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On intuitionistic fuzzy ψ closed sets

M. Parimala, C. Indirani, A. Selvakumar

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Abstract. This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper we discuss the notion of intuitionistic fuzzy ψ closed sets and derive some of its properties.

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Corresponding Author: A.Selvakumar (selvam_mphil@yahoo.com)

1. Introduction

Fuzzy set as proposed by Zadeh [12] in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. By adding the degree of non-membership to fuzzy set, Atanassov [1] proposed intuitionistic fuzzy set in 1986 which appeals more accurate to uncertainty quantification and provides the opportunity to precisely model the problem, based on the existing knowledge and observations. Coker [2] introduced the concept of intuitionistic fuzzy topological space in 1997. In 2008, Thakur and Chaturvedi [9] extended the concepts of fuzzy g-closed sets and fuzzy g-continuity in intuitionistic fuzzy topological spaces. In this paper, we discuss the notion of intuitionistic fuzzy ψ closed sets and introduce intuitionistic fuzzy ψ-$T_{1/2}$ spaces and intuitionistic fuzzy ψ-T_R spaces. We provide some characterizations of intuitionistic fuzzy ψ closed sets and establish the relationships with other classes of early defined forms of intuitionistic fuzzy sets.

2. Preliminaries

Let X be a nonempty fixed set. An intuitionistic fuzzy set [1] A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A : A \to [0,1]$ and $\nu_A : A \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of nonmembership $\nu_A(x)$ of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ is called a subset of an intuitionistic fuzzy set $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$. The intersection (respectively union) of any arbitrary family of intuitionistic fuzzy sets of X are the intuitionistic fuzzy set $A_i = \{\langle x, \mu_{A_i}(x), \nu_{A_i}(x)\rangle : x \in X, i \in \Lambda\}$ of X are the intuitionistic fuzzy set $\cap A_i = \{\langle x, \wedge \mu_{A_i}(x), \vee \nu_{A_i}(x)\rangle : x \in X, i \in \Lambda\}$ (resp. $\cup A_i = \{\langle x, \vee \mu_{A_i}(x), \wedge \nu_{A_i}(x)\rangle : x \in X, i \in \Lambda\}$). Two intuitionistic fuzzy sets $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$ are said to be q-coincident (AqB for short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. A family τ of intuitionistic fuzzy sets on a nonempty set X is called an intuitionistic fuzzy topology [2] on X if the intuitionistic fuzzy sets $0_\sim$ and $1_\sim \in \tau$ and τ is closed under arbitrary union and finite intersection. The ordered pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and each intuitionistic fuzzy set in τ is called an intuitionistic fuzzy open set. The complement of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set. The intersection of all intuitionistic fuzzy closed sets containing A is called the closure of A, which is denoted by $\mathrm{cl}(A)$. The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted by $\mathrm{int}(A)$ [2].

Lemma 2.1 ([2])**.** *Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space (X, τ). Then*

(1) $\neg(AqB) \Rightarrow A \subseteq B^c$.
(2) *A is an intuitionistic fuzzy closed set in $X \Leftrightarrow cl(A) = A$.*
(3) *A is an intuitionistic fuzzy open set in $X \Leftrightarrow int(A) = A$.*
(4) $cl(A^c) = (int(A))^c$.
(5) $int(A^c) = (cl(A))^c$.
(6) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$.
(7) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$.
(8) $cl(A \cup B) = cl(A) \cup cl(B)$.
(9) $int(A \cap B) = int(A) \cap int(B)$.

Definition 2.2 ([3])**.** Let X be a nonempty set and $c \in X$ is a fixed element. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha + \beta \leq 1$, then $c(\alpha, \beta) = \langle x, \alpha, 1 - \beta\rangle$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of $c(\alpha, \beta)$ and β denotes the degree of nonmembership of $c(\alpha, \beta)$.

