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On intuitionistic fuzzy soft $b-$closed sets in intuitionistic fuzzy soft topological spaces

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Abstract. The aim of this paper is to introduce and study some new concepts like $(IFSbCT)$, $(IFSbC)-$continuous, $(IFSbC)-$mapping, $(IFSbC)_f - SC$, $(IFSbC) - C$, $(IFSbC)-$ Ć. Furthermore, we generated $\xi_{H(A)}$ (induced fuzzy soft set by (H, A)) and $\eta_{H(A)}$ (induced intuitionistic fuzzy soft set by H_A), where (H, A) is a soft set over the universe of the given set with a fixed set of parameters and H_A is a fuzzy soft set . In another side $\eta \circ \xi_{H(A)}$ is an induced intuitionistic fuzzy soft set by (H, A). That means there are three main types of new ways are introduced in this work which are help us to find new classes association for sot sets. Moreover, some of its basic properties are given.

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1. Introduction

A fuzzy set is a class of objects with a continuum of grades of membership this concept is proposed by Zadeh [26] in 1965. After the introduction of fuzzy topology by Chang [9] in 1968, there have been several generalizations of notions of fuzzy set and fuzzy topology. By adding the degree of non-membership to fuzzy set, Atanassov [6] proposed intuitionistic fuzzy set in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [10] introduced the concept of intuitionistic fuzzy topological space. In 1999, Molodtsov ([18]) initiated a concept namely, soft set theory to solve complicated problems in engineering, physics, computer science, medical science etc. To improve this concept, many researchers applied this notion on group theory ([3]) , ring theory ([1]) , topological spaces [17] and also on decision making problem [14] . Moreover, in 2013, Li and Cui [13] introduced the fundamental concept of Intuitionistic fuzzy soft topology. In the recent years the concepts of b-closed set was generated by many mathematicians such as Binod, Diganta and others see [8, 5] . Also, the concept of soft b-closed sets is introduced and studied by Metin and Alkan [2] in 2014. In the present work the new concepts like (IFSbCT), (IFSbC)-continuous, $(IFSbCT)$, $(IFSbC)-$continuous, $(IFSbC)-$mapping, $(IFSbC)_f-SC$, $(IFSbC)-C$, $(IFSbC)-$ Ć, $\xi_{H(A)}$, $\eta_{H(A)}$ are introduced. Moreover, three main types of new ways are introduced in this work which are help us to find new classes association for sot sets. Also we gave some basic properties of these concepts.

2. DEFINITIONS AND NOTATIONS

The following definitions have been used to obtain the results and properties developed in this paper.

Definition 2.1 ([4, 23])**.** Let A be a subset of a topological space (X, T), then A is called a $b-$open set if $A \subseteq cl(int(A)) \cup int(cl(A))$. The complement of a $b-$open set is said to be $b-$closed. The intersection of all $b-$closed sets of containing A is called the $b-$closure of A and is denoted by $bint(A)$. The union of all $b-$open sets of X contained in A is called $b-$interior of A and is denoted by $bint(A)$.

Example 2.2. Let (X, T_1) and (X, T_2) be two topological spaces where $X = \{a, b, c, d\}$, $T_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $T_2 = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then $\{a\}$ is an open $b-$closed set in (X, T_1) but is not $b-$closed set in (X, T_2).

Definition 2.3 ([18, 21, 25])**.** A pair (F, A) is called a soft set (over U) where F is a mapping $F : A \longrightarrow P(U)$. In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(e)$, $e \in E$, from this family may be considered as the set of $e-$elements of the soft set (F, A), or as the set of $e-$approximate elements of the soft set. Clearly, a soft set is not a set. For two soft sets (F, A) and (G, B) over the common universe U, we say that $(F, A)$is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \subseteq (G, B)$. (F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A). Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A). A soft set (F, A) over U is called a null soft set, denoted by $\Phi = (\phi, \phi)$, if for each$F(e) = \phi, \forall e \in A$. Similarly, it is called universal soft set, denoted by (U, E), if for each $F(e) = U, \forall e \in A$ .The collection of soft sets (F, A) over a universe U and the parameter set A is a family of soft sets denoted by $SS(U_A)$.

Remark 2.4 ([11])**.** The Cardinality of $SS(U_A)$ is given by $n(SS(U_A)) = 2^{n(U)\times n(A)}$.

Example 2.5. If $U = \{c_1, c_2, c_3, c_4\}$ and $A = \{e_1, e_2\}$, then $n(SS(U_A)) = 2^{4\times 2} = 256$.

Definition 2.6 ([12, 22])**.** Let τ be the collection of soft sets over U. Then τ is called a soft topology on U if τ satisfies the following axioms:
(i) Φ, (U, E) belong to τ.
(ii) The union of any number of soft sets in τ belongs to τ .
(iii) The intersection of any two soft sets in τ belongs to τ .

The triplet (U, E, τ) is called a soft topological space over U. The members of τ are called soft open sets in U and complements of them are called soft closed sets in U.

Definition 2.7 ([15]). Let U be an initial universe set and let E be a set of parameters. Let I^U denotes the collection of all fuzzy subsets of U and $A \subseteq E$. Then the mapping $F_A : A \longrightarrow I^U$ defined by $F_A(e) = \mu^e_{F_A}$ (a fuzzy subset of U), is called a fuzzy soft set over (U, E), where $\mu^e_{F_A} = \bar{0}$ if $e \in E \backslash A$ and $\mu^e_{F_A} \neq \bar{0}$ if $e \in A$. The set of all fuzzy soft sets over (U, E) is denoted by $FS(U, E)$.

Definition 2.8 ([15]). The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be null fuzzy soft set and it is denoted by Φ, if for all $e \in E$, $F(e)$ is the null fuzzy set $\bar{0}$ of U, where $\bar{0}(x) = 0$ for all $x \in U$.

Definition 2.9 ([15]). Let $F_E \in FS(U, E)$ and $F_E(e) = \bar{1}$ for all $e \in E$, where $\bar{1}(x) = 1$ for all $x \in U$. Then F_E is called absolute fuzzy soft set. It is denoted by $\bar{E}$.

Definition 2.10 ([15]). A fuzzy soft set F_A is said to be a fuzzy soft subset of a fuzzy soft set G_B over a common universe U if $F_A(e) \subseteq G_B(e)$ for all $e \in E$, i.e., if $\mu^e_{F_A}(x) \leq \mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in E$.

