**A**nnals of **F**uzzy **M**athematics and **I**nformatics
Volume 10, No. 2, (August 2015), pp. 203–213
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr



# Notes on $\gamma$-soft operator and some counter examples on (supra) soft continuity

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Received 16 December 2014; Revised 4 February 2015; Accepted 23 February 2015

Abstract. Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [28] and easily applied to many problems having uncertainties from social life. The main purpose of this paper, is to introduce more properties of of $\gamma$-soft interior and $\gamma$-soft closure mentioned in [8, 12]. Also, counter examples of (supra) soft continuity, mentioned in [9, 12], are introduced.

2010 AMS Classification: 06D72, 54A40

Keywords: Soft set, Soft topological space, $\gamma$-operation, $\alpha$-open soft set, Semi-open soft set, $\beta$-open soft set, b-open soft sets, Continuous soft functions, pre-continuous soft functions, Semi-continuous soft functions, $\alpha$-continuous soft functions, $\beta$-continuous soft functions, Supra soft topological spaces.

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## 1. Introduction

The concept of soft sets was first introduced by Molodtsov [28] in 1999 as a general mathematical tool for dealing with uncertain objects. In [28, 27], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions,game theory, operations research,Riemann integration, Perron integration, probability, theory of measurement, and so on.
After presentation of the operations of soft sets [25], the properties and applications of soft set theory have been studied increasingly [4, 22, 27, 30]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [2, 3, 6, 11, 20, 23, 24, 25, 26, 27, 29, 33]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [7].

Recently, in 2011, Shabir and Naz [31] initiated the study of soft topological spaces. They defined soft topology on the collection $\tau$ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open and closed soft sets, soft subspace, soft closure, soft nbd of a point and soft separation axioms, which is extended in [5, 32]. In [12], Kandil et. al. introduced some soft operations such as semi open soft, pre open soft, $\alpha$-open soft and $\beta$-open soft and investigated their properties in detail. Kandil et al. [19] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al.[15]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al. [10, 13, 14, 16, 17, 18, 21].

The main purpose of this paper, is to introduce more properties of of $\gamma$-soft interior and $\gamma$-soft closure mentioned in [8, 12]. Also, counter examples of (supra) soft continuity, mentioned in [9, 12], are introduced.

## 2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

**Definition 2.1** ([28])**.** Let $X$ be an initial universe and $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and $A$ be a non-empty subset of $E$. A pair $(F, A)$ denoted by $F_A$ is called a soft set over $X$ , where $F$ is a mapping given by $F : A \to P(X)$. In other words, a soft set over $X$ is a parametrized family of subsets of the universe $X$. For a particular $e \in A$ , $F(e)$ may be considered the set of $e$-approximate elements of the soft set $(F, A)$ and if $e \notin A$, then $F(e) = \phi$ i.e
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets over $X$ denoted by $SS(X)_A$.

**Definition 2.2** ([32])**.** The soft set $(F, E) \in SS(X)_E$ is called a soft point in $X_E$ if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point $(F, E)$ is denoted by $x_e$.

**Definition 2.3** ([31])**.** Let $\tau$ be a collection of soft sets over a universe $X$ with a fixed set of parameters $E$, then $\tau \subseteq SS(X)_E$ is called a soft topology on $X$ if

- **(1):** $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,
- **(2):** the union of any number of soft sets in $\tau$ belongs to $\tau$,
- **(3):** the intersection of any two soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, \tau, E)$ is called a soft topological space over $X$.

**Definition 2.4** ([12])**.** Let $(X, \tau, E)$ be a soft topological space. A mapping $\gamma : SS(X)_E \to SS(X)_E$ is said to be an operation on $OS(X)$ if $F_E \tilde{\subseteq} \gamma(F_E)$ $\forall$ $F_E \in OS(X)$. The collection of all $\gamma$-open soft sets is denoted by $OS(\gamma) = \{F_E : F_E \tilde{\subseteq} \gamma(F_E), F_E \in SS(X)_E\}$. Also, the complement of $\gamma$-open soft set is called $\gamma$-closed soft set, i.e
$CS(\gamma) = \{F'_E : F_E\ is\ a\ \gamma-open\ soft\ set, F_E \in SS(X)_E\}$ is the family of all $\gamma$-closed soft sets.

