, E)$ for $i = 1, 2, ..., k$. Then we have that $P_x^{\lambda} \tilde{\in} \sqcap_{i=1}^{k} (S_i, E)$ and so by Proposition 3.19, a base element $(S, E)_{P_x^{\lambda}}$ such that $P_x^{\lambda} \tilde{\in} (S, E)_{P_x^{\lambda}} \sqsubseteq \sqcap_{i=1}^{k} (S_i, E)$. Thus $(F, E) = \sqcup_{P_x^{\lambda} \tilde{\in} (S,E)} (S, E)_{P_x^{\lambda}}$ i.e., (F, E) is a union of soft base elements. This implies $(F, E) \in \mathcal{T}_n$. Now, we show that an arbitrary union of soft sets in $\mathcal{T}_n$ belongs in $\mathcal{T}_n$. Let us take an indexed family $\{(F_i, E)\}_{i \in J}$ of elements of $\mathcal{T}_n$ and assume that $(F, E) = \sqcup_{i \in J} (F_i, E)$. If every (F_i, E) is a empty soft set, then $(F, E) \in \mathcal{T}_n$. If for at least $i_0 \in J$, $(F_{i_0}, E) \neq \Phi$ then it is a union of soft base elements. Since (F, E) is the union of all of the soft base elements constructing the $(F_i, E)' s$, it follows that $(F, E) \in \mathcal{T}_n$. ☐

Remark 4.3. Let $(X, \mathcal{T})$ be a soft topological space and $(X_n, \mathcal{T}_n)$ be a soft star topology on X_n. Corresponding to each parameter $\lambda \in E$, the topological space $\left(X_n, \mathcal{T}_n^{\lambda}\right)$ is a star topology on X_n. However, in general the reverse of this fact is not true.

Proposition 4.4. *Let $(X, \mathcal{T})$ be a soft topological space and $(X_n, \mathcal{T}_n)$ be a soft star topology on X_n. Then X is a soft subspace of X_n.*

Theorem 4.5. *Let $(X, \mathcal{T})$ be a soft topological space and $(X_n, \mathcal{T}_n)$ be a soft star topology on X_n, where $\mathcal{T}_n = \langle (F_1, E), (F_2, E), ..., (F_n, E)\rangle$. Then $(X_n, \mathcal{T}_n)$ is a soft compactification of $(X, \mathcal{T})$ iff*

(1) $\tilde{X} - \sqcup_{i=1}^{n} (F_i, E)$ *is soft compact for* $i = 1, 2, ..., n$;,
(2) (F_i, E) *is not contained in a soft compact and soft closed* (K, E) *of* X *i.e.,* $(F_i, E) \not\sqsubseteq (K, E)$ *for* $i = 1, 2, ..., n$.

Proof. $(\Rightarrow)$ Let $(X_n, \mathcal{T}_n)$ be a soft compactification of $(X, \mathcal{T})$ and suppose that $\mathcal{C}$ is a soft cover of $\tilde{X} - \sqcup_{i=1}^{n} (F_i, E)$ by soft open sets in X. Then $\mathcal{C}' = \mathcal{C} \cup \left\{ (F_i, E) \sqcup \left\{ P_y^{\mu} \right\} \right\}$ is a soft cover of X_n by soft open sets in X_n. Since X_n is a soft compact space, there exists a finite subcover of $\mathcal{C}$ covering the $\tilde{X} - \sqcup_{i=1}^{n} (F_i, E)$. Therefore every soft open cover of $\tilde{X} - \sqcup_{i=1}^{n} (F_i, E)$ by soft open sets in X has a finite subcover and so $\tilde{X} - \sqcup_{i=1}^{n} (F_i, E)$ is soft compact for $i = 1, 2, ..., n$.

