Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 9, No. 6, (June 2015), pp. 975–990
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Intuitionistic chaotic continuous functions

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Received 31 July 2014; Revised 9 December 2014; Accepted 9 January 2015

Abstract. In this paper, the concepts of intuitionistic periodic points, intuitionistic orbit sets, intuitionistic sensitive functions, intuitionistic clopen orbit sets, intuitionistic clopen chaotic sets and intuitionistic chaos spaces are introduced and studied. The concepts of intuitionistic chaotic continuous functions, intuitionistic chaotic* continuous functions, intuitionistic chaotic** continuous functions, intuitionistic chaotic*** continuous functions, intuitionistic chaotic normal spaces, intuitionistic chaotic T_1 spaces, intuitionistic chaotic regular spaces and intuitionistic chaotic extremally disconnected spaces are introduced and studied. Besides providing some interesting properties and interrelations are discussed.

2010 AMS Classification: 54A40, 54E55

Keywords: Intuitionistic periodic points, Intuitionistic orbit sets, Intuitionistic chaotic sets, Intuitionistic sensitive functions, Intuitionistic chaotic extremally disconnected spaces.

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1. Introduction

The concept of intuitionistic set was introduced by çoker [1]. In 1989, R. L. Devaney [3] defined chaotic function in general metric spaces. The concepts of intuitionistic periodic points, intuitionistic orbit sets, intuitionistic sensitive functions, intuitionistic clopen orbit sets, intuitionistic clopen chaotic sets and intuitionistic chaos spaces. The concepts of intuitionistic chaotic continuous functions, intuitionistic chaotic* continuous functions, intuitionistic chaotic** continuous functions and intuitionistic chaotic*** continuous functions are introduced and studied. Some interrelation are discussed with suitable examples.

Also the concepts of intuitionistic chaotic normal spaces, intuitionistic orbit normal spaces, intuitionistic chaotic T_1 spaces, intuitionistic orbit regular spaces, intuitionistic chaotic regular spaces, intuitionistic orbit compact spaces, intuitionistic chaotic locally indiscrete, intuitionistic chaotic Hausdorff spaces, intuitionistic orbit Hausdorff spaces, intuitionistic chaotic irresolute functions, intuitionistic chaotic compact spaces, intuitionistic chaotic connected, intuitionistic orbit T_1 spaces, intuitionistic orbit connected, intuitionistic orbit irresolute functions and intuitionistic chaotic extremally disconnected spaces are introduced and studied. Besides providing some interesting properties and interrelations are established.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a non empty set. An ***intuitionistic set*** (IS for short) A is an object having the form $A = \langle x, A^1, A^2\rangle$ where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A.

Definition 2.2 ([1])**.** Let X be a non empty set, $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, B^1, B^2\rangle$ be intuitionistic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A^i = \langle x, A^1_i, A^2_i\rangle$.

(i) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $A^2 \supseteq B^2$.
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(iii) $\overline{A} = \langle x, A^2, A^1\rangle$.
(iv)$A \cap B$=$\langle x, A^1 \cap B^1, A^2 \cup B^2\rangle$, $A \cup B$=$\langle x, A^1 \cup B^1, A^2 \cap B^2\rangle$.
(v) $\bigcup A_i = \langle x, \cup A^1_i, \cap A^2_i\rangle$.
(vi) $\bigcap A_i = \langle x, \cap A^1_i, \cup A^2_i\rangle$.
(vii)$A - B = A \cap \overline{B}$.
(viii) $\phi_\sim = \langle x, \phi, X\rangle$; $X_\sim = \langle x, X, \phi\rangle$.

Definition 2.3 ([2])**.** An intuitionistic topology (IT for short) on a nonempty set X is a family T of intuitionistic set in X satisfying the following axioms:

(i) $\phi_\sim$, $X_\sim \in T$.
(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$.
(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i : i \in J\} \subseteq T$.

In this case the pair (X, T) is called an ***intuitionistic topological space*** (ITS for short) and any intuitionistic set in T is called an ***intuitionistic open set***(IOS for short) in X. The complement $\overline{A}$ of an intuitionistic open set A is called an ***intuitionistic closed set*** (ICS for short) in X.

Definition 2.4 ([2])**.** Let (X, T) be an intuitionistic topological space and $A = \langle X, A^1, A^2\rangle$ be an intuitionistic set in X. Then the ***closure*** and ***interior*** of A are defined by

$cl(A) = \cap\{K : K$ is an intuitionistic closed set in X and $A \subseteq K\}$,
$int(A) = \cup\{G : G$ is an intuitionistic open set in X and $G \subseteq A\}$.

It can be also shown that $cl(A)$ is an intuitionistic closed set and $int(A)$ is an intuitionistic open set in X, and A is an intuitionistic closed set in X iff $cl(A) = A$; and A is an intuitionistic open set in X iff $int(A) = A$.

Definition 2.5 ([1])**.** (a) If $B = \langle y, B^1, B^2\rangle$ is an intuitionistic set in Y, then the ***preimage*** of B under f, denoted by $f^{-1}(B)$, is the intuitionistic set in X defined by $f^{-1}(B) = \langle x, f^{-1}(B^1), f^{-1}(B^2)\rangle$.

