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On soft fuzzy G_δ pre S-closed spaces in soft fuzzy topological spaces

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Abstract. In this paper the concept of soft fuzzy G_δ pre S-closedness and soft fuzzy G_δ pre semi S-closedness are introduced and studied. We give some characterizations of soft fuzzy G_δ pre S-closedness in terms of soft fuzzy G_δ pre semiopen soft fuzzy G_δ pre regular closed and soft fuzzy G_δ pre semiclosed.

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Keywords: Soft topological spaces, Soft fuzzy G_δ pre open set, Soft fuzzy G_δ pre continuous, Soft fuzzy G_δ pre compact space, Soft fuzzy G_δ pre almost compactness, Soft fuzzy G_δ pre S-closedness, Soft fuzzy G_δ pre semi S-closedness.

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1. Introduction

Zadeh introduced the fundamental concept of a fuzzy set in [15]. Chang in [2] introduced and developed the concept of fuzzy topological spaces. In Azad [1] some weaker forms of continuity, fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity were considered for the first time. S-closedness in topology has been studied by many researchers.[9, 11] A.Di Concilio and G.Gerla [3] and A.H.Eş [4, 5] introduced and studied almost compactness and S-closedness for fuzzy topological spaces. In 1999, D.Molodtsov [8] introduced the soft set theory to solve complicated problems in economics, engineering and environment. He has shown several applications of this theory in solving many practical problems. There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets etc. In 2001, P.K.Maji, A.R.Roy and R.Bisvas [7] also initiate the more generalized concept of fuzzy soft sets which is combination of fuzzy set and soft set. In 2012 S.Roy and T.K.Samanta redefined some definitions on fuzzy soft set in another form [10]. I.U.Tiryaki and L.M.Brown studied fuzzy sets over the poset I=[0,1] with the usual order. These form a canonical example of fuzzy sets over a poset discussed in [13]. Characterizations of these so called "soft fuzzy sets" are obtained, and soft fuzzy sets are shown to have a richer mathematical theory than classical I-fuzzy sets. The concept of soft fuzzy topological space is introduced by I.U.Tiryaki in [12] from a different view of point. Visalakshi, Uma and Roja [14] introduced concept of soft fuzzy G_δ pre continuity, soft fuzzy G_δ pre connected space and soft fuzzy G_δ pre compact. A.H.Eş [6] introduced the concept of soft fuzzy G_δ pre almost compactness in soft fuzzy topological spaces.

In this paper soft fuzzy G_δ pre S-closedness and soft fuzzy G_δ pre semi S-closedness are introduced and their properties are discussed.

2. Prelimininaries

Definition 2.1. ([14]). Let (X, τ) be a topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$, where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ.

Definition 2.2. ([14]). Let (X, τ) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy pre open set if $\lambda \leq int(cl(\lambda))$. The complement of a pre open set is pre closed.

Definition 2.3. ([12]). Let X be a set, μ be a fuzzy subset of X and $M \subseteq X$. Then, the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by $SF(X)$.

Proposition 2.4. ([12]). *If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j)|j \in J\}$ has a meet, that is greatest lower bound, in $(SF(X), \sqsubseteq)$, denoted by*

$$\sqcap_{j\in J}(\mu_j, M_j) \text{ such that } \sqcap_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where

$\mu(x) = \bigwedge_{j\in J} \mu_j(x), \forall x \in X.$
$M = \bigcap_{j\in J} M_j$

Proposition 2.5. ([12]). *If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j)|j \in J\}$ has a join, that is least upper bound, in $(SF(X), \sqsubseteq)$, denoted by*

$$\sqcup_{j\in J}(\mu_j, M_j) \text{ such that } \sqcup_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where

$\mu(x) = \bigvee_{j\in J} \mu_j(x), \forall x \in X.$
$M = \bigcup_{j\in J} M_j$

Definition 2.6. ([12]). The relation $\sqsubseteq$ on $SF(X)$ is given by
(1) $(\mu, M) \sqsubseteq (\lambda, N) \Leftrightarrow ((\mu(x) < \lambda(x))$ or $((\mu(x) = \lambda(x)$ and $x \notin M \setminus N), \forall x \in X$ and for all $(\mu, M), (\lambda, N) \in SF(X)$.
(2) $(\mu, M) = (\lambda, N) \Leftrightarrow \mu(x) = \lambda(x) \forall x \in X, M = N.$
(3) $(\mu, M)' = 1 - \mu(x), \forall x \in X, X \setminus M.$
(4) $(\mu, M) \sqcap (\lambda, N) = (\sigma, M \cap N)$, where $\sigma(x) = \mu(x) \wedge \lambda(x), \forall x \in X$ and and for all $(\mu, M), (\lambda, N) \in SF(X)$.
(5) $(\mu, M) \sqcup (\lambda, N) = (\sigma, M \cup N)$, where $\sigma(x) = \mu(x) \vee \lambda(x), \forall x \in X$ and and for all $(\mu, M), (\lambda, N) \in SF(X)$.

Definition 2.7. ([12]).

$$(0, \varnothing) = \{(\lambda, N) | \lambda = 0, N = \varnothing\}$$
$$(1, X) = \{(\lambda, N) | \lambda = 1, N = X\}$$

Definition 2.8. ([12]). For $(\mu, M) \in SF(X)$ the soft fuzzy set

$(\mu, M)' = (1 - \mu, X \setminus M)$ is called the complement of (μ, M).