Now, suppose there exists a soft closed and soft compact subset (K, E) of X such that $(F_i, E) \sqsubseteq K$ for at least $i \in I$. Then the soft set $((F_i, E) - (K, E)) \sqcup \left\{ P_y^{\mu} \right\} = \left\{ P_y^{\mu} \right\}$ is soft open in the soft star topology which contradicts the fact that $\overline{\tilde{X}} = \tilde{X}_n$.

$(\Leftarrow)$ Let $\tilde{X} - \sqcup_{i=1}^{n} (F_i, E)$ be soft compact. Then since $\tilde{X}_n = \left(\tilde{X} - \sqcup_{i=1}^{n} (F_i, E) \right) \sqcup \left(\sqcup_{i=1}^{n} \left\{ P_{y_i}^{\mu_i} \right\} \right) \sqcup \sqcup_{i=1}^{n} (F_i, E)$, it is clear that $\tilde{X}_n$ is soft compact. $\square$

Example 4.6. Let $X = \{x_1, x_2, x_3\}$, $E = \{\lambda_1, \lambda_2\}$ and

$$\mathcal{T} = \left\{ \Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E) \right\}$$

where $(F_1, E), (F_2, E)$ and (F_3, E) are soft open sets on X, defined as follows:

$$\begin{aligned} (F_1, E) &= \{(\lambda_1, \{x_1\}), (\lambda_2, \{x_2\})\}, \\ (F_2, E) &= \{(\lambda_1, \{x_3\}), (\lambda_2, \{x_1, x_3\})\}, \\ (F_3, E) &= \{(\lambda_1, \{x_1, x_3\}), (\lambda_2, X)\}. \end{aligned}$$

Then $(X, \mathcal{T})$ is a soft topological space. Consider the soft open subset (F_1, E) of X. Suppose that $X_1 = X \cup \{y_1\}$ and $E_1 = E \cup \{\mu\} = \{\lambda_1, \lambda_2, \mu\}$ and an absolute soft point $P_{y_1}^{e}$ in X_1 such that $P(e) = y_1$ for every $e \in E_1$. Then the soft topology $\mathcal{T}_1$ on X_1 generated by

$$\mathfrak{B} = \mathcal{T} \cup \left\{ ((F_1, E) - (F_3^c, E)) \sqcup \left\{ P_y^e \right\} : (F_3^c, E) \text{ soft closed and soft compact} \right\}$$

and so

$$\mathfrak{B} = \mathcal{T} \cup \{(G_1, E)\},$$

where

$$((F_1, E) - (F_3^c, E)) \sqcup \left\{ P_y^e \right\} = (G_1, E) = \{(\lambda_1, \{x_1, y_1\}), (\lambda_2, \{x_2, y_1\}), (\mu, \{y_1\})\}$$

is a soft star topology of $(X, \mathcal{T})$, where $\mathcal{T}_1 = \langle (F_1, E) \rangle$ and it is defined by

$$\mathcal{T}_1 = \left\{ \Phi, \tilde{X}_1, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E) \right\}$$

where

$$(F_2, E) \sqcup (G_1, E) = (G_2, E) = \{(\lambda_1, \{x_1, x_3, y_1\}), (\lambda_2, X_1), (\mu, \{y_1\})\}$$

Moreover, since

$$\mathcal{T}_1^{\lambda_1} = \{\varnothing, X_1, X, \{x_1\}, \{x_3\}, \{x_1, x_3\}, \{x_1, y_1\}, \{x_1, x_3, y_1\}\},$$

and

$$\mathcal{T}_1^{\lambda_2} = \{\varnothing, X_1, X, \{x_2\}, \{x_1, x_3\}, \{x_2, y_1\}\},$$

then $\mathcal{T}_1^{\lambda_1}$ is a star topology on X_1, where $\mathcal{T}_1^{\lambda_1} = \langle\{x_1\}\rangle$ by the base

$$\mathfrak{B}_1^{\lambda_1}= \{\varnothing, X_1, \{x_1\}, \{x_3\}, \{x_1, x_3\}, \{x_1, y_1\}\},$$

while $\mathcal{T}_1^{\lambda_2}$ is a star topology on X_1, where $\mathcal{T}_1^{\lambda_2} = \langle\{x_1\}\rangle$ by the base

$$\mathfrak{B}_1^{\lambda_2}= \{\varnothing, X_1, \{x_1\}, \{x_3\}, \{x_1, x_3\}, \{x_2, y_1\}\}.$$

