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On soft $\widetilde{I}$-baire spaces

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Abstract. In the present paper, we introduce nowhere *-soft dense sets and *-soft first category which is generalized of nowhere *-soft dense sets. Also, we define soft $\widetilde{I}$-Baire spaces and investigate some properties of these spaces. Then, we show that soft regular ideal topological space is a soft $\widetilde{I}$-Baire space.

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1. Introduction

There are uncertainty in many complicated problems in the fields of engineering, physics, computer science, medical science, social science and economics. These problems can not be solved by classical methods such as probability theory, fuzzy set theory, intitionistic fuzzy sets, rough set etc. In [14] Molodtsov initiated the concept of soft sets as a new mathematical tool for dealing with uncertainties.

Hence, soft set theory has successful applications in various fields. Ali et al [1] introduced some algebraic operations on soft sets. Kharal and Ahmad [9] gave the concept of a mapping on soft classes and studied propeties of images and inverse images of soft sets. Shabir and Naz [19] introduced the soft topological spaces and investigated its fundamental properties. Zorlutuna et all. [20] worked on soft topological spaces. Das and Samanta [2] defined soft metric spaces and studied some their properties. Then, they redefined the notion of soft point. In literature, studies on soft theory and its applications can be seen [3, 4, 12, 16].
The concept of ideal was introduced by Kuratowski [10]. In 1990, Janković and Hamlett [5] introduced another topology $\tau^*(I)$ by using a given ideal I for a given topology τ, which satisfies $\tau \subseteq \tau^*(I)$.

Baire spaces play an important role on classical topology and have various applications on completely metric spaces. Recently, Li and Lin [11] have studied I-Baire spaces by using ideal.

Sahin and Kucuk [18] introduced the notion of soft ideal. Then, the different version of soft ideal was given in [6, 15]. Also, Kandil et all. [6, 7, 8] obtained the concepts of soft *-topology, soft semi-I-compactness and soft connectedness via soft ideals.

In this paper, we present nowhere *-soft dense sets and we study some properties of *-soft dense and nowhere *-soft dense sets. Also, we introduce the notion of soft $\widetilde{I}$-Baire space by using these concepts and investigate relationships between soft ideal regular space and soft $\widetilde{I}$-Baire space.

2. Preliminaries

Throughout this paper, X will be a nonempty initial universal set and A will be a set of parameters. Let $\mathcal{P}(X)$ denote the power set of X and $SS(X)_A$ denote the set of all soft sets over X.

Definition 2.1 ([14])**.** A pair (F, A), where F is a mapping from A to $\mathcal{P}(X)$, is called a soft set over X.

Definition 2.2 ([13])**.** Let (F_1, A) and (F_2, A) be two soft sets over a common universe X. Then, (F_1, A) is said to be a soft subset of (F_2, A) if $F_1(\alpha) \subseteq F_2(\alpha)$, for all $\alpha \in A$. This is denoted by $(F_1, A)\widetilde{\subseteq}(F_2, A)$.

(F_1, A) is said to be soft equal to (F_2, A) if $F_1(\alpha) = F_2(\alpha)$, for all $\alpha \in A$. This is denoted by $(F_1, A) = (F_2, A)$.

Definition 2.3 ([1])**.** The complement of a soft set (F, A) is defined as $(F, A)^c = (F^c, A)$, where $F^c : A \to \mathcal{P}(X)$ is a mapping given by $F^c(\alpha) = X\backslash F(\alpha)$, for all $\alpha \in A$.

Definition 2.4 ([13])**.** Let (F, A) be a soft set over X.

(i) (F, A) is said to be a null soft set if $F(\alpha) = \varnothing$, for all $\alpha \in A$. This is denoted by $\widetilde{\varnothing}$.

(ii) (F, A) is said to be an absolute soft set if $F(\alpha) = X$, for all $\alpha \in A$. This is denoted by $\widetilde{X}$.

Clearly, we have $(\widetilde{X})^c = \widetilde{\varnothing}$ and $(\widetilde{\varnothing})^c = \widetilde{X}$.

Definition 2.5 ([19])**.** The difference (H, A) of two soft sets (F_1, A) and (F_2, A) over X, denoted by $(F_1, A)\backslash(F_2, A)$, is defined as $H(\alpha) = F_1(\alpha)\backslash F_2(\alpha)$, for all $\alpha \in A$.

Definition 2.6 ([13])**.** The union (H, A) of two soft sets (F, A) and (G, A) over a common universe X, denoted by $(F, A)\widetilde{\cup}(G, A)$ is defined as $H(\alpha) = F(\alpha) \cup G(\alpha)$, for all $\alpha \in A$.

The following definition of intersection of two soft sets is given as that of the bi-intersection in [3].

Definition 2.7 ([3])**.** The intersection (H, A) of two soft sets (F, A) and (G, A) over a common universe X, denoted by $(F, A)\widetilde{\cap}(G, A)$, is defined as $H(\alpha) = F(\alpha) \cap G(\alpha)$, for all $\alpha \in A$.

Definition 2.8 ([20]). Let I be an arbitrary index set and $\{(F_i, A)\}_{i\in I}$ be a nonempty family of soft sets over a common universe X.

(i) The union of these soft sets is the soft set (H, A), where $H(\alpha) = \bigcup_{i\in I} F_i(\alpha)$ for all $\alpha \in A$. This is denoted by $\widetilde{\bigcup}_{i\in I}(F_i, A) = (H, A)$.

(ii) The intersection of these soft sets is the soft set (H, A), where $H(\alpha) = \bigcap_{i\in I} F_i(\alpha)$ for all $\alpha \in A$. This is denoted by $\widetilde{\bigcap}_{i\in I}(F_i, A) = (H, A)$.

Definition 2.9 ([16, 2]).

(i) A soft set (E, A) over X is said to be a soft element (soft point) if there exists $\alpha \in A$ such that $E(\alpha)$ is a singleton, say $\{x\}$, and $E(\beta) = \varnothing$, for all $\beta(\neq \alpha) \in A$. Such a soft element (soft point) is denoted by E^x_α.

(ii)The soft element E^x_α is said to be in the soft set (G, A) denoted by $E^x_\alpha \widetilde{\in} (G, A)$, if $x \in G(\alpha)$.

(iii)Two soft elements $E^{x_1}_{\alpha_1}, E^{x_2}_{\alpha_2}$ are said to be equal if $\alpha_1 = \alpha_2$ and $x_1 = x_2$. Thus, $E^{x_1}_{\alpha_1} \neq E^{x_2}_{\alpha_2} \Leftrightarrow x_1 \neq x_2$ or $\alpha_1 \neq \alpha_2$.