Definition 2.9. ([12]). A subset $\tau \subseteq SF(X)$ is called an SF-topology on X if
(1) $(0, \varnothing)$ and $(1, X) \in \tau$
(2) $(\mu_j, M_j) \in \tau$, $j = 1, 2, \dots, n \Rightarrow \sqcap_{j=1}^{n} (\mu_j, M_j) \in \tau$
(3) $(\mu_j, M_j), j \in J \Rightarrow \sqcup_{j \in J} (\mu_j, M_j) \in \tau$. The elements of τ are called soft fuzzy open, and those of $\tau' = \{(\mu, M) | (\mu, M)' \in \tau\}$ soft fuzzy closed.
If τ is SF-topology on X we call the pair (X, τ) SF-topological space (in short, SFTS).

Definition 2.10. ([12]). The closure of a soft fuzzy set (μ, M) will be denoted by $\overline{(\mu, M)}$. It is given by

$$\overline{(\mu, M)} = \sqcap\{(\gamma, N) | (\mu, M) \sqsubseteq (\gamma, N), (\gamma, N) \in \tau'\}.$$

Likewise the interior is given by

$$(\mu, M)^{\circ} = \sqcup\{(\gamma, N) | (\gamma, N) \in \tau, (\gamma, N) \sqsubseteq (\mu, M)\}.$$

Note : $\overline{(\mu, M)} = cl(\mu, M)$ and $(\mu, M)^{\circ} = int(\mu, M)$.

Definition 2.11. ([12]). A soft fuzzy topological space (X, τ) is said to be a soft fuzzy compact if whenever $\sqcup_{i \in I} (\lambda_i, M_i) = (1, X), (\lambda_i, M_i) \in \tau, i \in I$, there is a finite subset J of I with $\sqcup_{j \in J} (\lambda_j, \mu_j) = (1, X)$.

Definition 2.12. ([14]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy G_δ set if $(\lambda, N) = \sqcap_{i=1}^{\infty} (\mu_i, M_i)$, where each (μ_i, M_i) is soft fuzzy open set. The complement of a soft fuzzy G_δ set is soft fuzzy F_σ.

Definition 2.13. ([14]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy pre open set if $(\lambda, N) \sqsubseteq int(cl(\lambda, N))$. The complement of a soft fuzzy pre open set is soft fuzzy pre closed.

Definition 2.14. ([14]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy G_δ pre open set if $(\lambda, N) = (\mu, M) \sqcap (\gamma, L)$, where (μ, M) is soft fuzzy G_δ set and (γ, L) is soft fuzzy pre open set. The complement of a soft fuzzy G_δ pre open set is soft fuzzy F_σ pre closed.

Definition 2.15. ([14]). Let (X, τ) be a soft fuzzy topological space and let (λ, M) be any soft fuzzy set in (X, τ). Then soft fuzzy G_δ pre interior of (λ, M) is defined as follows

SF G_δ pre int$(\lambda, M) = \sqcup\{(\mu, N) | (\mu, N)$ is SF G_δ pre open and $(\mu, N) \sqsubseteq (\lambda, M)\}$.

Proposition 2.16. *([14]). Let (X, τ) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, τ). Then SF G_δ pre int(λ, N) is a soft fuzzy G_δ pre open set in (X, τ).*

Proposition 2.17. *([14]). Let (X,τ) be any soft fuzzy topological space and (λ,M), (μ,N) be soft fuzzy sets in (X,τ). Then the following properties hold:*

(i) SF G_δ pre int$(\lambda,M)\sqsubseteq(\lambda,M)$.
(ii) $(\lambda,M)\sqsubseteq(\mu,N)\Rightarrow$ SF G_δ pre int$(\lambda,M)\sqsubseteq$ SF G_δ pre int(μ,N).
(iii) SF G_δ pre int(SF G_δ pre int(λ,M)) =SF G_δ pre int(λ,M).
(iv) SF G_δ pre int$((\lambda,M)\sqcap(\mu,N))\sqsubseteq$ SF G_δ pre int$(\lambda,M)\sqcap$ SF G_δ pre int(μ,N).
(v) SF G_δ pre int$(1,X)=(1,X)$.

Definition 2.18. ([14]). Let (X,τ) be any soft fuzzy topological space and let (λ,M) be any soft fuzzy set in (X,τ). Then soft fuzzy F_σ pre closure of (λ,M) is defined as follows

SF F_σ pre cl$(\lambda,M)=\sqcap\{(\mu,N)|(\mu,N)$ is SF F_σ pre closed and $(\lambda,M)\sqsubseteq(\mu,N)\}$.

Proposition 2.19. *([14]). Let (X,τ) be any soft fuzzy topological space. Let (λ,N) be any soft fuzzy set in (X,τ). Then SF F_σ pre cl(λ,N) is a soft fuzzy F_σ pre closed set in (X,τ).*

Proposition 2.20. *([14]). Let (X,τ) be any soft fuzzy topological space and (λ,M), (μ,N) be soft fuzzy sets in (X,τ). Then the following properties hold:*

(i) $(\lambda,M)\sqsubseteq$SF F_σ pre cl(λ,M).
(ii) $(\lambda,M)\sqsubseteq(\mu,N)\Rightarrow$ SF F_σ pre cl$(\lambda,M)\sqsubseteq$ SF F_σ pre cl(μ,N).
(iii) SF F_σ pre cl(SF F_σ pre cl(λ,M)) =SF F_σ pre cl(λ,M).
(iv) SF F_σ pre cl$((\lambda,M)\sqcup(\mu,N))=$ SF F_σ pre cl$(\lambda,M)\sqcup$ SF F_σ pre cl(μ,N).
(v) SF F_σ pre cl$(0,\varnothing)=(0,\varnothing)$.