Theorem 4.7. *Let $(X, \mathcal{T})$ be a soft topological space. Suppose that $(X_n, \mathcal{T}_n)$ is a soft star topological space on X_n. Then $(X_n, \mathcal{T}_n)$ is a soft Hausdorff compactification of $(X, \mathcal{T})$ iff*

(1) *$(X_n, \mathcal{T}_n)$ is a soft compactification.*
(2) *$(X, \mathcal{T})$ is a soft locally and soft Hausdorff topological space.*
(3) *There exists a soft compact (K, E) such that $(F_i, E) \sqcap (F_j, E) \sqsubseteq (K, E)$ for $i, j = 1, 2, ..., n$ and $i \neq j$.*

Proof. ($\Rightarrow$) Let $(X_n, \mathcal{T}_n)$ be a soft Hausdorff compactification of $(X, \mathcal{T})$. (1) It is clear that $(X_n, \mathcal{T}_n)$ is a soft compactification. (2) Suppose that X_n is a soft Hausdorff . Then X is soft Hausdorff since the property is hereditary by Proposition 3.14. Moreover, since $\tilde{X}_n - \{P_{x_1}^{\lambda_1}, P_{x_2}^{\lambda_2}, ...P_{x_n}^{\lambda_n}\} = \tilde{X}$ and $\{P_{x_1}^{\lambda_1}, P_{x_2}^{\lambda_2}, ...P_{x_n}^{\lambda_n}\}$ is a finite set in X and by Proposition 3.14, X is a soft open set in $\tilde{X}_n$ and so is a soft locally compact. (3) Let $P_x^\lambda \neq P_y^\mu$. Since $(X_n, \mathcal{T}_n)$ is a soft Hausdorff, there exist two soft open sets such that $P_x^\lambda \tilde{\in} (F_i, E)$, $P_y^\mu \tilde{\in} (F_j, E)$ and $((F_i, E) - (K, E)) \sqcap ((F_j, E) - (L, E)) = \varnothing$. Then this implies that $(F_i, E) \sqcap (F_j, E) \sqsubseteq (K, E) \sqcup (L, E)$. Thus since $(K, E) \sqcup (L, E)$ is a soft compact, this completes the proof.

($\Leftarrow$) Conversely, let $(X_n, \mathcal{T}_n)$ be a soft compactification. Suppose that $(X, \mathcal{T})$ is a soft locally and soft Hausdorff. To show that $(X_n, \mathcal{T}_n)$ is a soft Hausdorff we only show that there exist two disjoint sets separating points $P_x^\lambda, P_y^\mu \tilde{\in} \tilde{X}_n$ $\left(P_x^\lambda \neq P_y^\mu\right)$. We need to check three separate cases. First, If both P_x^λ and P_y^μ are soft points in $\tilde{X}$, it is clear that $\tilde{X}_n$ is soft Hausdorff since $\tilde{X}$ is soft Hausdorff. Second, assume that $P_x^\lambda, P_y^\mu \tilde{\in} \tilde{X}_n$ $\left(P_x^\lambda, P_y^\mu \tilde{\notin} \tilde{X}\right)$. Then by hypothesis, for some soft compact (K, E), there exist two soft open sets $(F, E), (G, E)$ such that

$$\left((F, E) - (K, E) \sqcup \{P_x^\lambda\}\right) \sqcap \left((G, E) - (K, E) \sqcup \{P_y^\mu\}\right) = \Phi.$$