Definition 2.11 ([15]). Two fuzzy soft sets F_A and G_B over a common universe U are said to be fuzzy soft equal if F_A is a fuzzy soft subset of G_B and G_B is a fuzzy soft subset of F_A.

Definition 2.12 ([15]). The union of two fuzzy soft sets F_A and G_B over the common universe U is the fuzzy soft set H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \bigcup B$. Here we write $H_C = F_A \coprod G_B$.

Definition 2.13 ([15]). Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \bigcap B$. Here we write $H_C = F_A \prod G_B$.

Definition 2.14 ([24]). Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:
(i) Φ, $\bar{E}$ belong to ψ.
(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ.
(iii) The intersection of any two fuzzy soft sets in ψ belongs to ψ.
The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.15 ([6]). An intuitionistic fuzzy (IF, in short) set A over the universe U can be defined as follows $A = \{(x, \mu_A(x), \nu_A(x))\colon x \in U\}$ where $\mu_A(x) : U \longrightarrow [0, 1]$, $\nu_A(x) : U \longrightarrow [0, 1]$ with the property $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\forall x \in U$. The values $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and non-membership of x to A respectively.

Definition 2.16 ([6, 19]). Let $A = \{(x, \mu_A(x), \nu_A(x)) : x \in U\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) : x \in U\}$ be intuitionistic fuzzy sets of U.
(1)$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in U$,

$(2) A \bigcap B = \{(x, min\{\mu_A(x), \mu_B(x)\}, max\{\nu_A(x), \nu_B(x)\}) : x \in U\}$,
$(3) A \bigcup B = \{(x, max\{\mu_A(x), \mu_B(x)\}, min\{\nu_A(x), \nu_B(x)\}) : x \in U\}$.

Definition 2.17 ([6, 19])**.** An intuitionistic fuzzy set A over the universe set U defined as $A = \{(x, 0, 1) : x \in U\}$ is said to be intuitionistic fuzzy null set and is denoted by $\bar{0}$.

Definition 2.18 ([6, 19])**.** An intuitionistic fuzzy set A over the universe set U defined as $A = \{(x, 0, 1) : x \in U\}$ is said to be intuitionistic fuzzy null set and is denoted by $\bar{0}$. An intuitionistic fuzzy set A over the universe U defined as $A = \{(x, 1, 0) : x \in U\}$ is said to be intuitionistic fuzzy absolute set and is denoted by $\bar{1}$.

Definition 2.19 ([16])**.** Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all IF subsets of U. Let $A \subseteq B$. A pair (F, A) is called an IF soft set over U where F is a mapping given by $F : A \longrightarrow IF^U$. In general, for every $e \in A$, $F(e)$ is an IF set of U and it is called IF value set of parameter e. Clearly, $F(e)$ can be written as an IF set such that $F(e) = \{(x, \mu_{F_e}(x), \nu_{F_e}(x) : x \in U\}$. The set of all IF soft sets over U with parameters from E is called an IF soft class and it is denoted by $IFS(U_E)$.

Definition 2.20 ([16])**.** The union of two IF soft sets (F, A) and(G, B) over the common universe U is the IF soft set $(H, C) = (F, A)\bar{\coprod}(G, B)$, where $C = A \cup B$ and for all $e \in C$,

$$H(C) = \begin{cases} \text{F(e)} & \text{if } e \in A - B, \\ \text{G(e)} & \text{if } e \in B - A, \\ \text{F(e)} \bigcup \text{G(e)} & \text{otherwise.} \end{cases}$$

Definition 2.21 ([16])**.** Let (F, A) and (G, B) be two IF soft set, then the intersection of (F, A) and (G, B) is a IF soft set $(F, A)\bar{\prod}(G, B) = (H, C)$. Where $C = A \bigcap B$ and $H(e) = F(e) \bigcap G(e)$ for all $e \in C$.

Definition 2.22 ([16])**.** A IF soft set (F, A) is said to be a IF soft subset of a IF soft set (G, B) over a common universe U if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$.

Definition 2.23 ([16])**.** The complement of an intuitionistic fuzzy soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \longrightarrow IF^U$ is a mapping given by $F^c(e) = [F(e)]^c$ for all $e \in A$. Thus if $F(e) = \{(x, \mu_{F(e)}(x), \nu_{F(e)}(x)) : x \in U\}$, then $\forall e \in A$, $F^c(e) = [F(e)]^c = \{(x, \nu_{F(e)}(x), \mu_{F(e)} (x)) : x \in U\}$.

Definition 2.24 ([16])**.** A IF soft set (F, A) over U is said to be absolute intuitionistic fuzzy soft set and is denoted by $\bar{A}$ if $\forall e \in A$, $F(e)$ is the absolute intuitionistic fuzzy set $\bar{1}$ of U where $\bar{1}(x) = 1; \forall x \in U$. We would use the notation $\bar{U}_A$ to represent the absolute intuitionistic fuzzy soft set with respect to the set of parameters A.

Definition 2.25 ([16])**.** A IF soft set (F, A) over U is said to be null intuitionistic fuzzy soft set and is denoted by $\bar{0}$ if $\forall e \in A$, $F(e)$ is the null intuitionistic fuzzy set $\bar{0}$ of U where $\bar{0}(x) = 0; \forall x \in U$, We would use the notation Φ_A to represent the null intuitionistic fuzzy soft set with respect to the set of parameters A.

Definition 2.26 ([13]). Let $\rho \subseteq IFS(U_E)$, then ρ is said to be a IF soft topology on U, if ρ satisfies the following axioms:
(i) $\phi_E, \bar{U}_E$ belong to ρ ,
(ii) The union of any number of IF soft sets in ρ belongs to ρ ,
(iii) The intersection of any two IF soft sets in ρ belongs to ρ .
ρ is called a IF soft topology on U and the pair (U_E, ρ) is called a IF soft topological space over U. Any member belongs to ρ is said to be IF soft open set $(IFSOS)$ in ρ. A IF soft set over U is said to be a IF soft closed set $(IFSCS)$ in U, if its complement $(F, A)^c$ belongs to ρ . Let us refer to IF soft topological space by $IFSTS$.

