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Note on rough multiset and its multiset topology

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Abstract. We show that an alleged properties stated in [4] are invalid in general, by giving a counter-examples.

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1. Introduction

Proposition 48 (vii) and Proposition 58 (vii) of [4] asserted that for any multiset, (mset [1] or bag [6], for short), relation R on a nonempty mset M the following conditions hold respectively:

(1) $R_L(M_1 \ominus M_2) = R_L(M_1) \ominus R_L(M_2)$,
(2) $R_U(M_1 \ominus M_2) \subseteq R_U(M_1) \ominus R_U(M_2)$.

These properties are incorrect in general. Moreover, Counter-examples are introduced to prove our claim.

2. Preliminaries

In this section, a brief survey of the notion of msets as introduced by Yager [6], Blizard [1, 2] and Jena et al. [5] have been mentioned. Moreover, rough msets and the basic definitions and notions of relations in mset context presented by Girish et al. [3, 4].

Definition 2.1. A collection of elements containing duplicates is called an mset. Formally, if X is a set of elements, an mset M drawn from the set X is represented by a function count M or C_M defined as $C_M : X \to \mathbb{N}$ where $\mathbb{N}$ represents the set of non negative integers.

Let M be an mset from the set $X = \{x_1, x_2, ..., x_n\}$ with x appearing n times in M. It is denoted by $x \in^n M$. The mset M drawn from the set X is denoted by $M = \{k_1/x_1, k_2/x_2, ..., k_n/x_n\}$ where M is an mset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on. In Definition 2.1, $C_M(x)$ is the number of occurrences of the element x in the mset M. However those elements which are not included in the mset M have zero count. An mset M is a set if $C_M(x) = 0$ or $1 \; \forall \, x \in X$.

Definition 2.2. A domain X, is defined as a set of elements from which msets are constructed. The mset space $[X]^m$ is the set of all msets whose elements are in X such that no element in the mset occurs more than m times. The set $[X]^\infty$ is the set of all msets over a domain X such that there is no limit on the number of occurrences of an element in an mset.

Let $M, N \in [X]^m$. Then the following are defined:

(1) M is a submset of N denoted by $(M \subseteq N)$ if $C_M(x) \leq C_N(x) \; \forall \, x \in X$.
(2) $M = N$ if $M \subseteq N$ and $N \subseteq M$.
(3) M is a proper submset of N denoted by $(M \subset N)$ if $C_M(x) \leq C_N(x) \; \forall \, x \in X$ and there exists at least one element $x \in X$ such that $C_M(x) < C_N(x)$.
(4) $P = M \cup N$ if $C_P(x) = \max\{C_M(x), C_N(x)\}$ for all $x \in X$.
(5) $P = M \cap N$ if $C_P(x) = \min\{C_M(x), C_N(x)\}$ for all $x \in X$.
(6) Addition of M and N results in a new mset $P = M \oplus N$ such that $C_P(x) = \min\{C_M(x) + C_N(x), m\}$ for all $x \in X$.
(7) Subtraction of M and N results in a new mset $P = M \ominus N$ such that $C_P(x) = \max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$, where $\oplus$ and $\ominus$ represent mset addition and mset subtraction, respectively.
(8) An mset M is empty if $C_M(x) = 0 \; \forall \, x \in X$.

Definition 2.3. Let $M \in [X]^m$. Then the complement M^c of M in $[X]^m$ is an element of $[X]^m$ such that

$$C_{M^c}(x) = m - C_M(x) \; \forall x \in X.$$

Definition 2.4. Let M_1 and M_2 be two msets drawn from a set X, then the Cartesian product of M_1 and M_2 is defined as

$$M_1 \times M_2 = \{(m/x, n/y)/mn : x \in^m M_1, \; y \in^n M_2\}.$$

Here the entry $(m/x, n/y)/mn$ in $M_1 \times M_2$ denotes x is repeated m times in M_1, y is repeated n times in M_2 and the pair (x, y) is repeated mn times in $M_1 \times M_2$.

The Cartesian product of three or more nonempty msets can be defined by generalizing the definition of the Cartesian product of two msets.

Definition 2.5. A submset R of $M_1 \times M_2$ is said to be an mset relation from M_1 to M_2 if every member $(m/x, n/y)$ of R has a count, the product of $C_1(x, y)$ and $C_2(x, y)$. m/x related to n/y is denoted by $(m/x)R(n/y)$.

Definition 2.6. Let R be an mset relation on M. The post-mset of $x \in^m M$ is defined as $(m/x)R = \{n/y : \exists$ some k with $(k/x)R(n/y)\}$ and the pre-mset of $x \in^r M$ is defined as $R(r/x) = \{p/y : \exists$ some q with $(p/y)R(q/x)\}$.

Definition 2.7. Let R be any binary mset relation on M in $[X]^m$. Then the mset $\langle n/y \rangle R$ is defined as the intersection of all post-msets containing y with nonzero multiplicity.

$$\text{(2.1)} \qquad i.e.,\ \langle n/y\rangle R = \cap\{(m/x)R : y \in^n (m/x)R\}.$$

Also, $R\langle n/y\rangle$is the intersection of all pre-msets containing y with nonzero multiplicity.

$$\text{(2.2)} \qquad i.e.,\ R\langle n/y\rangle = \cap\{R(m/x) : y \in^n R(m/x)\}.$$

Definition 2.8. Let R be a binary mset relation on M. For $N \subseteq M$, a pair of lower and upper mset approximations, $\underline{R}_L(N)$ and $\overline{R}_U(N)$, are defined respectively as

$$\text{(2.3)} \qquad \underline{R}_L(N) = \{m/x :\ \langle m/x\rangle_R \subseteq N\}.$$

$$\text{(2.4)} \qquad \overline{R}_U(N) = \{m/x :\ \langle m/x\rangle_R \cap N \neq \phi\}.$$

The pair $(\underline{R}_L(N), \overline{R}_U(N))$ is referred to as the rough multiset of N.