Thus $\tilde{X}_n$ is soft Hausdorff. Finally, suppose that $P_x^\lambda \tilde{\in}\ \tilde{X}$ and $P_y^\mu \tilde{\in} \tilde{X}_n$. Since X is soft locally compact, there is a soft open set (F, E) in X such that $P_x^\lambda \tilde{\in} (F, E)$ and $\overline{(F, E)}$ is soft compact and so is soft closed. Then (F, E) and $\tilde{X}_n - \overline{(F, E)}$ are two disjoint soft open sets in $\tilde{X}_n$ containing $P_x^\lambda$and P_y^μ respectively. Then $\tilde{X}_n$ is soft Hausdorff. □

Definition 4.8. Let $X = \{x_1, x_2, x_{3,}...\}$ be a initial universe and $E = \{\lambda_1, \lambda_2\}$. Consider the family $\mathcal{T} = \{(F_n, E) : (\lambda_1, \{x_1, x_2, ..., x_n\}), (\lambda_2, \{x_1, x_2, ..., x_n\}) : \lambda_i \in E$ and

$n \in \mathbb{N}$, $i = 1, 2\} \sqcup \left\{\tilde{X}, \Phi\right\}$, then the pair $(X, \mathcal{T})$ is a soft topological space. It is called the soft topology of nested soft sets.

Example 4.9. Let $(X, \mathcal{T})$ be the soft topology of nested soft sets. This space is not soft compact. The collection

$$\mathcal{A} = \{(F_n, E) : (F_n, E) = (\lambda_1, \{x_1, x_2, ..., x_n\}), (\lambda_2, \{x_1, x_2, ..., x_n\}) : n \in \mathbb{N} \text{ and } \lambda_i \in E\}$$

is a soft cover by sets that are soft open in X, i.e., $\tilde{X} = \sqcup_{n\in\mathbb{N}}(F_n, E)$. But there is not finite subcover of $\mathcal{A}$ that covers $\tilde{X}$, and therefore X with soft topology its is not soft compact. However all nontrivial soft open sets except X itself are soft compact because they are the form $(F_n, E) \sqsubseteq (F_m, E)$ for $n < m$. On the other hand, since a family $\mathcal{A}$ covering any soft closed set have not a finite subcover that also covers it, we can say that the only soft closed and soft compact set of X with soft topology is null soft set Φ.

Now, we show the soft star compactifications by soft points of the soft topological space $(X, \mathcal{T})$. Assume that $(F_1, E), (F_2, E)$ are two soft open sets in X such that $\tilde{X} - ((F_1, E) \sqcup (F_2, E))$ is a soft compact. Since the only soft closed and soft compact set of X is null soft set Φ, one of they should be $\tilde{X}$. Then for a soft open set $(F_n, E) : \{(\lambda_1, \{x_1, x_2, ..., x_n\}), (\lambda_2, \{x_1, x_2, ..., x_n\})\}$, this implies that the soft star compactifications by two soft points of X are mainly three types such that $\mathcal{T}_{2_1} = \left\langle \tilde{X}, \tilde{X} \right\rangle$, $\mathcal{T}_{2_2} = \left\langle (F_n, E), \tilde{X} \right\rangle$ and $\mathcal{T}_{2_3} = \left\langle \tilde{X}, (F_n, E) \right\rangle$. By select of (F_n, E), we also can say that $(X, \mathcal{T})$ has infinite soft star compactifications by two soft points of type $\mathcal{T}_{2_3}$.