Definition 2.27 ([7]). Let (F, A) and (G, B) be two IF soft sets over U. We define the difference of $(F, A)$and (G, B) as the IF soft (H, C) written as
$(F, A) - (G, B) = (H, C)$ where $C = A \bigcap B \neq \phi$ and $\forall e \in C, \forall x \in U$,
$\mu_{H(e)}(x) = min\{\mu_{H(e)}(x), \nu_{G(e)}(x)\}$,
$\nu_{H(e)}(x) = max\{\nu_{H(e)}(x), \mu_{G(e)}(x)\}$.
That means$(F, A)^c = \bar{U}_E - (F, A)$, $\phi_E^c = \bar{U}_E$ and $(\bar{U}_E)^c = \phi_E$.

Definition 2.28 ([20]). A IF soft set (F, A) is said to be a IF soft point, denoted by e_F, if for the element $e \in A$, $F(e) \neq \bar{0}$ and $F(\acute{e}) = \bar{0}, \forall \acute{e} \in A - \{e\}$.

Definition 2.29 ([20]). The complement of a IF soft point e_F is a IF soft point e_F^c, such that $F^c(e) = (F(e))^c$.

3. INTUITIONISTIC FUZZY SOFT $b-$CLOSED SETS

In this section, we introduce some new concepts like $(IFSbCT)$, $(IFSbC)-$continuous, $(IFSbC)-$mapping, $(IFSbC)_f - SC$, $(IFSbC) - C$, $(IFSbC)-\acute{C}$, $\xi_{H(A)}$, $\eta_{H(A)}$ and study some of their properties. Moreover, the current work is supported by a number of examples.

Definition 3.1. In a soft topological space (U, E, τ), a soft set (G, A) is said to be soft $b-$closed set if $cl(int(G, A)) \bigcap int(cl(G, A)) \subseteq (G, A)$.

Definition 3.2. A soft topological space (U, E, τ), is called soft $b-$closed topological space if for each $(G, A) \in \tau - \{\Phi, (U, E)\}$ is soft $b-$closed set.

Example 3.3. Let the set of students under consideration be $U = \{s_1, s_2, s_3\}$. Let $E = \{$pleasing personality (e_1); conduct (e_2); good result (e_3); sincerity $(e_4)\}$ be the set of parameters framed to choose the best student. Suppose that the soft set (F, A) describing the Mr.X opinion to choose the best student of an academic year was defined by
$A = \{e_1, e_2\}$
$F(e_1) = \{s_1\}$, $F(e_1) = \{s_1, s_2, s_3\}$ In addition, we assume that the (best student) in the opinion of another teacher, say Mr.Y, is described by the soft set (G, B), where
$B = \{e_1, e_3, e_4\}$
$G(e_1) = \{s_2, s_3\}$, $G(e_3) = \{s_1, s_2, s_3\}$, $G(e_4) = \{s_1, s_2, s_3\}$ Consider that:
$\tau = \{\Phi, (U, E), (F, A), (G, B)\}$. Then (U, E, τ) is soft $b-$closed topological space.

Definition 3.4. In a fuzzy soft topological space (U, E, Ψ), a fuzzy soft set $F_A \in FS(U, E)$ is said to be fuzzy soft $b-$closed set if $cl(int(F_A)) \prod int(cl(F_A)) \subseteq F_A$, i.e., if $\mu^e_{F_A}(x) \geq Min\{\mu^e_{cl(int(F_A))}(x), \mu^e_{int(cl(F_A))}(x)\}$ for all $e \in E$, $x \in U$.

Definition 3.5. A fuzzy soft topological space (U, E, Ψ), is called fuzzy soft $b-$closed topological space if for each $F_A \in \Psi - \{\Phi, \bar{E}\}$ is fuzzy soft $b-$closed set.

Example 3.6. Let $U = \{c_1, c_2, c_3\}$ be the set of three flats and $E = \{$ costly (e_1), modern (e_2), cheap $(e_3)\}$ be the set of parameters ,where $A = \{e_1, e_2\} \subset E$. consider that $\Psi = \{\bar{E}, \phi, F_A, G_A, H_A, K_A\}$ is a fuzzy soft topology over (U, E) where F_A, G_A, H_A, K_A are fuzzy soft sets over (U, E), defined as follows:
$\mu^{e_1}_{F_A} = \{.2, .4, .9\}$, $\mu^{e_2}_{F_A} = \{.75, .5, .2\}$
$\mu^{e_1}_{G_A} = \{.2, .4, .1\}$, $\mu^{e_2}_{G_A} = \{.25, .5, .2\}$
$\mu^{e_1}_{H_A} = \{.8, .6, .1\}$, $\mu^{e_2}_{H_A} = \{.25, .5, .8\}$
$\mu^{e_1}_{K_A} = \{.8, .6, .9\}$, $\mu^{e_2}_{K_A} = \{.75, .5, .8\}$
Then (U, E, Ψ) is fuzzy soft $b-$closed topological space.

Definition 3.7. ($IFSbCS$, in short) In an intuitionistic fuzzy soft topological space (U_E, ρ), an intuitionistic fuzzy soft set $(F, E) \in IFS(U_E)$ is said to be intuitionistic fuzzy soft $b-$closed set if $cl(int(F, E)) \prod int(cl(F, E)) \subseteq (F, E)$. Moreover, the complement of the intuitionistic fuzzy soft $b-$closed sets is called intuitionistic fuzzy soft $b-$open sets ($IFSbOS$, in short).

Definition 3.8. ($IFSbCT$, in short) An intuitionistic fuzzy soft topological space (U_E, ρ), is called intuitionistic fuzzy soft $b-$closed topological space if for each $(F, A) \in \rho - \{\Phi, U_E\}$ is intuitionistic fuzzy soft $b-$closed set.