Proposition 2.10 ([2]). *The union of any collection of soft elements can be considered as a soft set and every soft set can be expressed as union of all soft elements belonging to it, i.e.,* $(F, A) = \widetilde{\bigcup}_{E^x_\alpha \widetilde{\in} (F,A)} E^x_\alpha$.

Proposition 2.11 ([2]). *For two soft sets* (F, A) *and* (G, A), $(F, A) \widetilde{\subseteq} (G, A) \Leftrightarrow E^x_\alpha \widetilde{\in} (F, A) \Rightarrow E^x_\alpha \widetilde{\in} (G, A)$ *and hence* $(F, A) = (G, A)$ *if and only if* $E^x_\alpha \widetilde{\in} (F, A) \Leftrightarrow E^x_\alpha \widetilde{\in} (G, A)$.

Definition 2.12 ([19]). Let τ be a collection of soft sets over X. Then τ is said to be a soft topology on X if

(i) $\widetilde{\varnothing}, \widetilde{X} \in \tau$,

(ii) the union of any number of soft sets in τ belongs to τ,

(iii) the intersection of any two soft sets in τ belongs to τ.

The triple (X, τ, A) is called a soft topological space over X. The members of τ are said to be soft open sets in X. A soft set over X is said to be soft closed in X if its complement belongs to τ.

Definition 2.13. Let (X, τ, A) be a soft topological space, and let (G, A) be a soft set over X.

(i) [19] The soft closure of (G, A) is the soft set $cl((G, A)) = \widetilde{\bigcap}\{(S, A) : (S, A)$ is soft closed and $(G, A) \widetilde{\subseteq} (S, A)\}$,

(ii) [20] The soft interior of (G, A) is the soft set $int((G, A)) = \widetilde{\bigcup}\{(S, A) : (S, A)$ is soft open and $(S, A) \widetilde{\subseteq} (G, A)\}$.

(iii) [4] (G, A) is said to be soft dense if $cl(G, A) = \widetilde{X}$.

(iv) [6] (G, A) is said to be nowhere dense soft set if $int(cl(G, A)) = \widetilde{\varnothing}$.

Definition 2.14 ([16]). Let (X, τ, A) be a soft topological space. A soft set (F, A) is said to be a soft neighbourhood of the soft set (H, A) if there exists a soft set $(G, A) \in \tau$ such that $(H, A) \widetilde{\subseteq} (G, A) \widetilde{\subseteq} (F, A)$. If $(H, A) = E^x_\alpha$, then (F, A) is said to be a soft neighbourhood of the soft element E^x_α.

The soft neighbourhood system of a soft element E^x_α, denoted by $\mathcal{N}_\tau(E^x_\alpha)$, is the family of all its soft neighbourhoods.

The soft open neighbourhood system of a soft element E^x_α, denoted by $\mathcal{U}(E^x_\alpha)$, is the family of all its soft open neighbourhoods.

Proposition 2.15 ([16]). *A soft set (F,A) over X is a soft open if and only if (F,A) is a neighbourhood of each of its soft elements, i.e. $(F,A) \in \tau$ if and only if $(F,A) \in \mathcal{N}_\tau(E^x_\alpha)$, for all $E^x_\alpha \widetilde{\in} (F,A)$.*

Theorem 2.16 ([12]). *Let (X,τ,A) be a soft topological space. A soft element $E^x_\alpha \widetilde{\in} cl((F,A))$ if and only if each soft neighbourhood of E^x_α intersects (F,A).*

Proposition 2.17 ([16]). *Let (X,τ,A) be a soft topological space and (F,A) be a soft set over X.*
(i) (F,A) is soft closed if and only if $(F,A) = cl((F,A))$,
(ii) (F,A) is soft open if and only if $(F,A) = int((F,A))$.

Definition 2.18 ([19]). Let (X,τ,A) be a soft topological space over X and Y be a non-empty subset of X. Then,

$$\tau_Y = \{(F_Y, A) = \widetilde{Y} \widetilde{\cap} (F,A) : (F,A) \in \tau\}$$

is said to be the soft relative topology on Y and (Y, τ_Y, A) is called a soft subspace of (X,τ,A).

Theorem 2.19 ([19]). *Let (Y,τ_Y,A) be a soft subspace of a soft topological space (X,τ,A) and (F,A) be a soft set over X, then (F,A) is soft open in Y if and only if $(F,A) = \widetilde{Y} \widetilde{\cap} (G,A)$ for some $(G,A) \in \tau$.*

Definition 2.20 ([9]). Let $SS(X)_A$ and $SS(Y)_B$ be families of all soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : A \to B$ be two mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;

(i) If $(F,A) \in SS(X)_A$. Then the image of (F,A) under f_{pu}, written as $f_{pu}(F,A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(b) = \begin{cases} \bigcup_{a \in p^{-1}(b) \cap A} u(F(a)), & p^{-1}(b) \cap A \neq \varnothing \\ \varnothing, & \text{otherwise} \end{cases}$$

for all $b \in B$.

(ii) If $(G,B) \in SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f^{-1}_{pu}(G,B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f^{-1}_{pu}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B \\ \varnothing, & \text{otherwise} \end{cases}$$

for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective [20].

Theorem 2.21 ([9, 20]). *Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For a function $f_{pu} : SS(X)_A \to SS(Y)_B$, the following statements are true.*
(i) $f_{pu}(\widetilde{\varnothing}) = \widetilde{\varnothing}$, $f_{pu}(\widetilde{X}) \widetilde{\subseteq} \widetilde{Y}$, $f^{-1}_{pu}(\widetilde{\varnothing}) = \widetilde{\varnothing}$ and $f^{-1}_{pu}(\widetilde{Y}) = \widetilde{X}$.
(ii) $f_{pu}(F,A) \widetilde{\cap} f_{pu}(G,B) \widetilde{\subseteq} f_{pu}((F,A) \widetilde{\cap} (G,B))$.

(iii) $f_{pu}^{-1}(F,A)\widetilde{\cap}f_{pu}^{-1}(G,B) = f_{pu}^{-1}((F,A)\widetilde{\cap}(G,B))$.

(iv) $f_{pu}^{-1}((G,B)^c) = (f_{pu}^{-1}(G,B))^c$ *for any soft set* (G,B) *in* $SS(Y)_B$.

(v) $f_{pu}(f_{pu}^{-1}(G,B))\widetilde{\subseteq}(G,B)$ *for any soft set* (G,B) *in* $SS(Y)_B$. *If* f_{pu} *is surjective, then the equality holds.*

(vi) $(F,A)\widetilde{\subseteq}f_{pu}^{-1}(f_{pu}(F,A))$ *for any soft set* (F,A) *in* $SS(X)_A$. *If* f_{pu} *is injective, then the equality holds.*

Definition 2.22. [20] Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces, $f_{pu}: SS(X)_A \to SS(Y)_B$ be a function and $E_\alpha^x\widetilde{\in}\widetilde{X}$.

