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Fuzzy I_w-continuous mappings

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Abstract. In this paper we introduce the concept of fuzzy I_w-continuous mappings in fuzzy ideal topological spaces and obtain some of its basic properties and characterizations.

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1. Introduction

In 1945, R. Vaidyanathaswamy [15] introduced the concept of ideal topological spaces. Hayashi [4]defined the local function and studied some topological properties using local function in ideal topological spaces in 1964 . Since then many mathematicians studied various topological concepts in ideal topological spaces. After the introduction of fuzzy sets by Zadeh [18] in 1965 and fuzzy topology by Chang [1] in 1968 , several researches were conducted on the generalization of the notions of fuzzy sets and fuzzy topology . The hybridization of fuzzy topology and fuzzy ideal theory was initiated by Mahmoud [5] and Sarkar [9] independently in 1997. They ([5],[9]) introduced the concept of fuzzy ideal topological spaces as an extension of fuzzy topological spaces and ideal topological spaces . After introduction of fuzzy topological spaces many the concepts such as fuzzy semi$-I-$open sets [3] , fuzzy$-\alpha-I-$open sets [16], fuzzy $-\gamma-I-$open sets [2] , fuzzy pre$-I-$open sets [7] and fuzzy$-\delta-I-$open sets [17] , fuzzy I_g-closed sets [11], fuzzy I_w-closed sets [13] and fuzzy I_g-continuous mappings [12] have been introduced and studied in fuzzy ideal topological spaces . In the present paper we investigate and study a new class of mappings called fuzzy I_w-continuous mappings which contains the class of all fuzzy continuous mappings and contained in the class of all fuzzy I_g-continuous mappings.

2. Preliminaries

Let X be a nonempty set. A family τ of fuzzy sets of X is called a fuzzy topology [1] on X if the null fuzzy set 0 and the whole fuzzy set 1 belongs to τ and τ is closed with respect to any union and finite intersection. If τ is a fuzzy topology on X, then the pair (X, τ) is called a fuzzy topological space. The members of τ are called fuzzy open sets of X and their complements are called fuzzy closed sets. The closure of a fuzzy set A of X denoted by $Cl(A)$, is the intersection of all fuzzy closed sets which contains A. The interior [1] of a fuzzy set A of X denoted by $Int(A)$ is the union of all fuzzy subsets contained in A. A fuzzy set A in (X, τ) is said to be quasi-coincident with a fuzzy set B, denoted by AqB, if there exists a point $x \in X$ such that $A(x) + B(x) > 1$ [3]. A fuzzy set V in (X, τ) is called a $Q-$neighbourhood of a fuzzy point x_β if there exists a fuzzy open set U of X such that $x_\beta q U \leq V$ [3]. A fuzzy set A of fuzzy topological space (X, τ) is called fuzzy semi-open if $A \leq Cl(Int(A)$.The complement of a fuzzy semi-open set is called fuzzy semi-closed. A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy $g-$closed [10] (resp. fuzzy $w-$closed [14]) if $Cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open (resp. fuzzy semi-open). Every fuzzy closed set is fuzzy $w-$closed and every fuzzy $w-$closed set is fuzzy $g-$closed, but the converses may not be true [15]. Complement of a fuzzy $g-$closed (resp. fuzzy $w-$closed) set is called fuzzy $g-$open (resp. fuzzy $w-$open). A mapping f from a fuzzy topological space (X, τ) to another fuzzy topological space (Y, τ) is called fuzzy continuous [1] (resp. fuzzy $w-$continuous [14], fuzzy $g-$continuous [6]) if the inverse image of each fuzzy open set of Y is fuzzy open (resp. fuzzy $w-$open, fuzzy $g-$open) in X [14]. Every fuzzy continuous mapping is fuzzy $w-$continuous and every fuzzy $w-$continuous mapping is fuzzy $g-$continuous, but the converse may not be true [14].

