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Intuitionistic fuzzy Baire spaces

R.Dhavaseelan

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Abstract. In this paper the concepts of intuitionistic fuzzy Baire spaces are introduced and characterizations of intuitionistic fuzzy Baire spaces are studied. Several related examples are given.

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Corresponding Author: R.Dhavaseelan(dhavaseelan.r@gmail.com)

1. Introduction and Preliminaries

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L.A.Zadeh [14].The theory of fuzzy topological space was introduced and developed by C.L.Chang [6] and since then various notions in classical topology have been extended to fuzzy topological space.The idea of "intuitionistic fuzzy set" was first published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2, 3, 4].Later, this concept was generalized to "intuitionistic L - fuzzy sets"by Atanassov and Stoeva [5].The concept of somewhat fuzzy continuous functions and somewhat fuzzy open hereditarily irresolvable by G.Thangaraj and G.Balasubramanian in [11, 12]. The concept of fuzzy nowhere dense set in fuzzy topological space by G.Thangaraj and S.Anjalmose in [13]

In this paper the concepts of intuitionistic fuzzy Baire spaces are introduced and characterizations of intuitionistic fuzzy Baire spaces are studied. Several related examples are given.

Definition 1.1 ([3])**.** Let X be a nonempty fixed set.An intuitionistic fuzzy set(IFS for short) A is an object having the form $A = \{\langle x, \mu_A(x), \delta_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \to I$ and $\delta_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$)and the degree of nonmembership($\delta_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \delta_A(x) \leq 1$ for each $x \in X$.

Definition 1.2 ([7]). Let X be a nonempty set and the intuitionistic fuzzy sets A and B in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$,$B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(c) $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;
(f) $[\;]A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;
(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.3 ([7]). $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 1.4 ([7]). An intuitionistic fuzzy topology (IFT)on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq \tau$.

In this case the ordered pair (X, τ) or simply X is called an intuitionistic fuzzy topological space (IFTS) and each IFS in τ is called an intuitionistic fuzzy open set (IFOS). The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 1.5 ([7]). Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space X. Then

$int(A) = \bigcup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$ is called the intuitionistic fuzzy interior of A;

$cl(A) = \bigcap\{G \mid G$ is an IFCS in X and $G \supseteq A\}$ is called the intuitionistic fuzzy closure of A.

Definition 1.6 ([8]). An intuitionistic fuzzy set A in intuitionistic fuzzy topological space (X, T) is called intuitionistic fuzzy dense if there exists no intuitionistic fuzzy closed set B in (X, T) such that $A \subset B \subset 1_\sim$

Proposition 1.7 ([9]). *If A is an intuitionistic fuzzy nowhere dense set in (X, T), then $\overline{A}$ is an intuitionistic fuzzy dense set in (X, T).*

Proposition 1.8 ([9]). *Let A be an intuitionistic fuzzy set. If A is an intuitionistic fuzzy closed set in (X, T) with $IFint(A) = 0_\sim$, then A is an intuitionistic fuzzy nowhere dense set in (X, T).*

2. Intuitionistic Fuzzy Baire Spaces

Definition 2.1. Let (X, T) be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set A in (X, T) is called intuitionistic fuzzy first category if $A = \bigcup_{i=1}^{\infty} B_i$, where B_i's are intuitionistic fuzzy nowhere dense sets in (X, T). Any other intuitionistic fuzzy set in (X, T) is said to be of intuitionistic fuzzy second category.

Definition 2.2. An intuitionistic fuzzy topological space (X, T) is called intuitionistic fuzzy first category space if the intuitionistic fuzzy set $1_\sim$ is an intuitionistic fuzzy first category set in (X,T).That is, $1_\sim = \bigcup_{i=1}^{\infty} A_i$ where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T).Otherwise (X,T) will be called an intuitionistic fuzzy second category space.

Proposition 2.3. *If A be an intuitionistic fuzzy first category set in (X,T),then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $IFcl(B_i) = 1_\sim$.*

Proof. Let A be an intuitionsitic fuzzy first category set in (X,T). Then $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T). Now $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty}(\overline{A_i})$. Now A_i is an intuitionistic fuzzy nowhere dense set in (X,T). Then, by Proposition 1.7, we have $\overline{A_i}$ is an intuitionistic fuzzy dense set in (X,T). Let us put $B_i = \overline{A_i}$. Then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $IFcl(B_i) = 1_\sim$. □

Definition 2.4. Let A be an intuitionistic fuzzy first category set in (X,T). Then $\overline{A}$ is called an intuitionistic fuzzy residual set in (X,T).