(i) f_{pu} is soft pu-continuous at $E_\alpha^x\widetilde{\in}\widetilde{X}$if for each $(G,B)\in\mathcal{N}_{\tau_2}(f_{pu}(E_\alpha^x))$, there exists a $(H,A)\in\mathcal{N}_{\tau_1}(E_\alpha^x)$ such that $f_{pu}(H,A)\widetilde{\subseteq}(G,B)$.

(ii) f_{pu} is soft pu-continuous on $\widetilde{X}$ if f_{pu} is soft continuous at each soft point in $\widetilde{X}$.

Theorem 2.23 ([20]). *Let* (X,τ_1,A) *and* (Y,τ_2,B) *be soft topological spaces,* $f_{pu}: SS(X)_A \to SS(Y)_B$ *be a function and* $E_\alpha^x\widetilde{\in}\widetilde{X}$. *Then the following statements are equivalent.*

(i) f_{pu} *is soft pu-continuous at* E_α^x.

(ii) For each $(G,B)\in\mathcal{N}_{\tau_2}(f_{pu}(E_\alpha^x))$, *there exists a* $(H,A)\in\mathcal{N}_{\tau_1}(E_\alpha^x)$ *such that* $(H,A)\widetilde{\subseteq}f_{pu}^{-1}(G,B)$.

(iii) For each $(G,B)\in\mathcal{N}_{\tau_2}(f_{pu}(E_\alpha^x))$, $f_{pu}^{-1}(G,B)\in\mathcal{N}_{\tau_1}(E_\alpha^x)$.

Theorem 2.24 ([20]). *Let* (X,τ_1,A) *and* (Y,τ_2,B) *be soft topological spaces,* $f_{pu}: SS(X)_A \to SS(Y)_B$ *be a function. Then the following statements are equivalent.*

(i) f_{pu} *is soft pu-continuous.*

(ii) For each $(H,B)\in\tau_2$, $f_{pu}^{-1}(H,B)\in\tau_1$.

(iii) For each soft closed set (F,B) *over* Y, $f_{pu}^{-1}(F,B)$ *is soft closed over* X.

Definition 2.25 ([7]). Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces, $f_{pu}: SS(X)_A \to SS(Y)_B$ be a function. Then the function f_{pu} is called open soft if $f_{pu}(G,A)\in\tau_2$ for each $(G,A)\in\tau_1$.

Definition 2.26 ([17]). Let (X,τ,A) be a soft topological space and let (G,A) be a soft closed set in X and soft point $E_\alpha^x\widetilde{\notin}(G,A)$. If there exist soft open sets (F_1,A) and (F_2,A) such that $E_\alpha^x\widetilde{\in}(F_1,A)$, $(G,A)\widetilde{\subseteq}(F_2,A)$ and $(F_1,A)\widetilde{\cap}(F_2,A)=\widetilde{\varnothing}$, then (X,τ,A) is called a soft regular space.

Definition 2.27 ([6, 15]). A soft ideal $\widetilde{I}$ over X is a collection of soft sets over X which satisfies the following properties:

(i) If $(F,A)\in\widetilde{I}$ and $(G,A)\in\widetilde{I}$, then $(F,A)\widetilde{\cup}(G,A)\in\widetilde{I}$,

(ii)If $(F,A)\in\widetilde{I}$ and $(G,A)\widetilde{\subseteq}(F,A)$, then $(G,A)\in\widetilde{I}$.

Definition 2.28 ([6]). Let (X,τ,A) be a soft topological space and $\widetilde{I}$ be a soft ideal over X with the same set of parameters A. Then

$$(F,A)^*(\widetilde{I},\tau) = \widetilde{\bigcup}\{E_\alpha^x\in\widetilde{X} : (U,A)\widetilde{\cap}(F,A)\notin\widetilde{I}, \forall(U,A)\in\tau \text{ containing } E_\alpha^x\}$$

is called the soft local function of (F,A) with respect to $\widetilde{I}$ and τ.

Throughout the paper, we will say that $(X, \tau, A, \widetilde{I})$ is a soft ideal topological space instead of (X, τ, A) is a soft topological space and $\widetilde{I}$ be a soft ideal over X with the same set of parameters A.

Theorem 2.29 ([6]). *Let $(X, \tau, A, \widetilde{I})$ and $(X, \tau, A, \widetilde{J})$ be two soft ideal topological spaces and $(F, A), (G, A) \in SS(X)_A$.*
(i) If $(F, A)\widetilde{\subseteq}(G, A)$, then $(F, A)^*\widetilde{\subseteq}(G, A)^*$.
(ii)If $\widetilde{I} \subseteq \widetilde{J}$, then $(F, A)^*(\widetilde{J})\widetilde{\subseteq}(F, A)^*(\widetilde{I})$.
(iii) $((F, A)^*)^*\widetilde{\subseteq}(F, A)^*$
(iv) $((F, A)\widetilde{\cup}(G, A))^* = (F, A)^*\widetilde{\cup}(G, A)^*$
(v) If $(G, A) \in \tau$, then $(G, A)\widetilde{\cap}(F, A)^*\widetilde{\subseteq}((G, A)\widetilde{\cap}(F, A))^*$.

Theorem 2.30 ([6]). *Let $(X, \tau, A, \widetilde{I})$ be a soft ideal topological space. Then the operator $cl^* : SS(X)_A \to SS(X)_A$ defined by*

$$cl^*(F, A) = (F, A)\widetilde{\cup}(F, A)^*$$

is a soft closure operator. Then there exists a unique soft topology over X with the same set of parameters A, finer than τ, called the $*$-soft topology, denoted by $\tau^*(\widetilde{I})$, given by

$$\tau^*(\widetilde{I}) = \{(F, A) \in SS(X)_A : cl^*((F, A)^c) = (F, A)^c\}$$

In this paper, we will give the definition of $int^*(G, A)$ as

$$int^*(G, A) = \widetilde{X}\backslash cl^*(\widetilde{X}\backslash(G, A))$$

for each $(G, A) \in SS(X)_A$. Obviously, $(G, A) \in \tau^*$ if and only if $int^*(G, A) = (G, A)$.

Corollary 2.31 ([6]). *Let $(X, \tau, A, \widetilde{I})$ be a soft ideal topological space. Then $\tau \subseteq \beta(\widetilde{I}, \tau) \subseteq \tau^*(\widetilde{I})$.*

It is easily seen that

$$int(G, A) \subseteq int^*(G, A) \subseteq (G, A) \subseteq cl^*(G, A) \subseteq cl(G, A)$$

for all $(G, A) \in SS(X)_A$.