**Definition 2.5** ([12]). Let $(X, \tau, E)$ be a soft topological space. Different cases of $\gamma$-operations on $SS(X)_E$ are as follows:

- **(1):** If $\gamma = int(cl)$, then $\gamma$ is called pre open soft operator. We denote the set of all pre open soft sets by $POS(X, \tau, E)$, or when there can be no confusion by $POS(X)$ and the set of all pre-closed soft sets by $PCS(X, \tau, E)$, or $PCS(X)$.
- **(2):** If $\gamma = int(cl(int))$, then $\gamma$ is called $\alpha$-open soft operator. We denote the set of all $\alpha$-open soft sets by $\alpha OS(X, \tau, E)$, or $\alpha OS(X)$ and the set of all $\alpha$-closed soft sets by $\alpha CS(X, \tau, E)$, or $\alpha CS(X)$.
- **(3):** If $\gamma = cl(int)$, then $\gamma$ is called semi open soft operator. We denote the set of all semi open soft sets by $SOS(X, \tau, E)$, or $SOS(X)$ and the set of all semi closed soft sets by $SCS(X, \tau, E)$, or $SCS(X)$.
- **(4):** If $\gamma = cl(int(cl))$, then $\gamma$ is called $\beta$-open soft operator. We denote the set of all $\beta$-open soft sets by $\beta OS(X, \tau, E)$, or $\beta OS(X)$ and the set of all $\beta$-closed soft sets by $\beta CS(X, \tau, E)$, or $\beta CS(X)$.

**Definition 2.6** ([12]). Let $(X, \tau, E)$ be a soft topological space, $(F, E) \in SS(X)_E$ . Then, the pre soft interior (resp. semi soft interior, $\alpha$-soft interior, $\beta$-soft interior) of $(F, E)$ is denoted by $PSint(F, E)$ (resp. $SSint(F, E)$, $\alpha Sint(F, E)$, $\beta Sint(F, E)$ ), which is the soft union of all pre open (resp. semi open, $\alpha$-open, $\beta$-open) soft sets contained in $(F, E)$.

**Definition 2.7** ([12]). Let $(X, \tau, E)$ be a soft topological space, $(F, E) \in SS(X)_E$ and $x_e \in SS(X)_E$. Then

- **(1):** $x_e$ is called $\gamma$- interior soft point of $(F, E)$ if $\exists$ $(G, E) \in OS(\gamma)$ such that $x_e \in (G, E) \tilde{\subseteq} (F, E)$, the set of all $\gamma$-interior soft points of $(F, E)$ is called the $\gamma$-soft interior of $(F, E)$ and is denoted by $\gamma Sint(F, E)$ consequently, $\gamma Sint(F, E) = \tilde{\bigcup}\{(G, E) : (G, E) \tilde{\subseteq} (F, E),\ (G, E) \in OS(\gamma)\}$.
- **(2):** $x_e$ is called $\gamma$-closure soft point of $(F, E)$ if $(F, E) \tilde{\cap} (H, E) \neq \tilde{\phi}$ $\forall$ $\gamma$-open soft set $(H, E)$ containing $x_e$. The set of all $\gamma$-closure soft points of $(F, E)$ is called $\gamma$-soft closure of $(F, E)$ and is denoted by $\gamma Scl(F, E)$ consequently, $\gamma Scl(F, E) = \tilde{\bigcap}\{(H, E) : (H, E) \in CS(\gamma),\ (F, E) \tilde{\subseteq} (H, E)\}$.