(b) If $A = \langle x, A^1, A^2\rangle$ is an intuitionistic set in X, then the ***image*** of A under f, denoted by $f(A)$, is the intuitionistic set in Y defined by $f(A) = \langle y, f(A^1), f\_(A^2)\rangle$, where $f\_(A^2) = Y - f(X - A^2)$.

Definition 2.6 ([2]). Let (X, T) and (Y, S) be any two intuitionistic topological spaces and let $f : X \to Y$ be a function. Then f is said to be ***continuous*** if and only if the preimage of each intuitionistic set in S is an intuitionistic set in T.

Definition 2.7 ([2]). (i) If a family $\{\langle X, G_i^1, G_i^2\rangle : i \in J\}$ of intuitionistic open set in (X, T) satisfies the condition $A \subseteq \cup\{\langle X, G_i^1, G_i^2\rangle : i \in J\}$, then it is called an ***open cover*** of A. A finite subfamily of an open cover $\{\langle X, G_i^1, G_i^2\rangle : i \in J\}$ of A, which is also an intuitionistic open cover of A, is called a ***finite subcover*** of $\{\langle X, G_i^1, G_i^2\rangle : i \in J\}$.
(ii) An intuitionistic set $A = \langle X, A^1, A^2\rangle$ is an intuitiinustic topological space (X, T) is called ***compact*** iff each open cover of A has a finite sub cover.

Definition 2.8 ([2]). Let (X, T) be an intuitionistic topological space. A family $\beta \subseteq T$ is called a ***base*** for (X, T) iff each member of T can written as a union of elements of β.

Definition 2.9 ([2]). Let (X, T) and (Y, S) be two intuitionistic topological spaces and let $f : (X, T) \to (Y, S)$ be a function. Then f is said to be ***open*** iff the image of each intuitionistic set in T is an intuitionistic set in S.

Definition 2.10 ([3]). x in X is called a ***periodic point*** of f if $f^n(x) = x$, for some $n \in Z_+$. Smallest of these n is called period of x.

3. Characterizations of intuitionistic chaotic continuous functions

Notation 3.1. Let (X, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set of X.

(i) $Icl(A)$ denotes intuitionistic closure of A.
(ii) $Iint(A)$ denotes intuitionistic interior of A.
(iii) Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set of X. $x \in A$ denotes $x \in A^1$ and $x \notin A^2$.
(iv) $IK(X)$ denotes the collection of all non empty intuitionistic compact sets of X.
(v) Let $f : (X, T) \to (X, T)$ be a function and $U = \langle x, U^1, U^2\rangle$ be an intuitionistic set of X. $f(x) \in U$ denotes $f(x) \in U^1$ and $f(x) \notin U^2$.

Definition 3.2. Let (X, T) be an intuitionistic topological space. An ***orbit*** of a point x in X under the function $f : (X, T) \to (X, T)$ is denoted and defined as $O_f(x) = \{x, f^1(x), f^2(x), ...f^n(x)\}$ for $x \in X$ and $n \in Z_+$.

Example 3.3. Let $X = \{a, b, c\}$. Let $f : X \to X$ be a function defined by $f(a) = b$, $f(b) = c$, and $f(c) = a$. If $n = 1$, then the orbit points $O_f(a) = \{a, b\}$, $O_f(b) = \{b, c\}$ and $O_f(c) = \{a, c\}$. If $n = 2$, then the orbit points $O_f(a) = X_\sim$, $O_f(b) = X_\sim$ and $O_f(c) = X_\sim$.

Definition 3.4. Let (X,T) be an intuitionistic topological space. An ***intuitionistic orbit set*** in X under the function $f:(X,T)\to(X,T)$ is denoted and defined as $IO_f(x)=\langle x,O_f(x),X-O_f(x)\rangle$ for $x\in X$.

Example 3.5. Let $X=\{a,b,c,d\}$. Let $f:X\to X$ be a function defined by $f(a)=b$, $f(b)=d$, $f(c)=a$ and $f(d)=c$. If $n=1$, then the intuitionistic orbit sets $O_f(a)=\langle x,\{a,b\},\{c,d\}\rangle$, $O_f(b)=\langle x,\{b,d\},\{a,c\}\rangle$, $O_f(c)=\langle x,\{a,c\},\{b,d\}\rangle$ and $O_f(d)=\langle x,\{c,d\},\{a,b\}\rangle$. If $n=2$, then the intuitionistic orbit sets $O_f(a)=\langle x,\{a,b,d\},\{c\}\rangle$, $O_f(b)=\langle x,\{b,c,d\},\{a\}\rangle$, $O_f(c)=\langle x,\{a,b,c\},\{d\}\rangle$ and $O_f(d)=\langle x,\{a,c,d\},\{b\}\rangle$. If $n=3$, then the intuitionistic orbit sets $O_f(a)=X_\sim$, $O_f(b)=X_\sim$, $O_f(c)=X_\sim$ and $O_f(d)=X_\sim$.