Lemma 2.32 ([8]). *Let $(X, \tau, A, \widetilde{I})$ be a soft ideal topological space and Y be a non null subset of X. Then, $\widetilde{I}_Y = \{\widetilde{Y}\widetilde{\cap}(I, A) : (I, A) \in \widetilde{I}\}$ is a soft ideal over Y.*

Theorem 2.33 ([7]). *Let $(X, \tau, A, \widetilde{I})$ be a soft ideal topological space, (Y, τ_2, B) be a soft topological space and $f_{pu} : (X, \tau_1, A, \widetilde{I}) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}(\widetilde{I}) = \{f_{pu}((F, A)) : (F, A) \in \widetilde{I}\}$ is a soft ideal on Y.*

3. *-Soft Dense and Nowhere *-Soft Dense Sets

Lemma 3.1. *Let $(X, \tau, A, \widetilde{I})$ be a soft ideal topological space and $(F, A) \in SS(Y)_A$ where Y is non null subset of X. Then, $(F, A)^*(\widetilde{I}_Y, \tau_Y) = (F, A)^*\ (\widetilde{I}, \tau)\widetilde{\cap}\widetilde{Y}$.*

Proof. Let $E^x_\alpha\widetilde{\notin}(F, A)^*(\widetilde{I}, \tau)\widetilde{\cap}\widetilde{Y}$. Then, we have $E^x_\alpha\widetilde{\notin}(F, A)^*(\widetilde{I}, \tau)$ or $E^x_\alpha\widetilde{\notin}\widetilde{Y}$. If $E^x_\alpha\widetilde{\notin}(F, A)^*(\widetilde{I}, \tau)$, then there exists a soft open set (U, A) on X containing E^x_α such that $(U, A)\widetilde{\cap}(F, A) \in \widetilde{I}$. Thus, $((U, A)\widetilde{\cap}\widetilde{Y})\widetilde{\cap}(F, A) \in \widetilde{I}_Y$. This implies, $E^x_\alpha\widetilde{\notin}(F, A)^*(\widetilde{I}_Y, \tau_Y)$. If $E^x_\alpha\widetilde{\notin}\widetilde{Y}$, then $E^x_\alpha\widetilde{\notin}(F, A)^*(\widetilde{I}_Y, \tau_Y)$ since $(F, A)^*(\widetilde{I}_Y, \tau_Y)\widetilde{\subseteq}\widetilde{Y}$. Hence, $(F, A)^*$

$(\widetilde{I}_Y, \tau_Y)\widetilde{\subseteq}(F,A)^*(\widetilde{I},\tau)\widetilde{\cap}\widetilde{Y}$. Conversely, let $E^x_\alpha\widetilde{\notin}(F,A)^*(\widetilde{I}_Y,\tau_Y)$. Then, there exists a soft open set (G,A) on Y containing E^x_α such that $(G,A)\widetilde{\cap}(F,A)\in\widetilde{I}_Y$. By the definition of τ_Y; there exists a soft open set (V,A) on X containing E^x_α such that $(G,A)=(V,A)\widetilde{\cap}\widetilde{Y}$. Thus, $(V,A)\widetilde{\cap}(F,A)\in\widetilde{I}$ implies $E^x_\alpha\widetilde{\notin}(F,A)^*(\widetilde{I},\tau)$. Furthermore, $(F,A)^*(\widetilde{I},\tau)\widetilde{\cap}\widetilde{Y}\widetilde{\subseteq}(F,A)^*(\widetilde{I}_Y,\tau_Y)$. □

Theorem 3.2. *Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space and $(F,A)\in SS(Y)_A$ where Y is a non null subset of X. Then,the operator $cl^*_Y : SS(Y)_A \to SS(Y)_A$ defined by $cl^*_Y(F,A)=(F,A)\widetilde{\cup}(F,A)^*(\widetilde{I}_Y,\tau_Y)$ is a soft closure operator.*

Proof. The proof is obvious by Theorem 2.29. □

Lemma 3.3. *Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space and $(F,A)\in SS(X)_A$.*

(i) If $(U,A)\in\tau$, then $(U,A)\widetilde{\cap}cl^*(F,A)\widetilde{\subseteq}cl^*((U,A)\widetilde{\cap}(F,A))$.

(ii) If $(F,A)\in SS(Y)_A$ and $Y\subseteq X$, then $cl^*_Y(F,A)=cl^*(F,A)\widetilde{\cap}\widetilde{Y}$.

Proof.

(i) Let $(U,A)\in\tau$. Then, $(U,A)\widetilde{\cap}cl^*(F,A)=(U,A)\widetilde{\cap}((F,A)\widetilde{\cup}\ (F,A)^*(\widetilde{I},\tau))=((U,A)\widetilde{\cap}(F,A))\widetilde{\cup}((U,A)\widetilde{\cap}(F,A)^*(\widetilde{I},\tau))\widetilde{\subseteq}((U,A)\widetilde{\cap}(F,A))\widetilde{\cup}((U,A)\widetilde{\cap}(F,A))^*(\widetilde{I},\tau)=cl^*((U,A)\ \widetilde{\cap}(F,A))$ by Theorem 2.29(v).

(ii) $cl^*_Y(F,A)=(F,A)\widetilde{\cup}(F,A)^*(\widetilde{I}_Y,\tau_Y)=(F,A)\widetilde{\cup}((F,A)^*(\widetilde{I},\tau)\widetilde{\cap}\widetilde{Y})$ by Lemma 3.1. Since $(F,A)\in SS(Y)_A$, we have $(F,A)\widetilde{\cup}((F,A)^*(\widetilde{I},\tau)\widetilde{\cap}\widetilde{Y})=cl^*(F,A)\widetilde{\cap}\widetilde{Y}$. □

Definition 3.4. Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space. A soft set (F,A) over X is called

(i) [8] *-soft dense if $cl^*(F,A)=\widetilde{X}$.

(ii) nowhere *-soft dense if $int^*(cl(F,A))=\widetilde{\varnothing}$.

The family of all nowhere *-soft dense sets over X is denoted by $\widetilde{\mathcal{N}}^*(X)$ or $\widetilde{\mathcal{N}}^*$.

Proposition 3.5. *Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space and $(F,A),(G,A)\in SS(X)_A$ such that $(G,A)\widetilde{\subseteq}(F,A)$.*

*(i) If (F,A) is *-soft dense in X, then (F,A) is soft dense in X.*

*(ii) If (F,A) is nowhere *-soft dense in X, then (F,A) is nowhere soft dense in X.*

(iii) If $(F,A)\in\widetilde{\mathcal{N}}^*$, then $(G,A)\in\widetilde{\mathcal{N}}^*$.

*(iv) If (G,A) is *-soft dense, then (F,A) is *-soft dense.*

(v) If $(F,A)\in\widetilde{\mathcal{N}}^*$ if and only if $cl(F,A)\in\widetilde{\mathcal{N}}^*$.