3. Counter-examples

The following example shows that $R_L(M_1) \ominus R_L(M_2) \nsubseteq R_L(M_1 \ominus M_2)$ and $R_U(M_1 \ominus M_2) \nsubseteq R_U(M_1) \ominus R_U(M_2)$.

Example 3.1. Let $M = \{3/x, 2/y, 4/z, 5/r\}$ and $R = \{(3/x, 3/x)/9, (2/y, 2/y)/4, (4/z, 4/z)/16, (5/r, 5/r)/25, (3/x, 2/y)/6, (3/x, 4/z)/12, (4/z, 3/x)/12, (5/r, 2/y)/10, (4/z, 2/y)/8\}$. Hence

$$\begin{array}{ll} (3/x)R = \{3/x, 2/y, 4/z\} & \langle 3/x\rangle_R = \{3/x, 2/y, 4/z\} \\ (2/y)R = \{2/y\} & \langle 2/y\rangle_R = \{2/y\} \\ (4/z)R = \{3/x, 2/y, 4/z\} & \langle 4/z\rangle_R = \{3/x, 2/y, 4/z\} \\ (5/r)R = \{5/r, 2/y\} & \langle 5/r\rangle_R = \{5/r, 2/y\}. \end{array}$$

Let $M_1 = \{3/x, 2/y, 4/z\}$ and $M_2 = \{3/x, 2/y, 5/r\}$ be two submsets of M. Then $R_L(M_1) = \{3/x, 2/y, 4/z\}$ and $R_L(M_2) = \{2/y, 5/r\}$. Thus, $R_L(M_1) \ominus R_L(M_2) = \{3/x, 4/z\}$. Moreover, $M_1 \ominus M_2 = \{4/z\}$. So, $R_L(M_1 \ominus M_2) = \phi$. Thus, $R_L(M_1) \ominus R_L(M_2) \nsubseteq R_L(M_1 \ominus M_2)$. Also, if $M_1 = \{3/x, 2/y\}$ and $M_2 = \{2/y, 4/z\}$ be two submsets of M. Then $R_U(M_1) = M$ and $R_U(M_2) = M$. Thus, $R_U(M_1) \ominus R_U(M_2) = \phi$. Moreover, $M_1 \ominus M_2 = \{3/x\}$ and $R_U(M_1 \ominus M_2) = \{3/x, 4/z\}$. So, $R_U(M_1 \ominus M_2) \nsubseteq R_U(M_1) \ominus R_U(M_2)$.

The following example shows that $R_L(M_1 \ominus M_2) \nsubseteq R_L(M_1) \ominus R_L(M_2)$.

Example 3.2. Let $M = \{2/x, 3/y, 5/z, 7/r\}$ and $R = \{(2/x, 2/x)/4, (2/x, 3/y)/6, (3/y, 3/y)/9, (5/z, 5/z)/25, (5/z, 3/y)/15, (7/r, 2/x)/14, (7/r, 5/z)/35\}$. Hence

$$\begin{array}{ll} (2/x)R = \{2/x, 3/y\} & \langle 2/x\rangle_R = \{2/x\} \\ (3/y)R = \{3/y\} & \langle 3/y\rangle_R = \{3/y\} \\ (5/z)R = \{3/y, 5/z\} & \langle 5/z\rangle_R = \{5/z\} \\ (7/r)R = \{2/x, 5/z\} & \langle 7/r\rangle_R = \phi. \end{array}$$

Let $M_1 = \{2/x, 3/y, 7/r\}$ and $M_2 = \{2/x, 7/r\}$ be two submsets of M. Then $R_L(M_1) = \{2/x, 3/y, 7/r\}$ and $R_L(M_2) = \{2/x, 7/r\}$. Thus, $R_L(M_1) \ominus R_L(M_2) = \{3/y\}$. Moreover, $M_1 \ominus M_2 = \{3/y\}$. So, $R_L(M_1 \ominus M_2) = \{3/y, 7/r\}$. Thus, $R_L(M_1 \ominus M_2) \nsubseteq R_L(M_1) \ominus R_L(M_2)$.

The following proposition presents the relationship between $R_U(M_1) \ominus R_U(M_2)$ and $R_U(M_1 \ominus M_2)$.

Proposition 3.3. *Let R be an mset relation on a nonempty mset M. Then*
$$R_U(M_1) \ominus R_U(M_2) \subseteq R_U(M_1 \ominus M_2).$$

Proof.
$$\begin{aligned} R_U(M_1 \ominus M_2) &= \{m/x : \langle m/x\rangle_R \cap (M_1 \ominus M_2) \neq \phi\} \\ &\supseteq \{m/x : \langle m/x\rangle_R \cap M_1 \neq \phi\} \ominus \{m/x : \langle m/x\rangle_R \cap M_2 \neq \phi\} \\ &= R_U(M_1) \ominus R_U(M_2). \end{aligned}$$
□

4. Conclusions

In this paper, we introduce counter-examples to show that some properties stated in [4] are wrong in general. Furthermore, we establish the relationships between these properties.

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