Theorem 4.10. *Let $(X, \mathcal{T})$ and $(Y, \mathcal{V})$ be two soft topological space and $\varphi_\psi : (X_n, \mathcal{T}_n) \to (Y_n, \mathcal{V}_n)$ be a soft continuous such that $\varphi : (X_n - X) \to (Y_n - Y)$ is injective.*

(1) *If $\mathcal{V}_n$ is a soft star topology on Y, where $\mathcal{V}_n = \langle (G_1, E), (G_2, E), ..., (G_n, E) \rangle$, then $\varphi_\psi^{-1}(\mathcal{V}_n)$ is a soft star topology on X, where*

$$\varphi_\psi^{-1}(\mathcal{V}_n) = \left\langle \varphi_\psi^{-1}((G_1, E)), \varphi_\psi^{-1}((G_2, E)), ..., \varphi_\psi^{-1}((G_n, E)) \right\rangle.$$

(2) *If $\mathcal{V}_n$ is a soft star compactifications of Y, then $\varphi_\psi^{-1}(\mathcal{V}_n)$ is a soft star compoctification of X.*

Proof. (1) It is clear from Theorem 3.23.

(2) Suppose that $\mathcal{V}_n$ is a soft star compactifications of Y, where

$$\mathcal{V}_n = \langle (G_1, E), (G_2, E), ..., (G_n, E) \rangle.$$

Then $\tilde{Y} - \sqcup_{i=1}^n (G_i, E)$ is soft compact for $i = 1, 2, ..., n$. Then, the set

$$\varphi_\psi^{-1}\left(\tilde{Y} - \sqcup_{i=1}^n (G_i, E)\right) = \tilde{X} - \sqcup_{i=1}^n \varphi_\psi^{-1}((G_i, E))$$

is soft compact in X. On the other hand, since $(F_i, E) \not\sqsubseteq (K, E)$ for every soft closed and soft compact (K, E) in Y, we have that $\varphi_\psi^{-1}((F_i, E)) \not\sqsubseteq \varphi_\psi^{-1}((K, E))$. Then $\varphi_\psi^{-1}(\mathcal{V}_n)$ is a soft star compactification of X. □

Theorem 4.11. *Let $(X, \mathcal{T})$ be a soft topological space and $(X_n, \mathcal{T}_{n_1})$ be a soft star topology on X_n, where $\mathcal{T}_{n_1} = \langle (F_1, E), (F_2, E), ..., (F_n, E) \rangle$ such that $(F_i, E) = \tilde{X}$ for each $i = 1, 2, ..., n$. If $(X_n, \mathcal{T}_{n_2})$ is an arbitrary soft star topology on X_n, where $\mathcal{T}_{n_2} = \langle (G_1, E), (G_2, E), ..., (G_n, E) \rangle$, then $\mathcal{T}_{n_1} \subseteq \mathcal{T}_{n_2}$.*

Proof. Let $(F,E) = ((F_i,E) - (K,E)) \sqcup \{P_y^\mu\}$ be a soft open set in $\mathcal{T}_{n_1}$, where (K,E) is a soft closed and soft compact of X. Then

$$\begin{aligned}(F,E) &= ((F_i,E) - (K,E)) \sqcup \{P_y^\mu\} \\ &= ((F_i,E) - (K,E)) \sqcup ((G_i,E) - (K,E)) \sqcup \{P_y^\mu\} \in \mathcal{T}_{n_2}\end{aligned}$$

because of $((F_i,E) - (K,E)) = \left(\tilde{X} - (K,E)\right) \in \mathcal{T}$ and $((G_i,E) - (K,E)) \sqcup \{P_y^\mu\} \in \mathcal{T}_{n_2}$. Hence $(F,E) \in \mathcal{T}_{n_2}$ and so $\mathcal{T}_{n_1} \subseteq \mathcal{T}_{n_2}$. □

Definition 4.12. If take $n = 1$ and $\mathcal{T}_1 = \left\langle \tilde{X} \right\rangle$, then the soft topology $\mathcal{T}_1$ generated by

$$\mathfrak{B} = \mathcal{T} \sqcup \left\{ \left(\tilde{X} - (K,E)\right) \sqcup \{P_y^\mu\} : (K,E) \sqsubseteq \tilde{X} \text{ soft closed and soft compact} \right\}$$

is called an Alexandroff soft compactification of $(X,\mathcal{T})$ by one soft point.