Definition 2.3. An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an

(a) intuitionistic fuzzy pre open set [5] (IFPOS in short) if $A \subseteq \mathrm{int}(\mathrm{cl}(A))$.
(b) intuitionistic fuzzy regular open set [5] (IFROS in short) if $A = \mathrm{int}(\mathrm{cl}(A))$.
(c) intuitionistic fuzzy α-open set [5] (IFαOS in short) if $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$.
(d) intuitionistic fuzzy semi open set [5] (IFSOS in short) if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$.
(e) intuitionistic fuzzy semi-preopen set [11] (IFSPOS in short) if there exists an intuitionistic fuzzy preopen set B such that $B \subseteq A \subseteq \mathrm{cl}(B)$.

An IFS A is called an intuitionistic fuzzy semi closed set, intuitionistic fuzzy α-closed set, intuitionistic fuzzy pre-closed set, intuitionistic fuzzy regular closed set, intuitionistic fuzzy semi pre-closed set (IFSCS , IFαCS, IFPCS, IFRCS and IFSPCS resp.), if the complement A^c is an IFSOS, IFαOS, IFPOS, IFROS and IFSPOS respectively.

Definition 2.4. If A is an intuitionistic fuzzy set in intuitionistic fuzzy topological space (X, τ), then

(a) $\mathrm{scl}(A) = \cap\{F : A \subseteq F, F$ is intuitionistic fuzzy semi-closed set$\}$ [5].
(b) $\mathrm{sint}(A) = \cup\{F : A \supseteq F, F$ is intuitionistic fuzzy semi-open set$\}$ [5].
(c) $\alpha\mathrm{cl}(A) = \cap\{F : A \subseteq F, F$ is intuitionistic fuzzy α-closed set$\}$ [5].
(d) $\mathrm{pcl}(A) = \cap\{F : A \subseteq F, F$ is intuitionistic fuzzy pre-closed set$\}$ [5].
(e) $\mathrm{spcl}(A) = \cap\{F : A \subseteq F, F$ is intuitionistic fuzzy semi pre-closed set$\}$ [11].

Definition 2.5. An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an

(a) intuitionistic fuzzy sg-closed (IFSGCS in short) [6] if $\mathrm{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy semi open in X.
(b) intuitionistic fuzzy gs-closed (IFGSCS in short) [8] if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy semi open in X.
(c) intuitionistic fuzzy gsp-closed (IFGSPCS in short) [7] if $\mathrm{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy open in X.

An IFS A is called an intuitionistic fuzzy sg-open set, intuitionistic fuzzy gs-open set and intuitionistic fuzzy gsp-open set (IFSGOS , IFGSOS and IFGSPOS resp.), if the complement A^c is an IFSGCS, IFGSCS and IFGSPCS respectively.

Definition 2.6 ([6])**.** An intuitionistic fuzzy topological space (X, τ) is said to be an intuitionistic fuzzy semi-$T_{1/2}$ space if every IFSGCS in X is an IFSCS in X.

Definition 2.7 ([8])**.** An intuitionistic fuzzy topological space (X, τ) is said to be an intuitionistic fuzzy ${}_cT_{1/2}$ space if every IFGSCS in X is an IFCS in X.

Definition 2.8 ([10])**.** Two intuitionistic fuzzy sets A and B in an intuitionistic fuzzy topological space (X, τ) are called q-separated if $\mathrm{cl}(A) \cap B = 0_\sim = A \cap \mathrm{cl}(B)$.

3. Properties of Intuitionistic Fuzzy ψ closed sets

In this section, we derive some properties of intuitionistic fuzzy ψ closed sets.

Definition 3.1. An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is said to be an

(a) intuitionistic fuzzy ψ closed set (IFψCS in short) [4] if $\mathrm{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is IFSGOS in X.

(b) intuitionistic fuzzy $\alpha\psi$ closed set (IF$\alpha\psi$CS in short) [4] if ψcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is IFαOS in X.

An IFS A is called an intuitionistic fuzzy ψ-open set and intuitionistic fuzzy $\alpha\psi$-open set (IFψOS and IF$\alpha\psi$OS resp.), if the complement A^c is an IFψCS and IF$\alpha\psi$CS respectively.

Example 3.2. Let $X = \{a, b\}$ and V $= \langle x, (0.3, 0.4), (0.2, 0.5)\rangle$.Then $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X is and the IFS A $= \langle x, (0.1, 0.4), (0.6, 0.5)\rangle$ is an IFψCS in (X, τ).