**Theorem 2.8** ([12]). *Let $(X, \tau, E)$ be a soft topological space and $(F, E) \in SS(X)_E$. Then*

- **(1):** $(F, E) \in SOS(X)$ *if and only if* $cl(F, E) = cl(int(F, E))$.
- **(2):** *If* $(G, E) \in OS(X)$, *then* $(G, E) \tilde{\cap} cl(F, E) \tilde{\subseteq} cl((F, E) \tilde{\cap} (F, E))$.
- **(3):** *If* $(H, E) \in CS(X)$, *then* $int[(G, E) \tilde{\cup} (H, E)] \tilde{\subseteq} int(G, E) \tilde{\cup} (H, E)$.

## 3. More properties of $\gamma$-soft interior and $\gamma$-soft closure

**Theorem 3.1** ([8]). *Let $(X, \tau, E)$ be a soft topological space. Then, the following properties are satisfied:*

- **(1):** $PScl(F, E) = (F, E) \tilde{\cup} cl(int(F, E))$.
- **(2):** $PSint(F, E) = (F, E) \tilde{\cap} int(cl(F, E))$.
- **(3):** $\alpha Scl(F, E) = (F, E) \tilde{\cup} cl(int(cl(F, E)))$.
- **(4):** $\alpha Sint(F, E) = (F, E) \tilde{\cap} int(cl(int(F, E)))$.
- **(5):** $SScl(F, E) = (F, E) \tilde{\cup} int(cl(F, E))$.

**(6):** $SSint(F,E) = (F,E)\tilde{\cap}cl(int(F,E))$.
**(7):** $\beta Scl(F,E) = (F,E)\tilde{\cup}int(cl(int(F,E)))$.
**(8):** $\beta Sint(F,E) = (F,E)\tilde{\cap}cl(int(cl(F,E)))$.

*Proof.* We shall prove only the first statement, the other cases are similar. Since $cl(int[(F,E)\tilde{\cup}cl(int(F,E))])\tilde{\subseteq}cl[int(F,E)\tilde{\cup}cl(int(F,E))] = cl(int(F,E))\tilde{\cup}cl(int(F,E)) = cl(int(F,E))\tilde{\subseteq}(F,E)\tilde{\cup}cl(int(F,E))$ from Theorem 2.8 (3). This means that, $(F,E)\tilde{\cup}cl(int(F,E))$ is a pre closed soft set containing $(F,E)$. So, $PScl(F,E)\tilde{\subseteq}(F,E)\tilde{\cup}cl(int(F,E))$. On the other hand, $PScl(F,E)$ is pre closed soft. So, we have $cl(int(F,E))\tilde{\subseteq}cl(int(PScl(F,E)))\tilde{\subseteq}PScl(F,E)$. Hence, $(F,E)\tilde{\cup}cl(int(F,E))\tilde{\subseteq}PScl(F,E)$. Therefore, $PScl(F,E) = (F,E)\tilde{\cup}cl(int(F,E))$. The rest of the proof by a similar way. □

**Theorem 3.2.** *Let $(X,\tau,E)$ be a soft topological space. Then, the following properties are satisfied:*

**(1):** $PScl(PSint(F,E)) = PSint(F,E)\tilde{\cup}cl(int(F,E))$.
**(2):** $SScl(Sint(F,E)) = SSint(F,E)\tilde{\cup}cl(int(cl(F,E)))$.

