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Intuitionistic fuzzy WO-connectedness between intuitionistic fuzzy sets

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Abstract. The aim of this paper is to introduce and discuss the concepts of intuitionistic fuzzy set WO - connectedness between intuitionistic fuzzy sets and intuitionistic fuzzy set WO - connected mappings in intuitionistic fuzzy topological spaces.

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1. Introduction

Fuzzy set as proposed by Zadeh[18] in 1965 represents a degree of membership for each member of universe of discourse to subset of it. Fuzzy set is a powerful tool to deal with vagueness. In 1968, Chang [4] extended the concept of point set topology to fuzzy sets and short the foundation as well as the basement of fuzzy topology. By adding the degree of non membership to fuzzy set, Atanassov [1, 2] introduced intuitionistic fuzzy set in 1983. After the introduction of intuitionistic fuzzy topology by Coker [5] in 1997, many fuzzy topological concepts have been generalized for intuitionistic fuzzy topological spaces. Connectedness is one of the basic notions in topology. The concept of connectedness between sets was first introduced by Kuratowski [8] in topology. A space X is said to connected between subset A and B if and only if there is no closed-open set F in X such that $A \subseteq F$ and $A \bigcap F = \phi$ [Kuratowski 1968]. Since then various weak and strong form of connectedness between sets such as s - connectedness between sets [6], p -connectedness between sets[10], GO -connectedness between sets [19] have been introduced and studied in general topology. In 1993 Maheshwari, Thakur and Malviya[9] extended the notions of connectedness between sets in Fuzzy topology. In another paper Chae

Thakur and Malviya [3] introduced the concept of fuzzy set connected mappings. In 2009, Thakur and Thakur [11] extended these concepts in intuitionistic fuzzy topology. Recently the authors of this paper study the concepts of intuitionistic fuzzy GO-connectedness between set[15] and intuitionistic fuzzy set GO - connected mappings[13] in intuitionistic fuzzy topological spaces. In the present paper we introduced and study a new form of intuitionistic fuzzy connectedness between set called intuitionistic fuzzy WO - connectedness between intuitionistic fuzzy sets and a new class of mappings called intuitionistic fuzzy set WO -connected mappings in intuitionistic fuzzy topological spaces.

2. Preliminaries

Let X be a nonempty fixed set. An intuitionistic fuzzy set [1] A in X is an object having the form

$$A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership ($namely\ \mu_A(x)$) and the degree of non membership ($namely\ \gamma_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each element $x \in X$. The intutionistic fuzzy sets $\tilde{\mathbf{0}} = \{< x, 0, 1 >: x \in X\}$ and $\tilde{\mathbf{1}}$ $= \{< x, 1, 0 >: x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$, is called a subset of an intuitionistic fuzzy set $B = \{< x, \mu_B(x), \gamma_B(x) >: x \in X\}$ (for short $A \subseteq B$) if $\{\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B X\}$. The complement of an intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ is the intuitionistic fuzzy set

$$A^c = \{< x, \gamma_A(x), \mu_A(x) >: x \in X\}$$

The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{< x, \mu_{Ai}(x), \gamma_{Ai}(x) >: x \in X,\ (\ i \in \wedge)\ \}$ of X be the intuitionistic fuzzy set

$$\cap A_i = \{< x, \wedge\mu_{Ai}(x), \vee\gamma_{Ai}(x) >: x \in X\}$$

(resp. $\cup A_i = \{< x, \vee\mu_{Ai}(x), \wedge\gamma_{Ai}(x) >: x \in X\}$). A family $\Im$ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology[5] on X if the intuitionistic fuzzy sets $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}} \in \Im$ and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair$(X, \Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in$\Im$ is called an intuitionistic fuzzy open set. The complement of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set .The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted by $cl(A)$. The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted $int(A)$ [5].Two intuitionistic fuzzy sets A and B of X are said to be q-coincident (A_qB for short) if and only if there exits an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. For any two intuitionistic fuzzy sets A and B of X, $\rceil(A_qB)$ if and only if $A \subseteq B^c$ [5].

