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γ-Synchronized fuzzy automata and their applications

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Abstract. γ-Synchronized fuzzy automaton were introduced by V. Karthikeyan and M. Rajasekar in [5]. In this paper we introduce γ-synchronization degree of a fuzzy automaton and procedure is given to find γ-synchronized words. Using this concept, we introduce an application related to petrol passing through two different pipelines in 4 cities. The objective is to find shortest optimum sequence of pipelines to cover all cities (we can start anywhere in 4 cities) such that the petrol reaches each city at least once and ends in a particular city with minimal flow capacity and minimum maintenance cost.

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1. INTRODUCTION

The concept of fuzzy set was introduced by L. A. Zadeh in 1965 [7]. The mathematical formulation of a fuzzy automaton was first proposed by W.G. Wee in 1967 [6]. γ-synchronized fuzzy automaton were introduced by V. Karthikeyan and M. Rajasekar in [5]. In this paper, we introduce γ-synchronization degree of a fuzzy automaton and procedure is given to find γ-synchronized words. Using this concept, we introduce an application related to petrol passing through two different pipelines in n cities. Flow capacity of the pipelines is given. Each pair of city is connected by pipelines with one other. Let C_{ij} be the maintenance cost for the pipeline that connects from i^{th} city to j^{th} city. The objective is to find the shortest optimum sequence of pipelines to cover all cities (we can start anywhere in n cities) such that the petrol reaches each city at least once and ends in a particular city with minimal flow capacity and minimum maintenance cost.

2. Preliminaries

Definition 2.1 ([8]). Let X denote a universal set. Then a fuzzy set A in X is set of ordered pairs: $A = \{(x,\ \mu_A(x)|x\ \in\ X\}$, $\mu_A(x)$ is called the membership function or grade of membership of x in A which maps X to the membership space $[0, 1]$.

Definition 2.2 ([4]). A finite fuzzy automaton is a system of 5 tuples, $M = (Q,\ \Sigma\ f_M,\ q_0,\ F)$,
where,
Q - set of states $\{q_0,\ q_1,\ q_2, ...,\ q_n\}$,
Σ - alphabets (or) input symbols,
f_M - function from $Q \times \Sigma \times Q \to [0, 1]$,
q_0 initial state and $q_0 \in Q$, and
$F \subseteq Q$ set of final states.
$f_M(q_i, a, q_j) = \mu$, $0 \le \mu \le 1$, means that when M is in state q_i and reads the input a will move to the state q_j with weight function μ.
f_M can be extended to $Q \times \Sigma^* \times Q \to [0, 1]$ by,

$$f_M(q_i,\ \epsilon,\ q_j) = \begin{cases} 1 & \text{if } q_i = q_j \\ 0 & \text{if } q_i \neq q_j \end{cases}$$

$f_M(q_i, w, q_m) = Max\{Min\{f_M(q_i,\ a_1,\ q_1), f_M(q_1,\ a_2,\ q_2), ..., f_M(q_{m-1},\ a_m,\ q_m)\}\}$ for $w\ =\ a_1\ a_2\ a_3\ ...\ a_m\ \in\ \Sigma^*$, where Max is taken over all the paths from q_i to q_m. We can represent transitions more conveniently by the matrix notation as follows:
For each $a \in \Sigma$, we can form a $n \times n$ matrix $F(a)$ whose $(i, j)^{th}$ element is $f_M(q_i,\ a,\ q_j) > 0$.

Note

Throughout this paper, we consider the fuzzy automaton $M = (Q, \Sigma, f_M)$. That is, we consider a fuzzy automaton without initial state and final state.

Definition 2.3 ([3]). A fuzzy automaton $M = (Q,\ \Sigma,\ f_M)$ is called deterministic if for each $a \in \Sigma$ and $q_i \in Q$, there exists a unique state q_a such that $f_M(q_i,\ a,\ q_a) > 0$ otherwise it is called nondeterministic.

Definition 2.4. A relation R on a set Q is said to be an equivalence relation if it is reflexive, symmetric and transitive.

Definition 2.5 ([1, 2]). Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automaton. An equivalence relation R on Q in M is called a congruence relation if for all $q_i, q_j \in Q$ and $a\ \in\ \Sigma$, $q_i\ R\ q_j$ implies that, then there exists $q_l, q_k \in Q$ such that $f_M(q_i, a, q_l) > 0$, $f_M(q_j, a, q_k) > 0$ and $q_l R q_k$.