A nonempty collection of fuzzy sets I of a set X satisfying the conditions (i) if $A \in I$ and $B \leq A$, then $B \in I$(heredity), (ii) if $A \in I$ and $B \in I$ then $A \bigcup B \in I$ (finite additivity) is called a fuzzy ideal on X. The triplex (X, τ, I) denotes a fuzzy ideal topological space with a fuzzy ideal I and fuzzy topology τ ([5],[9]). The local function for a fuzzy set A of X with respect to τ and I denoted by $A^*(\tau, I)$ (briefly A^*) in a fuzzy ideal topological space (X, τ, I) is the union of all fuzzy points x_β such that if U is a $Q-$neighbourhood of x_β and $E \in I$ then for at least one point $y \in X$ for which $U(y) + A(y) - 1 > E(y)$ [8] . The $*-$closure operator of a fuzzy set A denoted by $Cl^*(A)$ in (X, τ, I) defined as $Cl^*(A) = A \bigcup A^*$ [8]. In (X, τ, I), the collection $\tau^*(I)$ is an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta = \{U - E : U \in \tau, E \in I\}$ as a base [8]. A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called fuzzy I_g-closed [11](resp. fuzzy I_w-closed [13]) if $Cl^*(A) \leq U$ whenever $A \leq U$ and U is fuzzy open (resp. fuzzy semi-open). Every fuzzy closed set is fuzzy I_w-closed and every fuzzy I_w-closed set is fuzzy I_g-closed, but the converses may not be true [13]. The complement of a fuzzy I_g-closed [11] (resp. fuzzy I_w-closed [13]) set is called fuzzy I_g-open (resp. fuzzy I_w-open). A mapping $f : (X, \tau, I) \to (Y, \sigma)$ is called fuzzy I_g-continuous if the inverse image of every fuzzy closed set of Y is fuzzy I_g-closed in X [12].

3. Fuzzy I_w-continuous mappings

Definition 3.1. A mapping f from a fuzzy ideal topological space (X, τ, I) to a fuzzy topological space (Y, σ) is said to be fuzzy I_w-continuous if the inverse image of every fuzzy closed set of Y is fuzzy I_w-closed in X.

Theorem 3.2. *A mapping $f : (X, \tau, I) \to (Y, \sigma)$ is fuzzy I_w-continuous if and only if the inverse image of every fuzzy open set of Y is fuzzy I_w-open in X.*

Proof. It is obvious because $f^{-1}(1-U) = 1-f^{-1}(U)$ for every fuzzy set U of Y. □

Remark 3.3. Every fuzzy continuous mapping is fuzzy I_w-continuous, but the converse may not be true. For,

Example 3.4. Let $X = \{a, b\}$ and $Y = \{x, y\}$ and the fuzzy sets U and V are defined as follows:

$$U(a) = 0.5, U(b) = 0.4$$

$$V(x) = 0.5, V(y) = 0.5$$

Let $\tau = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be fuzzy topologies on X and Y respectively and $I = \{0\}$ be the fuzzy ideal on X. Then the mapping $f : (X, \tau, I) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is fuzzy I_w-continuous but not fuzzy continuous.

Remark 3.5. Every fuzzy I_w-continuous mapping is fuzzy I_g-continuous but the converse may not be true. For,

Example 3.6. Let $X = \{a, b\}$ and $Y = \{x, y\}$ and the fuzzy sets U and V are defined as follows:

$$U(a) = 0.7, U(b) = 0.6$$

$$V(x) = 0.6, V(y) = 0.7$$

Let $\tau = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be fuzzy topologies on X and Y respectively and $I = \{0\}$ be the fuzzy ideal on X. Then the mapping $f : (X, \tau, I) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is fuzzy I_g-continuous but not fuzzy I_w-continuous.