Theorem 3.3. *Let (X, τ) be an intuitionistic fuzzy topological space. Then the following are hold:*

(a) *Every IFCS in X is an IFψCS in X.*
(b) *Every IFRCS in X is an IFψCS in X.*
(c) *Every IFαCS and hence IFSCS in X is an IFψCS in X.*
(d) *Every IFψCS in X is an IFSPCSin X.*
(e) *Every IFψCS in X is an IFGSPCS in X.*
(f) *Every IFψCS in X is an IFGSCS and hence IFSGCS in X.*
(g) *Every IFψCS in X is an IF$\alpha\psi$CS in X.*

Proof. Let (X, τ) be an intuitionistic fuzzy topological space.

(a) It is obvious.
(b) It follows from the fact that every IFRCS is an IFCS in X.
(c) Let A be an IFαCS and hence IFSCS in X. Let $A \subseteq U$ and U is an IFSGOS in X. By hypothesis scl$(A) = A$, hence scl$(A) \subseteq U$. Therefore A is an IFψCS in X.
(d) It is obvious.
(e) Let A be an IFψCS in X. Let $A \subseteq U$ and U is an IFOS in X and hence U is an IFSOS in X. Since A is an IFψCS, scl$(A) \subseteq U$ which implies spcl$(A) \subseteq U$. Therefore A is an IFGSPCS in X.
(f) Let A be an IFψCS in X. Let $A \subseteq U$ and U is an IFOS in X and hence U is an IFSOS in X. Since A is an IFψCS, scl$(A) \subseteq U$. Therefore A is an IFGSCS and hence IFSGCS in X.
(g) Let A be an IFψCS in X. Let $A \subseteq U$ and U is an IFαOS in X and hence U is an IFSOS in X. Since A is an IFψCS, scl$(A) \subseteq U$ which implies ψcl$(A) \subseteq U$. Therefore A is an IF$\alpha\psi$CS in X.

The converse of the above theorem need not be true as shown by the following examples. □

Example 3.4. Let (X, τ) be an intuitionistic fuzzy topological space.

(a) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.4, 0.3), (0.5, 0.3)\rangle$. Let $A = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$ be any IFS in X. Here scl$(A) \subseteq V$, whenever $A \subseteq V$ for all IFSGOS V in X. Therefore A is IFψCS in X, since cl$(A) = V^c \neq A$.
(b) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.2, 0.2), (0.4, 0.5)\rangle$. Take $A = \langle x, (0.1, 0.1), (0.5, 0.5)\rangle$ be any IFS in X. Clearly scl$(A) \subseteq V$ whenever $A \subseteq V$ for all IFSGOS V in X. Therefore A is an IFψCS, but not an IFRCS in X.

(c) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.2, 0.2), (0.4, 0.5)\rangle$. Take $A = \langle x, (0.1, 0.1), (0.5, 0.5)\rangle$ be any IFS in X. Here $\mathrm{scl}(A) = V$, clearly $\mathrm{scl}(A) \subseteq V$ whenever $A \subseteq V$ for all IFSGOS V in X. Therefore A is an IFψCS, but not IFSCS and hence IFαCS in X, since $\mathrm{scl}(A) \nsubseteq A$.

(d) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.5, 0.5), (0.3, 0.2)\rangle$. Let $A = \langle x, (0.5, 0.5), (0.3, 0.3)\rangle$ be any IFS in X. Clearly $\mathrm{scl}(A) \subseteq V$ whenever $A \subseteq V$ for all IFSGOS V in X. Therefore A is an IFψCS, but not IFSPCS in X, since $\mathrm{spcl}(A) \nsubseteq A$.

(e) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Take $A = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$ be any IFS in X. Here IFOS $V_1 = \langle x, (0.7, 0.6), (0.3, 0.2)\rangle$, clearly $A \subseteq V_1$. Therefore A is an IFGSPCS in X, but not IFψCS in X, since $\mathrm{scl}(A) \nsubseteq V_1$.