Definition 2.1. An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X, \Im)$ is called :

(a) intuitionistic fuzzy g - closed[11] if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.

(b) intuitionistic fuzzy g - open [11] if its complement A^c is intuitionistic fuzzy g - closed.

(c) intuitionistic fuzzy w - closed [14] If $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.

(d) intuitionistic fuzzy w - open [14] if its complement A^c is intuitionistic fuzzy w -closed.

Remark 2.2 ([14]). Every intuitionistic fuzzy closed (resp. intuitionistic fuzzy open) set is intuitionistic fuzzy w - closed (resp. intuitionistic fuzzy w -open) and every intuitionistic fuzzy w-closed set (resp. intuitionistic fuzzy w-open) is intuitionistic fuzzy g-closed (resp. intuitionistic fuzzy g-open) but the converse may not be true.

Definition 2.3 ([14]). Let $(X, \Im)$ be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set in X. Then the w-interior and w-closure of A are defined as follows:

wcl(A) = $\cap$ {K : K is an intuitionistic fuzzy w-closed set in X and $A \subseteq K$},

wint(A) = $\cup$ {G : G is an intuitionistic fuzzy w-open set in X and $G \subseteq A$ }.

Definition 2.4. An intuitionistic fuzzy topological space $(X, \Im)$ is said to be :

(a) intuitionistic fuzzy C_5-connected[17] if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy open and intuitionistic fuzzy -closed.

(b) intuitionistic fuzzy GO - connected[11] if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy g- open and intuitionistic fuzzy g -closed.

(c) Intuitionistic fuzzy w - connected[14] if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy w- open and intuitionistic fuzzy w - closed.

Definition 2.5. An intuitionistic fuzzy topological space $(X, \Im)$ is said to be

(a) intuitionistic fuzzy connected between intuitionistic fuzzy sets[12] A and B if there is no intuitionistic fuzzy closed open set F in X such that $A \subseteq F$ and $\rceil(FqB)$.

(b) intuitionistic fuzzy GO -connected between intuitionistic fuzzy sets[15] A and B if there is no intuitionistic fuzzy g - closed g -open set F in X such that $A \subseteq F$ and $\rceil(FqB)$.

Definition 2.6. A mapping $f : (X, \Im) \to (Y, \sigma)$ is said to be

(a) intuitionistic fuzzy set connected [12] provided that, if X is intuitionistic fuzzy connected between intuitionistic fuzzy sets A and B , $f(X)$ is intuitionistic fuzzy connected between $f(A)$ and $f(B)$ with respect to relative intuitionistic fuzzy topology.

(b) intuitionistic fuzzy set GO - connected[13] provided that , if X is intuitionistic fuzzy GO -connected between intuitionistic fuzzy sets A and B, $f(X)$ is intuitionistic fuzzy GO - connected between $f(A)$ and $f(B)$ with respect to relative intuitionistic fuzzy topology.

Definition 2.7. A mapping $f : (X, \Im) \to (Y, \sigma)$ is said to be

(a) Intuitionistic fuzzy continuous [7] if the pre image of each intuitionistic fuzzy open set in Y is an intuitionistic fuzzy open set in X.

(b) Intuitionistic fuzzy w - irresolute [16] if pre image of every intuitionistic fuzzy w -closed set of Y is intuitionistic fuzzy w-closed in X

3. Intuitionistic fuzzy WO - connectedness between intuitionistic fuzzy sets

Definition 3.1. An intuitionistic fuzzy topological space$(X, \Im)$ is said to be intuitionistic fuzzy WO -connected between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy w - closed w -open set F in X such that $A \subseteq F$ and $\rceil(FqB)$.