*Proof.* **(1):** Since $cl(int[PSint(F,E)\tilde{\cup}cl(int(F,E))])\tilde{\subseteq}cl[int(PSint(F,E))\tilde{\cup}cl(int(F,E))] = cl(int(PSint(F,E)))\tilde{\cup}cl(int(F,E)) = cl(int(F,E))\tilde{\subseteq}PSint(F,E)\tilde{\cup}cl(int(F,E))$ from Theorem 2.8 (3). This means that, $PSint(F,E)\tilde{\cup}cl(int(F,E))$ is a pre closed soft set containing $PSint(F,E)$. So, $PScl(PSint(F,E))\tilde{\subseteq}PSint(F,E)\tilde{\cup}cl(int(F,E))$. On the other hand, $PScl(PSint(F,E))$ is the largest pre closed soft set containing $PSint(F,E)$. Hence, $PSint(F,E)\tilde{\cup}cl(int(F,E))\tilde{\subseteq}PScl(PSint(F,E))$. Therefore, $PScl(PSint(F,E)) = PSint(F,E)\tilde{\cup}cl(int(F,E))$.
**(2):** By a similar way.

□

**Definition 3.3** ([8]). Let $(X,\tau,E)$ be a soft topological space and $(F,E)\in SS(X)_E$. Then, $(F,E)$ is called a b-open soft set if $(F,E)\tilde{\subseteq}cl(int(F,E))\tilde{\cup}int(cl(F,E))$. The set of all b-open soft sets is denoted by $BOS(X,\tau,E)$, or $BOS(X)$ and the set of all b-closed soft sets is denoted by $BCS(X,\tau,E)$, or $BCS(X)$.

**Remark 3.4** ([8]). It is obvious that, $POS(X)\cup SOS(X)\subseteq BOS(X)\subseteq \beta OS(X)$.

**Theorem 3.5.** *Let $(X,\tau,E)$ be a soft topological space. Then, the following are equivalent:*

**(1):** *$(F,E)$ is a b-open soft set.*
**(2):** $(F,E) = PSint(F,E)\tilde{\cup}Sint(F,E)$.
**(3):** $(F,E)\tilde{\subseteq}PScl(PSint(F,E))$.

*Proof.* (1) $\Rightarrow$ (2)**:** Let $(F,E)$ be a b-open soft set. Then, $(F,E)\tilde{\subseteq}cl(int(F,E))\tilde{\cup}int(cl(F,E))$. By Theorem 3.1, $PSint(F,E)\tilde{\cup}Sint(F,E) = [(F,E)\tilde{\cap}int(cl(F,E))]\tilde{\cup}[(F,E)\tilde{\cap}cl(int(F,E))] = (F,E)\tilde{\cap}[int(cl(F,E))\tilde{\cup}\ cl(int(F,E))] = (F,E)$.
(2) $\Rightarrow$ (3)**:** $(F,E) = PSint(F,E)\tilde{\cup}Sint(F,E) = PSint(F,E)\tilde{\cup}[(F,E)\tilde{\cap}cl(int(F,E))]\tilde{\subseteq}PSint(F,E)\tilde{\cup}cl(int(F,E)) = PScl(PSint(F,E))$, from Theorem 3.1 (3) and Theorem 3.2 (1).
(3) $\Rightarrow$ (1)**:** $(F,E)\tilde{\subseteq}PScl(PSint(F,E)) = PSint(F,E)\tilde{\cup}cl(int(F,E))\tilde{\subseteq}int(cl(F,E))\tilde{\cup}cl(int(F,E))$, from Theorem 3.1 (1) and Theorem 3.2 (1).

□

**Theorem 3.6.** *Let $(X, \tau, E)$ be a soft topological space and $(F, E) \in SS(X)_E$. Then, the following properties are satisfied:*