(f) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Take $A = \langle x, (0.6, 0.6), (0.2, 0.2)\rangle$ be any IFS in X. Here IFOS $V_1 = \langle x, (0.7, 0.6), (0.2, 0.2)\rangle$, clearly $A \subseteq V_1$. Therefore A is an IFGSCS and hence IFSGCS in X, but not IFψCS in X, since $\mathrm{scl}(A) \nsubseteq V_1$.

(g) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Take $A = \langle x, (0.6, 0.6), (0.2, 0.2)\rangle$ be any IFS in X. Here IFαOS $V_1 = \langle x, (0.7, 0.6), (0.2, 0.2)\rangle$, clearly $\psi\mathrm{cl}(A) \subseteq V_1$. Therefore A is an IF$\alpha\psi$CS in X, but not IFψCS in X, since $\mathrm{scl}(A) \nsubseteq V_1$.

Remark 3.5. The concepts of IFP closedness and IFψ closedness are independent of each other as seen from the following two examples.

Example 3.6. Let $X = \{a, b\}$ and $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Take $A = \langle x, (0.5, 0.4), (0.3, 0.4)\rangle$ be any IFS in X. Here the only IFSOS are $V_1 = \{0_\sim,\ 1_\sim,\ V\}$. Therefore A is an IFPCS in X, but not IFψCS in X, since $\mathrm{scl}(A) \nsubseteq V_1$.

Example 3.7. Let $X = \{a, b\}$ and $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Take $A = \langle x, (0.6, 0.6), (0.2, 0.2)\rangle$ be any IFS in X.Clearly $\mathrm{scl}(A) \subseteq V$ whenever $A \subseteq V$ for all IFSGOS V in X. Therefore A is an IFψCS, but not IFPCS in X, since $\mathrm{pcl}(A) \nsubseteq A$.

Remark 3.8. The union of any two IFψCS's need not be an IFψCS in general as seen from the following example.

Example 3.9. Let $X = \{a, b\}$ and $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.5, 0.7), (0.4, 0.3)\rangle$. Here $A = \langle x, (0.2, 0.7), (0.8, 0.3)\rangle$ and $B = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ are IFψCS's in X, but $A \cup B$ is not an IFψCS in X.

Remark 3.10. From the Theorem 3.3 and Example 3.4, we have the following diagram of implication.

$$\begin{array}{ccccc}
\mathbf{IF}\alpha CS & \rightarrow & \mathbf{IFSCS} \rightarrow \mathbf{IFGSCS} \\
\uparrow & \searrow & \downarrow \quad \nearrow \quad \downarrow \\
\mathbf{IFRCS} & \rightarrow & \mathbf{IF}\psi\mathbf{CS} \rightarrow \mathbf{IFSGCS} \\
\downarrow & \swarrow & \downarrow \quad \searrow \quad \downarrow \\
\mathbf{IF}\alpha\psi\mathbf{CS} & & \mathbf{IFGSPCS} \leftarrow \mathbf{IFSPCS}
\end{array}$$

Theorem 3.11. *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is IFψ-open if and only if $B \subseteq sint(A)$ whenever B is an IFSGCS and $B \subseteq A$.*

Proof. **Necessity:** Let A be IFψ-open in X. Let B be an IFSGCS in X such that $B \subseteq A$. Then B^c is an IFSGOS in X such that $A^c \subseteq B^c$. Now by hypothesis A^c is an IFψCS, we have $\text{scl}(A^c) \subseteq B^c$. But $\text{scl}(A^c) = (\text{sint}(A))^c$. Hence $(\text{sint}(A))^c \subseteq B^c$, which implies $B \subseteq \text{sint}(A)$.
Sufficiency: Let F be an IFSGOS in X such that $A^c \subseteq F$. Then F^c is an IFSGCS in X and $F^c \subseteq A$. Therefore by hypothesis $F^c \subseteq \text{sint}(A)$. This implies that $\text{scl}(A^c) = (\text{sint}(A))^c \subseteq F$. Hence A^c is an IFψCS and A is an IFψOS in X. ☐

Theorem 3.12. *If the concepts of IFSCS and IFGOS are coincide, then every intuitionistic fuzzy subset of X is an IFψCS.*