Theorem 3.2. *If an intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B , then it is intuitionistic fuzzy connected between A and B.*

Proof. If $(X, \Im)$ is not intuitionistic fuzzy connected between A and B , then there exists an intuitionistic fuzzy closed open set F in X such that $A \subseteq F$ and $\rceil(FqB))$. Then by Remark 2.2 ,F is an intuitionistic fuzzy w -closed w - open set in X such that $A \subseteq F$ and $\rceil(FqB)$. Hence $(X, \Im)$ is not intuitionistic fuzzy WO - connected between A and B , which contradicts our hypothesis. □

Remark 3.3. Converse of Theorem 3.1 may be false. As the following example shows

Example 3.4. Let $X = \{$ a , b $\}$ and $U = \{ \langle$ a , 0.5, 0.4 $\rangle$,$\langle$ b , 0.6 ,0.4 $\rangle \}$ A = $\{ \langle$ a, 0.2, 0.7 $\rangle$,$\langle$ b, 0.3, 0.6 $\rangle \}$ and $B = \{ \langle$ a, 0.5, 0.4 $\rangle$, $\langle$ b, 0.4, 0.5 $\rangle \}$ be intuitionistic fuzzy sets on X .let $\Im = \{ \tilde{\mathbf{0}}$, U, $\tilde{\mathbf{1}} \}$ be an intuitionistic fuzzy topology on X. Then $(X, \Im)$ is intuitionistic fuzzy connected between A and B but it is not intuitionistic fuzzy WO - connected between A and B.

Theorem 3.5. *If an intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy GO - connected between intuitionistic fuzzy sets A and B , then it is intuitionistic fuzzy WO-connected between A and B.*

Proof. If $(X, \Im)$ is not intuitionistic fuzzyWO- connected between A and B , then there exists an intuitionistic fuzzy w-closed$w$- open set F in X such that $A \subseteq F$ and $\rceil(FqB))$. Then by Remark 2.2 ,F is an intuitionistic fuzzy g -closed g - open set in X such that $A \subseteq F$ and $\rceil(FqB)$. Hence $(X, \Im)$ is not intuitionistic fuzzy GO - connected between A and B , which contradicts our hypothesis. □

Remark 3.6. Converse of Theorem 3.5 may be false. As the following example shows

Example 3.7. Let $X = \{$ a , b $\}$ and $U = \{ \langle$ a , 0.4, 0.3 $\rangle$,$\langle$ b , 0.5 ,0.4 $\rangle \}$ A = $\{ \langle$ a, 0.2, 0.6 $\rangle$,$\langle$ b, 0.3, 0.5 $\rangle \}$ and $B = \{ \langle$ a, 0.4, 0.3 $\rangle$, $\langle$ b, 0.4, 0.6 $\rangle \}$ be intuitionistic fuzzy sets on X .let $\Im = \{ \tilde{\mathbf{0}}$, U, $\tilde{\mathbf{1}} \}$ be an intuitionistic fuzzy topology on X. Then $(X, \Im)$ is intuitionistic fuzzy WO- connected between A and B but it is not intuitionistic fuzzy GO - connected between A and B.

Theorem 3.8. *An intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B if and only if there is no intuitionistic fuzzy w - closed w - open set F in X such that $A \subseteq F \subseteq B^c$.*

Proof. Necessity: Let $(X, \Im)$ is intuitionistic fuzzy WO - connected between intuitionist ic fuzzy sets A and B. Suppose on the contrary , that F is an intuitionistic fuzzy w -closed w -open set in X such that$A \subseteq F \subseteq B^c$. Now $F \subseteq B^c$ which implies that $\rceil(FqB)$. Therefore F is an intuitionistic fuzzy w - closed w - open set in X such that $A \subseteq F$ and $\rceil(FqB)$. Hence $(X, \Im)$ is not intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B , which is a contradiction.

Sufficiency: Suppose on the contrary, that $(X, \Im)$ is not intuitionistic fuzzy WO -connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy w-closed w-open set F in X such that $A \subseteq F$ and $\rceil(FqB)$. Now, $\rceil(FqB)$ which implies that $F \subseteq B^c$. Therefore F is an intuitionistic fuzzy w -closed w -open set in X such that $A \subseteq F \subseteq B^c$, which contradicts our assumption. □

Theorem 3.9. *If an intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy WO -connected between intuitionistic fuzzy sets A and B ,then A and B are non- empty.*

