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A characterization of fuzzy self centered graphs

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Abstract. The distance and related concepts like eccentricity, radius, diameter, center, periphery, etc. are already defined and used in many applications of graph theory. In this paper, we define a new distance called fuzzy distance in fuzzy graphs. Using this distance, fuzzy eccentricity, fuzzy center, etc. are defined and the relation between fuzzy radius and fuzzy diameter is established. Also the concept of self centered graphs is generalized to fuzzy self centered graphs and a necessary condition for a fuzzy graph to be a fuzzy self centered graph is obtained. The max-max composition of the fuzzy distance matrix with itself is introduced, and present an easy check to see whether a given fuzzy graph is fuzzy self centered or not. The central properties of a tree are also discussed in the fuzzy sense.

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1. Introduction

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving a combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. Most important thing which is to be noted is that, when we have uncertainty regarding either the set of vertices or edges or both, the model becomes a fuzzy graph. Distance and central concepts play important roles in applications related with graphs and fuzzy graphs. In this paper, the authors introduced the concept of a new distance called fuzzy distance in fuzzy graphs. In the already existing distances, we simply sum the weights of edges in a geodesic between the given pair of nodes. The fuzzy distance between a given pair of nodes actually represents the amount of flow between these nodes. So this distance concept is more relevant in flow problems, where we model the network as a fuzzy network. In order to minimize the amount of money, time, energy ,..., the location of the center of a fuzzy graph is always needy and useful in real life problems. The center of a fuzzy graph, which is presented in this paper is with respect to this fuzzy distance in fuzzy graphs. The discussions about the important structures namely fuzzy trees and complement of a fuzzy graph are also relevant. By a geodesic between a given pair of nodes, we simply mean a shortest path between these nodes. This means a path between these nodes whose weight sum (sum of the weights of all arcs present in the path) is less than all the other paths between these nodes. But in this article, the path which is considered for calculating the fuzzy distance between a given pair of nodes may not be the same as the geodesic path between the nodes. Several authors including Rosenfeld, Bhutani[3, 4, 5, 6, 9] and Sunil Mathew and Sunitha[5, 11, 13, 14, 15, 16, 17, 18, 19] introduced many connectivity concepts in fuzzy graphs following the works of Zadeh[24, 25, 26] and Rosenfeld [9]. More related work can be seen in[1, 2, 7, 8, 10, 20, 21, 22, 23].

A fuzzy graph (f-graph for short)[8] is a pair $G : (\sigma, \mu)$ where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. We assume that S is finite and nonempty, μ is reflexive and symmetric. In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \;\epsilon S : \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \;\epsilon SXS : \mu(u, v) > 0\}$. A fuzzy graph $H : (\tau, \nu)$ is called a partial fuzzy subgraph of $G : (\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for every u and $\nu(u, v) \leq \mu(u, v)$ for every pair of nodes u and σ^*. In particular we call $H : (\tau, \nu)$ a fuzzy subgraph of $G^* : (\sigma^*, \mu^*)$ if $\tau(u) = \sigma(u)$ for every $u \;\epsilon\tau^*$ and $\nu(u, v) = \mu(u, v)$ for every $(u, v) \;\epsilon\nu^*$. A path P of length n is a sequence of distinct nodes $u_0, u_1, ...u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, ..., n$ and the degree of membership of a weakest edge is defined as its strength. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$. An $x - y$ path P is called a strongest $x - y$ path if its strength equals $CONN_G(x, y)$[9, 10]. An f-graph $G : (\sigma, \mu)$ is connected if for every x, y in $\sigma^*, CONN_G(x, y) > 0$. Through out, we assume that G is connected.

The following are some existing classical results in graphs. Let $G : (V, E)$ be a crisp graph. Then the distance between two nodes u and v of G is defined as the length of a $u - v$ geodesic [7]. The length of a path is the number of edges present in the path[7]. A shortest $u-v$ path is called a $u-v$ geodesic[7]. The eccentricity of a node u is defined and denoted by $e(u) = max\{d(u, v)/v \in V\}$[7]. Among the eccentricities of all the nodes in a graph, the minimum is called the radius of the graph and the maximum is called the diameter of the graph[7]. A node with minimum eccentricity is called a central or radial node and a node with maximum eccentricity is called a diametral or peripheral node[7].