*(vi) If (F,A) is *-soft dense if and only if $cl^*(F,A)$ is *-soft dense.*

Proof. Proofs are clear from Corollary 2.31 and properties of the soft closure operator cl^*. □

The following example shows that the converse implications of (i), (iii) and (iv) of Proposition 3.5 do not hold in general.

Example 3.6. Let $X=\{a,b,c\}$, $A=\{\alpha,\beta\}$,

$\tau=\{\widetilde{\varnothing},\widetilde{X},\{(\alpha,\{a\}),(\beta,\{b\})\},\{(\alpha,\{a,c\}),(\beta,\{b,c\})\},\{(\alpha,\{a,b\}),(\beta,\{a,b\})\}$ and

$I=\{\widetilde{\varnothing},\{(\alpha,\{b\}),(\beta,\varnothing)\},\{(\alpha,\varnothing),(\beta,\{a\})\},\{(\alpha,\{b\}),(\beta,\{a\})\}\}$.

Then $\tau^* = \{\widetilde{\varnothing}, \widetilde{X}, \{(\alpha,\{a\}),(\beta,\{b\})\}, \{(\alpha,\{a,c\}),(\beta,\{b,c\})\}, \{(\alpha,\{a,b\}),(\beta,\{a,b\}),$ $\{(\alpha,\{a\}),(\beta,\{a,b\})\}, \{(\alpha,\{a,b\}),(\beta,\{b\})\}, \{(\alpha,\{a,c\}),(\beta,X)\}, \{(\alpha,X),(\beta,\{b,c\})\}\}$.
(a) For $(F,A) = \{(\alpha,X),(\beta,\varnothing)\}$, $cl(F,A) = \widetilde{X}$ and $cl^*(F,A) = \{(\alpha,X),(\beta,\{a\})\}$. So (F,A) is soft dense but but it is not *-soft dense.
(b) For $(G,A) = \{(\alpha,\varnothing),(\beta,\{c\})\} \subset (H,A) = \{(\alpha,\{a,c\}),(\beta,\{c\})\}$, $int^*(cl(G,A)) = \widetilde{\varnothing}$ and $int^*(cl(H,A)) = \widetilde{X}$. So (G,A) is nowhere *-soft dense but (H,A) is not nowhere *-soft dense.
(c) For $(U,A) = \{(\alpha,\{b,c\}),(\beta,\{c\})\} \subset (V,A) = \{(\alpha,\{b,c\}),(\beta,X)\}$, $cl^*(U,A) = (U,A)$ and $cl^*(V,A) = \widetilde{X}$. So (U,A) is not *-soft dense but (V,A) is *-soft dense.

The following example shows that the converse implications of (ii) of Proposition 3.5 does not hold in general.

Example 3.7. Let $X = \{a,b,c\}$, $A = \{\alpha,\beta\}$,
$\tau = \{\widetilde{\varnothing}, \widetilde{X}, \{(\alpha,\{a\}),(\beta,\{b\})\}, \{(\alpha,\{a,c\}),(\beta,\{b,c\})\}, \{(\alpha,\{a,b\}),(\beta,\{a,b\})\}$ and $I = \{\widetilde{\varnothing}, \{(\alpha,\{a\}),(\beta,\varnothing)\}, \{(\alpha,\varnothing),(\beta,\{b\})\}, \{(\alpha,\{a\}),(\beta,\{b\})\}\}$. Then

$$\begin{aligned}\tau^* = \{&\widetilde{\varnothing}, \widetilde{X}, \{(\alpha,\{a\}),(\beta,\{b\})\}, \{(\alpha,\{a,c\}),(\beta,\{b,c\})\},\\ &\{(\alpha,\{a,b\}),(\beta,\{a,b\}), \{(\alpha,\varnothing),(\beta,\{b\})\}, \{(\alpha,\{c\}),(\beta,\{c\})\},\\ &\{(\alpha,\{b\}),(\beta,\{a\})\}, \{(\alpha,\{a\}),(\beta,\varnothing)\}, \{(\alpha,\{a,c\}),(\beta,\{c\})\},\\ &\{(\alpha,\{a,b\}),(\beta,\{a\})\}, \{(\alpha,\{c\}),(\beta,\{b,c\})\}, \{(\alpha,\{b\}),(\beta,\{a,b\})\},\\ &\{(\alpha,\{b,c\}),(\beta,\{a,c\})\}, \{(\alpha,X),(\beta,\{a,c\})\}, \{(\alpha,\{b,c\}),(\beta,X)\}\}.\end{aligned}$$

For $(F,A) = \{(\alpha,\{c\})), (\beta,\varnothing)\}$, $int(cl(F,A)) = \widetilde{\varnothing}$ and

$$int^*(cl(F,A)) = \{(\alpha,\{c\}),(\beta,\{c\})\}.$$

So (F,A) is nowhere soft dense but it is not nowhere *-soft dense.

Proposition 3.8. *Let $(X,\tau,A,\widetilde{I})$ and $(X,\tau,A,\widetilde{J})$ be two soft ideal topological spaces such that $\widetilde{I} \subseteq \widetilde{J}$ and let $(F,A) \in SS(X)_A$. If (F,A) is *-soft dense in X with $\widetilde{J}$, then (F,A) is *-soft dense in X with $\widetilde{I}$.*

Proof. Let (F,A) be *-soft dense in X with $\widetilde{J}$. Then, we have

$$cl^*(F,A) = (F,A)\widetilde{\cup}(F,A)^*(\widetilde{J}) = \widetilde{X}.$$

Since $\widetilde{I} \subseteq \widetilde{J}$ implies $(F,A)^*(\widetilde{J})\widetilde{\subseteq}(F,A)^*(\widetilde{I})$ by Theorem 2.29 (ii), we get $(F,A)\widetilde{\cup}$ $(F,A)^*(\widetilde{I}) = \widetilde{X}$. Thus, (F,A) is *-soft dense in X with $\widetilde{I}$. □

Proposition 3.9. *Let $(X,\tau,A,\widetilde{I})$ and $(X,\tau',A,\widetilde{I})$ be two soft ideal topological spaces such that $\tau \subseteq \tau'$ and let $(F,A) \in SS(X)_A$. If (F,A) is *-soft dense in (X,τ',A), then (F,A) is *-soft dense in (X,τ,A).*

Proof. Since $\tau \subseteq \tau'$ implies that $(F,A)^*(\widetilde{I},\tau')\widetilde{\subseteq}(F,A)^*(\widetilde{I},\tau)$, the proof is obvious. □

Lemma 3.10. *Let $(X,\tau_1,A,\widetilde{I})$ be a soft ideal topological space, (Y,τ_2,B) be a soft topological space and $f_{pu} : (X,\tau_1,A,\widetilde{I}) \to (Y,\tau_2,B)$ be a bijective soft function. If f_{pu} is soft pu-continuous, then $f_{pu}(cl^*(F,A))\widetilde{\subseteq}\, cl^*(f_{pu}(F,A))$ for each $(F,A) \in SS(X)_A$.*