Proof. Let intuitionistic fuzzy set A is empty, then A is an intuitionistic fuzzy w-closed w-open set in X and $A \subseteq B$. Now we claim that $\rceil(AqB)$. If AqB ,then there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$ .But $\mu_A(x) = \tilde{\mathbf{0}}$ and $\gamma_A(x) = \tilde{\mathbf{1}}$ for all $x \in X$. Therefore no point $x \in X$ for which $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$, which is a contradiction. Hence $\rceil(AqB)$ and $(X, \Im)$ is not intuitionistic fuzzy WO - connected between A and B. □

Theorem 3.10. *An intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B if and only if it is intuitionistic fuzzy WO - connected between $wcl(A)$ and $wcl(B)$.*

Proof. Necessity: Follows from Theorem 3.9, because $A \subseteq wcl(A)$ and $B \subseteq wcl(B)$.

Sufficiency: Suppose $(X, \Im)$ is not intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B, Then there is an intuitionistic fuzzy w - closed w-open set F of X such that $A \subseteq F$ and $\rceil(FqB)$. Since F is intuitionistic fuzzy w-closed and $A \subseteq F$, $wcl(A) \subseteq F$. Now,$\rceil(FqB)$ which implies that $F \subseteq B^c$. Therefore $F = wint(F) \subseteq wint(B^c)$= $(wcl(B))^c$. Hence $\rceil(Fqwcl(B))$ and X is not intuitionistic fuzzy WO-connected between $wcl(A)$ and $wcl(B)$. □

Theorem 3.11. *Let $(X, \Im)$ be an intuitionistic fuzzy topological space and A and B be two intuitionistic fuzzy sets in X. If AqB then $(X, \Im)$ is intuitionistic fuzzy WO -connected between intuitionistic fuzzy sets A and B.*

Proof. If F is any intuitionistic fuzzy w-closed w-open set of X such that $A \subseteq F$, then AqB hence FqB. Therefore, there is no intuitionistic fuzzy w-closed w-open set F in X such that $A \subseteq F$ and $\rceil(FqB)$. Therefore $(X, \Im)$ is intuitionistic fuzzy WO-connected between intuitionistic fuzzy sets A and B. □

Remark 3.12. The converse of Theorem 3.11 may not be true. As the following example shows.

Example 3.13. Let $X = \{$ a , b $\}$ and $U = \{ \langle$ a , 0.2, 0.6 $\rangle$,$\langle$ b , 0.3 , 0.5 $\rangle \}$ $A = \{ \langle$ a, 0.4, 0.3 $\rangle$,$\langle$ b, 0.3, 0.6 $\rangle \}$ and $B = \{ \langle$ a, 0.2, 0.5 $\rangle$,$\langle$ b, 0.5, 0.4 $\rangle \}$ be intuitionistic fuzzy sets on X .let $\Im = \{ \tilde{\mathbf{0}}$, U, $\tilde{\mathbf{1}} \}$ be an intuitionistic fuzzy topology on X. Then $(X, \Im)$ is intuitionistic fuzzy connected WO connected between intuitionistic fuzzy sets A and B but$\rceil(AqB)$

Theorem 3.14. *An intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy W -connected if and only if it is intuitionistic fuzzy WO-connected between every pair of its non- empty intuitionistic fuzzy sets.*

Proof. Necessity: Let A and B be any pair of intuitionistic fuzzy sets of X. Suppose $(X, \Im)$ is not intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy w-closed w-open set F of X such that $A \subseteq F$ and $\rceil(FqB)$. Since intuitionistic fuzzy sets A and B are non- empty, it follows that F is a non- empty proper intuitionistic fuzzy w-closed w-open set of X. Hence $(X, \Im)$ is not intuitionistic fuzzy W - connected.

Sufficiency: Suppose $(X, \Im)$ is not intuitionistic fuzzy W -connected. Then there exists a non-empty proper intuitionistic fuzzy w-closed w-open set F of X. Consequently X is not intuitionistic fuzzy WO-connected between $F$and F^c, a contradiction. □

Remark 3.15. If an intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy WO-connected between a pair of its intuitionistic fuzzy subsets it is not necessarily that $(X, \Im)$ is intuitionistic fuzzy WO-connected between every pair of its intuitionistic fuzzy subsets and so is not necessarily intuitionistic fuzzy W-connected. As the following example shows.