Definition 2.6. Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automaton. The quotient fuzzy automaton determined by the congruence $\cong$ is a fuzzy automaton $M/\cong\ =\ (Q/\cong,\ \Sigma,\ f_{M/\cong})$, where $Q/\cong\ =\ \{Q_i = [q_i]\}$ and
$f_{M/\cong}(Q_1, a, Q_2) = Min\,\{f_M(q_1, a, q_2) > 0\ /\ q_1\ \in\ Q_1,\ q_2\ \in\ Q_2\ \text{ and } a \in \Sigma\}$.

Definition 2.7 ([5]). Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automaton. We say that two states $q_i, q_j \in Q$ are stability related and denoted by $q_i \equiv q_j$, if for any word

$u \in \Sigma^*$, there exists a word $w \in \Sigma^*$ and $q_k \in Q$ such that $f_M(q_i, uw, q_k) > 0 \Leftrightarrow f_M(q_i, uw, q_k) > 0$.

Definition 2.8 ([5]). Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automaton. We say that a fuzzy automaton is γ-synchronized at the state q_j if there exist a real number γ with $0 < \gamma \leq 1$, and a word $w \in \Sigma^*$ that takes each state q_i of Q into q_j such that $f_M(q_i, w, q_j) \geq \gamma$, where γ = minimal membership value in a fuzzy automaton.

Definition 2.9. Let $M = (Q,\ \Sigma,\ f_M)$ be a fuzzy automaton and let$P \subseteq Q$.
The γ-synchronization degree is defined as,
$\theta_M = Min_{w\ \in\ \Sigma^*} \{card(P)\ /\ Min\{f_M(Q, w, P) > 0, q \in Q, p \in P\}\}$.
A fuzzy automaton is γ-synchronized if and only if θ_M is equal to 1.

3. Procedure to Find γ-Synchronized Words

Let $M = (Q, \Sigma, f_M)$ be a fuzzy automaton. We define another fuzzy automaton M_S as follows:
$M_S = (2^Q, \Sigma, f_{M_S}, Q, F \subseteq Q)$, where,

Q is called initial state on M_S,
F is called set of all final states on M_S,
f_{M_S} is the transition function on M_S.

The transition function f_{M_S} is defined by,
$f_{M_S}(P, a, T) = Min\{f_M(p, a, t) > 0, p \in P, t \in T\}$, $P,\ T\ \in\ 2^Q$ for $a \in \Sigma$.
Clearly, M_S is a deterministic fuzzy automaton and more over a word w is γ-synchronized in M if and only if there exists a singleton subsets $S \in 2^Q$ such that $f_{Ms}(Q, w, S) = \gamma$.