2. Fuzzy distance

In this section, we define a new type of distance in fuzzy graphs.

Definition 2.1. Let $G : (\sigma, \mu)$ be a fuzzy graph, then the fuzzy distance between two nodes u and v in G is defined and denoted by $d_f(u, v) = \wedge_P\{l(P) * S(P)/P$ is a $u - v$ path, $l(P)$ is the length and $S(P)$ is the strength of the $P\}$. $\wedge$ represents the minimum and $*$ represents ordinary product.

Remark 2.2. The above distance satisfies all four properties of a metric.
1. $d_f(u, v) \geq 0$ for all $u, v \in \sigma^*$
2. $d_f(u, v) = 0$ for all $u = v$
3. $d_f(u, v) = d_f(v, u)$ for all $u, v \in \sigma^*$ [symmetry].
4. $d_f(u, v) \leq d_f(u, w) + d_f(w, v)$ for all $u, v, w \in \sigma^*$ [triangle inequality].

The distance d_f is a metric and $(\sigma^*, d_f)$is a metric space.

Example 2.3. Consider the following graph on four vertices (Figure 1).

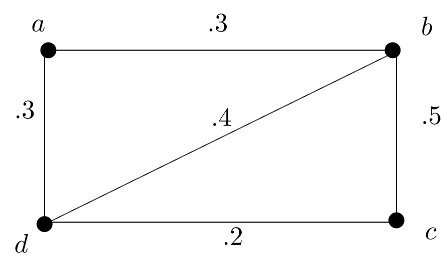

Figure 1: Distance in a fuzzy graph

In this graph, $d_f(a, b) = \wedge\{1 * .3, 2 * .3, 3 * .2\} = .3, d_f(a, c) = \wedge\{2 * .3, 3 * .3, 3 * .2, 2 * .2\} = .4, d_f(a, d) = \wedge\{1 * .3, 2 * .3, 3 * .2\} = .3, d_f(b, c) = \wedge\{1 * .5, 2 * .2, 3 * .2\} = .4, d_f(b, d) = \wedge\{1 * .4, 2 * .2, 2 * .3\} = .4$ and $d_f(c, d) = \wedge\{1 * .2, 2 * .4, 2 * .3\} = .2$.

Definition 2.4. Let $G : (\sigma, \mu)$ be a fuzzy graph. Then the fuzzy eccentricity of a node $u \in V(G)$ is defined and denoted by $e_w(u) = \vee_{v \in V}\{d_f(u, v)\}$, where $\vee$ represents the maximum.

Definition 2.5. The minimum of the fuzzy eccentricities of all nodes is called the fuzzy radius of the graph G. It is denoted as $r_f(G)$. Thus $r_f(G) = \wedge_{u \in V}\{e_f(u)\}$.

Definition 2.6. The maximum of the fuzzy eccentricities of all the nodes is called the fuzzy diameter of the graph G. It is denoted as $d_f(G)$.
That is, $d_f(G) = \vee_{u \in V}\{e_f(u)\}$. In example 2.3(Figure 1), $e_f(a) = e_f(b) = e_f(c) = e_f(d) = .4$.

Remark 2.7. For any fuzzy graph G, Since the weights are less than 1, $d(u, v) > d_f(u, v)$, where $d(u, v)$ is the distance between u and v in the underlying crisp graph of G.

Definition 2.8. A node $v \in V(G)$ is called a fuzzy eccentric node of another node u if $e_f(u) = d_f(u, v)$.

The set of all fuzzy eccentric nodes of a node u is denoted by u_f^*

Definition 2.9. Nodes with minimum fuzzy eccentricity are called fuzzy central nodes or fuzzy radial nodes, and nodes with maximum fuzzy eccentricity are called fuzzy diametral nodes or fuzzy peripheral nodes.