Proposition 2.21. *([14]). For any soft fuzzy set (λ,M) in a soft fuzzy topological space (X,τ) the following hold:*

(i) SF F_σ pre cl$((1,X)-(\lambda,M))=(1,X)-$SF G_δ pre int(λ,M).
(ii) SF G_δ pre int$((1,X)-(\lambda,M))=(1,X)-$SF F_σ pre cl(λ,M).

Definition 2.22. ([14]). Let (X,τ) be a soft fuzzy topological space. Let (λ,N) be a soft fuzzy set in (X,τ). Then (λ,N) is said to be soft fuzzy regular G_δ pre open if (λ,N) =SF G_δ pre int(SF F_σ pre cl(λ,N)).

Definition 2.23. ([14]). Let (X,τ) be a soft fuzzy topological space. Let (λ,N) be a soft fuzzy set in (X,τ). Then (λ,N) is said to be soft fuzzy regular F_σ pre closed if (λ,N) =SF F_σ pre cl(SF G_δ pre int(λ,N)).

Proposition 2.24. *([14]). (i) The soft fuzzy F_σ pre closure of a soft fuzzy G_δ pre open set is soft fuzzy regular F_σ pre closed.*
(ii) The soft fuzzy G_δ pre interior of a soft fuzzy F_σ pre closed set is soft fuzzy regular G_δ pre open.

Definition 2.25. ([14]). Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\tau^*)$ is said to be soft fuzzy G_δ pre continuous, if the inverse image of every soft fuzzy open set in (Y,τ^*) is soft fuzzy G_δ pre open in (X,τ).

Remark 2.26. Every soft fuzzy continuous function is soft fuzzy G_δ pre continuous. The converse of the above property need not be true as shown in the following example.

Example 2.27. $X = \{a, b, c, d\}, \tau = \{(0, \varnothing), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0, 1]$ for $i = 1, 2, 3, 4, 5$ and $M_i \subseteq X$, for $i = 1, 2, 3, 4, 5$ are defined as follows:

$\lambda_1(a) = 0, \lambda_1(b) = 0.4, \lambda_1(c) = 0, \lambda_1(d) = 0.3;$
$\lambda_2(a) = 0.7, \lambda_2(b) = 0, \lambda_2(c) = 0.8, \lambda_2(d) = 0;$
$\lambda_3(a) = 0.7, \lambda_3(b) = 0.4, \lambda_3(c) = 0.8, \lambda_3(d) = 0.3;$
$\lambda_4(a) = 0.7, \lambda_4(b) = 1, \lambda_4(c) = 0.8, \lambda_4(d) = 1;$
$\lambda_5(a) = 1, \lambda_5(b) = 0.4, \lambda_5(c) = 1, \lambda_5(d) = 0;$
$M_1 = \{b\}, M_2 = \{c\}, M_3 = \{b, c\}, M_4 = \{a, b, c\}, M_5 = \{d, b, c\}.$

Then (X, τ) is a soft fuzzy topological space.
Let $Y = \{p, q, r\}$, $\tau^* = \{(0, \varnothing), (1, Y), (\mu_1, N_1), (\mu_2, N_2)\}$ where $\mu_i : Y \to [0, 1]$ for $i = 1, 2$ are defined as follows:

$$\mu_1(p) = 0.4, \mu_1(q) = 0, \mu_1(r) = 0.7; \ \mu_2(p) = 0.4, \mu_2(q) = 0.3, \mu_2(r) = 0.7;$$

$N_1 = \{r\}, N_2 = \{q, r\}$. Then (Y, τ^*) is a soft fuzzy topological space.
Let $f : (X, \tau) \to (Y, \tau^*)$ be a function defined as
$f(a) = p, f(b) = q, f(c) = r, f(d) = q.$
Then f is soft fuzzy G_δ pre continuous ([14]), but not soft fuzzy continuous.
Consider the soft fuzzy open set (μ_1, N_1) in (Y, τ^*), $f^{-1}(\mu_1, N_1)$ is not soft fuzzy open set in (X, τ).

Definition 2.28. ([14]). Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy G_δ pre irresolute, if the inverse image of every soft fuzzy G_δ pre open set in (Y, τ^*) is soft fuzzy G_δ pre open in (X, τ).

Definition 2.29. ([14]). A soft fuzzy topological space (X, τ) is said to be a soft fuzzy G_δ pre compact if whenever $\sqcup_{i\in I}(\lambda_i, M_i) = (1, X)$, (λ_i, M_i) is soft fuzzy G_δ pre open, $i \in I$, there is a finite subset J of I with $\sqcup_{i\in J}(\lambda_i, M_i) = (1, X)$.