**(1):** $bScl(F, E) = Scl(F, E) \tilde{\cap} PScl(F, E)$.
**(2):** $bSint(F, E) = Sint(F, E) \tilde{\cup} PSint(F, E)$.

*Proof.* **(1):** Since $bScl(F, E)$ is a b-closed soft set. Then, $cl(int(bScl(F, E))) \tilde{\cap} int(cl(bScl(F, E))) \tilde{\subseteq} bScl(F, E)$. It follows that, $cl(int(F, E)) \tilde{\cap} int(cl(F, E)) \tilde{\subseteq} bScl(F, E)$. So, $(F, E) \tilde{\cup} [cl(int(F, E)) \tilde{\cap} int(cl(F, E))] \tilde{\subseteq} (F, E) \tilde{\cup} bScl(F, E) = bScl(F, E)$. Hence, $[(F, E) \tilde{\cup} cl(int(F, E))] \tilde{\cap} [(F, E) \tilde{\cup} int(cl(F, E))] = Scl(F, E) \tilde{\cap} PScl(F, E)$, from Theorem 3.1. This means that, $Scl(F, E) \tilde{\cap} PScl(F, E) \tilde{\subseteq} bScl(F, E)$. The reverse inclusion is obvious from Remark 3.4.

**(2):** By a similar way.

□

## 4. Counter examples on soft continuity in (supra) soft topological spaces

**Remark 4.1.** The converse of [ [12], Theorem 5.3] is not true in general, as shown in the following examples.

**Examples 4.2.** **(1):** Let $X = \{a, b, c, d\}$, $Y = \{x, y, z, w\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = x$, $u(b) = z$ , $u(c) = y$ , $u(d) = w$,
$p(e_1) = k_1$, $p(e_2) = k_2$. Let $(X, \tau_1, A)$ be a soft topological space over $X$ where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_2, A), (F_3, A)\}$, where $(F_1, A), (F_2, A), (F_3, A)$ are soft sets over $X$ defined as follows:
$F_1(e_1) = \{d\}$, $F_1(e_2) = \{a, b\}$.
$F_2(e_1) = \{a, b\}$, $F_2(e_2) = \{d\}$.
$F_3(e_1) = \{a, b, d\}$, $F_3(e_2) = \{a, b, d\}$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x\}$, $G(k_2) = \{z\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a\}), (e_2, \{b\})\}$ is a pre (resp. $\beta$-) open soft set, but not open soft. Hence, $f_{pu}$ is a pre- (resp. $\beta$-) continuous soft function, but not continuous soft.

**(2):** Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = x$, $u(b) = z$ , $u(c) = y$,
$p(e_1) = k_2$, $p(e_2) = k_1$. Let $(X, \tau_1, A)$ be a soft topological space over $X$ where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, (F, A)\}$, where $(F, A)$ is a soft set over $X$ defined as follows:
$F(e_1) = \{b\}$, $F(e_2) = \{a\}$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:

$G(k_1) = \{x\}, \quad G(k_2) = \{x, z\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a, b\}), (e_2, \{b\})\}$ is a semi (resp. $\alpha$-) open soft set, but not open soft set. Hence, $f_{pu}$ is a semi- (resp. $\alpha$-) continuous soft function, but not continuous soft.

**Remark 4.3.** The converse of [ [12], Theorem 5.4] is not true in general, as shown in the following examples.

**Examples 4.4.** **(1):** Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = x, \ u(b) = z \ , \ u(c) = z$,
$p(e_1) = k_1, \ p(e_2) = k_2$. Let $(X, \tau_1, A)$ be a soft topological space over $X$ where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_2, A), (F_3, A)\}$, where $(F_1, A), (F_2, A), (F_3, A)$ are soft sets over $X$ defined as follows:
$F_1(e_1) = \{a\}, \quad F_1(e_2) = \{a\}$.
$F_2(e_1) = \{b\}, \quad F_2(e_2) = \{b\}$.
$F_3(e_1) = \{a, b\}, \quad F_3(e_2) = \{a, b\}$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{z\}, \quad G(k_2) = \{z\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}$ is a semi open soft set, but not $\alpha$-open soft. Hence, $f_{pu}$ is a semi-continuous soft function, but not $\alpha$-continuous soft.

**(2):** In Examples 4.2 (1), let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x\}, \quad G(k_2) = \{y\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a\}), (e_2, \{c\})\}$ is a $\beta$-open soft set, but not semi open soft. Hence, $f_{pu}$ is a $\beta$-continuous soft function, but not semi-continuous soft.