Consider the following example,

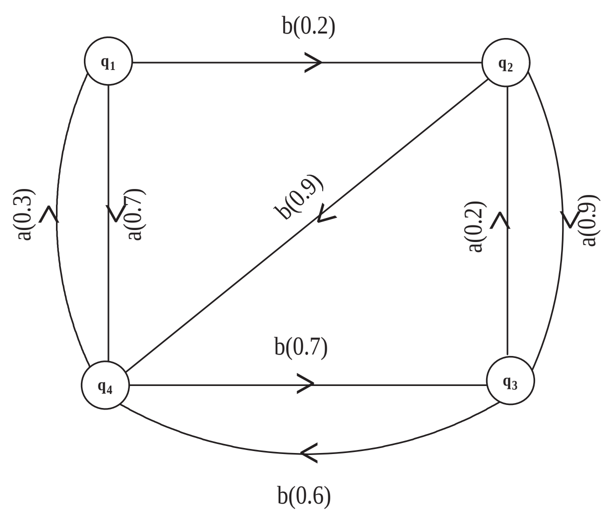

Fig - 3.1

Fuzzy Automaton M_S

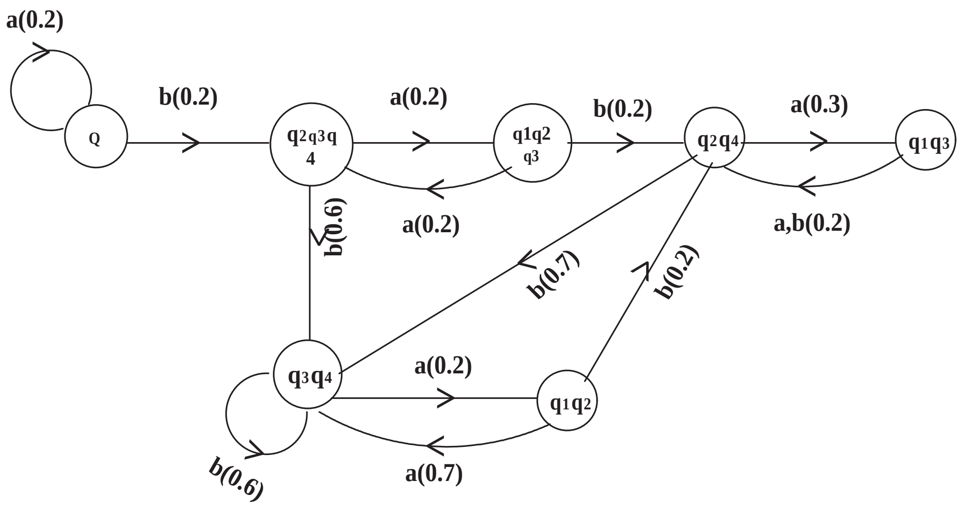

Fig - 3.2

M_S contains two elements subsets of Q and hence, γ-synchronization degree is 2. Hence, M is not γ-synchronized.

Consider the following example,

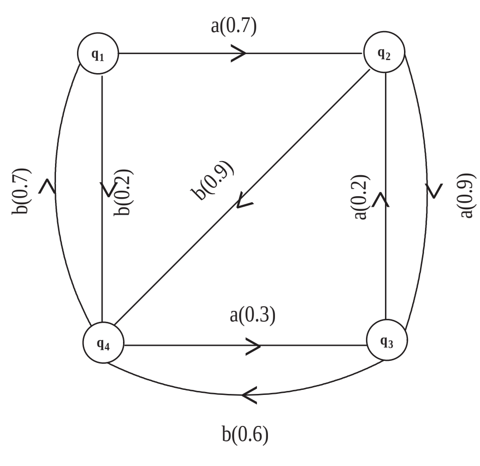

Fig -3.3

Fuzzy Automaton M_S

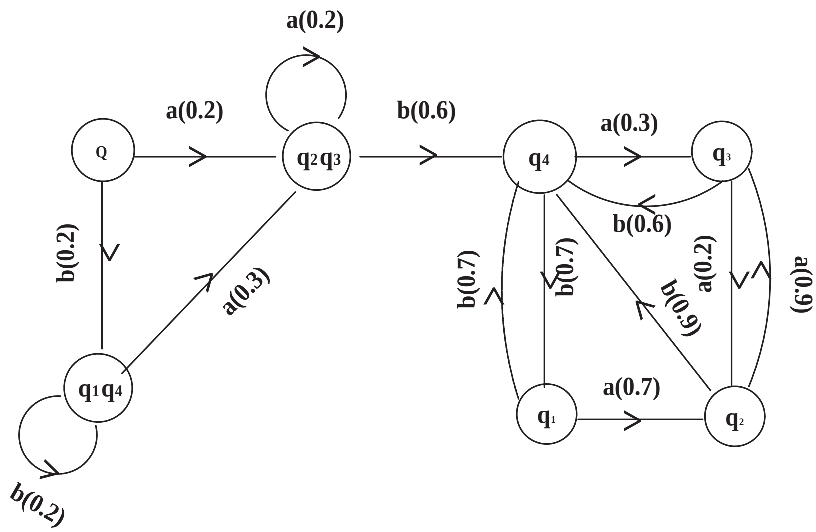

Fig - 3.4

In M_S, $S = \{q_4\}$ is a singleton subset of Q. Hence, the fuzzy automaton is γ-synchronized with a word ab and $\gamma = 0.2$

4. Applications

Assume that there are 4 cities and two different pipelines that are leaving from each city. Assume that the pipelines are of different width and the particular petrol agency decided to supply petrol through two pipelines. Flow capacity of the pipelines is given. Each pair of city is connected by pipelines with one other. Let C_{ij} be the maintenance cost for the pipeline that connects from i^{th} city to j^{th} city.