Proposition 2.30. *([14]). Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre continuous bijection and (X, τ) is soft fuzzy G_δ pre compact, then (Y, τ^*) is soft fuzzy compact.*

Proposition 2.31. *([14]). Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre irresolute bijection and (X, τ) is soft fuzzy G_δ pre compact, then (Y, τ^*) is soft fuzzy G_δ pre compact.*

Definition 2.32. ([6]). A soft fuzzy topological space (X, τ) is said to be a soft fuzzy G_δ pre almost compact if whenever $\sqcup_{i\in I}(\lambda_i, M_i) = (1, X)$, (λ_i, M_i) is soft fuzzy G_δ pre open, $i \in I$, there is a finite subset J of I with $\sqcup_{i\in J}\mathrm{SF}\ F_\sigma$ pre $\mathrm{cl}(\lambda_i, M_i) = (1, X)$.

Definition 2.33. ([6]). Let (X, τ) be a soft fuzzy topological space. If (λ_i, M_i), $i \in I$, of soft fuzzy sets in (X, τ) satisfies the finite intersection property (FIP for short) iff every finite subfamily (λ_i, M_i), $i = 1, 2, \ldots, n$ of the family satisfies the condition $\sqcap_{i=1}^{n}(\lambda_i, M_i) \neq (0, \varnothing)$.

Theorem 2.34. *([6]). A soft fuzzy topological space (X, τ) is fuzzy G_δ pre almost compact iff for every collection of soft fuzzy G_δ pre open sets (λ_i, M_i) of $SF(X)$ having the finite intersection property we have $\sqcap_{i \in I} SF\ F_\sigma$ pre $cl(\lambda_i, M_i) \neq (0, \varnothing)$.*

Definition 2.35. ([14]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be a soft fuzzy set in (X, τ). Then (λ, N) is said to be soft fuzzy α open, (β open) if $(\lambda, N) \sqsubseteq \text{int}(\text{cl}(\text{int}(\lambda, N)))$ ($(\lambda, N) \sqsubseteq \text{cl}(\text{int}(cl(\lambda, N)))$). The complement of a soft fuzzy α open (β open) set is soft fuzzy α closed (β closed).

Definition 2.36. ([14]). Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, τ). Then (λ, N) is said to be soft fuzzy G_δ β open set if $(\lambda, N) = (\mu, M) \sqcap (\gamma, L)$, where (μ, M) is soft fuzzy G_δ set and (γ, L) is soft fuzzy β open set.

Remark 2.37. ([14]). Every soft fuzzy pre open set is soft fuzzy β open set.

Proposition 2.38. *([14]). Every soft fuzzy G_δ pre open set is soft fuzzy G_δ β open set.*

Remark 2.39. ([14]). The converse of the property need not be true as shown in the following example.

Example 2.40. ([14]) $X = \{a, b, c, d\}, \tau = \{(0, \varnothing), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0, 1]$ for $i = 1, 2, 3, 4, 5$ and $M_i \subseteq X$, for $i = 1, 2, 3, 4, 5$ are defined as follows:

$\lambda_1(a) = 0.6, \lambda_1(b) = 0, \lambda_1(c) = 0.2, \lambda_1(d) = 0;$
$\lambda_2(a) = 0, \lambda_2(b) = 0.5, \lambda_2(c) = 0, \lambda_2(d) = 0.1;$
$\lambda_3(a) = 0.6, \lambda_3(b) = 0.5, \lambda_3(c) = 0.2, \lambda_3(d) = 0.1;$
$\lambda_4(a) = 0.6, \lambda_4(b) = 1, \lambda_4(c) = 0.2, \lambda_4(d) = 1;$
$\lambda_5(a) = 1, \lambda_5(b) = 0.5, \lambda_5(c) = 1, \lambda_5(d) = 0.1;$
$M_1 = \{a\}, M_2 = \{c\}, M_3 = \{a, c\}, M_4 = \{a, d, c\}, M_5 = \{a, b, c\}.$
Then (X, τ) is a soft fuzzy topological space.
Consider the soft fuzzy set (λ, M) where
$\lambda : X \to [0, 1]$ and $M \subseteq X$ are defined as
$\lambda(a) = 0.4, \lambda(b) = 0.5, \lambda(c) = 0.2, \lambda(d) = 0.3;$
and M={c}. Now $cl(int(cl(\lambda, M))) \sqsupseteq (\lambda, M)$. Thus (λ, M) is soft fuzzy β open set. Consider the soft fuzzy G_δ set (λ_4, M_4). Now $(\lambda_4, M_4) \sqcap (\lambda, M) = (\lambda, M)$ is a soft fuzzy G_δ β open set. But (λ, M) is not soft fuzzy pre open. Thus (λ, M) is a soft fuzzy G_δ β open set and it is not a soft fuzzy G_δ pre open set.

3. Soft Fuzzy G_δ pre S-Closed Spaces

Definition 3.1. Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy pre semiopen set if there exists a soft fuzzy pre open set (μ, M) of $SF(X)$ such that $(\mu, M) \sqsubseteq (\lambda, N) \sqsubseteq \text{cl}(\text{int}(\mu, M))$.

Definition 3.2. Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy G_δ pre semiopen set if $(\lambda, N) = (\mu, M) \sqcap (\gamma, L)$, where (μ, M) is soft fuzzy G_δ set and (γ, L) is soft fuzzy pre semiopen set. The complement of a soft fuzzy G_δ pre semiopen set is soft fuzzy F_δ pre semiclosed.