In Example 2.3, $r_f(G) = .4, d_f(G) = .4$. Here all nodes are both fuzzy central as well as fuzzy diametral. Now consider the following example (Figure 2).

Example 2.10.

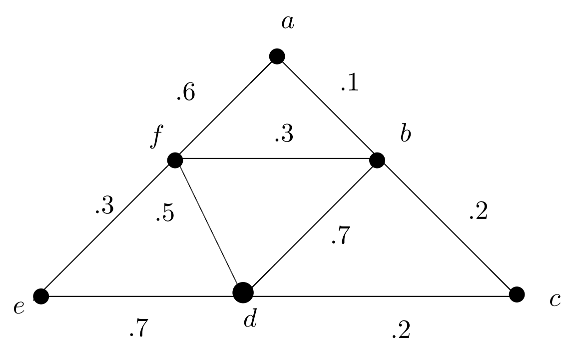

Figure 2: Eccentricity and Center

In this graph, $d_f(a, b) = .1, d_f(a, c) = .2, d_f(a, d) = .2, d_f(a, e) = .3, d_f(a, f) = .2, d_f(b, c) = .2, d_f(b, d) = .3, d_f(b, e) = .3, d_f(b, f) = .2, d_f(c, d) = .2, d_f(c, e) = .4, d_f(c, f) = .3$ and $d_f(d, e) = .4, d_f(d, f) = .3, d_f(e, f) = .3$.

Here, $e_f(a) = e_f(b) = e_f(f) = .3, e_f(c) = e_f(d) = e_f(e) = .4$ and $r_f(G) = .3, d_f(G) = .4$. Note that a, b and f are not fuzzy eccentric nodes of any other nodes.

Similar to the classical distance in graphs, we have the following result.

Theorem 2.11. *In any connected fuzzy graph G, $r_f(G) \leq d_f(G) \leq 2r_f(G)$.*

Proof. The first inequality follows from the definition itself. To prove the other, let u and v be any two nodes such that $d_f(u, v) = d_f(G)$. Let w be any fuzzy central node of G. Then by triangle inequality, $d_f(u, v) \leq d_f(u, w) + d_f(w, v)$. But $d_f(u, w) \leq r_f(G)$ and $d_f(G) \leq r_f(G)$.
Thus $d_f(u, v) \leq r_f(G) + r_f(G) = 2r_f(G)$. □

Definition 2.12. The subgraph induced by the set of all fuzzy central nodes is called the center of the fuzzy graph G and the subgraph induced by the set of all fuzzy diametral nodes is called the periphery of the fuzzy graph G.

Remark 2.13. The fuzzy center of a fuzzy graph need not be the same as the center of its underlying graph.

3. Fuzzy self centered graphs

In this section, we shall discuss the properties of self centered graphs with respect to the new distance. First, we have the following definition.

Definition 3.1. A fuzzy graph G is called fuzzy self centered, if it is isomorphic with its fuzzy center.

Example 3.2. A fuzzy graph and its underlying graph are shown below (Figure 3).

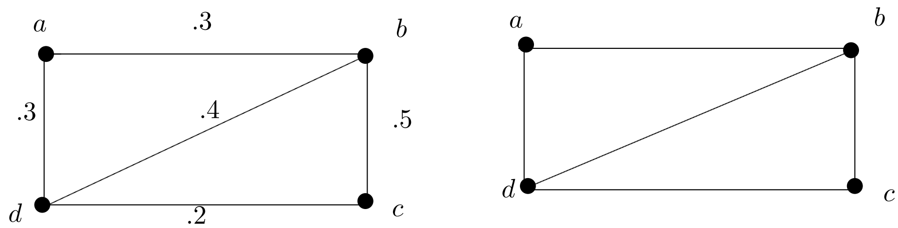

Figure 3: A fuzzy graph and its underlying graph

In this fuzzy graph, $d_f(a,b) = 0.3, d_f(a,c) = 0.4, d_f(a,d) = 0.3, d_f(b,c) = 0.4, d_f(b,d) = 0.4, d_f(c,d) = 0.2, d(a,b) = 1, d(a,c) = 2, d(a,d) = 1, d(b,c) = 1, d(b,d) = 1, d(c,d) = 1$.