Example 3.16. Let $X = \{$ a , b $\}$ and $U = \{ \langle$ a , 0.5, 0.4 $\rangle$,$\langle$ b , 0.6 , 0.4 $\rangle \}$ $A = \{ \langle$ a, 0.4, 0.3 $\rangle$,$\langle$ b, 0.6, 0.4 $\rangle \}$,$B = \{ \langle$ a, 0.5, 0.2 $\rangle$,$\langle$ b, 0.4, 0.4 $\rangle \}$, $C = \{ \langle$ a, 0.2, 0.7 $\rangle$,$\langle$ b, 0.3, 0.6 $\rangle \}$ and $D = \{ \langle$ a, 0.5, 0.4 $\rangle$,$\langle$ b, 0.4, 0.5 $\rangle \}$ be intuitionistic fuzzy sets on X .let $\Im = \{ \tilde{\mathbf{0}}$, U, $\tilde{\mathbf{1}} \}$ be an intuitionistic fuzzy topology on X. Then $(X, \Im)$ is intuitionistic fuzzy connected WO connected between intuitionistic fuzzy sets A and B but it is not intuitionistic fuzzy WO -connected between intuitionistic fuzzy sets C and D. Also $(X, \Im)$ is not intuitionistic fuzzy w-connected.

Theorem 3.17. *Let $(Y, \Im_Y)$ be a subspace of a intuitionistic fuzzy topological space $(X, \Im)$ and A , B be intuitionistic fuzzy subsets of Y. If $(Y, \Im_Y)$ is intuitionistic fuzzy WO -connected between A and B then so is $(X, \Im)$.*

Proof. Suppose, on the contrary, that $(X, \Im)$ is not intuitionistic fuzzy WO- connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy w-closed w-open set F of X such that $A \subseteq F$ and $\rceil(FqB)$. Put $F_Y = F \cap Y$. Then F_Y is intuitionistic fuzzy w-closed w-open set in Y such that $A \subseteq F_Y$ and $\rceil(F_Y qB)$. Hence $(Y, \Im_Y)$ is not intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B , a contradiction. □

Theorem 3.18. *Let $(Y, \Im_Y)$ be an intuitionistic fuzzy closed open subspace of a intuitionistic fuzzy topological space $(X, \Im)$ and A , B be intuitionistic fuzzy subsets of Y. If $(X, \Im)$ is intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B , then so is $(Y, \Im_Y)$.*

Proof. If $(Y, \Im_Y)$ is not intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B, then there exists an intuitionistic fuzzy w-closed w-open set F of Y such that $A \subseteq F$ and $\rceil(FqB)$. Since Y is intuitionistic fuzzy closed open in X, F is an intuitionistic fuzzy w-closed w-open set in X. Hence X cannot be intuitionistic fuzzy WO-connected between intuitionistic fuzzy sets A and B, a contradiction. □

4. Intuitionistic fuzzy set WO-connected mappings

Definition 4.1. A mapping$f : (X, \Im) \to (Y, \sigma)$ is said to be intuitionistic fuzzy set WO - connected provided that, if X is intuitionistic fuzzy WO -connected between intuitionistic fuzzy sets A and B, $f(X)$ is intuitionistic fuzzy WO - connected between $f(A)$ and $f(B)$ with respect to relative intuitionistic fuzzy topology.

Theorem 4.2. *A mapping $f : (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy set WO-connected if and only if $f^{-1}(F)$ is a intuitionistic fuzzy w - closed w - open set of X for every intuitionistic fuzzy w -closed w-open set F of $f(X)$.*

Proof. Necessity: Let F be an intuitionistic fuzzy w - closed w - open set of $f(X)$. Suppose $f^{-1}(F)$ is not intuitionistic fuzzy w - closed w -open set of X. Then X is intuitionistic fuzzy WO - connected between $f^{-1}(F)$ and $(f^{-1}(F))^c$. Therefore $f(X)$ is intuitionistic fuzzy WO - connected between $f(f^{-1}(F))$ and $f((f^{-1}(F))^c)$. But , $f((f^{-1}(F)) = F \cap f(X) = F \cap f((f^{-1}(F))^c) = f(X) \cap F^c = F^c$ which implies that F is not intuitionistic fuzzy w-closed w-open set in X, a contradiction.