Proof. Let f_{pu} be soft pu-continuous and $E^y_\beta \widetilde{\notin} cl^*(f_{pu}(F,A))$ for $(F,A) \in SS(X)_A$. Since f_{pu} is surjective, there exists $E^x_\alpha \widetilde{\in} \widetilde{X}$ such that $f_{pu}(E^x_\alpha) = E^y_\beta$. Then,

$$f_{pu}(E^x_\alpha) \widetilde{\notin} f_{pu}(F,A) \text{ and } f_{pu}(E^x_\alpha) \widetilde{\notin} (f_{pu}(F,A))^*(f_{pu}(\widetilde{I}), \tau_2).$$

Thus, $E^x_\alpha \widetilde{\notin} (F,A)$ and there exists a soft open set (U,B) containing E^y_β such that $(U,B) \widetilde{\cap} f_{pu}(F,A) \in f_{pu}(\widetilde{I})$. Since f_{pu} is soft pu-continuous and injective, $f^{-1}_{pu}(U,B)$ is soft open set containing E^x_α and $f^{-1}_{pu}(U,B) \widetilde{\cap} (F,A) \in \widetilde{I}$. This implies

$$E^x_\alpha \widetilde{\notin} (F,A)^*(\widetilde{I}, \tau_1).$$

Hence, $E^x_\alpha \widetilde{\notin} cl^*(F,A)$. So, we have $E^y_\beta \widetilde{\notin} f_{pu}(cl^*(F,A))$. □

Theorem 3.11. *Let $(X, \tau_1, A, \widetilde{I})$ be a soft ideal topological space, (Y, τ_2, B) be a soft topological space and $f_{pu} : (X, \tau_1, A, \widetilde{I}) \to (Y, \tau_2, B)$ be a bijective soft pu-continuous function. If (F,A) is *-soft dense in X, then $f_{pu}(F,A)$ is *-soft dense in Y for each $(F,A) \in SS(X)_A$.*

Proof. Let (F,A) be *-soft dense in X. Then, we have $cl^*(F,A) = \widetilde{X}$. By Lemma 3.10, we get $f_{pu}(cl^*(F,A)) = f_{pu}(\widetilde{X}) \widetilde{\subseteq} cl^*(f_{pu}(F,A))$. Since f_{pu} is surjective, then $\widetilde{Y} \widetilde{\subseteq} cl^*(f_{pu}(F,A))$. This implies that $cl^*(f_{pu}(F,A)) = \widetilde{Y}$. Thus, $f_{pu}(F,A)$ is *-soft dense in Y. □

Lemma 3.12. *Let $(X, \tau_1, A, \widetilde{I})$ be a soft ideal topological space, (Y, τ_2, B) be a soft topological space and $f_{pu} : (X, \tau_1, A, \widetilde{I}) \to (Y, \tau_2, B)$ be a bijective soft function. If f_{pu} is open soft, then $int^*(f^{-1}_{pu}(G,B)) \widetilde{\subseteq} f^{-1}_{pu}(int^*(G,B))$ for each $(G,B) \in SS(Y)_B$.*

Proof. Let $E^x_\alpha \widetilde{\in} int^*(f^{-1}(G,B))$. Then, $E^x_\alpha \widetilde{\notin} cl^*(\widetilde{X} \setminus f^{-1}_{pu}(G,B))$. If $E^x_\alpha \widetilde{\notin} \widetilde{X} \setminus f^{-1}_{pu}(G,B)$, then $f_{pu}(E^x_\alpha) \widetilde{\notin} \widetilde{Y} \setminus (G,B)$. If $E^x_\alpha \widetilde{\notin} (\widetilde{X} \setminus f^{-1}_{pu}(G,B))^*(\widetilde{I}, \tau_1)$, then there exists a soft open set (U,A) on X containing E^x_α such that $(U,A) \widetilde{\cap} (\widetilde{X} \setminus f^{-1}_{pu}(G,B)) \in \widetilde{I}$. Then, this implies that $f_{pu}(E^x_\alpha) \widetilde{\notin} (\widetilde{Y} \setminus (G,B))^*(f_{pu}(\widetilde{I}), \tau_2)$ since f_{pu} is bijective open soft function. Thus, $f_{pu}(E^x_\alpha) \widetilde{\in} int^*(G,B)$. Hence, we have $E^x_\alpha \widetilde{\in} f^{-1}_{pu}(int^*(G,B))$. □

Theorem 3.13. *Let $(X, \tau_1, A, \widetilde{I})$ be a soft ideal topological space, (Y, τ_2, B) be a soft topological space and $f_{pu} : (X, \tau_1, A, \widetilde{I}) \to (Y, \tau_2, B)$ be a bijective open soft function. If (G,B) is *-soft dense in Y, then $f^{-1}_{pu}(G,B)$ is *-soft dense in X for each $(G,B) \in SS(Y)_B$.*

Proof. Let (G,B) be *-soft dense in Y. That is, $cl^*(G,B) = \widetilde{Y}$. Then, we have $int^*(\widetilde{Y} \setminus (G,B)) = \widetilde{\varnothing}$. By Lemma 3.12, we have

$$int^*(f^{-1}(\widetilde{Y} \setminus (G,B))) \widetilde{\subseteq} f^{-1}_{pu}(int^*(\widetilde{Y} \setminus (G,B))) = \widetilde{\varnothing}.$$

Thus, $int^*(f^{-1}(\widetilde{Y} \setminus (G,B))) = \widetilde{\varnothing}$. This implies that $cl^*(f^{-1}_{pu}(G,B)) = \widetilde{X}$. Hence, $f^{-1}_{pu}(G,B)$ is *-soft dense in X. □

Theorem 3.14. *Let $(X, \tau, A, \widetilde{I})$ be a soft ideal topological space and $(F,A), (G,A) \in SS(X)_A$. If $(F,A), (G,A) \in \widetilde{\mathcal{N}}^*$, then $(F,A) \widetilde{\cup} (G,A) \in \widetilde{\mathcal{N}}^*$.*

Proof. Let $(F,A),(G,A) \in \widetilde{\mathcal{N}}^*$. Then, we have $int^*(cl(F,A)) = int^*(cl(G,A)) = \widetilde{\varnothing}$. Thus, $int^*(cl((F,A)\widetilde{\cup}(G,A))) = \widetilde{X}\backslash cl^*(\widetilde{X}\backslash cl((F,A)\widetilde{\cup}(G,A))) = \widetilde{X}\backslash cl^*(\widetilde{X}\backslash (cl(F,A)\widetilde{\cup}cl(G,A)) = \widetilde{X}\backslash cl^*((\widetilde{X}\backslash cl(F,A))\widetilde{\cap}(\widetilde{X}\backslash cl(G,A))) \widetilde{\subseteq}\widetilde{X}\backslash ((\widetilde{X}\backslash cl(F,A))\widetilde{\cap} cl^*(\widetilde{X}\backslash cl(G,A))) = cl(F,A)\widetilde{\cup}int^*(cl(G,A)) = cl(F,A)$ by Lemma 3.3 (i) and hypothesis. This implies that $int^*(int^*(cl((F,A)\widetilde{\cup}(G,A)))) \widetilde{\subseteq} int^*(cl(F,A)) = \widetilde{\varnothing}$. Thus, $int^*(cl((F,A)\widetilde{\cup}(G,A))) = \widetilde{\varnothing}$. Hence, $(F,A)\widetilde{\cup}\ (G,A) \in \widetilde{\mathcal{N}}^*$. □

The following corollary follows from Proposition 3.5 (iii) and Theorem 3.14.