Remark 3.3. Every soft fuzzy open set is soft fuzzy G_δ pre open set. Every soft fuzzy open set is soft fuzzy G_δ pre semiopen set. The converse of the property need not be true as shown in the following example.

Example 3.4. (i) Let $X = \{a, b, c, d\}, \tau = \{(0, \varnothing), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0, 1]$ for $i = 1, 2, 3, 4, 5$ and $M_i \subseteq X$, for $i = 1, 2, 3, 4, 5$ are defined as follows:

$\lambda_1(a) = 0, \lambda_1(b) = 0.3, \lambda_1(c) = 0.7, \lambda_1(d) = 0.3;$
$\lambda_2(a) = 0.7, \lambda_2(b) = 0, \lambda_2(c) = 0.8, \lambda_2(d) = 0;$
$\lambda_3(a) = 0.7, \lambda_3(b) = 0.3, \lambda_3(c) = 0, \lambda_3(d) = 0.3;$
$\lambda_4(a) = 0.7, \lambda_4(b) = 1, \lambda_4(c) = 0.8, \lambda_4(d) = 1;$
$\lambda_5(a) = 1, \lambda_5(b) = 0.3, \lambda_5(c) = 1, \lambda_5(d) = 0.3;$
$M_1 = \{b\}, M_2 = \{c\}, M_3 = \{b, c\}, M_4 = \{a, b, c\}, M_5 = \{b, c, d\}.$
Then (X, τ) is a soft fuzzy topological space.
Consider the soft fuzzy set (λ, M) where $\lambda : X \to [0, 1]$ and $M \subseteq X$ are defined as
$\lambda(a) = 0.4, \lambda(b) = 0, \lambda(c) = 0.7, \lambda(d) = 0;$
and M={c}. Now $int(cl(\lambda, M))) \sqsupseteq (\lambda, M)$. Thus (λ, M) is soft fuzzy pre open set. Consider the soft fuzzy G_δ set (λ_2, M_2). Now $(\lambda_2, M_2) \sqcap (\lambda, M) = (\lambda, M)$ is a soft fuzzy G_δ pre open set. But (λ, M) is not soft fuzzy open set.

(ii) Let $X = \{a, b, c, d\}, \tau = \{(0, \varnothing), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0, 1]$ for $i = 1, 2, 3, 4, 5$ and $M_i \subseteq X$, for $i = 1, 2, 3, 4, 5$ are defined as follows:

$\lambda_1(a) = 0, \lambda_1(b) = 0.4, \lambda_1(c) = 0, \lambda_1(d) = 0.3;$
$\lambda_2(a) = 0.7, \lambda_2(b) = 0, \lambda_2(c) = 0.8, \lambda_2(d) = 0;$
$\lambda_3(a) = 0.7, \lambda_3(b) = 0.4, \lambda_3(c) = 0.8, \lambda_3(d) = 0.2;$
$\lambda_4(a) = 0.7, \lambda_4(b) = 1, \lambda_4(c) = 0.8, \lambda_4(d) = 1;$
$\lambda_5(a) = 1, \lambda_5(b) = 0.4, \lambda_5(c) = 1, \lambda_5(d) = 0.3;$
$M_1 = \{b\}, M_2 = \{c\}, M_3 = \{b, c\}, M_4 = \{a, b, c\}, M_5 = \{b, c, d\}.$
Then (X, τ) is a soft fuzzy topological space.
Consider the soft fuzzy set (λ, M) where $\lambda : X \to [0, 1]$ and $M \subseteq X$ are defined as
$\lambda(a) = 1, \lambda(b) = 0.4, \lambda(c) = 1, \lambda(d) = 0.2;$
and $M = \{a, b, c\}$. Now $int(cl(\lambda, M)) = int(\lambda_1, M_1)' = (\lambda_5, M_5) \sqsupseteq (\lambda, M)$. Thus (λ, M) is soft fuzzy pre open set. Consider the soft fuzzy G_δ set (λ_5, M_5). Now $(\lambda_5, M_5) \sqcap (\lambda, M) = (\sigma, M_3)$. where (σ, M_3) defined as
$\sigma(a) = 1, \sigma(b) = 0.4, \sigma(c) = 1, \sigma(d) = 0.2;$
and $M = \{b, c\}$. Now, $(\lambda, M) \sqsubseteq (\lambda, M) \sqsubseteq cl(int(\lambda, M)) = (\lambda_1, M_1)'$. Therefore (λ, M) is a soft fuzzy pre semiopen set. Thus (λ, M) is a soft fuzzy G_δ pre semiopen set. But it is not a soft fuzzy open set.

Remark 3.5. Every soft fuzzy G_δ pre semiopen set is soft fuzzy G_δ β open set. The converse of the property need not be true as shown in the following example.

Example 3.6. We consider Example 2.40. The soft fuzzy set (λ, M) defined as $\lambda : X \to [0,1]$ and $M \subseteq X$,

$\lambda(a) = 0.4, \lambda(b) = 0.5, \lambda(c) = 0.2, \lambda(d) = 0.3$; and $M = \{c\}$. Now $cl(int(cl(\lambda, M)))$ $= cl(\lambda_2, M_2)) \sqsupseteq (\lambda, M)$. Thus (λ, M) is soft fuzzy β open set. Consider the soft fuzzy G_δ set (λ_4, M_4). Now $(\lambda_4, M_4) \sqcap (\lambda, M) = (\lambda, M)$ is a soft fuzzy G_δ β open set([14]).But (λ, M) is not soft fuzzy pre semiopen set.
Since $int(cl(\lambda, M)) = (\lambda_2, M_2)) \not\sqsupseteq (\lambda, M)$ and $(\lambda, M) \sqsubseteq (\lambda, M) \not\sqsubseteq cl(int(\lambda, M))$, (λ, M) is not a soft fuzzy G_δ pre semiopen set.