Sufficiency: Let X be intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B. Suppose $f(X)$ is not intuitionistic fuzzy WO - connected between $f(A)$ and $f(B)$. Then by theorem 3.8 there exist an intuitionistic fuzzy w - closed w - open set F in $f(X)$ such that $f(A) \subseteq F \subseteq (f(B))^c$. By hypothesis $f^{-1}(F)$ is intuitionistic fuzzy w - closed w - open set of X and $A \subseteq f - 1(F) \subseteq B^c$. Therefore X is not intuitionistic fuzzy WO - connected between intuitionistic fuzzy sets A and B, a contradiction. Hence f is intuitionistic fuzzy set WO - connected mapping. □

Theorem 4.3. *If $f : (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy set WO - connected then $f^{-1}(F)$ is an intuitionistic fuzzy w - closed w - open set of X for any intuitionistic fuzzy w - closed w -open set of Y .*

Proof. Obvious. □

Theorem 4.4. *Every intuitionistic fuzzy w - irresolute mapping is intuitionistic fuzzy set WO - connected.*

Proof. Let $f : (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy w - irresolute. Let F is intuitionistic fuzzy w -closed w -open set of Y, then $f^{-1}(F)$ is intuitionistic fuzzy w - closed w -open set of X . Hence by theorem 4.3 f is intuitionistic fuzzy set WO - connected. □

Remark 4.5. The converse of Theorem 4.3 may not be true. As the following example shows.

Example 4.6. Let X = {a, b}, Y = {p, q} and $U = \{\langle a, 0.3, 0.6\rangle, \langle b, 0.4, 0.5\rangle\}$, $V = \{\langle p, 0.4, 0.6\rangle, \langle q, 0.5, 0.4\rangle\}$ be the intuitionistic fuzzy sets. Let $\Im$ = { $\tilde{\mathbf{0}}$, U, $\tilde{\mathbf{1}}$ } and σ = { $\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$ } be the intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ is defined by $f(a) = p$, $f(b) = q$ is intuitionistic fuzzy set WO - connected but not intuitionistic fuzzy w - irresolute.

Theorem 4.7. *Every mapping $f : (X, \Im) \to (Y, \sigma)$, such that $f(X)$ is intuitionistic fuzzy W - connected is an intuitionistic fuzzy set WO - connected mapping.*

Proof. Let $f(X)$ be intuitionistic fuzzy W - connected. Then no nonempty proper intuitionistic fuzzy set of $f(X)$ which is both intuitionistic fuzzy w -closed and w - open. Hence f is intuitionistic fuzzy set WO - connected. □

Theorem 4.8. *Let $f : (X, \Im) \to (Y, \sigma)$, be an intuitionistic fuzzy set WO - connected mapping. If X is intuitionistic fuzzy W - connected then $f(X)$ is intuitionistic fuzzy W -connected.*

Proof. Suppose $f(X)$ is not intuitionistic fuzzy W - connected. Then there is a nonempty proper intuitionistic fuzzy w - closed w - open set F of $f(X)$. Since f is intuitionistic fuzzy set WO - connected by theorem 4.2, $f^{-1}(F)$ is a nonempty proper intuitionistic fuzzy w - closed w - open set of X. Consequently X is not intuitionistic fuzzy W - connected. □

Theorem 4.9. *Let $f : (X, \Im) \to (Y, \sigma)$ be a surjective intuitionistic fuzzy set WO-connected and $g : (Y, \sigma) \to (Z, \phi)$ a intuitionistic fuzzy set WO- connected mapping. Then $gof : (X, \Im) \to (Z, \phi)$ is intuitionistic fuzzy set WO-connected.*