Also note that $e_f(a) = e_f(b) = e_f(c) = e_f(d) = 0.4$ and $e(a) = 2, e(b) = 1, e(c) = 2, e(d) = 1$.

This fuzzy graph is fuzzy self centered but the underlying graph is not self centered. In the following theorem we present a necessary condition for a fuzzy graph G to be fuzzy self centered.

Theorem 3.3. *If a connected fuzzy graph G is fuzzy self centered, then each node of G is fuzzy eccentric.*

Proof. Assume that the fuzzy graph G is fuzzy self centered. We have to prove that each node of G is fuzzy eccentric. Let u be any arbitrary node of G and let $v \in u_f^*$. By the definition of a fuzzy eccentric node, $e_f(u) = d_f(u,v)$. But since G is fuzzy self centered, $e_f(u) = e_f(v)$ and hence $e_f(v) = d_f(u,v) = d_f(v,u)$. Thus u is a fuzzy eccentric node of v. Hence every node of G is fuzzy eccentric. □

The next result is also a necessary condition for fuzzy self centered graphs.

Theorem 3.4. *If a connected fuzzy graph G is fuzzy self centered then for every pair of nodes u, v such that whenever u is a fuzzy eccentric node of v, then v should be one of the fuzzy eccentric nodes of u.*

Proof. Assume that G is fuzzy self centered. Also assume that u is a fuzzy eccentric node of v. This means, $e_f(v) = d_f(v, u)$. Since G is fuzzy self centered, all nodes will be having the same fuzzy eccentricity. Therefore $e_f(v) = e_f(u)$.
From the above two equations, $e_f(u) = d_f(v, u) = d_f(u, v)$. Thus $e_f(u) = d_f(u, v)$. That is, v is a fuzzy eccentric node of u. □

This result is not sufficient, as we are not able to prove the fuzzy eccentricity of a third node is equal to the common fuzzy eccentricity of the selected pair of nodes.

In the following theorem [Theorem 3.5] , it is proved that, from any fuzzy graph we can construct a new fuzzy graph such that the former is the fuzzy center of the latter.

Theorem 3.5. *Every fuzzy graph G is the fuzzy center of some fuzzy connected graph H.*

Proof. Construct H from G by inserting 4 more nodes, say, u, v, w, and x. Construct H as the sequential join $u+v+G+w+x$. Assign the least weight among the weights of edges of G to all the new edges of H. Let that minimum weight be k.

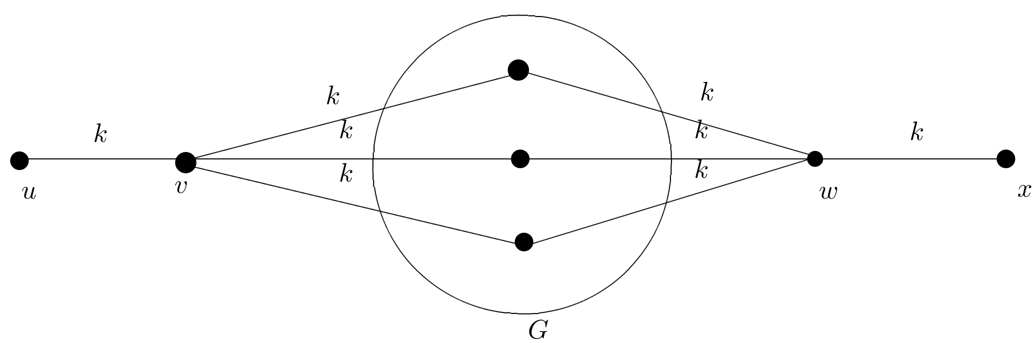

Figure 4: Construction.

Thus, $e_f(u) = e_f(x) = 4k, e_f(v) = e_f(w) = 3k, e_f(y) = 2k$ for every $y \in V(G)$.