Proposition 3.7. *Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, τ). Then SFS F_σ pre $cl(\lambda, N)$ is a soft fuzzy F_σ pre semi closed set in (X, τ).*

Proof. It is easy to prove from the definition of SFS F_σ pre semi closure of a soft fuzzy set. □

Definition 3.8. A soft fuzzy topological space (X, τ) is said to be a soft fuzzy G_δ pre S-closed if whenever $\sqcup_{i \in I}(\lambda_i, \mu_i) = (1, X)$, (λ_i, μ_i) is soft fuzzy G_δ pre semiopen, $i \in I$, there is a finite subset F of I with $\sqcup_{i \in F}$ SFS $F_\sigma$pre cl$(\lambda_i, \mu_i) = (1, X)$.

Theorem 3.9. *A soft fuzzy topological space (X, τ) is soft fuzzy G_δ pre S-closed iff for every collection (λ_i, μ_i), $i \in I$ of soft fuzzy F_σ pre semiclosed sets such that*

$$\sqcap_{i \in I} SF\ S\ G_\delta \text{ pre } int(\lambda_i, \mu_i) = (0, \varnothing),$$

there exists a finite subset F of I with

$$\sqcap_{i \in F} SF\ S\ G_\delta pre\ int(\lambda_i, \mu_i) = (0, \varnothing).$$

Proof. This follows form the above definitions. □

Theorem 3.10. *A soft fuzzy topological space (X, τ) is soft fuzzy G_δ pre S-closed iff every soft fuzzy G_δ β open cover of $SF(X)$ has a finite subcollection whose closures cover $SF(X)$.*

Proof. Let (λ_i, M_i), $i \in I$ be a soft fuzzy G_δ β open cover of $SF(X)$. From

$$(\lambda_i, M_i) \sqsubseteq \text{SF } F_\sigma \text{ pre cl(SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}((\lambda_i, M_i))),$$

we deduce

$$\text{SF } F_\sigma \text{ pre cl}(\lambda_i, M_i) \sqsubseteq \text{SF } F_\sigma \text{ pre cl(SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}((\lambda_i, M_i)))$$

and hence

$$(\text{SF } F_\sigma \text{ pre cl}(\lambda_i, M_i)),\ i \in I \text{ is a soft fuzzy } G_\delta \text{ pre semiopen cover of } SF(X).$$

From the soft fuzzy G_δ pre S-closedness, it follows that there exists a finite subset F of I such that

$$\sqcup_{i \in F} \text{ SF } F_\sigma \text{ pre cl } (\lambda_i, M_i) = (1, X).$$

The converse is obvious from the fact that every soft fuzzy G_δ pre semiopen set is soft fuzzy G_δ β open set. □

Definition 3.11. Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, τ). Then (λ, N) is said to be soft fuzzy extremally disconnected if

$$\text{SF } F_\sigma \text{ pre cl } (\lambda, N) \in (X, \tau) \text{ for every } (\lambda, N) \in (X, \tau).$$

Theorem 3.12. *Let (X, τ) be a soft fuzzy G_δ extremally disconnected space. Then (X, τ) is soft fuzzy G_δ pre S-closed iff (X, τ) is soft fuzzy G_δ pre almost compact.*

Proof. Let (X, τ) be a soft fuzzy almost compact and $(\lambda_i, M_i), i \in I$ a soft fuzzy G_δ pre semiopen cover of $SF(X)$. Then there exists a $(\mu_i, N_i) \in (X, \tau)$ such that

$$(\mu_i, N_i) \sqsubseteq (\lambda_i, M_i) \sqsubseteq \text{SF } F_\sigma \text{ pre cl}((\mu_i, N_i),\ i \in I,$$

and so SF F_σ pre cl$((\mu_i, N_i) \sqsubseteq$ SF F_σ pre cl (λ_i, M_i). Since (X, τ) is soft fuzzy G_δ extremally disconnected, then $\sqcup_{i \in I}$ SF F_σ pre cl$((\mu_i, N_i) = (1, X)$. From the soft fuzzy G_δ pre almost compactness it follows that there exists a finite subset F of I such that

$$\sqcup_{i \in F} \text{ SF } F_\sigma \text{ pre cl } (\mu_i, N_i) {=} \sqcup_{i \in F} \text{ SF } F_\sigma \text{ pre cl } (\lambda_i, M_i) = (1, X).$$

The converse is obvious, since every soft fuzzy G_δ pre open set is soft fuzzy G_δ pre semiopen set. □

Definition 3.13. Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy G_δ pre almost open, if we have

$$f^{-1}(SFF_\sigma \text{ pre cl } (\lambda, M)) \sqsubseteq \text{SF } F_\sigma \text{ pre cl } f^{-1}(\lambda, M))$$

for each soft fuzzy G_δ open $(\lambda, M) \in SF(Y)$.