The main objective is to find the shortest optimum sequence of pipelines to cover all cities (we can start anywhere in n cities) such that the petrol reaches each city atleast once through two different pipelines and ends in a particular city with minimal flow capacity and minimum maintenance cost.

Consider the following example, we consider the cities as states, the two different pipelines as alphabets and flow capacity as a membership value. We can construct the given problem as a fuzzy automaton.

Example 4.1

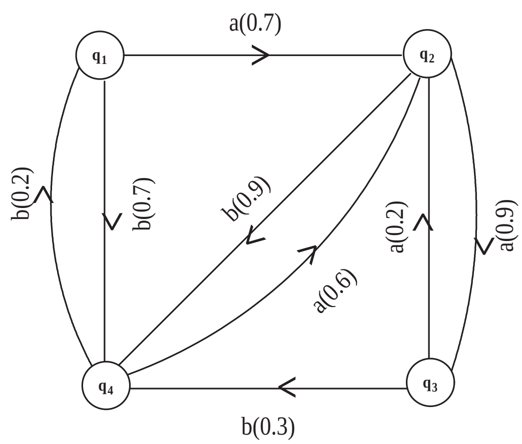

Fig - 4.1

Cost sheet for pipelines "a" and "b" as follows.

pipeline a	q_1	q_2	q_3	q_4
q_1	–	05	13	16
q_2	03	–	11	06
q_3	09	13	–	12
q_4	07	14	02	–

pipeline b	q_1	q_2	q_3	q_4
q_1	–	03	09	12
q_2	14	–	01	13
q_3	04	14	–	15
q_4	10	16	01	–

Fuzzy Automaton M_S

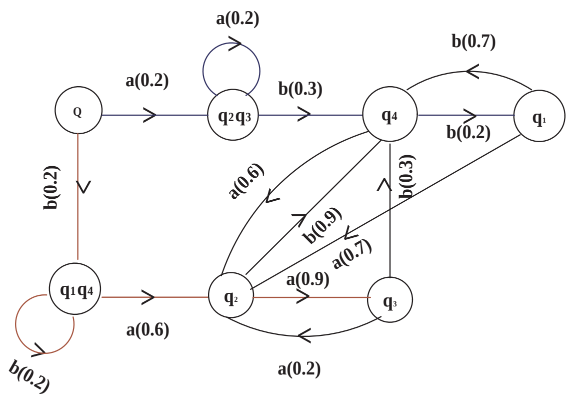

Fig - 4.2

Shortest optimum sequence to cover all cities with minimal flow capacity

1) $aabb$

2) $bbaa$ such that

$$\text{(i) } F(aabb) = \begin{pmatrix} 0.2 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{(ii)} F(bbaa) = \begin{pmatrix} 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0 \end{pmatrix}$$

Minimum cost calculation

1) For the sequence $aabb$

(i) $f_M(q_1, aabb, q_1) = 0.2 > 0$ the cost is $05 + 11 + 15 + 10 = 41$

(ii)$f_M(q_2, aabb, q_1) = 0.2 > 0$ the cost is $11 + 13 + 13 + 10 = 47$

(iii)$f_M(q_3, aabb, q_1) = 0.2 > 0$ the cost is $13 + 11 + 15 + 10 = 49$

(iv)$f_M(q_4, aabb, q_1) = 0.2 > 0$ the cost is $14 + 11 + 15 + 10 = 50$

2) For the sequence $bbaa$

(i) $f_M(q_1, bbaa, q_3) = 0.2 > 0$ the cost is $12 + 10 + 05 + 11 = 38$

(ii)$f_M(q_2, bbaa, q_3) = 0.2 > 0$ the cost is $13 + 10 + 05 + 11 = 39$

(iii)$f_M(q_3, bbaa, q_3) = 0.2 > 0$ the cost is $15 + 10 + 05 + 11 = 41$

(iv)$f_M(q_4, bbaa, q_3) = 0.2 > 0$ the cost is $10 + 12 + 14 + 11 = 47$

The optimum sequence is

$f_M(q_1, bbaa, q_3) = 0.2 > 0$ such that the minimal cost is Rs. 38.