Corollary 4.13. *Let $(X,\mathcal{T})$ be a soft topological space and $(X_1,\mathcal{T}_1)$ is the Alexandroff soft compactification of $(X,\mathcal{T})$ by one soft point. Then we say that for each parameter $\lambda \in E$, the star topological space $\left(X_1,\mathcal{T}_1^\lambda\right)$ is an Alexandroff compactification of $\left(X,\mathcal{T}^\lambda\right)$ by one point.*

Theorem 4.14. *Let $(X,\mathcal{T})$ be a soft topological space, and $(X_n,\mathcal{T}_{n_1})$ be a soft star compactification of $(X,\mathcal{T})$, where $\mathcal{T}_{n_1} = \left\langle \tilde{X}, \tilde{X}, ..., \tilde{X} \right\rangle$. Suppose that $(X_n,\mathcal{T}_{n_2})$ is another soft star compactifications of $(X,\mathcal{T})$, where $\mathcal{T}_{n_2} = \langle (F_1,E),(F_2,E),...,(F_n,E)\rangle$ such that $(F_1,E) = (F_2,E) =,...,= (F_n,E)$. Then $\mathcal{T}_{n_1} = \mathcal{T}_{n_2}$.*

Proof. By Theorem 4.11, we have $\mathcal{T}_{n_1} \subseteq \mathcal{T}_{n_2}$. We show that $\mathcal{T}_{n_2} \subseteq \mathcal{T}_{n_1}$. For some $i_0 \in \{1,2...,n\}$, suppose that $(F,E) = ((F_{i_0},E) - (K,E)) \sqcup \{P_y^\mu\}$ is a soft open set in $\mathcal{T}_{n_2}$, where (K,E) is a soft closed and soft compact of X. Moreover, $\tilde{X} - \sqcup_{i=1}^n (F_i,E) = \tilde{X} - (F_{i_0},E)$ is a soft closed and soft compact set in X. Then

$$((F_{i_0},E) - (K,E)) \sqcup \{P_y^\mu\} = \left(\tilde{X} - \left((K,E) \sqcup \left(\tilde{X} - (F_{i_0},E)\right)\right)\right) \sqcup \{P_y^\mu\} \in \mathcal{T}_{n_2},$$

where $(K,E) \sqcup \left(\tilde{X} - (F_{i_0},E)\right)$ is a soft closed and soft compact set in X and therefore $\mathcal{T}_{n_2} \subseteq \mathcal{T}_{n_1}$. □

Theorem 4.15. *Let $(X,\mathcal{T})$ be a soft topological space. Assume that $(X_n,\mathcal{T}_{n_1})$ and $(X_n,\mathcal{T}_{n_2})$ are two soft star topologies on X_n, where*

$$\mathcal{T}_{n_1} = \langle (F_1,E),(F_2,E),...,(F_n,E)\rangle \text{ and } \mathcal{T}_{n_2} = \langle (G_1,E),(G_2,E),...,(G_n,E)\rangle .$$

If $(G_i,E) \sqsubseteq (F_i,E)$ for every $i = 1,2,...,n$, then $\mathcal{T}_{n_1} \subseteq \mathcal{T}_{n_2}$.