Example 3.9. Suppose that U is the set of men whose looking for job under consideration, say $U = \{x_1, x_2, x_3\}$. Let E be the set of some attributes of such men, say $E = \{e_1, e_2, e_3, e_4\}$, where e_1, e_2, e_3, e_4 stand for the attributes (young), (speaking English well), (qualification), (honest), respectively. Let F, G, H, K be the mappings from A to IF^U defined by,
$F(e_1) = (x_1, .2, .6), F(e_1) = (x_2, .7, .5), F(e_1) = (x_3, .1, .8),$
$F(e_2) = (x_1, .3, .5), F(e_2) = (x_2, .1, .3), F(e_2) = (x_3, .9, .4),$
$F(e_3) = (x_1, .2, .5), F(e_3) = (x_2, .6, .3), F(e_3) = (x_3, .5, .22),$
$F(e_4) = (x_1, .7, .1), F(e_4) = (x_2, .2, .4), F(e_4) = (x_3, .5, .8),$
$G(e_1) = (x_1, .6, .2), G(e_1) = (x_2, .5, .7), G(e_1) = (x_3, .8, .1),$
$G(e_2) = (x_1, .5, .3), G(e_2) = (x_2, .3, .1), G(e_2) = (x_3, .4, .9),$
$G(e_3) = (x_1, .5, .2), G(e_3) = (x_2, .3, .6), G(e_3) = (x_3, .22, .5),$
$G(e_4) = (x_1, .1, .7), G(e_4) = (x_2, .4, .2), G(e_4) = (x_3, .8, .5),$
$H(e_1) = (x_1, .2, .6), H(e_1) = (x_2, .5, .7), H(e_1) = (x_3, .1, .8),$
$H(e_2) = (x_1, .3, .5), H(e_2) = (x_2, .1, .3), H(e_2) = (x_3, .4, .9),$
$H(e_3) = (x_1, .2, .5), H(e_3) = (x_2, .3, .6), H(e_3) = (x_3, .22, .5),$
$H(e_4) = (x_1, .1, .7), H(e_4) = (x_2, .4, .2), H(e_4) = (x_3, .8, .5),$
$K(e_1) = (x_1, .6, .2), K(e_1) = (x_2, .7, .5), K(e_1) = (x_3, .8, .1),$
$K(e_2) = (x_1, .3, .5), K(e_2) = (x_2, .3, .1), K(e_2) = (x_3, .4, .9),$
$K(e_3) = (x_1, .5, .2), K(e_3) = (x_2, .6, .3), K(e_3) = (x_3, .5, .22),$
$K(e_4) = (x_1, .7, .1), K(e_4) = (x_2, .4, .2), K(e_4) = (x_3, .8, .5).$
Then $\rho = \{\bar{U}_E, \phi_E, (F, E), (G, E), (H, E), (K, E)\}$ is a IF soft topology over U. Moreover, any member in $\{(F, E), (G, E), (H, E), (K, E)\}$ is IF soft $b-$closed. Hence (U_E, ρ) is $IFSbCT$.

Definition 3.10. (($IFSbC$)$-$continuous). Let $\partial : (U_E, \rho_1) \longrightarrow (W_D, \rho_2)$ be a mapping from $IFSTS$ (U_E, ρ_1) into $IFSTS$ (W_D, ρ_2). Then ∂ is called an intuitionistic fuzzy soft $b-$closed continuous mapping if $\partial^{-1}(F, E) \in \rho_1$ and is a $IFSbCS$ in U_E for each $IFSbCS$ $(F, E) \in \rho_2$ in W_D.

Definition 3.11. (($IFSbC$)$-$mapping). Let $\partial : (U_E, \rho_1) \longrightarrow (W_D, \rho_2)$be a mapping from $IFSTS$ (U_E, ρ_1) into $IFSTS$ (W_D, ρ_2). Then ∂ is called an intuitionistic fuzzy soft $b-$closed mapping if $\partial(F, E) \in \rho_2$ and is a $IFSbCS$ in W_D for each $IFSbCS(F, E) \in \rho_1$ in U_E.

Theorem 3.12. *Let $\partial : (U_E, \rho_1) \longrightarrow (W_D, \rho_2)$be a mapping from $IFSTS$ (U_E, ρ_1) into $IFSTS(W_D, \rho_2)$. Then the following statements are equivalent.*
(1) *∂ is an intuitionistic fuzzy soft $b-$closed continuous mapping.*
(2) *$\partial^{-1}(H, C) \in \rho_1$ is a $IFSbOS$ in U_E for every $IFSbOS$ $(H, C) \in \rho_2$ in W_D*

Proof. Suppose that ∂ is an intuitionistic fuzzy soft $b-$closed continuous mapping and let $(H, C) \in \rho_2$ be a $IFSbOS$ in W_D. Then $(H, C)^c$ is $IFSbCS$ in W_D, but ∂ is an intuitionistic fuzzy soft $b-$closed continuous mapping. Hence $\partial^{-1}((H, C)^c)$ is $IFSbCS$ and open in U_E. However, $\partial^{-1}((H, C)^c) = [\partial^{-1}(H, C)]^c$, thus $\partial^{-1}(H, C)$ is a $IFSbOS$ in U_E. Conversely, assume that $\partial^{-1}(H, C)$ is a $IFSbOS$ in X for every $IFSbOS$ (H, C) in W_D. Now, for any $IFSbCS$ (H, C) in W_D we obtain $(H, C)^c$ is $IFSbOS$ in W_D. Hence $\partial^{-1}((H, C)^c)$ is $IFSbOS$ in U_E. However, $\partial^{-1}((H, C)^c) = [\partial^{-1}(H, C)]^c$, thus $\partial^{-1}(H, C)$ is a $IFSbCS$ in U_E. Then ∂ is an intuitionistic fuzzy soft $b-$closed continuous mapping. □

Definition 3.13. A family $\Re$ of IF soft open sets is an open cover [($IFSO$)$-C$, in short] of a IF soft set (H, C) if $(H, C) \subseteq \bar{\coprod}\{(F_i, A)|(F_i, A) \in \Re, i \in \triangle\}$. A subcover of (H, C) is a subfamily of $\Re$ which is also a cover. A subcover Γ of $\Re$ is called IF soft $b-$closed subcover [($IFSbC$)$-SC$, in short] if each member of Γ is a $IFSbCS$. Moreover, we refer to Γ by $(IFSbC)_f - SC$, if Γ has finite members.

Definition 3.14. Let (U_E, ρ) be $IFSTS$ and $(H, A) \in IFS(U_E)$. A IF soft set (H, A) is called IF soft $b-$closed compact [($IFSbC$)$-C$, in short], if each ($IFSO$)$-C$ of (H, A) has a $(IFSbC)_f - SC$. Also IFSTS (U_E, ρ) is called IF soft $b-$closed compact [($IFSbC$)$-C$, in short], if each ($IFSO$) $-C$ of U_E has a $(IFSbC)_f - SC$.

Example 3.15. Let (U_E, ρ) be $IFSTS$ with finite universe set U, then (U_E, ρ) is ($IFSbC$) $-C$.