Proof. Let F be an intuitionistic fuzzy w - closed w -open set of $g(Y)$. Then $g^{-1}(F)$ is an intuitionistic fuzzy w - closed w - open set of $Y = f(X)$. And so $f^{-1}(g^{-1}(F))$ is an intuitionistic fuzzy w - closed w - open fuzzy set in X . Now $(gof)(X) = g(Y)$ and $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$, by theorem 4.2, gof is intuitionistic fuzzy set WO - connected. □

Theorem 4.10. *Let $f : (X, \Im) \to (Y, \sigma)$ be a mapping and $g : (X, \Im) \to (X, \Im) \times (Y, \sigma)$ be the graph mapping of f defined by $g(x) = (x, f(x))$ for each $x \in X$. If g is intuitionistic fuzzy set WO - connected. Then f is intuitionistic fuzzy set WO - connected.*

Proof. Let F be an intuitionistic fuzzy w - closed w - open set of the subspace $f(X)$ of Y . Then $X \times F$ is an intuitionistic fuzzy w - closed w - open set of subspace $X \times f(X)$ of the intuitionistic fuzzy product space $X \times Y$. Since $g(X)$ is a subset of $X \times f(X)$, $(X \times F) \cap g(X)$ is an intuitionistic fuzzy w - closed w -open set of the subspace $g(X)$ of $X \times$ Y. By theorem 4.2, $g^{-1}(X \times F) \cap g(X)$ is an intuitionistic fuzzy w - closed w - open set of X . It follows from $g^{-1}((X \times F) \cap g(X) = g^{-1}(X \times F)$ $= f^{-1}(F)$ that $f^{-1}(F)$ is an intuitionistic fuzzy w - closed w - open set of X. Hence by theorem 4.2, f is intuitionistic fuzzy set WO- connected. □

Definition 4.11. An intuitionistic fuzzy topological space $(X, \Im)$ is said to be intuitionistic fuzzy WO - extremely disconnected if the w - closure of every intuitionistic fuzzy w - open set of X is intuitionistic fuzzy w - open in X.

Theorem 4.12. *Let $(X, \Im)$ be an intuitionistic fuzzy topological space , Then the following conditions are equivalent:*

(a) X is intuitionistic fuzzy WO - extremely disconnected.

(b) For each intuitionistic fuzzy w - closed set A , $wint(A)$ is intuitionistic fuzzy w -closed.

(c) For each intuitionistic fuzzy w - open set A, $wcl(A) = [wcl(wcl(A))^c]^c$

(d) For each pair of intuitionistic fuzzy w -open sets A and B such that $wcl(A)$ $= B^c$, $wcl(A) = (wcl(B))^c$

Proof. (a)$\Rightarrow$ (b) : Let A be an intuitionistic fuzzy w-closed set. Then A^c is intuitionistic fuzzy w-open set,so by(a) $wcl(A^c)$ is intuitionistic fuzzy w - open in X. Now $wcl(A^c) = (wint(A))^c$, therefore $(wint(A))^c$ intuitionistic w - open in X which implies that $wint(A)$ is intuitionistic fuzzy w-closed.

(b) $\Rightarrow$ (c): Let A be an intuitionistic fuzzy w - open set, we have $wcl(wcl(A))^c = wcl(wint(A^c)) \Rightarrow [wcl(wcl(A))^c]^c = [wcl(wint(A^c))]^c$, since A is intuitionistic fuzzy w - open , A^c is intuitionistic fuzzy w - closed and so by (b) $wint(A^c)$ is intuitionistic fuzzy w - closed. Therefore, $wcl(wint(A^c)) = wint(A^c)$. Hence we have $[wcl(wcl(A))^c]^c = [wcl(wint(A^c))]^c = [wint(A^c)]^c = [(wcl(A))^c]^c = wcl(A)$.

(c) $\Rightarrow$ (d): Let A and B be any two intuitionistic fuzzy w - open set in X such that $wcl(A) = B^c$. Then by (c), $wcl(A) = [wcl(wint(A^c))]^c = [wcl((wcl(A))^c]^c = [(wcl((B^c))^c]^c = [wcl(B)]^c$.