Proof. Suppose that (F,E) is a soft open set of $\mathcal{T}_{n_1}$ and $P_y^\mu \tilde{\in} (F,E)$. Then there exists a soft base element (S_1,E) in $\mathfrak{B}_1$, where $\{P_y^\mu\} \tilde{\in} (S_1,E) = ((F_i,E) - (K,E)) \sqcup \{P_y^\mu\} \sqsubseteq (F,E)$ for any soft closed and soft compact set (K,E) and $i = 1,2...,n$. Since $(G_i,E) \sqsubseteq (F_i,E)$ for every $i = 1,2,...,n$, we have that

$$\{P_y^\mu\} \tilde{\in} ((G_i,E) - (K,E)) \sqcup \{P_y^\mu\} \sqsubseteq ((F_i,E) - (K,E)) \sqcup \{P_y^\mu\} \sqsubseteq (F,E).$$

Then $\{P_y^\mu\} \tilde{\in} ((G_i,E) - (K,E)) \sqcup \{P_y^\mu\} \sqsubseteq (F,E)$ and so $(F,E) \in \mathcal{T}_{n_2}$. Thus, $\mathcal{T}_{n_1} \subseteq \mathcal{T}_{n_2}$. □

Theorem 4.16. *Let $(X, \mathcal{T})$ be a soft topological space. Assume that $(X_n, \mathcal{T}_{n_1})$ and $(X_n, \mathcal{T}_{n_2})$ are two soft star topologies on X_n, where*

$$\mathcal{T}_{n_1} = \langle (F_1, E), (F_2, E), ..., (F_n, E) \rangle \text{ and } \mathcal{T}_{n_2} = \langle (G_1, E), (G_2, E), ..., (G_n, E) \rangle,$$

respectively. Then $(X_n, \mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2})$ is soft star topology on X_n, where

$$\mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2} = \langle (F_1, E) \sqcup (G_1, E), (F_2, E) \sqcup (G_2, E), ..., (F_n, E) \sqcup (G_n, E) \rangle.$$

Proof. Suppose that (F, E) is a soft open set of $\mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2}$ and $P_y^\mu \tilde{\in} (F, E)$. Then $P_y^\mu \tilde{\in} (F, E) \tilde{\in} \mathcal{T}_{n_1}$ and $P_y^\mu \tilde{\in} (F, E) \tilde{\in} \mathcal{T}_{n_2}$, and so there exist soft base elements (S_1, E) in $\mathfrak{B}_1$ and (S_2, E) in $\mathfrak{B}_2$, where $P_y^\mu \tilde{\in} (S_1, E) = A \sqcup \left((F_i, E) - (K, E) \sqcup \left\{ P_y^\mu \right\} \right) \sqsubseteq (F, E)$ and $P_y^\mu \tilde{\in} (S_2, E) = B \sqcup \left((G_i, E) - (L, E) \sqcup \left\{ P_y^\mu \right\} \right) \sqsubseteq (F, E)$ for $A, B \in \mathcal{T}$ and two soft closed and soft compact sets (K, E) and (L, E), respectively. On the other hand, since

$$\begin{aligned}
& P_y^\mu \tilde{\in} (S_1, E) \sqcup (S_2, E) \\
& = A \sqcup B \sqcup \left[(F_i, E) - (K, E) \sqcup \left\{ P_y^\mu \right\} \right] \sqcup \left[(G_i, E) - (L, E) \sqcup \left\{ P_y^\mu \right\} \right] \\
& = [((F_i, E) \sqcup (G_i, E)) - ((K, E) \sqcup (L, E))] \\
& \sqcup \left[((F_i, E) - (K, E)) \sqcup ((G_i, E) - (L, E)) \sqcup \left\{ P_y^\mu \right\} \right] \sqsubseteq (F, E),
\end{aligned}$$

where $((F_i, E) \sqcup (G_i, E)) - ((K, E) \sqcup (L, E)) \sqsubseteq ((F_i, E) - (K, E)) \sqcup ((G_i, E) - (L, E))$. Here, if we take

$$C = [((F_i, E) - (K, E)) \sqcup ((G_i, E) - (L, E))]$$

then C is a soft open set in X. So $P_y^\mu \tilde{\in} C \sqcup [((F_i, E) \sqcup (G_i, E)) - ((K, E) \sqcup (L, E))] \sqcup \left\{ P_y^\mu \right\} \sqsubseteq (F, E)$. Then $\mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2}$ is soft star topology on X_n, where

$$\mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2} = \langle (F_1, E) \sqcup (G_1, E), (F_2, E) \sqcup (G_2, E), ..., (F_n, E) \sqcup (G_n, E) \rangle.$$