Definition 2.5. Let (X,T) be an intuitionistic fuzzy topological space. Then (X,T) is said to intuitionistic fuzzy Baire space if $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T).

Example 2.6. Let $X = \{a,b,c\}$. Define the intuitionistic fuzzy sets A,B,C and D as follows : $A = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.5})\rangle$, $B = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3})\rangle$, $C = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.4})\rangle$ and $D = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7})\rangle$. Then the family $T = \{0_\sim, 1_\sim, A\}$ is an intuitionistic fuzzy topologies on X. Thus, (X,T) is an intuitionistic fuzzy topological spaces. Now $\overline{A}, \overline{B}, C$ and D are intuitionistic fuzzy nowhere dense sets in (X,T) . Also $IFint(\overline{A} \cup \overline{B} \cup C \cup D) = 0_\sim$. Hence (X,T) is an intuitionistic fuzzy Baire space.

Proposition 2.7. *If $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$ where $IFint(A_i) = 0_\sim$ and $A_i \in T$, then (X,T) is an intuitionistic fuzzy Baire space.*

Proof. Now $A_i \in T$ implies that A_i is an intuitionistic fuzzy closed set in (X,T). Since $IFint(A_i) = 0_\sim$.By Proposition 1.8, A_i is an intuitionistic fuzzy nowhere dense set in (X,T). Therefore $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$. where A_i's are intuitionistic fuzzy nowhere dense set in (X,T). Hence (X,T) is an intuitionistic fuzzy Baire space. □

Proposition 2.8. *IF $IFcl(\bigcap_{i=1}^{\infty} A_i) = 1_\sim$ where A_i's are intuitionistic fuzzy dense and intuitionistic fuzzy open sets in (X,T), then (X,T) is an intuitionistic fuzzy Baire Space.*

Proof. Now $IFcl(\bigcap_{i=1}^{\infty} A_i) = 1_\sim$ implies that $\overline{IFcl(\bigcap_{i=1}^{\infty} A_i)} = 0_\sim$. Then we have $IFint(\overline{\bigcap_{i=1}^{\infty} A_i}) = 0_\sim$. Which implies that $IFint(\bigcup_{i=1}^{\infty} \overline{A_i}) = 0_\sim$. Let $B_i = \overline{A_i}$. Then $IFint(\bigcup_{i=1}^{\infty} B_i) = 0_\sim$. Now $A_i \in T$ implies that $\overline{A_i}$ is an intuitionistic fuzzy closed set in (X,T) and hence B_i is an intuitionistic fuzzy closed and $IFint(B_i) = IFint(\overline{A_i}) = \overline{IFcl(A_i)} = 0_\sim$. Hence By Proposition 1.8, B_i is an intuitionistic fuzzy nowhere dense set in (X,T). Hence $IFint(\bigcup_{i=1}^{\infty} B_i) = 0_\sim$ where B_i's are intuitionistic fuzzy nowhere dense sets, implies that (X,T) is an intuitionistic fuzzy Baire space. □

Proposition 2.9. *Let (X,T) be an intuitionistic fuzzy topological space. Then the following are equivalent*

(i) (X,T) *is an intuitionistic fuzzy Baire space.*
(ii) $IFint(A) = 0_\sim$*, for every intuitionistic fuzzy first category set* A *in* (X,T)*.*
(iii) $IFcl(B) = 1_\sim$*, for every intuitionistic fuzzy residual set* B *in* (X,T)*.*

Proof. $(i) \Rightarrow (ii)$ Let A be an intuitionistic fuzzy first category set in (X,T). Then $A = (\bigcup_{i=1}^{\infty} A_i)$ where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T). Now $IFint(A) = IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$. Since (X,T) is an intuitionistic fuzzy Baire space. Therefore $IFint(A) = 0_\sim$.

$(ii) \Rightarrow (iii)$ Let B be an intuitionistic fuzzy residual set in (X,T). Then $\overline{B}$ is an intuitionistic fuzzy first category set in (X,T). By hypothesis $IFint(\overline{B}) = 0_\sim$ which implies that $\overline{IFcl(A)} = 0_\sim$. Hence $IFcl(A) = 1_\sim$.