**(3):** Let $X = \{a, b, c, d\}$, $Y = \{x, y, z, w\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = x, \ u(b) = z \ , \ u(c) = w \ , \ u(d) = z$,
$p(e_1) = k_1, \ p(e_2) = k_2$. Let $(X, \tau_1, A)$ be a soft topological space over $X$ where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$, where $(F_1, A), (F_2, A)$, $(F_3, A), (F_4, A)$ are soft sets over $X$ defined as follows:
$F_1(e_1) = \{a\}, \quad F_1(e_2) = \{b\}$.
$F_2(e_1) = \{b\}, \quad F_2(e_2) = \{a\}$.
$F_3(e_1) = \{a, b\}, \quad F_3(e_2) = \{a, b\}$.
$F_4(e_1) = \{a, b, d\}, \quad F_4(e_2) = \{a, b, d\}$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x, z\}, \quad G(k_2) = \{y, z\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) =$

$\{(e_1, \{a, d\}), (e_2, \{b, d\})\}$ is a $\beta$-open soft set, but not pre open soft. Hence, $f_{pu}$ is a $\beta$-continuous soft function, but not pre-continuous soft.

**(4):** In (1), let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{z\}, \quad G(k_2) = \{x\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{b, c\}), (e_2, \{a\})\}$ is a pre open soft set, but not $\alpha$-open soft set. Hence, $f_{pu}$ is a pre-continuous soft function, but not $\alpha$-continuous soft.

**Remark 4.5.** The converse of [ [9], Theorem 6.2] is not true in general, as shown in the following examples.

**Examples 4.6.** **(1):** Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = x, \ u(b) = z \ , \ u(c) = y$,
$p(e_1) = k_2, \ p(e_2) = k_1$. Let $(X, \tau_1, A)$ be a soft topological space over $X$ where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A)\}$, where $(F_1, A)$ is a soft set over $X$ defined as follows:
$F(e_1) = \{a, b\}, \quad F(e_2) = \{a, b\}$.
The supra soft topology $\mu_1$ is defined as follows, $\mu_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_2, A), (F_3, A)\}$, where $(F_1, A), (F_2, A), (F_3, A)$ are soft sets over $X$ defined as follows:
$F_1(e_1) = \{a\}, \quad F_1(e_2) = \{a\}$.
$F_2(e_1) = \{a, b\}, \quad F_2(e_2) = \{a, b\}$.
$F_3(e_1) = \{b, c\}, \quad F_2(e_2) = \{b, c\}$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x, y\}, \quad G(k_2) = \{x, y\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$ is a supra pre (resp. $\beta$-) open soft set, but not supra open soft set. Hence, $f_{pu}$ is a supra pre- (resp. $\beta$-) continuous soft function, but not supra continuous soft.

**(2):** Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = x, \ u(b) = z \ , \ u(c) = y$,
$p(e_1) = k_2, \ p(e_2) = k_1$. Let $(X, \tau_1, A)$ be a soft topological space over $X$ where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A)\}$, where $(F_1, A)$ is a soft set over $X$ defined as follows:
$F(e_1) = \{a, b\}, \quad F(e_2) = \{a\}$.
The supra soft topology $\mu_1$ is defined as follows, $\mu_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A), (F_2, A)\}$, where $(F_1, A), (F_2, A)$ are soft sets over $X$ defined as follows:
$F_1(e_1) = \{a, b\}, \quad F_1(e_2) = \{a\}$.
$F_2(e_1) = \{a, c\}, \quad F_2(e_2) = \{b, c\}$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x, y\}, \quad G(k_2) = \{x, z\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) =$

$\{(e_1, \{a, b\}), (e_2, \{a, c\})\}$ is a supra semi (resp. $\alpha$-) open soft set, but not supra open soft set. Hence, $f_{pu}$ is a supra semi- (resp. $\alpha$-) continuous soft function, but not supra continuous soft.