□

Theorem 4.17. *Let $(X, \mathcal{T})$ be a soft topological space. Assume that $(X_n, \mathcal{T}_{n_1})$ and $(X_n, \mathcal{T}_{n_2})$ are two soft star compactifications of $(X, \mathcal{T})$ by n soft point, where $\mathcal{T}_{n_1} = \langle (F_1, E), (F_2, E), ..., (F_n, E) \rangle$ and $\mathcal{T}_{n_2} = \langle (G_1, E), (G_2, E), ..., (G_n, E) \rangle$, respectively. Then $(X_n, \mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2})$ is soft star compactification of $(X, \mathcal{T})$, where*

$$\mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2} = \langle (F_1, E) \sqcup (G_1, E), (F_2, E) \sqcup (G_2, E), ..., (F_n, E) \sqcup (G_n, E) \rangle.$$

Proof. We know that $\mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2}$ is a soft star topology on X_n. For every $i = 1, 2..., n$ and every soft closed and soft compact (K, E) of X, since $(F_1, E) \not\sqsubseteq (K, E)$, it follows that $(F_i, E) \sqcup (G_i, E) \not\sqsubseteq (K, E)$. On the other hand,

$$\begin{aligned}
\tilde{X} - \sqcup_{i=1}^n ((F_i, E) \sqcup (G_i, E)) &= \tilde{X} - ((\sqcup_{i=1}^n (F_i, E)) \sqcup (\sqcup_{i=1}^n (G_i, E))) \\
&= \left(\tilde{X} - \sqcup_{i=1}^n (F_i, E) \right) \sqcap \left(\tilde{X} - \sqcup_{i=1}^n (G_i, E) \right)
\end{aligned}$$

where $\tilde{X} - \sqcup_{i=1}^n (F_i, E)$ and $\tilde{X} - \sqcup_{i=1}^n (G_i, E)$ are soft compact subsets of X. Then

$$\tilde{X} - \sqcup_{i=1}^n ((F_i, E) \sqcup (G_i, E))$$

is a soft compact and so $\mathcal{T}_{n_1} \sqcap \mathcal{T}_{n_2}$ is a soft star compactification of $(X, \mathcal{T})$. □

5. Conclusions and in future work

One of the most important theories of general topology is compact spaces theory. This theory is important not only in sub-branches of mathematics but also in physics especially in quantum physics. The theory in analysis has been based upon the studies about continuity since 1800s makes it more important. Therefore topologists researched that under which conditions non-compact space can be taken as a sub-space of a compact space and they presented a large number of interesting and useful approaches. These studies are quite useful for creating of geometric models in solution of many engineering problems as well as in general topology and fuzzy topology. Some of these important approaches are Alexandroff (one point) compactification, Wallman compactification, Fan-Gottesman compactification and Stone-Cech compactification. The aim of this study is to create a structure which will be the base to compactifications process of non-compact soft topological spaces. We firstly discussed some related properties of concept of soft base. By adding $n-$soft points, we extended the soft topological spaces to soft star topological spaces and obtained some significant results. Also we showed that under which conditions the soft star topological spaces is a soft compactification. Based on the choose of soft sets, the soft compactification can be reduced the Alexandroff soft compactification by one soft point and also Alexandroff compactification in topological spaces which is a parameterized family. In the future, by extending the concept of the well-known normal base of general topology into soft topolojical spaces, the idea of the soft compactification will be expanded to Fan-Gottesman compactification being Wallman–type compactification and Wallman compactification via the soft normal base.

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Ridvan Şahin (mat.ridone@gmail.com)
Department of Mathematics, Faculty of Education, Bayburt University, Bayburt, 69000, Turkey