Proposition 3.16. *Let (H, A) be a $IFSCS$ in (U_E, ρ) ($IFSbC$)$-SC$. Then (H, A) is also ($IFSbC$)$-SC$.*

Proof. Let $\Re = \{(F_i, A)|(F_i, A) \in \Re, i \in \triangle\}$ be ($IFSO$)$-C$ of (H, A) But $\Re$ with $(H, A)^c$ is a ($IFSO$)$-C$ of U_E, since $(H, A)^c$ is a $IFSOS$, say $\acute{}\Re = \{\Re, (H, A)^c\}$ is a ($IFSO$)$-C$ of U_E. That means $U_E \subseteq \{\bar{\coprod}_{i\in\triangle}(F_i, A)\}\bar{\coprod}(H, A)^c$. However, (U_E, ρ) is ($IFSbC$)$-C$, thus U_E has a $(IFSbC)_f - SC$, say $\acute{}\Gamma$ of $\acute{}\Re$ such that :
$\acute{}\Gamma = \{(F_i, A) : i = 1, 2, \ldots, n\}$ if $(H, A)^c$ is not ($IFSbCS$) or $\acute{}\Gamma = \{\{(F_i, A) : i = 1, 2, \ldots, n\}, (H, A)^c\}$ if $(H, A)^c$ is ($IFSBCS$). Now, it's clearly if $(H, A)^c$ is not ($IFSbCS$). Then (H, A) is ($IFSbC$)$-C$. Moreover, if $(H, A)^c$ is ($IFSbCS$). Then we have $U_E \subseteq \bar{\coprod}_{i=1}^{n}\{(F_i, A)\}\bar{\coprod}(H, A)^c$.This implies that $(H, A) \subseteq \bar{\coprod}_{i=1}^{n}\{F_i, A)\}\bar{\coprod}(H,$

$A)^c$. Now, for each IF soft point $e_H \in (H,A)$ we have $e_H \notin (H,A)^c$. That means $(H,A)^c$ does not cover its complement in any IF soft point. Hence $\{(F_i,A) : i = 1,2,\ldots,n\}$ is $(IFSbC)_f - SC$ of $\Re$ such that $(H,A) \subseteq \bar{\coprod}_{i=1}^{n}\{(F_i,A)\}$. Then (H,A) is $(IFSbC)-C$. □

Theorem 3.17. *Let $\partial : (U_E,\rho_1) \longrightarrow (W_D,\rho_2)$ be a $(IFSbC)-$continuous mapping from $IFSTS$ (U_E,ρ_1) onto $IFSTS$ (W_D,ρ_2). If (U_E,ρ_1) is $((IFSbC)-C$, then (W_D,ρ_2) is verifies the same property.*

Proof. Let $\Re = \{(F_i,A)|(F_i,A) \in \Re, i \in \triangle\}$ be $(IFSO)-C$ of (W_D,ρ_2); i.e $W_D \subseteq \bar{\coprod}_{i\in\triangle}(F_i,A)$. Therefore $\partial^{-1}(W_D) \subseteq \partial^{-1}(\bar{\coprod}_{i\in\triangle}(F_i,A))$, this implies that $U_E \subseteq \bar{\coprod}_{i\in\triangle}\partial^{-1}((F_i,A))$. So $\partial^{-1}((F_i,A)) \in \rho_1$ for all $i \in \triangle$ (since ∂ is $(IFSbC)-$continuous). However, (U_E,ρ_1) is $(IFSbC)-C$, thus (U_E,ρ_1) has a $(IFSbC)_f - SC$ say $\{\partial^{-1}((F_i,A)) |i = 1,2,\ldots,n\}$ of $\{\partial^{-1}((F_i,A))|i \in \triangle\}$. AS $U_E \subseteq \bar{\coprod}_{i=1}^{n}\partial^{-1}((F_i,A))$, this implies that $W_D = \partial(U_E) \subseteq \partial(\bar{\coprod}_{i=1}^{n}\partial^{-1}((F_i,A))) = \bar{\coprod}_{i=1}^{n}\partial\partial^{-1}((F_i,A)) = \bar{\coprod}_{i=1}^{n}(F_i,A)$ (since ∂ is onto). So we have (W_D,ρ_2) has a $(IFSbC)_f - SC$ of $\Re$. Hence (W_D,ρ_2) is $(IFSbC)-C$. □

Definition 3.18. Let (U_E,ρ) be a $IFSTS$ over U. Then (U_E,ρ) is said to be IF soft $b-$closed disconnected [$(IFSbC)-$ Ć, in short], if there exists a pair (H,E), (G,E) of no-null $(IFSbCS)$ each one of them belongs to ρ and such that $\bar{U}_E = (H,E)\bar{\coprod}(G,E)$, $(H,E)\bar{\prod}(G,E) = \Phi_E$.

Example 3.19. Let us consider the $IFSTS$ (U_E,ρ) that is given in Example **??**, we have there does not exist a pair (H,E), (G,E) of no-null $(IFSbCS)$ each one of them belongs to ρ such that $\bar{U}_E = (H,E)\bar{\coprod}(G,E)$ and $(H,E)\bar{\prod}(G,E) = \Phi_E$. Then (U_E,ρ) is $(IFSbC)-C$.

Remark 3.20. Let (U_E,ρ) be a $IFSTS$ over U. Then, for any $(H,E) \in IFS(U_E)$ is both $(IFSbCS)$ and $(IFSbOS)$ if (H,E) and its complement belong to ρ.

Theorem 3.21. *Let (U_E,ρ) be a $IFSTS$ over U. Then (U_E,ρ) is $(IFSbC)-C$ if and only if there is no proper $(IFSbCS)$ that is both $(IFSO)$ and $(IFSC)$.*

Proof. Let (U_E,ρ) be a $(IFSbC)-C$ and (H,E) be a proper $(IFSbC)-C$ that is both $(IFSO)$ and $(IFSC)$. Clearly, $(H,E)^c \in \rho$ and is a $(IFSbCS)$ different from Φ_E and $\bar{U}_E$. Also, $\bar{U}_E = (H,E)\bar{\coprod}(H,E)^c$, $(H,E)\bar{\prod}(H,E)^c = \Phi_E$. Therefore we have (U_E,ρ) is a $(IFSbC)-$Ć. This is a contradiction. Hence Φ_E and $\bar{U}_E$ the only $(IFSbCS)$ are both $(IFSO)$ and $(IFSC)$.