Proof. Let A be an intuitionistic fuzzy subset of X, such that $A \subseteq U$, where U is an IFSGOS. Then U is IFSCS such that $\text{sc1}(A) \subseteq \text{scl}(U) = U$. Hence $\text{sc1}(A) \subseteq U$. Therefore A is an IFψCS. ☐

Theorem 3.13. *Let A be an IFψCS subset of (X, τ). Then $scl(A) - A$ does not contain any non-empty IFSGCS.*

Proof. Assume that A is IFψCS. Let F be an non-empty IFSGCS, such that $F \subseteq \text{scl}(A) - A = \text{scl}(A) \cap A^c$. (i.e) $F \subseteq \text{scl}(A)$ and $F \subseteq A^c$. Therefore $A \subseteq F^c$. Since F^c is an IFSGOS, $\text{scl}(A) \subseteq F^c$ implies $F \subseteq (\text{scl}(A))^c$. But we have, $F \subseteq \text{scl}(A) - A$. So $F \subseteq (\text{scl}(A) - A) \cap$ $(\text{scl}(A))^c \subseteq \text{scl}(A) \cap$ $(\text{scl}(A))^c$. (i.e) $F \subseteq \phi$. Therefore F is empty. ☐

Theorem 3.14. *Let A be an IFψCS in an intuitionistic fuzzy topological space (X, τ) and $A \subseteq B \subseteq scl(A)$. Then B is an IFψCS in X.*

Proof. Let A be an IFψCS in an intuitionistic fuzzy topological space (X, τ) such that $A \subseteq B \subseteq \text{scl}(A)$. Let U be an IFSGOS such that $B \subseteq U$. Then $A \subseteq U$ and since A is an IFψCS, we have $\text{scl}(A) \subseteq U$. Now $B \subseteq \text{scl}(A) \Rightarrow \text{scl}(B) \subseteq \text{scl}(\text{scl}(A)) \subseteq \text{scl}(A) \subseteq U$. Therefore, B is an IFψCS in X. ☐

Theorem 3.15. *If A is an IFSGOS and IFψCS in intuitionistic fuzzy topological space (X, τ), then A is an IFSCS and hence IFSP clopen.*

Proof. Suppose that A is an IFSGOS and IFψCS in X. Since $A \subseteq A$, we have $\text{scl}(A) \subseteq A$. Also $A \subseteq \text{scl}(A)$. Therefore $\text{scl}(A) = A$. Hence A is an IFSCS in X.

Now A is an IFSGOS, then A is an IFSPOS and A is an IFψCS. Therefore A is an IFSPCS and hence A ia an IFSP clopen. □

Theorem 3.16. *Let (X, τ) be an intuitionistic fuzzy topological space. Then A is an IFψCS if and only if $\neg(AqF) \Rightarrow \neg(scl(A)qF)$ for every IFSGCS F of X.*

Proof. **Necessity:** Let A be an IFψCS and F be an IFSGCS of X such that $\neg(AqF)$. Then by Lemma 2.1(1), $A \subseteq F^c$ and F^c is an IFSGOS in X. Therefore, $\mathrm{scl}(A) \subseteq F^c$ because A is an IFψCS. Hence by Lemma 2.1(1), $\neg(\mathrm{scl}(A)qF)$.
Sufficiency: Let U be an IFSGOS of X such that $A \subseteq U$. (i.e.) $A \subseteq (U^c)^c$. Then by Lemma 2.1(1), $\neg(AqU^c)$ and U^c is an IFSGCS in X. Hence by hypothesis $\neg(\mathrm{scl}(A)qU^c)$. Therefore by Lemma 2.1(1), $\mathrm{scl}(A) \subseteq (U^c)^c$. (i.e.) $\mathrm{scl}(A) \subseteq U$. Hence A is an IFψCS in X. □

Theorem 3.17. *Let A be an IFψCS in an intuitionistic topological space (X, τ) and $c(\alpha, \beta)$ be an IF point of X, such that $c(\alpha, \beta)q$ $cl(int(A))$ then $cl(int(c(\alpha, \beta))qA$.*