Theorem 3.7. *If $f : (X, \tau, I) \to (Y, \sigma)$ is fuzzy I_w-continuous then for each fuzzy point x_β of X and each fuzzy open set V of Y such that $f(x_\beta) \in V$ then there exists a fuzzy I_w-open set U of X such that $x_\beta \in U$ and $f(U) \leq V$.*

Proof. Let x_β be a fuzzy point of X and V is fuzzy open set of Y such that $f(x_\beta) \in V$, put $U = f^{-1}(V)$. Then by hypothesis U is fuzzy I_w-open set of X such that $x_\beta \in U$ and $f(U) = f(f^{-1}(V)) \leq V$. □

Theorem 3.8. *If $f : (X, \tau, I) \to (Y, \sigma)$ is fuzzy I_w-continuous then for each fuzzy point x_β of X and each fuzzy open set V of Y such that $f(x_\beta)qV$. Then there exists a fuzzy I_w-open set U of X such that $x_\beta qU$ and $f(U) \leq V$.*

Proof. Let x_β be a fuzzy point of X and V is fuzzy open set of Y such that $f(x_\beta)qV$. Put $U = f^{-1}(V)$.Then by hypothesis U is fuzzy I_w-open set of X such that $x_\beta qU$ and $f(U) = f(f^{-1}(V)) \leq V$. □

Definition 3.9. Let (X, τ, I) be a fuzzy ideal topological space. The I_w-closure of a fuzzy set A of X denoted by $I_wcl(A)$ is defined as:

$$I_wcl(A) = inf\{B : B \geq A,\ B\ is\ fuzzy\ I_w - closed\ set\ of\ (X, \tau, I)\}.$$

Remark 3.10. It is clear that $A \leq gcl(A) \leq wcl(A) \leq I_wcl(A) \leq Cl(A)$ for any fuzzy set A of X.

Theorem 3.11. *A mapping $f : (X, \tau, I) \to (Y, \sigma)$ is fuzzy I_w-continuous then $f(I_wcl(A)) \leq Cl(f(A))$ for every fuzzy set A of X.*

Proof. Let A be a fuzzy set of X. Then $Cl(f(A))$ is a fuzzy closed set of Y. Since f is fuzzy I_w-continuous, $f^{-1}(Cl(f(A)))$ is fuzzy I_w-closed in X. Clearly, $A \leq f^{-1}(Cl(f(A)))$. Therefore $I_wcl(A) \leq I_wcl(f^{-1}(Cl(f(A)))) = f^{-1}(Cl(f(A)))$. Hence $f(I_wcl(A)) \leq Cl(f(A))$. □

Remark 3.12. The converse of above Theorem may not be true. For,

Example 3.13. Let $X = \{a, b, c\},\ Y = \{x, y, z\}$ and the fuzzy set U and V are defined as:

$$U(a) = 1, U(b) = 0, U(c) = 0$$
$$V(x) = 1, V(y) = 0, V(z) = 1$$

Let $\tau = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be fuzzy topologies on X and Y respectively and $I = \{0\}$ be a fuzzy ideal on X. Let the mapping $f : (X, \tau, I) \to (Y, \sigma)$ defined by $f(a) = y, f(b) = x, f(c) = z$. Then $f(I_wcl(A)) \leq Cl(f(A))$ holds for every fuzzy set A of X, but f is not fuzzy I_w-continuous.

Definition 3.14. A fuzzy ideal topological space (X, τ, I) is fuzzy I_w-continuous is said to be fuzzy $I_w - T_{1/2}$ if every fuzzy I_w-closed set in X is fuzzy semi-closed in X.

Theorem 3.15. *A mapping f from a fuzzy $I_w - T_{1/2}$ space (X, τ, I) to a fuzzy topological space (Y, σ) is fuzzy continuous if and only if it is fuzzy I_w-continuous.*

Proof. Obvious. □

Remark 3.16. The composition of two fuzzy I_w-continuous mappings may not be fuzzy I_w-continuous. For,

Example 3.17. Let $X = \{a, b\}, Y = \{x, y\}, Z = \{p, q\}$ and the fuzzy sets U, V and W defined as follows:

$$U(a) = 0.5, U(b) = 0.4$$
$$V(x) = 0.5, V(y) = 0.3$$
$$W(p) = 0.6, W(q) = 0.4$$

Let $\tau = \{0, U, 1\},\ \sigma = \{0, V, 1\}$ and $\eta = \{0, W, 1\}$ be the fuzzy topologies on X, Y and Z respectively and $I_1 = \{0\}$ be fuzzy ideal on X and $I_2 = \{0\}$ be fuzzy ideal on Y. Then the mapping $f : (X, \tau, I_1) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ and the mapping $g : (Y, \sigma, I_2) \to (Z, \eta)$ defined by $g(x) = p$ and $g(y) = q$ are fuzzy I_w-continuous but gof is not fuzzy I_w-continuous.