Theorem 3.14. *Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre weakly continuous and soft fuzzy G_δ pre almost open surjective and (X, τ) is soft fuzzy G_δ pre S-closed, then (Y, τ^*) is soft fuzzy G_δ pre S-closed.*

Proof. This result follows from Theorem 3.12 and Definition 3.13. □

Definition 3.15. A soft fuzzy topological space (X, τ) is said to be soft fuzzy G_δ pre S-regular iff each fuzzy pre open set (λ, M) of $SF(X)$ is a union of soft fuzzy G_δ pre semiopen sets of (λ_i, M_i) of $SF(X)$ such that SF F_σ pre cl$((\lambda_i, M_i) \sqsubseteq (\lambda, M)$ for each $i \in I$.

Theorem 3.16. *If (X, τ) is soft fuzzy G_δ pre S-regular and G_δ pre S-closed, then (X, τ) is G_δ pre compact.*

Proof. Let (λ_i, M_i), $i \in I$ be a soft fuzzy G_δ pre open cover of $SF(X)$. From the G_δ pre S-regularity of $SF(X)$, it follows that

$$(\lambda_j, M_j) = \sqcup_{i \in J_i}(\lambda_{j_i}, M_{j_i}) \text{ where}$$

(λ_{j_i}, M_{j_i}) is a soft fuzzy G_δ pre semiopen set such that

$$\text{SF } F_\sigma \text{ pre cl } (\lambda_{j_i}, M_{j_i}) \sqsubseteq (\lambda_j, M_j).$$

Then $\sqcup_{j\in J}(\lambda_j, M_j) = \sqcup_{j\in J} \sqcup_{i\in J_i} (\lambda_{j_i}, M_{j_i})$ follows. Since (X,τ) is soft fuzzy G_δ pre S-closed, there exist a finite subset F of J and $F_j \subset J_i$ such that

$$\sqcup_{j\in F}\sqcup_{i\in F_j} \text{ SF } F_\sigma \text{ pre cl } (\lambda_{j_i}, M_{j_i}) = (1, X).$$

Hence $\sqcup_{j\in F}(\lambda_j, M_j) = (1, X)$. □

4. Soft Fuzzy Semi S-Closed Spaces

Definition 4.1. Let (X,τ) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy G_δ semiregular if it is both soft fuzzy G_δ pre semi open and soft fuzzy F_σ pre semiclosed.

Definition 4.2. Let (X,τ) be any soft fuzzy topological space and (λ, M) be any soft fuzzy set in (X,τ). Then soft fuzzy G_δ pre semi interior of (λ, M) is defined as follows
SFS G_δ pre int $(\lambda, M) = \sqcup \{(\mu, N)|(\mu, N)$ is SFS G_δ pre open and $(\mu, N) \sqsubseteq (\lambda, M)\}$.

Definition 4.3. Let (X,τ) be any soft fuzzy topological space and let (λ, M) be any soft fuzzy set in (X,τ). Then soft fuzzy F_σ pre semi closure of (λ, M) is defined as follows
SFS F_σ pre cl $(\lambda, M) = \sqcap \{(\mu, N)|(\mu, N)$ is SFS F_σ pre closed and $(\lambda, M) \sqsubseteq (\mu, N)\}$.

Proposition 4.4. *If (λ, M) is soft fuzzy G_δ pre semiopen set in (X,τ), then SFS F_σ pre cl (λ, M) is soft fuzzy G_δ semi regular.*

Proof. Since SFS F_σ pre cl (λ, M) is SF F_σ pre closed, we must show that

SFS F_σ pre cl (λ, M) is SF G_δ pre semiopen in $SF(X)$.

Since (λ, M) is SF G_δ pre semiopen in $SF(X)$, $(\mu, N) \sqsubseteq (\lambda, M) \sqsubseteq$ SF F_σ pre cl (μ, N) holds for some $(\mu, N) \in X, \tau)$. Therefore, we have

$$(\mu, N) \sqsubseteq \text{SFS } F_\sigma \text{ pre cl } (\mu, N) \sqsubseteq \text{SFS } F_\sigma \text{ pre cl } (\lambda, M) \sqsubseteq \text{SF } F_\sigma \text{ pre cl } (\mu, N),$$

and hence (λ, M) is SF G_δ pre semiopen. □

Definition 4.5. A soft fuzzy topological space (X,τ) is said to be a soft fuzzy G_δ pre semi S-closed, if whenever $\sqcup_{i\in I}(\lambda_i, M_i) = (1, X)$, $(\lambda_i, M_i), i \in I$ is soft fuzzy G_δ pre semiopen, there is a finite subset F of I with $\sqcup_{i\in F}$ SFS F_σ pre cl $(\lambda_i, M_i) = (1, X)$.

Definition 4.6. Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. A function $f : (X,\tau) \to (Y,\tau^*)$ is said to be soft fuzzy G_δ pre semicontinuous, if the inverse image of every soft fuzzy G_δ pre open set in (Y,τ^*) is soft fuzzy G_δ pre semiopen in (X,τ).