Conversely, assume that (U_E,ρ) is a $(IFSbC)-$Ć, then there exists a pair (H,E), (G,E) of no-null $(IFSbCS)$ each one of them belongs to ρ and such that $\bar{U}_E = (H,E)\bar{\coprod}(G,E)$, $(H,E)\bar{\prod}(G,E) = \Phi_E$. Let $(H,E) = \bar{U}_E$, thus $(G,E) = \Phi_E$ (but this is a contradiction). So $(H,E) \neq \bar{U}_E$. Hence $(H,E) = (G,E)^c$. That means (H,E) is both $(IFSO)$ and $(IFSC)$ different from Φ_E and $\bar{U}_E$. That is a contradiction. Then (U_E,ρ) is $(IFSbC)-C$. □

Definition 3.22. Let (H,A) be a soft set over U. Then $\xi_{H(A)}$ is called an induced fuzzy soft set over (U,E), where $\xi\colon SS(U_A) \longrightarrow FS(U,E)$ is a mapping which is given as: for $(H,A) \in SS(U_A)$ the image of (H,A) under ξ denoted by $\xi_{H(A)}$, is defended as following: $\xi_{H(A)}(e) = \mu^e_{H(A)}$ (a fuzzy subset of U), where $\mu^e_{H(A)}(x) = \bar{0}$ if $e \in E \backslash A$ and

$$\mu^e_{H(A)}(x) = \begin{cases} 1 & \text{if } x \neq H(e) \ \& \ x \in H(e), \\ 1/Z & \text{if } x = H(e) \ , \\ Z/K & \text{if } x \neq H(e) \ \& \ x \notin H(e) \neq \phi, \\ 0 & \text{if } H(e) = \phi. \end{cases}$$

For all $x \in U$, if $e \in A$, where Z=$n(SS(U_A))$ & K=$n(SS(U_E))$.

Example 3.23. Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3, e_4\}$ that be given in (Example 3.3) and let (F, A) be a soft set over U where $A = \{e_1, e_3, e_4\}$, $F(e_1) = \{x_1\}$, $F(e_3) = \{x_1, x_2\}$, $F(e_4) = \phi$. Then $\xi_{F(A)} = \{F(e_1) = (1/Z, Z/K, Z/K), F(e_2) = (0, 0, 0), F(e_3) = (1, 1, Z/K), F(e_4) = (0, 0, 0)\}$.Where, $Z = n(SS(U_A)) = 2^9$ and $K = n(SS(U_E)) = 2^{12}$.

Remark 3.24. It's clearly for any soft set (F, A) subset of soft set (G, B) we can consider that $n(SS(U_A)) \leq n(SS(U_B))$.

Definition 3.25. Let H_A be a fuzzy soft set over (U, E). Then $\eta_{H(A)}$ is called an induced intuitionistic fuzzy soft set over U, where $\eta : FS(U, E) \longrightarrow IFS(U_E)$ is a mapping which is given as: for $H_A \in FS(U, E)$ the image of H_A under η denoted by $\eta_{H(A)}$, is defended as following: $\eta_{H(A)} = \{(x, \mu^e_{H_A}(x), 1 - \mu^e_{H_A}(x); x \in U\}$, for all $e \in A$.

Remark 3.26. For any soft set we can generated IF soft set by using the composition of two mappings ξ: $SS(U_A) \longrightarrow FS(U, E)$ and $\eta : FS(U, E) \longrightarrow IFS(U_E)$.

Proposition 3.27. *Let U be an initial universe set and let E be a set of parameters. Then the following statements are hold:*
(1) *The image of Null soft set $\Phi = (\phi, \phi)$ under ξ is Null fuzzy soft set.*
(2) *The image of Null fuzzy soft set under η is Null intuitionistic fuzzy soft set Φ_E.*
(3) *The image of Null soft set $\Phi = (\phi, \phi)$ under the composition $\eta \circ \xi$ is Null intuitionistic fuzzy soft set Φ_E.*
(4) *The image of Absolute soft set (U, E) under ξ is Absolute fuzzy soft set $\bar{E}$.*
(5) *The image of Absolute fuzzy soft set $\bar{E}$ under η is Absolute intuitionistic fuzzy soft set $\bar{U}_E$.*
(6) *The image of Absolute soft set (U, E) under the composition $\eta \circ \xi$ is Absolute intuitionistic fuzzy soft set $\bar{U}_E$.*

Proof. Follows from the definitions. □

Definition 3.28. Let (U, E, τ) be a soft topological space over U and (U, E, Ψ) be a fuzzy soft topological space. Then (U, E, Ψ) is said to be an induced fuzzy soft topological space by (U, E, τ) if and only if $\xi(\tau) = \Psi$.

Definition 3.29. Let (U, E, Ψ) be a fuzzy soft topological space over U and (U_E, ρ) be a IF soft topological space. Then (U_E, ρ) is said to be an induced IF soft topological space by (U, E, Ψ) if and only if $\eta(\Psi) = \rho$.

Definition 3.30. Let (U, E, τ) be a soft topological space over U and (U_E, ρ) be a IF soft topological space. Then (U_E, ρ) is said to be an induced IF soft topological space by (U, E, τ) if and only if $\eta \circ \xi(\tau) = \rho$.

Proposition 3.31. *Let U be an initial universe set and let E be a set of parameters. For any pair of soft sets (F, A) and (G, B), the following statements are hold:*
(i) If $(F, A) \subseteq (G, B)$, then $\xi_{F(A)} \subseteq \xi_{G(B)}$,
(ii) $\xi(int(F, A)) = int(\xi_{F(A)})$,
(iii) $\xi(cl(F, A)) = cl(\xi_{F(A)})$.