Theorem 3.18. *If $f : (X, \tau, I) \to (Y, \sigma)$ is fuzzy I_w-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is fuzzy continuous. Then $gof : (X, \tau, I) \to (Z, \eta)$ is fuzzy I_w-continuous.*

Proof. Let A be a fuzzy closed in Z then $f^{-1}(A)$ is fuzzy closed in Y, because g is fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy I_w-closed in X. Hence gof is fuzzy I_w-continuous. □

Theorem 3.19. *If $f : (X, \tau, I) \to (Y, \sigma)$ is fuzzy I_w-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is fuzzy $g-$continuous and (Y, σ) is fuzzy $I_w - T_{1/2}$, then $gof : (X, \tau, I) \to (Z, \eta)$ is fuzzy I_w-continuous.*

Proof. Let A be a fuzzy closed set in Z then $f^{-1}(A)$ is fuzzy $g-$closed set in Y because g is $g-$continuous. Since Y is fuzzy $I_w - T_{1/2}$, $g^{-1}(A)$ is fuzzy closed in Y. And so, $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy I_w-closed in X. Hence $gof : (X, \tau, I) \to (Z, \eta)$ is fuzzy I_w-continuous. □

Theorem 3.20. *If mappings $f : (X, \tau, I) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ are fuzzy I_w-continuous and fuzzy $I_w - T_{1/2}-$space then $gof : (X, \tau, I) \to (Z, \eta)$ is fuzzy I_w-continuous.*

Proof. The proof is obvious. □

Definition 3.21 ([13])**.** A collection $\{A_i : i \in \wedge\}$ of fuzzy I_w-open sets in a fuzzy ideal topological space (X, τ, I) is called a fuzzy I_w-open cover of a fuzzy set B of X if $B \leq \bigcup\{A_i : i \in \wedge\}$.

Definition 3.22 ([13])**.** A fuzzy ideal topological space (X, τ, I) is said to be fuzzy $IGO-$compact if every fuzzy I_g-open cover of X has a finite subcover.

Definition 3.23 ([13])**.** A fuzzy set B of a fuzzy ideal space (X, τ, I) is said to be fuzzy $IWO-$compact if for every collection $\{A_i : i \in \wedge\}$ of fuzzy I_w-open subsets of X such that $B \leq \bigcup\{A_i : i \in \wedge\}$ there exists a finite subset $\wedge_0$ and $\wedge$ such that $B \leq \{A_i : i \in \wedge_0\}$.

Definition 3.24 ([13])**.** A crisp subset B of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy $IWO-$compact if B is fuzzy $IWO-$compact as a fuzzy ideal subspace of X.

Theorem 3.25. *A fuzzy I_w-continuous image of a fuzzy $IWO-$compact space is fuzzy compact.*

Proof. Let $f : (X, \tau, I) \to (Y, \sigma)$ is a fuzzy I_w-continuous mapping from a fuzzy $IWO-$compact space (X, τ, I) onto a fuzzy topological space (Y, σ). Let $\{A_i : i \in \wedge\}$ be a fuzzy open cover of Y. Then $\{f^{-1}(A_i : i \in \wedge)\}$ is a fuzzy I_w-open cover of X. Since X is fuzzy $IWO-$compact it has a finite fuzzy sub cover say $\{f^{-1}(A_1), f^{-1}(A_2), ..., f^{-1}(A_n)\}$. Since f is on to $\{A_1, A_2, ..., A_n\}$ is an open cover of Y. Hence (Y, σ) is fuzzy compact. □

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