Hence each node in G is a fuzzy central node of H and hence G is the fuzzy center of H. □

4. The fuzzy distance matrix

In this section, we present an easy check for a fuzzy graph G to find whether it is fuzzy self centered or not.

Definition 4.1. Let $G : (\sigma, \mu)$ be a connected fuzzy graph with n nodes. The fuzzy distance matrix $D_f = (d_{i,j})$ is a square matrix of order n and is defined by $(d_{i,j}) = d_f(v_i, v_j)$. Note that the fuzzy distance matrix is a symmetric matrix.

Definition 4.2. The max-max composition of a square matrix with itself is again a square matrix of the same order whose $(i,j)^{th}$ entry is given by $d_{i,j} = \max\{\max(d_{i1}, d_{1,j}), \max(d_{i2}, d_{2j}), ..., \max(d_{in}, d_{nj})\}$

In the following example (Figure 5), we give the fuzzy distance matrix and its max-max composition.

Example 4.3.

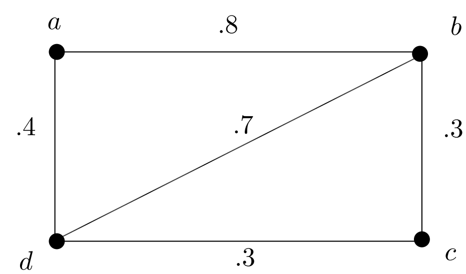

Figure 5: fuzzy distance matrix

The fuzzy distance matrix and its max-max composition are given below.

$$D_f = \begin{pmatrix} 0 & .8 & .6 & .4 \\ .8 & 0 & .3 & .6 \\ .6 & .3 & 0 & .3 \\ .4 & .6 & .3 & 0 \end{pmatrix}, \quad D_f o D_f = \begin{pmatrix} .8 & .8 & .8 & .8 \\ .8 & .8 & .8 & .8 \\ .8 & .8 & .6 & .6 \\ .8 & .8 & .6 & .6 \end{pmatrix}.$$

Next we have a theorem regarding the fuzzy eccentricities of nodes using the max-max composition of the fuzzy distance matrices.

Theorem 4.4. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph. The diagonal elements of the max-max composition of the fuzzy distance matrix of G with itself are the fuzzy eccentricities of the nodes.*

Proof. Let $d_f = (d_{i,j})$ be the fuzzy distance matrix of G.
Then $(d_{i,j}) = d_f(v_i, v_j)$. In the max-max composition, $D_f o D_f$, the i^{th} diagonal entry, $d_{i,i} = \max\{\max(d_{i,1}, d_{1,i}), \max(d_{i,2}, d_{2,i}), \max(d_{i,3}, d_{3,i}), ..., \max(d_{i,n}, d_{n,i})\} = \max\{d_{i,1}, d_{i,2}, d_{i,3}, ..., d_{i,n}\} = \max\{d_f(v_i, v_1), d_f(v_i, v_2), d_f(v_i, v_3), ..., d_f(v_i, v_n)\} = e_f(v_i)$. This completes the proof of the theorem. □

Theorem 4.5. A connected fuzzy graph $G : (\sigma, \mu)$ is fuzzy self centered if and only if all the entries in the principal diagonal of the max-max composition of the fuzzy distance matrix with itself are the same.

Proof. As proved in theorem 4.1, the principal diagonal entries in the max - max composition of the fuzzy distance matrix with itself are the fuzzy eccentricities of the nodes. If they are same, this means $e_f(u)$ is the same for all u in G. Then G is fuzzy self centered. Hence the proof is completed. □

We illustrate the above theorem in the following examples (Example 4.6 and Example 4.7.

Example 4.6.

Consider the following fuzzy graph (Figure 6).