Example 4.2

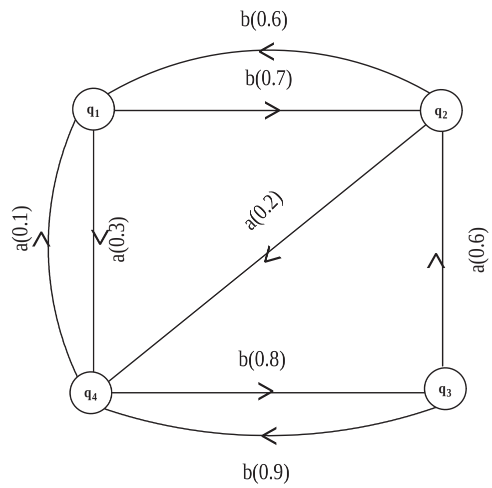

Fig - 4.3

Cost sheet for pipelines "a" and "b" are as follows.

pipeline a	q_1	q_2	q_3	q_4
q_1	–	02	03	06
q_2	03	–	01	06
q_3	09	05	–	04
q_4	07	08	02	–

pipeline b	q_1	q_2	q_3	q_4
q_1	–	03	09	02
q_2	04	–	01	03
q_3	04	09	–	05
q_4	10	06	07	–

Fuzzy Automaton M_s

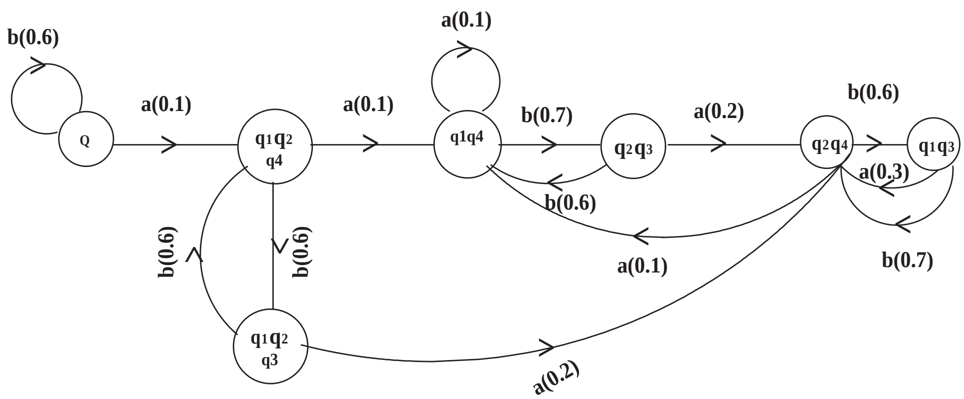

Fig - 4.4

In M_S contains two elements subsets of Q and hence $\gamma-$ synchronization degree is 2. Hence M is not $\gamma-$ synchronized.

In the above fuzzy automaton (Fig - 4.3), for any word $u \in \Sigma^*$, there exists a word $ba \in \Sigma^*$ and $q_k \in Q$ such that $f_M(q_1, uba, q_k) > 0 \Leftrightarrow f_M(q_2, uba, q_k) > 0$. Therefore, the states q_1 and q_2 are stability related.
For any word $u \in \Sigma^*$, there exists a word $aa \in \Sigma^*$ and $q_l \in Q$ such that $f_M(q_3, uaa, q_l) > 0 \Leftrightarrow f_M(q_4, uaa, q_l) > 0$.
Therefore, the states q_3 and q_4 are stability related.

The Quotient Fuzzy Automaton F

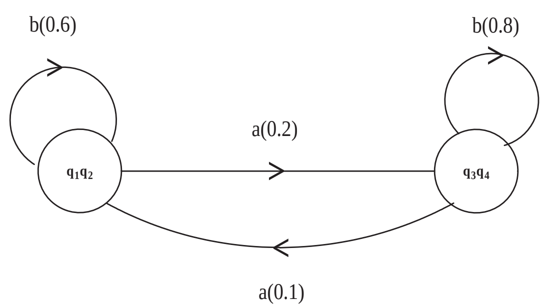

Fig - 4.5

The Relabeled Quotient Fuzzy Automaton F'