Corollary 3.15. *Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space. $\widetilde{\mathcal{N}}^*$ is also a soft ideal over X.*

4. Soft $\widetilde{I}$-Baire Spaces

Definition 4.1. Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space. A soft set (F,A) over X is called

(i) *-soft first category in X if there exists a soft sequence $\{(F_n,A) : n \in \mathbb{N}\}$ consisting of nowhere *-soft dense sets over X such that $(F,A) = \widetilde{\bigcup}_{n\in\mathbb{N}}(F_n,A)$.

(ii) *-soft residual in X if $\widetilde{X}\backslash(F,A)$ is *-soft first category in X.

The family of all *-soft first category sets over X is denoted by $\widetilde{\mathcal{M}^*}$.

Remark 4.2. Every nowhere *-soft dense set is a *-soft first category.

The following example shows that the converse of Remark 4.2 is not true, in general.

Example 4.3. Let $X = \mathbb{R}$, $A = \{\alpha\}$, $\tau = \{\varnothing, \widetilde{X}\} \cup \{(F,A) : F(\alpha) = (a,\infty), a \in \mathbb{R}\}$ and $\widetilde{I} = \{(G,A) : G(\alpha)$ is finite $\}$. Then, $(\mathbb{N},A)$ is a *-soft first category but it is not a nowhere *-soft dense set.

Proposition 4.4. *Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space. If $(F_n,A) \in \widetilde{\mathcal{M}^*}$ for each $n \in \mathbb{N}$, then $\widetilde{\bigcup}_{n\in\mathbb{N}}(F_n,A) \in \widetilde{\mathcal{M}^*}$.*

Proof. It is obvious. □

Definition 4.5. Let $(X,\tau,A,\widetilde{I})$ be a soft ideal topological space. X is called soft $\widetilde{I}$-Baire if for any sequence $\{(G_n,A)\}$ consisting of soft open and *-soft dense subsets of X, $\widetilde{\bigcap}_{n\in\mathbb{N}}(G_n,A)$ is soft dense in X.

Example 4.6. Let $X = \mathbb{N}$, $A = \{e_1,e_2\}$, $\tau = \{(F,A)\widetilde{\subseteq}\widetilde{X} : E^1_{e_1}, E^1_{e_2}\widetilde{\in}(F,A)\}\ \widetilde{\cup}\{\widetilde{\varnothing}\}$ and $\widetilde{I} = \{(G,A)\widetilde{\subseteq}\widetilde{X} : 1 \notin G(e_1), 1 \notin G(e_2)\}$. Let $(H_n,A) = E^1_{e_1}\widetilde{\cup}E^1_{e_2}\widetilde{\cup}(G_n,A)$ where $(G_n,A) \in \widetilde{I}$. Then, (H_n,A) is a soft open and *-soft dense set. We have $\widetilde{\bigcap}_{n\in\mathbb{N}}(H_n,A)\widetilde{\supseteq}E^1_{e_1}\widetilde{\cup}E^1_{e_2}$. Note that $cl(E^1_{e_1}\widetilde{\cup}E^1_{e_2}) = \widetilde{X}$. Then, $\widetilde{\bigcap}_{n\in\mathbb{N}}(H_n,A) = \widetilde{X}$. So, $(X,\tau,A,\widetilde{I})$ is soft $\widetilde{I}$-Baire.

Theorem 4.7. *Let $(X,\tau,A,\widetilde{I})$ and $(X,\tau,A,\widetilde{J})$ be two soft ideal topological spaces such that $\widetilde{I} \subseteq \widetilde{J}$. If $(X,\tau,A,\widetilde{I})$ is soft $\widetilde{I}$-Baire, then $(X,\tau,A,\widetilde{J})$ is soft $\widetilde{J}$-Baire.*

Proof. It is clear by Proposition 3.8. □

Lemma 4.8. *$(X, \tau, A, \widetilde{I})$ be soft $\widetilde{I}$-Baire space if and only if given any countable collection $\{(F_n, A)\}$ of nowhere*-soft dense subsets of $\widetilde{X}$, their soft union $\widetilde{\bigcup}_{n\in\mathbb{N}}(F_n, A)$ has soft empty interior in X.*

Proof. Let $\{(F_n, A) : n \in \mathbb{N}\}$ be nowhere*-soft dense subsets of $\widetilde{X}$. Then,

$$int^*(cl(F_n, A)) = \widetilde{\varnothing} \text{ for each } n \in \mathbb{N}.$$

So, $cl^*(int(F_n^c, A)) = \widetilde{X}$ where $int(F_n^c, A)$ is a soft open and *-soft dense set. By the hypothesis, $cl(\widetilde{\bigcap}_{n\in\mathbb{N}}(F_n^c, A)) = \widetilde{X}$. Hence, we obtain $int(\widetilde{\bigcup}_{n\in\mathbb{N}}(F_n, A)) = \widetilde{\varnothing}$. Let $\{(F_n, A) : n \in \mathbb{N}\}$ be soft open and *-soft dense subsets of $\widetilde{X}$. Then, $cl^*(F_n, A) = \widetilde{X}$ for each $n \in \mathbb{N}$. So, (F_n^c, A) is a nowhere *- soft dense set for each $n \in \mathbb{N}$. By the hypothesis, $int(\widetilde{\bigcup}_{n\in\mathbb{N}}(F_n^c, A)) = \widetilde{\varnothing}$. Hence, we obtain $cl(\widetilde{\bigcap}_{n\in\mathbb{N}}(F_n, A)) = \widetilde{\varnothing}$. □

Theorem 4.9. *Let $(X, \tau, A, \widetilde{I})$ be a soft ideal topological space. Then the followings are equivalent.*
(i) X is soft $\widetilde{I}$-Baire space.
*(ii) $int(F, A) = \widetilde{\varnothing}$ for every *-soft first category set (F, A) in X.*
*(iii) $cl(G, A) = \widetilde{X}$ for every *-soft residual set (G, A) in X.*

Proof. $(i) \Rightarrow (ii)$ Let (F, A) be a *-soft first category set. By the definition of *-soft first category, we have $(F, A) = \widetilde{\bigcup}_{n\in\mathbb{N}}(F_n, A)$ where (F_n, A) is a nowhere *- soft dense set for each $n \in \mathbb{N}$. Since X is a soft $\widetilde{I}$-Baire space, $int(\widetilde{\bigcup}_{n\in\mathbb{N}}(F_n, A)) = \widetilde{\varnothing}$. Then, we obtain $int(F, A) = \widetilde{\varnothing}$.