Proof. If $\neg\mathrm{cl}(\mathrm{int}(c(\alpha, \beta)))qA$ then by Lemma 2.1(a), $\mathrm{cl}(\mathrm{int}(c(\alpha, \beta))) \subseteq A^c$ which implies that $A \subseteq (\mathrm{cl}(\mathrm{int}(c(\alpha, \beta))))^c$ and so $\mathrm{cl}(A) \subseteq (\mathrm{cl}(\mathrm{int}(c(\alpha, \beta))))^c \subseteq (c(\alpha, \beta))^c$, because A is an IFψCS in X. Hence by Lemma 2.1(a), $\neg(c(\alpha, \beta)q\ \mathrm{cl}(\mathrm{int}(A)))$, a contradiction □

4. Application of Intuitionistic Fuzzy ψ closed sets

In this section, we introduce IFψ-$T_{1/2}$ space and IFψ-T_R space, which utilizes IFψCS and its characterizations are proved.

Definition 4.1. An intuitionistic fuzzy topological space (X, τ) is called an

(a) IFψ-$T_{1/2}$ space if every IFψCS is an IFSCS.
(b) IFψ-T_R space if every IFψCS is an IFRCS.

Theorem 4.2. *For an intuitionistic fuzzy topological space (X, τ), the following statements are hold:*

(a) *Every IFψ-T_R space is an IFψ-$T_{1/2}$space.*
(b) *Every IF Semi-$T_{1/2}$ space is an IFψ-$T_{1/2}$ space.*
(c) *Every IF ${}_cT_{1/2}$ space is an IFψ-$T_{1/2}$ space.*

Proof. For an intuitionistic fuzzy topological space (X, τ)

(a) Let A be an IFψCS in (X, τ). Since X is an IFψ-T_R space, then A is an IFRCS in X which implies A is an IFSCS in X. Hence (X, τ) be an IFψ-$T_{1/2}$ space.
(b) Let A be an IFψCS in (X, τ), then A is an IFSGCS in X. Since X is an IF Semi-$T_{1/2}$ space which implies A is an IFSCS in X. Hence (X, τ) be an IFψ-$T_{1/2}$ space.
(c) Let A be an IFψCS in (X, τ), then A is an IFGSCS in X. Since X is an IF ${}_cT_{1/2}$ space, then A is an IFCS in X which implies A is an IFSCS. Hence (X, τ) be an IFψ-$T_{1/2}$ space.

The converse of the above theorem need not be true as shown by the following examples. □

Example 4.3. For an intuitionistic fuzzy topological space (X, τ)

(a) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.3, 0.4), (0.3, 0.4)\rangle$. Let $A = \langle x, (0.3, 0.4), (0.3, 0.4)\rangle$ be any IFS in X.

(b) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.3, 0.4), (0.2, 0.5)\rangle$. Let $A = \langle x, (0.1, 0.4), (0.6, 0.5)\rangle$ be any IFS in X.

(c) Let $X = \{a, b\}$ and let $\tau = \{0_\sim, V, 1_\sim\}$ is an IFT on X, where $V = \langle x, (0.3, 0.4), (0.2, 0.5)\rangle$. Let $A = \langle x, (0.1, 0.4), (0.6, 0.5)\rangle$ be any IFS in X.

Theorem 4.4. *An intuitionistic fuzzy topological space (X, τ) is an IFψ-$T_{1/2}$ space if and only if IFSOS(X) = IFψOS(X).*

Proof. **Necessity.** Let A be an IFψOS in X, then A^c is an IFψCS in X. By hypothesis A^c is an IFSCS of X and therefore A is an IFSOS of X. Hence IFSOS(X) = IFψOS(X).
Sufficiency. Let A be an IFψCS in X, then A^c is an IFψOS of X. By hypothesis A^c is an IFSOS in X which implies A is an IFSCS in X. Hence (X, τ) is an IFψ-$T_{1/2}$ space. □

Theorem 4.5. *An intuitionistic fuzzy topological space (X, τ) is an IFψ-T_R space if and only if IFROS(X) = IFψOS(X).*