(d) $\Rightarrow$ (a): Let A be any intuitionistic fuzzy w - open set . Put $B = (wcl(A))^c$. Then $wcl(A) = B^c$, so by (d) $wcl(A) = (wcl(B))^c$, so that $wcl(A)$ is intuitionistic fuzzy w - open set. Hence, X is intuitionistic fuzzy WO - extremely disconnected. □

Definition 4.13. A mapping $f : (X, \Im) \to (Y, \sigma)$ is said to be intuitionistic fuzzy weakly w - irresolute if $f^{-1}(B) \subseteq wint(f^{-1}(wcl(B)))$ for each intuitionistic fuzzy w - open set B of Y.

Definition 4.14. A mapping$f : (X, \Im) \to (Y, \sigma)$ is said to be intuitionistic fuzzy almost w -irresolute if $f^{-1}(B) \subseteq wint(f^{-1}(wint(wcl(B)))$ for each intuitionistic fuzzy w - open set B of Y .

Theorem 4.15. *Let (Y, σ) be an intuitionistic fuzzy WO - extremely disconnected space. If a mapping $f : (X, \Im) \to (Y, \sigma)$ is an intuitionistic fuzzy set WO - connected, then f is intuitionistic fuzzy almost W - irresolute.*

Proof. Let V is an intuitionistic fuzzy w - open set of Y. Then $wcl(V)$ is an intuitionistic fuzzy w -closed w -open set in Y. Since f is intuitionistic fuzzy set WO - connected $f^{-1}(wcl(V))$ is intuitionistic fuzzy w - closed w - open set of X . Therefore $f^{-1}(V) \subseteq f^{-1}(wcl(V)) = wint(f^{-1}(wcl(V))) = wint(f^{-1}(wint(wcl(V))))$. Hence f is intuitionistic fuzzy almost w-irresolute. □

Corollary 4.16. *Let (Y, σ) be an intuitionistic fuzzy WO - extremely disconnected space. If a mapping $f : (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy set WO - connected then f is intuitionistic fuzzy weakly w -irresolute.*

Theorem 4.17. *Let (Y, σ) be an intuitionistic fuzzy WO - extremely disconnected and $f : (X, \Im) \to (Y, \sigma)$ be a surjective mapping. Then the following conditions are equivalent:*

(a) f is intuitionistic fuzzy set WO - connected.
(b) f is intuitionistic fuzzy almost w -irresolute .
(c) f is intuitionistic fuzzy weakly w -irresolute .

Proof. (a)$\Rightarrow$ (b) follows from theorem 4.15.

(b) $\Rightarrow$ (c) Let B is intuitionistic fuzzy w - open set of Y . Then since f is intuitionistic fuzzy almost w -irresolute $f^{-1}(B) \subseteq wint(f^{-1}(wint(wcl(B)))$. Since Y is intuitionistic fuzzy WO - extremely disconnected space, $wcl(B)$ is intuitionistic fuzzy w-open in Y, which implies that $wint(wcl(B)) = wcl(B)$. Hence we have $f^{-1}(B) \subseteq wint(f^{-1}(wcl(B))$ for every intuitionistic fuzzy set B of Y. Therefore f is intuitionistic fuzzy weakly w - irresolute.

(c)$\Rightarrow$ (a) Let f is intuitionistic fuzzy weakly w - irresolute. Let B is intuitionistic fuzzy w- closed w -open set of Y . Then $wcl(B)$ is intuitionistic fuzzy w -closed w-open set of Y, because (Y, σ) be an intuitionistic fuzzy WO - extremely disconnected . Now $wcl(B) = B$ implies that $f^{-1}(wcl(B)) = f^{-1}(B)$. Since f is intuitionistic fuzzy weakly w -irresolute $f^{-1}(B) \subseteq wint(f^{-1}(wcl(B)))$. Hence $f^{-1}(wcl(B)) \subseteq f^{-1}(B) \subseteq wint(f^{-1}(wcl(B)))$. Thus $wint(f^{-1}(wcl(B))) = f^{-1}(wcl(B))$. Therefore $f^{-1}(wcl(B))$ is intuitionistic fuzzy w -closed w-open set of X, By theorem 4.3 f is intuitionistic fuzzy set WO -connected mapping. □

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