$(iii) \Rightarrow (i)$ Let A be an intuitionistic fuzzy first category set in (X,T). Then $A = (\bigcup_{i=1}^{\infty} A_i)$ where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T). Now A is an intuitionistic fuzzy first category set implies that $\overline{A}$ is an intuitionistic fuzzy residual set in (X,T). By hypothesis, we have $IFcl(\overline{A}) = 1_\sim$, which implies that $\overline{IFint(A)} = 1_\sim$. Hence $IFint(A) = 0_\sim$. That is, $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X,T). Hence (X,T) is an intuitionsitic fuzzy Baire space. □

Proposition 2.10. *An intuitionistic fuzzy topological space* (X,T) *is an intuitionistic fuzzy Baire space if and only if* $(\bigcup_{i=1}^{\infty} A_i) = 1_\sim$*, where* A_i*'s is an intuitionistic fuzzy closed set in* (X,T) *with* $IFint(A_i) = 0_\sim$*, implies that* $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$*.*

Proof. Let (X,T) be an intuitionistic fuzzy Baire space. Now A_i is an intuitionistic fuzzy closed in (X,T) and $IFint(A_i) = 0_\sim$, implies that A_i is an intuitionistic fuzzy nowhere dense set in (X,T). Now $\bigcup_{i=1}^{\infty} A_i = 1_\sim$ implies that $1_\sim$ is an intuitionistic fuzzy first category set in (X,T). Since (X,T) is an intuitionistic fuzzy Baire space space, by Proposition 2.9, $IFint(1_\sim) = 0_\sim$. That is, $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$.

Conversely suppose that $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$ where A_i. By Proposition 1.8, A_i is an intuitionistic fuzzy nowhere dense set in (X,T). Hence $IFint(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$ implies that (X,T) is an intuitionistic fuzzy Baire space. □

Definition 2.11. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. A map $f : (X,T) \to (Y,S)$ is said to be an intuitionistic fuzzy open if the image of every intuitionistic fuzzy open set A in (X,T) is intuitionistic fuzzy open $f(A)$ in (Y,S).

Definition 2.12 ([10])**.** Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. A map $f : (X,T) \to (Y,S)$ is called intuitionistic fuzzy contra continuous(in short IF contra continuous)if the inverse image of every intuitionistic fuzzy open set in (Y,S) is intuitionistic fuzzy closed in (X,T).

Proposition 2.13. *Let* (X,T) *and* (Y,S) *be any two intuitionistic fuzzy topological spaces. If* $f : (X,T) \to (Y,S)$ *is an onto intuitionsitic fuzzy contra continuous and intuitionistic fuzzy open then* (Y,S) *is an intuitionistic fuzzy Baire space.*

Proof. Let A be an intuitionistic fuzzy first category set in (Y,S). Then $A = (\bigcup_{i=1}^{\infty} A_i)$ where A_i are intuitionistic fuzzy nowhere dense sets in (Y,S). Suppose

$IFint(A) \neq 0_\sim$. Then there exists an intuitionistic fuzzy open set $B \neq 0_\sim$ in (Y, S), such that $B \subseteq A$. Then $f^{-1}(B) \subseteq f^{-1}(A) = f^{-1}(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} f^{-1}(A_i)$. Hence

$$f^{-1}(B) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(IFcl(A_i)). \tag{2.1}$$

Since f is intuitionistic fuzzy contra continuous and $IFcl(A_i)$ is an intuitionistic fuzzy closed set in (Y, S), $f^{-1}(IFcl(A_i))$ is an intuitionistic fuzzy open in (X, T). From (2.1) we have

$$f^{-1}(B) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(IFcl(A_i)) = \bigcup_{i=1}^{\infty} IFint(f^{-1}(IFcl(A_i)). \tag{2.2}$$

Since f is intuitionsitic fuzzy open and onto, $IFint(f^{-1}(A_i)) \subseteq f^{-1}(IFint(A_i))$. From 2.2, we have $f^{-1}(B) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(IFintIFcl(A_i)) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(0_\sim) = 0_\sim$. Since A_i is an intuitionistic fuzzy nowhere dense. That is, $f^{-1}(B) \subseteq 0_\sim$ and hence $f^{-1}(B) = 0_\sim$ which implies that $B = 0_\sim$, which is a contradiction to $B \neq 0_\sim$. Hence $Ifint(A) = 0_\sim$ where A is an intuitionsitic fuzzy first category set in (Y, S). Hence by Proposition 2.9, (Y, S) is an intuitionistic fuzzy Baire space. □

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R.Dhavaseelan (dhavaseelan.r@gmail.com)
Department of mathematics,Sona College of Technology, Salem-636005,Tamil Nadu ,India