Proof. **Necessity.** Let A be an IFψOS in X, then A^c is an IFψCS in X. By hypothesis A^c is an IFRCS of X and therefore A is an IFROS of X. Hence IFROS(X) = IFψOS(X).
Sufficiency. Let A be an IFψCS in X, then A^c is an IFψOS of X. By hypothesis A^c is an IFROS in X which implies A is an IFRCS in X. Hence (X, τ) is an IFψ-T_R space. □

Theorem 4.6. *Let (X, τ) be an intuitionistic fuzzy topological space and X is an IFψ-$T_{1/2}$ space, then the following conditions are equivalent:*

$$(a)\ A \in IF\psi OS(X) \quad (b)\ A \subseteq cl(int(A)) \quad (c)\ A \in IFRCS(X)$$

Proof. (a) ⇒ (b) Let A be an IFψOS in X. Since X is an IFψ-$T_{1/2}$ space, A is an IFSOS in X. Hence by definition of IFSOS, $A \subseteq \mathrm{cl}(\mathrm{int}(A))$.
(b) ⇒ (c) Assume that $A \subseteq \mathrm{cl}(\mathrm{int}(A))$, then $A = \mathrm{cl}(\mathrm{int}(A))$. Hence A is an IFRCS and $A \in$ IFRCS(X). **(c) ⇒ (a)** Assume that $A \in$ IFRCS(X), then $A = \mathrm{cl}(\mathrm{int}(A))$. Since $A \subseteq \mathrm{cl}(\mathrm{int}(A))$, A is an IFSOS and hence A is an IFψOS, therefore $A \in$ IFψOS(X). □

Theorem 4.7. *Let (X, τ) be an IFψ-T_R space. Then every singleton set of X is either an IFSGCS or IFSGOS.*

Proof. Assume that (X, τ) is an IFψ-T_R space. Suppose that $\{x\}$ is not an IFSGCS for some $x \in X$. Then $X - \{x\}$ is not IFSGOS and hence X is the only IFSGOS containing $X - \{x\}$. Therefore, $X - \{x\}$ is an IFψCS in X. Since (X, τ) is an IFψ-T_R space, then $X - \{x\}$ is an IFRCS and hence $X - \{x\}$ is an IFSGCS or equivalently $\{x\}$ is an IFSGOS. □

Theorem 4.8. *Let A and B be q-separated IFψO sets in an intuitionistic fuzzy topological space (X, τ), then $A \cup B$ is also an IFψOS if X is an IFψ-$T_{1/2}$ space.*

Proof. Let F be an IFSGCS such that $F \subseteq A \cup B$. Then $F\cap$ cl$(A) \subseteq A$, since $B\cap$ cl$(A) = 0_\sim$. Since A is an IFSOS in X, $F\cap$ cl$(A) \subseteq$ sint(A). Similarly $F\cap$ cl$(B) \subseteq$ sint(B). Now $F = F \cap (A \cup B) \subseteq F\cap$ cl$(A) \cup F\cap$ cl$(B) \subseteq$ sint$(A)\cup$ sint$(B) \subseteq$ sint$(A \cup B)$. Therefore $F \subseteq$ sint$(A \cup B)$ implies $A \cup B$ is an IFψOS in X. □

Theorem 4.9. *Let A and B be two IFψCS of an intuitionistic fuzzy topological space (X, τ) and suppose that A^c and B^c are q-separated. Then $A \cap B$ is an IFψCS if X is an IFψ-$T_{1/2}$ space.*

Proof. Assume that A and B are IFψCS, then A^c and B^c are q-separated IFψOS. By theorem 4.8, $A^c \cup B^c$ is an IFψOS. Hence $(A \cap B)^c$ is an IFψOS which implies that $A \cap B$ is an IFψCS in X. □

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M.Parimala (riswanthpari@gmail.com)
Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam, Erode, Tamilnadu. India

C.Indirani (indirani2012@gmail.com)
Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam, Erode, Tamilnadu, India

A.Selvakumar (selvam_mphil@yahoo.com)
Department of Mathematics, Info Institute of Engineering, Coimbatore, Tamilnadu, India