**Remark 4.7.** The converse of [ [9], Theorem 6.3] is not true in general, as shown in the following examples.

**Examples 4.8.** **(1):** Let $X = \{a, b, c, d\}$, $Y = \{x, y, z, w\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = y$, $u(b) = x$ , $u(c) = x$, $u(d) = x$,
$p(e_1) = k_1$, $p(e_2) = k_2$. Let $(X, \tau_1, A)$ be a soft topological space over $X$ where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A)\}$, where $(F_1, A)$ is a soft set over $X$ defined as follows:
$F(e_1) = \{a, b\}$, $F(e_2) = \{a, b\}$.
The supra soft topology $\mu_1$ is defined as follows, $\mu_1 = \{\tilde{X}, \tilde{\phi}, (F_1, A),$ $(F_2, A), (F_3, A), (F_4, A)\}$, where $(F_1, A), (F_2, A), (F_3, A), (F_4, A)$ are soft sets over $X$ defined as follows:
$F_1(e_1) = \{a\}$, $F_1(e_2) = \{a\}$.
$F_2(e_1) = \{a, b\}$, $F_2(e_2) = \{a, b\}$.
$F_3(e_1) = \{b, c\}$, $F_2(e_2) = \{b, c\}$.
$F_4(e_1) = \{a, b, c\}$, $F_4(e_2) = \{a, b, c\}$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x, y\}$, $G(k_2) = \{x, w\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{b, c, d\}), (e_2, \{b, c, d\})\}$ is a supra semi open soft set, but not supra $\alpha$-open soft set. Hence, $f_{pu}$ is a supra semi-continuous soft function, but not supra $\alpha$-continuous soft.

**(2):** In (1), let $u : X \to Y$ defined as follows:
$u(a) = y$, $u(b) = x$ , $u(c) = y$ , $u(d) = y$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x, y\}$, $G(k_2) = \{x, w\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{b\}), (e_2, \{b\})\}$ is a supra $\beta$-open soft set, but not supra semi open soft set. Hence, $f_{pu}$ is a supra $\beta$-continuous soft function, but not supra semi-continuous soft.

**(3):** In (1), let $u : X \to Y$ defined as follows:
$u(a) = x$, $u(b) = y$ , $u(c) = y$ , $u(d) = x$.
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x, z\}$, $G(k_2) = \{x, w\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a, d\}), (e_2, \{a, d\})\}$ is a supra $\beta$-open soft set, but not supra pre open soft set. Hence, $f_{pu}$ is a supra $\beta$-continuous soft function, but not supra pre-continuous soft.

**(4):** In (1), let $u : X \to Y$ defined as follows:
$u(a) = x,\ u(b) = y\ ,\ u(c) = x\ ,\ u(d) = y.$
Let $(Y, \tau_2, B)$ be a soft topological space over $Y$ where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, (G, B)\}$, where $(G, B)$ is a soft set over $Y$ defined by:
$G(k_1) = \{x, z\}, \quad G(k_2) = \{x, w\}.$
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$ is a supra pre open soft set, but not supra $\alpha$-open soft set. Hence, $f_{pu}$ is a supra pre-continuous soft function, but not supra $\alpha$-continuous soft.

## 5. Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [28] and easily applied to many problems having uncertainties from social life. The main purpose of this paper, is to introduce more properties of of $\gamma$-soft interior and $\gamma$-soft closure mentioned in [8, 12]. Also, counter examples of (supra) soft continuity, mentioned in [9, 12], are introduced. In the next study, we extend the notion of b-open soft sets to supra soft topological spaces and and other topological properties. Also, we will use some topological tools in soft set application, like rough sets.

## 6. Acknowledgements

The authors express their sincere thanks to the reviewers for their valuable suggestions. The authors are also thankful to the editors-in-chief and managing editors for their important comments which helped to improve the presentation of the paper.

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