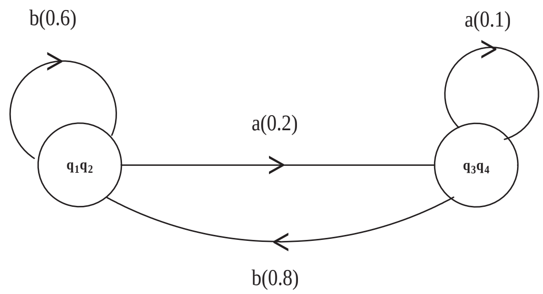

Fig - 4.6

The Relabeled Fuzzy Automaton M_1

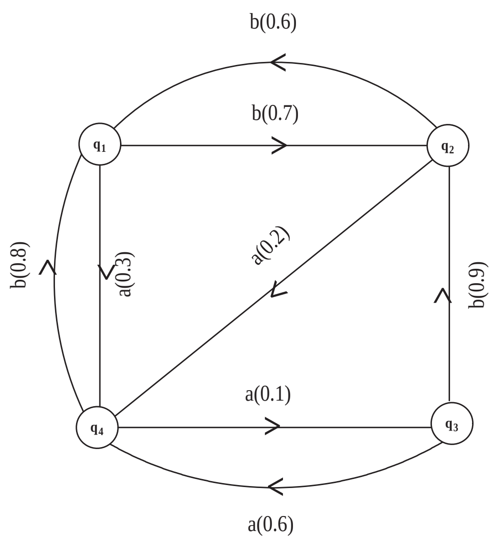

Fig - 4.7

Fuzzy Automaton M_S

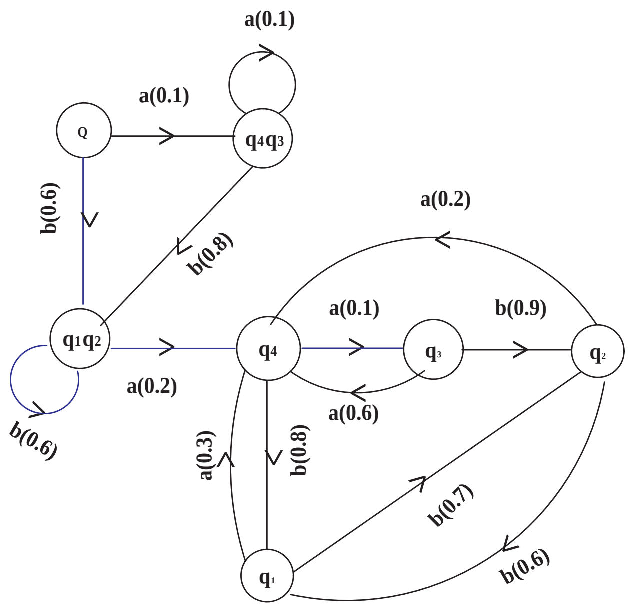

Fig - 4.8

Shortest optimum sequence to cover all cities with minimal flow capacity
1) $bbaa$ such that

$$\text{(i)}F(bbaa) = \begin{pmatrix} 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0 \end{pmatrix}$$

Minimum cost calculation

1) For the sequence $bbaa$

(i) $f_{M_1}(q_1, bbaa, q_3) = 0.1 > 0$ the cost is $03 + 04 + 06 + 02 = 15$
(ii)$f_{M_1}(q_2, bbaa, q_3) = 0.1 > 0$ the cost is $04 + 03 + 06 + 02 = 15$
(iii)$f_{M_1}(q_3, bbaa, q_3) = 0.1 > 0$ the cost is $09 + 04 + 06 + 02 = 21$
(iv)$f_{M_1}(q_4, bbaa, q_3) = 0.1 > 0$ the cost is $10 + 03 + 06 + 02 = 21$

The optimum sequence is

$f_{M_1}(q_1, bbaa, q_3) = 0.1 > 0$ and $f_{M_1}(q_2, bbaa, q_3) = 0.1 > 0$ such that the minimal cost is Rs. 15.

5. Conclusion

In this paper, we introduce γ-synchronization degree of a fuzzy automaton and procedure is given to find γ-synchronized words. We introduce an application related to petrol passing through two different pipelines in 4 cities. The objective is to find shortest optimum sequence of pipelines to cover all cities (we can start anywhere in 4 cities) such that the petrol reaches each city at least once and ends in a particular city with minimal flow capacity and minimum maintenance cost.

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