$(ii) \Rightarrow (iii)$ Let (G, A) be a *-soft residual set in X. Then, (G^c, A) is a *-soft first category. By (ii), $int(G^c, A) = \widetilde{\varnothing}$. Then, we obtain $cl(G, A) = \widetilde{X}$.

$(iii) \Rightarrow (i)$ Let $\{(F_n, A) : n \in \mathbb{N}\}$ be nowhere *- soft dense subsets of $\widetilde{X}$. Then, $(F, A) = \widetilde{\bigcup}_{n\in\mathbb{N}}(F_n, A)$ is a *-soft first category. So, (F^c, A) is a *-soft residual. By hypothesis, $cl(F^c, A) = \widetilde{X}$. Then, we obtain $int(\widetilde{\bigcup}_{n\in\mathbb{N}}(F_n, A)) = \widetilde{\varnothing}$. By Lemma 4.8, X is a soft $\widetilde{I}$-Baire space. □

Theorem 4.10. *Let $(X, \tau_1, A, \widetilde{I})$ be a soft ideal topological space, (Y, τ_2, B) be a soft topological space and $f_{pu} : (X, \tau_1, A, \widetilde{I}) \to (Y, \tau_2, B)$ be a bijective soft pu-continuous and open soft function. Then, if X is a soft $\widetilde{I}$-Baire space, Y is also a soft $f_{pu}(\widetilde{I})$-Baire space.*

Proof. Let $\{(G_n, B) : n \in \mathbb{N}\}$ be a family of soft open and *-soft dense sets in Y. Since f_{pu} is open soft, $f_{pu}^{-1}((G_n, B))$ is a *-soft dense set in X for each n $\in \mathbb{N}$ by Theorem 3.13. Also f_{pu} is soft pu-continuous, $f_{pu}^{-1}((G_n, B))$ is soft open in X for each $n \in \mathbb{N}$. Since X is a soft $\widetilde{I}$- Baire space, $\widetilde{\bigcap}_{n\in\mathbb{N}} f_{pu}^{-1}(G_n, B)$ is a *-soft dense set in X, i.e., $cl^*(\widetilde{\bigcap}_{n\in\mathbb{N}} f_{pu}^{-1}(G_n, B)) = \widetilde{X}$. By Lemma 3.10 and f_{pu} is bijective, $\widetilde{Y} = f_{pu}(cl^*(\widetilde{\bigcap}_{n\in\mathbb{N}} f_{pu}^{-1}(G_n, B))) \widetilde{\subseteq} cl^*(f_{pu}(\widetilde{\bigcap}_{n\in\mathbb{N}} f_{pu}^{-1}(G_n, B))) = \widetilde{\bigcap}_{n\in\mathbb{N}}(G_n, B)$. Hence, Y is a soft $f_{pu}(\widetilde{I})$-Baire space.

□

Theorem 4.11. *Let (X, τ, A) be a soft topological space. Then, (X, τ, A) is a soft regular space if and only if for each $x \in X$ and soft closed set (F, A) not containing E^x_α, there exists a soft open set (V, A) containing E^x_α such that $cl(V, A)\widetilde{\cap}(F, A) = \widetilde{\varnothing}$.*

Proof. Let (F, A) be a soft closed on X such that $E^x_\alpha \widetilde{\notin}(F, A)$. Then, there exist disjoint soft open sets (V, A) and (W, A) such that $E^x_\alpha \widetilde{\in}(V, A)$ and $(F, A)\widetilde{\subseteq}(W, A)$. Now $(V, A)\widetilde{\cap}(W, A) = \widetilde{\varnothing}$ implies that $(V, A)\widetilde{\subseteq}\widetilde{X}\backslash(W, A)$ and so $cl(V, A)\widetilde{\subseteq}\widetilde{X}\backslash(W, A)$. Hence, $cl(V, A)\widetilde{\cap}(F, A) = \widetilde{\varnothing}$. Let (F, A) be a soft closed on X such that $E^x_\alpha \widetilde{\notin}(F, A)$. Then there exists a soft open set (V, A) containing E^x_α such that $cl(V, A)\widetilde{\cap}(F, A) = \widetilde{\varnothing}$. Then $(F, A)\widetilde{\subseteq}\widetilde{X}\backslash cl(V, A)$. (V, A) and $\widetilde{X}\backslash cl(V, A)$ are the required disjoint soft open sets such that $E^x_\alpha \widetilde{\in}(V, A)$ and $(F, A)\widetilde{\subseteq}(\widetilde{X}\backslash cl(V, A))$. Hence (X, τ, A) is a soft regular. □

Theorem 4.12. *Let $(X, \tau, A, \widetilde{I})$ be a soft regular ideal topological space, then X is a soft $\widetilde{I}$-Baire space.*

Proof. Given a countable collection $\{(G_n, A) : n \in \mathbb{N}\}$ of nowhere *- soft dense set in X. We want to show that their soft union $\widetilde{\bigcup}_{n\in\mathbb{N}}(G_n, A)$ has soft empty interior. Given the nonempty soft open set (U_0, A). Since (G_n, A) is nowhere *- soft dense set for $n \in \mathbb{N}$, $int^*(cl(G_n, A)) = \widetilde{\varnothing}$. Then, $int(cl(G_n, A)) = \widetilde{\varnothing}$. By hypothesis, $cl(G_n, A)$ does not contain (U_0, A). Therefore, we may choose a soft point $E^{y_0}_\alpha \widetilde{\in}(U_0, A)$ that is not in $cl(G_n, A)$. X is soft regular space, there exists a soft open set (U_1, A) of $E^{y_0}_\alpha$ such that $cl(U_1, A)\widetilde{\cap}cl(G_n, A) = \widetilde{\varnothing}$ and $cl(U_1, A)\widetilde{\subseteq}(U_0, A)$. Therefore, (G_n, A) does not contain (U_0, A) for $n \in \mathbb{N}$. Then, $int(\widetilde{\bigcup}_{n\in\mathbb{N}}(G_n, A)) = \widetilde{\varnothing}$. □

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