Proof. (i) Since $(F, A) \subseteq (G, B)$, then $A \subseteq B$ and $F(e) \subseteq G(e)$, $\forall e \in A$, we consider that $\xi_{F(A)} = \xi_{G(B)}$ whenever $e \in E \backslash A$ (since $\mu^e_{F(A)}(x) = \bar{0}(x) = \mu^e_{G(B)}(x)$, $\forall x \in U$). If $e \in A$, then there are four cases as following:
Case (1): $\mu^e_{F(A)}(x) = 1$, if $x \neq F(e)$ & $x \in F(e)$. However, $F(e) \subseteq G(e)$ this implies that $F(e) = G(e)$. So $\mu^e_{F(A)}(x) = 1 = \mu^e_{G(B)}(x)$, if $x \neq F(e)$ & $x \in F(e)$.
Case (2): $\mu^e_{F(A)}(x) = 1/n(SS(U_A))$, if $x = F(e)$. However, $F(e) \subseteq G(e)$ this implies that either $F(e) = G(e)$ or $F(e) \subset G(e)$, if $F(e) = G(e)$ we have $\mu^e_{F(A)}(x) = 1/n(SS(U_A)) = \mu^e_{G(B)}(x)$. Also, if $F(e) \subset G(e)$ this implies that $x \neq G(e)$ & $x \in G(e)$. Then $\mu^e_{G(B)}(x) = 1$, thus $\mu^e_{F(A)}(x) < \mu^e_{G(B)}(x)$, if $x = F(e)$.
Case (3): $\mu^e_{F(A)}(x) = n(SS(U_A))/n(SS(U_E))$, if $x \neq F(e)$ & $x \notin F(e) \neq \phi$. However, $F(e) \subseteq G(e)$ this implies that $x \neq G(e)$ & $G(e) \neq \phi$. Now, if $x \in G(e)$ we consider that $\mu^e_{F(A)}(x) < \mu^e_{G(B)}(x) = 1$ and if $x \notin G(e)$ we have $\mu^e_{F(A)}(x) = n(SS(U_A))/n(SS(U_E)) \leq n(SS(U_B))/n(SS(U_E)) = \mu^e_{G(B)}(x)$. Therefore $\mu^e_{F(A)}(x) \leq \mu^e_{G(B)}(x)$, if $x \neq F(e)$ and $x \notin F(e) \neq \phi$.
Case (4): $\mu^e_{F(A)}(x) = 0$, if $F(e) = \phi$. That means $\mu^e_{F(A)}(x) \leq \mu^e_{G(B)}(x)$, for all $x \in U$. Finally we consider that $\mu^e_{F(A)}(x) \leq \mu^e_{G(B)}(x)$, for all $e \in A$ and $x \in U$. Then $\xi_{F(A)} \subseteq \xi_{G(B)}$.

(ii) Since $int(F, A) = \bigcup\{(G, B) | (G, B) \in \tau$ & $(G, B) \subseteq (F, A)\}$. Hence $\xi(int(F, A))$ $= \xi(\bigcup\{(G, B) | (G, B) \in \tau$ & $(G, B) \subseteq (F, A)\}) = \coprod\{\xi_{G(B)} | \xi_{G(B)} \in \Psi$ & $\xi_{G(B)} \subseteq \xi_{F(A)}\}$
Where (U, E, Ψ) is an induced fuzzy soft topological space by (U, E, τ). However, $int(\xi_{F(A)}) = \coprod\{\xi_{G(B)} | \xi_{G(B)} \in \Psi$ & $\xi_{G(B)} \subseteq \xi_{F(A)}\}$. Then $\xi(int(F, A)) = int(\xi_{F(A)})$.

(iii) Since $cl(F, A) = \bigcap\{(G, B) | (G, B)^c \in \tau$ & $(F, A) \subseteq (G, B)\}$. Hence $\xi(cl(F, A))$ $= \xi(\bigcap\{(G, B) | (G, B)^c \in \tau$ & $(F, A) \subseteq (G, B)\}) = \prod\{\xi_{G(B)} | \xi^c_{G(B)} \in \Psi$ & $\xi_{F(A)} \subseteq \xi_{G(B)}\}$. Where (U, E, Ψ) is an induced fuzzy soft topological space by (U, E, τ). However, $cl(\xi_{F(A)}) = \prod\{\xi_{G(B)} | \xi^c_{G(B)} \in \Psi$ & $\xi_{F(A)} \subseteq \xi_{G(B)}\}$. Then $\xi(cl(F, A)) = cl(\xi_{F(A)})$. □

Proposition 3.32. *Let (U, E, τ) be a soft $b-$closed topological space, then the induced fuzzy soft topological space by (U, E, τ) is a fuzzy soft $b-$closed topological space.*

Proof. Let (U, E, ψ) be an induced fuzzy soft topological space by (U, E, τ). Then $\xi(\tau) = \psi$. That means for any soft set $(F, A) \in \tau$ we have $\xi_{F(A)} \in \xi(\tau) = \psi$.

Also, for any fuzzy soft set $F_A \in \psi$, there exists soft set $(F, A) \in \tau$ such that $\xi((F, A)) = \xi_{F(A)} = F_A$. Now we want to show that for any $F_A \in \psi$ satisfies that $cl(int(F_A)) \prod int(cl(F_A)) \subseteq F_A$. However, $F_A \in \psi$ this implies that $cl(int(F_A)) \prod int(cl(F_A)) = cl(F_A) \prod int(cl(F_A)) = int(cl(F_A))$. Since (U, E, τ) is a soft $b-$closed topological space, thus $(F, A) \in \tau$ such that $cl(int(F, A)) \prod int(cl(F, A)) = int(cl(F, A)) \subseteq (F, A)$. Hence $\xi(int(cl(F, A))) = int(cl(\xi_{F(A)}) \subseteq \xi_{F(A)}$ (by Proposition **??**), but $\xi_{F(A)} = F_A$. Then for any $F_A \in \psi$ satisfies that $cl(int(F_A)) \prod int(cl(F_A)) \subseteq F_A$. Therefore the induced fuzzy soft topological space by (U, E, τ) is a fuzzy soft $b-$closed topological space. □

Proposition 3.33. *Let U be an initial universe set and let E be a set of parameters. For any pair of fuzzy soft sets F_A and G_B , the following statements are hold:*
(1) If $F_A \subseteq G_B$, then $\eta_{F(A)} \subseteq \eta_{G(B)}$,
(2) $\eta(int(F(A)) = int(\eta_{F(A)})$,
(3) $\eta(cl(F(A)) = cl(\eta_{F(A)})$.