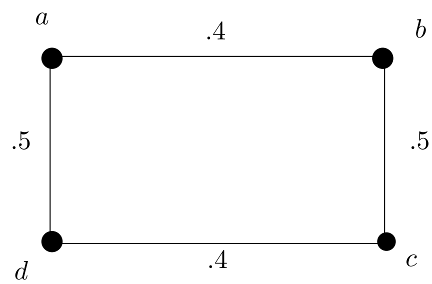

Figure 6: A fuzzy self centered graph

The fuzzy distance matrix and the max-max composition are given below.

$$D_f = \begin{pmatrix} 0 & .4 & .8 & .5 \\ .4 & 0 & .5 & .8 \\ .8 & .5 & 0 & .4 \\ .5 & .8 & .4 & 0 \end{pmatrix}, \quad D_f o D_f = \begin{pmatrix} .8 & .8 & .8 & .8 \\ .8 & .8 & .8 & .8 \\ .8 & .8 & .8 & .8 \\ .8 & .8 & .8 & .8 \end{pmatrix}$$

The graph is fuzzy self centered.

Example 4.7.
Consider the fuzzy graph shown in the following weighted graph (Figure 7).

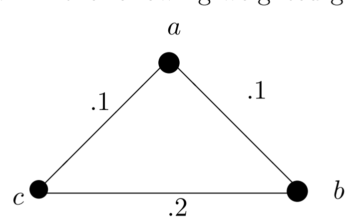

Figure 7: Graph not fuzzy self centered

The fuzzy distance matrix and the max-max composition are given below.

$$D_f = \begin{pmatrix} 0 & .1 & .1 \\ .1 & 0 & .2 \\ .1 & .2 & 0 \end{pmatrix}, \quad D_f o D_f = \begin{pmatrix} .1 & .2 & .2 \\ .2 & .2 & .2 \\ .2 & .2 & .2 \end{pmatrix}$$

Clearly all diagonal elements in the composition are not same, and hence the fuzzy graph is not fuzzy self centered.

5. Fuzzy center of trees

In this section, we make a comparison between the fuzzy center of a fuzzy tree and the center of its underlying tree.

Remark 5.1. In ordinary trees (trees in which each edge is assigned with unit weight), the center is either K_1 or K_2. But in fuzzy trees, the fuzzy center need not be K_1 or K_2.

Example 5.2. Consider the following tree.

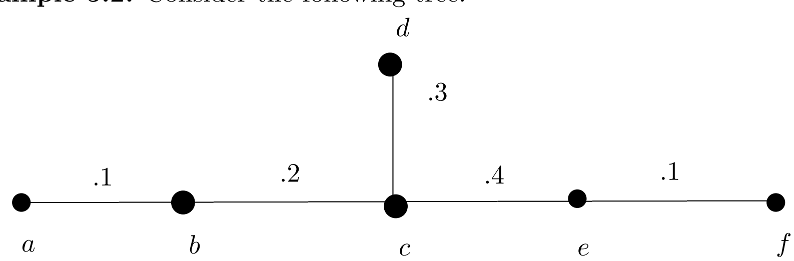

Figure 8: A fuzzy tree

In this fuzzy tree (Figure 8), $d_f(a,b) = .1 = d_f(e,f), d_f(a,c) = d_f(b,c) = d_f(c,f) = .2, d_f(a,d) = d_f(a,e) = d_f(b,f) = d_f(c,d) = d_f(d,f) = .3, d_f(a,f) = d_f(b,d) = d_f(b,e) = d_f(c,e) = .4, d_w(c,e) = .6$.

The fuzzy eccentricities are, $e_f(a) = e_f(b) = e_f(c) = e_f(f) = 4, e_f(d) = e_f(e) = .6$. The fuzzy center is shown below. It is neither K_1 nor K_2 (Figure 9).

Example 5.3.

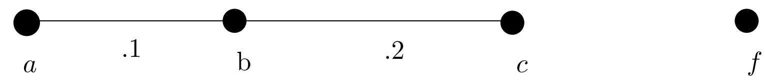

Figure 9: Center of a fuzzy tree

Remark 5.4. The fuzzy center of a fuzzy tree can be disconnected.