Theorem 4.7. *Let (X,τ) and (Y,τ^*) be any two soft fuzzy topological spaces. If $f : (X,\tau) \to (Y,\tau^*)$ is soft fuzzy G_δ pre semicontinuous bijection and (X,τ) is soft fuzzy G_δ pre semi S-closed, then (Y,τ^*) is soft fuzzy G_δ pre almost compact.*

Proof. Let $\{ (\lambda_i, M_i), i \in I \}$ be a soft fuzzy G_δ pre open cover of $SF(Y)$. Then $\{f^{-1}(\lambda_i, M_i)\}_{i\in I}$ is a soft fuzzy G_δ pre semiopen cover of $SF(X)$. From the soft fuzzy G_δ pre semi S-closedness it follows that there exist a finite subset F of I such that

$$\sqcup_{i\in F} \text{ SFS } F_\sigma \text{ pre cl } (\lambda_i, M_i) = (1, X).$$

From the bijectivity of f,
$f(1,X) = (1,Y) = f(\sqcup_{i\in F} \text{SFS } F_\sigma \text{ pre cl}(f^{-1}(\lambda_i, M_i))) \sqsubseteq \sqcup_{i\in F} \text{SF } F_\sigma \text{ pre cl}(f(f^{-1}(\lambda_i, M_i))) = \sqcup_{i\in F} \text{ SF } F_\sigma \text{ pre cl}(\lambda_i, M_i)$.
Hence (Y, τ^*) is soft fuzzy G_δ pre almost compact. □

Theorem 4.8. *Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre irresolute bijection and (X, τ) is soft fuzzy G_δ pre semi S-closed, then (Y, τ^*) is soft fuzzy G_δ pre semi S-closed.*

Proof. Proof is similar to the Theorem 4.7. □

Definition 4.9. A soft fuzzy topological space (X, τ) is said to be a soft fuzzy G_δ pre S-compact if whenever $\sqcup_{i\in I}(\lambda_i, M_i) = (1, X)$, (λ_i, M_i) is soft fuzzy G_δ pre semiopen, $i \in I$, there is a finite subset F of I with $\sqcup_{i\in F}(\lambda_i, M_i) = (1, X)$.
Obviously every soft fuzzy G_δ pre S-compact fuzzy topological space is soft fuzzy G_δ pre S-closed.

Corollary 4.10. *A soft fuzzy topological space is soft fuzzy G_δ pre S-compact iff $\sqcap_{i\in I} (\lambda_i, M_i) \neq (0, \varnothing)$ holds for every collection of soft fuzzy F_σ pre semiclosed sets (λ_i, M_i), $i \in I$ with the finite intersection property.*

Theorem 4.11. *Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre semicontinuous bijection and (X, τ) is soft fuzzy G_δ pre S-compact, then (Y, τ^*) is soft fuzzy G_δ pre compact.*

Proof. Proof is obvious. □

Definition 4.12. Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy G_δ pre quasi irresolute, if the inverse image of every soft fuzzy G_δ pre semiregular set in (Y, τ^*) is soft fuzzy G_δ pre semiregular set in (X, τ).

Theorem 4.13. *Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. If $f : (X, \tau) \to (Y, \tau^*)$ is soft fuzzy G_δ pre quasi irresolute bijection and (X, τ) is soft fuzzy G_δ pre semi S-closed space, then (Y, τ^*) is also soft fuzzy G_δ pre semi S-closed.*

Proof. Let $\{(\lambda_i, M_i)\}_{i\in I}$ be a soft fuzzy G_δ pre semiopen cover of $SF(Y)$. By proposition 4.4, the collection $\{\text{SFS } F_\sigma \text{ pre cl}(\lambda_i, M_i)\}_{i\in I}$ is a soft fuzzy G_δ semi regular cover of $SF(Y)$. Hence $\sqcup_{i\in I} [f^{-1}(\text{ SFS } F_\sigma \text{ pre cl } (\lambda_i, M_i))] = (1, X)$. Since (X, τ) is soft fuzzy G_δ pre semi S-closed space, there exists a finite subset F of I such that

$$f(\sqcup_{i\in F} f^{-1}(\text{ SFS } F_\sigma \text{ pre cl } (\lambda_i, M_i))) = \sqcup_{i\in F} f(f^{-1}(\text{ SFS } F_\sigma \text{ pre cl } (\lambda_i, M_i))$$
$$= \sqcup_{i\in F} \text{ SFS } F_\sigma \text{ pre cl } (\lambda_i, M_i)) = f(1, X) = (1, Y).$$
□

Definition 4.14. Let (X, τ) and (Y, τ^*) be any two soft fuzzy topological spaces. A function $f : (X, \tau) \to (Y, \tau^*)$ is said to be soft fuzzy G_δ pre semi weakly continuous, if the inverse image of every soft fuzzy G_δ pre semiopen (λ, M) in (Y, τ^*), we have

$$f^{-1}(\lambda, M) \sqsubseteq \text{ SFS } G_\delta \text{ pre int } (f^{-1}(\text{ SFS } F_\sigma \text{ pre cl } (\lambda, M)).$$

Theorem 4.15. *Let* (X, τ) *and* (Y, τ^*) *be any two soft fuzzy topological spaces. If* $f : (X, \tau) \to (Y, \tau^*)$ *is soft fuzzy* G_δ *pre semi weakly continuous bijection and* (X, τ) *is soft fuzzy* G_δ *pre S-compact, then* (Y, τ^*) *is soft fuzzy* G_δ *pre semi S-closed.*

Proof. Proof is similar to the Theorem 4.13. □

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