Proof. (1)Since $F_A \subseteq G_B$, then $\mu^e_{F_A}(x) \leq \mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in E$, and this implies that $1-\mu^e_{F_A}(x) \geq 1-\mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in E$. Hence $\eta_{F(A)} \subseteq \eta_{G(B)}$.
(2) Since $int(F_A) = \coprod \{G_B | G_B \in \Psi \ \& \ G_B \subseteq F_A\}$. Hence $\eta(int(F_A)) = \eta(\coprod \{G_B | G_B \in \Psi \ \& \ G_B \subseteq F_A\}) = \bar{\coprod}\{\eta_{G(B)} | \eta_{G(B)} \in \rho \ \& \ \eta_{G(B)} \subseteq \eta_{F(A)}\}$. Where (U_E, ρ) is an induced intuitionistic fuzzy soft topological space by (U, E, Ψ). However, $int(\eta_{F(A)}) = \bar{\coprod}\{\eta_{G(B)} | \eta_{G(B)} \in \rho \ \& \ \eta_{G(B)} \subseteq \eta_{F(A)}\}$. Then $\eta(int(F(A)) = int(\eta_{F(A)})$.
(3) Since $cl(F_A) = \prod \{G_B | G^c_B \in \Psi \ \& \ F_A \subseteq G_B\}$. Hence $\eta(cl(F_A)) = \eta(\prod \{G_B | G^c_B \in \Psi \ \& \ F_A \subseteq G_B\}) = \bar{\prod}\{\eta_{G(B)} | \eta^c_{G(B)} \in \rho \ \& \ \eta_{F(A)} \subseteq \eta_{G(B)}\}$. Where (U_E, ρ) is an induced intuitionistic fuzzy soft topological space by (U, E, Ψ). However, $cl(\eta_{F(A)}) = \bar{\prod}\{\eta_{G(B)} | \eta^c_{G(B)} \in \rho \ \& \ \eta_{F(A)} \subseteq \eta_{G(B)}\}$. Then $\eta(cl(F(A)) = cl(\eta_{F(A)})$. □

Proposition 3.34. *Let (U, E, Ψ) be a fuzzy soft $b-$closed topological space, then the induced fuzzy soft topological space by (U, E, Ψ) is a $(IFSbCT)$.*

Proof. Let (U_E, ρ) be an induced intuitionistic fuzzy soft topological space by (U, E, Ψ). Then $\eta(\Psi) = \rho$. That means for any fuzzy soft set $F_A \in \rho$ we have $\eta_{F(A)} \in \eta(\Psi) = \rho$. Also, for any intuitionistic fuzzy soft set $(F, A) \in \rho$, there exists fuzzy soft set $F_A \in \Psi$ such that $\eta(F_A) = \eta_{F(A)} = (F, A)$. Now we want to show that for any $(F, A) \in \rho$ satisfies that $cl(int(F, A))\bar{\prod}int(cl(F, A)) \subseteq (F, A)$. However, $(F, A) \in \rho$ this implies that $cl(int(F, A))\bar{\prod}int(cl(F, A)) = cl(F, A)\bar{\prod}int(cl(F, A)) = int(cl(F, A))$. Since (U, E, Ψ) is a fuzzy soft $b-$losed topological space, thus $F_A \in \Psi$ such that $cl(int(F_A)) \prod int(cl(F_A)) = int(cl(F_A)) \subseteq F_A$. Hence $\eta(int(cl(F_A))) = int(cl(\eta_{F(A)})) \subseteq \eta_{F(A)}$ (by Proposition **??**), but $\eta_{F(A)} = (F, A)$. Then for any $(F, A) \in \rho$ satisfies that $cl(int(F, A))\bar{\prod}int(cl(F, A)) \subseteq (F, A)$. Therefore the induced intuitionistic fuzzy soft topological space by (U, E, Ψ) is a $(IFSbCT)$. □

Proposition 3.35. *Let $\partial_1(U_E, \rho_1) \longrightarrow (W_D, \rho_2)$ be a $(IFSbC)-$mapping from $IFSTS$ (U_E, ρ_1) into $IFSTS$ (W_D, ρ_2) and let $\partial_2(U_E, \rho_1) \longrightarrow (T_N, \rho_3)$be a $(IFSbC)$ $-$continuous mapping from $IFSTS$ (U_E, ρ_1) into induced $IFSTS$ (T_N, ρ_3) by fuzzy* *soft $b-$closed space (T, N, Ψ). Then for any fuzzy soft set $F_A \in \Psi$ there exists $IFSbCS$ (G, B) in W_D such that$(G, B) \in \rho_2$.*

Proof. Suppose that $F_A \in \Psi$. Then F_A is a fuzzy soft $b-$closed set (since (T, N, Ψ) is fuzzy soft $b-$closed space). However, $\eta_{F(A)} \in \eta(\Psi) = \rho_3$ (since (T_N, ρ_3) is an induced by (T, N, Ψ). Moreover, $\eta_{F(A)}$ is a $IFSbCS$ in T_N (by Proposition **??**). Further, $\partial_2^{-1}(\eta_{F(A)}) \in \rho_1$ and $\partial_2^{-1}(\eta_{F(A)})$ is a $IFSbCS$ in U_E (since ∂_2 is a $(IFSbC)-$continuous). This implies that $\partial_1(\partial_2^{-1}(\eta_{F(A)})) \in \rho_2$ and $\partial_1(\partial_2^{-1}(\eta_{F(A)}))$ is a $IFSbCS$ in W_D (since ∂_1 is a $(IFSbC)-$mapping). Assume that $\partial_1(\partial_2^{-1}(\eta_{F(A)}))$ $= (G, B)$. Then for any fuzzy soft set $F_A \in \Psi$ there exists $IFSbCS$ (G, B) in W_D such that $(G, B) \in \partial_2$. □

4. Conclusions

In this work, we introduced some new concepts like $(IFSbCT)$, $(IFSbC)-$conti -nuous, $(IFSbC)-$mapping, $(IFSbC)_f - SC$, $(IFSbC) - C$, $(IFSbC)-$ Ć, $\xi_{H(A)}$ and $\eta_{H(A)}$. Also we gave some basic properties of these concepts. Moreover, it is interesting to work on the compositions of soft sets and Intuitionistic fuzzy sets. Also, The composition of two mappings η and ξ can be tried with other forms of soft sets like soft $b-$closed (F, A) and check $\eta \circ \xi((F, A))$ is need to be $(IFSbCS)$. In another side let (U, E, τ) be a soft topological space over U, then the induced IF soft topological space by (U, E, τ) is need to be $(IFSbCS)$.

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