Remark 5.5. In an ordinary tree with number of vertices greater than or equal to 3, an end node cannot be a central node. But in a fuzzy tree, an end node can be a central node.

Remark 5.6. Let the minimum fuzzy edge in a fuzzy tree be attached to a pendent node v and the minimum weight be k. Then $e_f(v) = nk$, where n is the length of the longest path from v in the fuzzy tree.

6. Fuzzy center of the complement of a fuzzy graph

In this section, we discuss complement of a fuzzy graph and its central properties.

Definition 6.1. Let $G : (\sigma, \mu)$ be a fuzzy graph. Then the complement of G is denoted as $G^c : (\sigma^c, \mu^c)$, where $\sigma^c = \sigma$ and $\mu^c(x, y) = \wedge\{\sigma(x), \sigma(y)\} - \mu(x, y)$

In the following example (Figure 10), all the nodes are of weight unity.
Example 6.2.

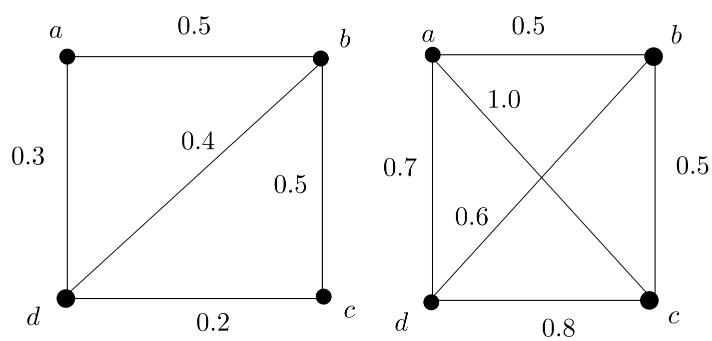

Figure 10: A fuzzy graph and its complement

For the fuzzy graph and complement in Example 6.2 (Figure 10), the fuzzy distance matrix and its composition are given below

$$D_f(G) = \begin{pmatrix} 0 & .3 & .4 & .3 \\ .3 & 0 & .4 & .4 \\ .4 & .4 & 0 & .2 \\ .3 & .4 & .2 & 0 \end{pmatrix} \quad D_f o D_f = \begin{pmatrix} .4 & .4 & .4 & .4 \\ .4 & .4 & .4 & .4 \\ .4 & .4 & .4 & .4 \\ .4 & .4 & .4 & .4 \end{pmatrix}.$$

$$D_f(G^c) = \begin{pmatrix} 0 & .5 & 1 & .7 \\ .5 & 0 & .5 & .6 \\ 1 & .5 & 0 & .8 \\ .7 & .6 & .8 & 0 \end{pmatrix} \quad D_f o D_f = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & .7 & 1 & .8 \\ 1 & 1 & 1 & 1 \\ 1 & .8 & 1 & .8 \end{pmatrix}.$$

Remark 6.3. From the above figure it is clear that sum of the weights of all edges in G and $G_c = nC_2k$ where n is the number of nodes in G and k is the highest weight of edges in G.

Remark 6.4. The complement of a fuzzy self centered graph may not be fuzzy self centered. In example 6.1, G is fuzzy self centered with 4 as the fuzzy eccentricity of all the four nodes. But in G_c, $e_f(a) = e_f(c) = e_f(d) = .3$ and $e_f(b) = .2$, which proves G_c is not fuzzy self centered.

7. Conclusion

Fuzzy graphs are precise models of all kinds of networks. Distance is an important concept in the entire graph theory. In this paper, the authors made an attempt to generalize the concept of distance. The concept of fuzzy distance is relevant as it represents the net flow between a given pair of nodes of a fuzzy graph. The concept of fuzzy center, fuzzy self centered graphs, complement of a fuzzy graph are also introduced. Facility location in a fuzzy network model can be made easy by using the concept of fuzzy center and fuzzy distance matrix. It is seen that the complement of a fuzzy self centered graph need not be fuzzy self centered. The fuzzy center of trees are also discussed. A characterization of fuzzy self centered graphs is obtained.

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