Definition 3.6. Let (X,T) be an intuitionistic topological space and $f:(X,T)\to(X,T)$ be an intuitionistic continuous function. Then f is said to be ***intuitionistic sensitive*** at $x\in X$ if given any intuitionistic open set $U=\langle x,U^1,U^2\rangle$ containing x there exists an intuitionistic open set $V=\langle x,V^1,V^2\rangle$ such that $f^n(x)\in V$, $f^n(y)\notin Icl(V)$ and $y\in U$, $n\in Z_+$. We say that f is intuitionistic sensitive on an intuitionistic compact set $F=\langle x,F^1,F^2\rangle$ if $f|F$ is intuitionistic sensitive at every point of F.

Example 3.7. Let $X=\{a,b,c,d\}$. Then the intuitionistic sets A, B, C and D are defined by $A=\langle x,\{a,b\},\{c,d\}\rangle$, $B=\langle x,\{a\},\{b,c,d\}\rangle$, $C=\langle x,\{c,d\},\{a,b\}\rangle$ and $D=\langle x,\{a,c,d\},\{b\}\rangle$. Then the family $T=\{X_\sim,\phi_\sim,A,B,C,D\}$ is an intuitionistic topology on X. Clearly, (X,T) is an intuitionistic topological space. Let $f:(X,T)\to(X,T)$ be a function defined by $f(a)=c$, $f(b)=d$, $f(c)=b$ and $f(d)=a$. Let $x=a$ and $y=d$. If $n=1,3,5$, then the intuitionistic open set $D=\langle x,\{a,c,d\},\{b\}\rangle$ containing x there exists an intuitionistic open set $C=\langle x,\{c,d\},\{a,b\}\rangle$ such that $f^n(x)\in C$, $f^n(y)\notin Icl(C)$ and $y\in D$. Hence the function f is called intuitionistic sensitive.

Definition 3.8. Let (X,T) be an intuitionistic topological space and $F=\langle x,F^1,F^2\rangle$ be an intuitionistic set of X. Then $T_F=\{A\cap F\mid A=\langle x,A^1,A^2\rangle\in T\}$ is an intuitionistic topology on F and is called the induced* intuitionistic topology. The pair (F,T_F) is called an intuitionistic subspace* of (X,T).

Notation 3.9. Let (X,T) be an intuitionistic topological space. Let $F=\langle x,F^1,F^2\rangle\subseteq X_\sim$ then $S(F)=\langle x,S(F)^1,S(F)^2\rangle$ where

$$S(F)^1=\{f:(F,T_F)\to(F,T_F)\mid f\text{ is intuitionistic sensitive on }F\}$$

and

$$S(F)^2=\{f\mid f\text{ is not intuitionistic sensitive on }F\}.$$

Definition 3.10. Let (X,T) be an intuitionistic topological space. Let $f:(X,T)\to(X,T)$ be a function. An ***intuitionistic periodic set*** is denoted and defined as $IP_f(x)=\langle x,\{x\in X\mid f^n(x)=x\},X-\{x\in X\mid f^n(x)=x\}\rangle$.

Example 3.11. Let $X=\{a,b,c\}$. Let $f:X\to X$ be a function defined by $f(a)=a$, $f(b)=c$ and $f(c)=b$. If $n=1$, then the intuitionistic periodic set $IP_f(a)=\langle x,\{a\},\{b,c\}\rangle$. If $n=2$, then the intuitionistic periodic sets $IP_f(b)=\langle x,\{b\},\{a,c\}\rangle$, $IP_f(c)=\langle x,\{c\},\{a,b\}\rangle$.

Definition 3.12. Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ of X is said to be an ***intuitionistic dense*** in X, if $Icl(A) = X$.

Definition 3.13. Let (X, T) be an intutionistic topological space and $F = \langle x, F^1, F^2\rangle \in IK(X)$. Let $f : (F, T_F) \to (F, T_F)$ be an intuitionistic continuous function. Then f is said to be ***intuitionistic chaotic on*** F if

(i) $Icl(IO_f(x))_T = F$ for some $x \in F$.
(ii) intuitionistic periodic points of f are intuitionistic dense in F. That is, $Icl(IP_f(x))_T = F$.
(iii) $f \in S(F)$.

Notation 3.14. Let (X, T) be an intutionistic topological space then

$$C(F) = \langle x, C(F)^1, C(F)^2\rangle$$

where

$$C(F)^1 = \{f : (F, T_F) \to (F, T_F) \mid f \text{ is intuitionistic chaotic on } F\}$$

and

$$C(F)^2 = \{f : (F, T_F) \to (F, T_F) \mid f \text{ is not intuitionistic chaotic on } F\}.$$

Notation 3.15. Let (X, T) be an intutionistic topological space then

$$CH(X) = \{F = \langle x, F^1, F^2\rangle \in IK(X) \mid C(F) \neq \phi\}.$$

Definition 3.16. An intuitionistic topological space (X, T) is called an ***intuitionistic chaos space*** if $CH(X) \neq \phi$. The members of $CH(X)$ are called intuitionistic chaotic sets.

Definition 3.17. Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ is intuitionistic clopen if it is both intuitionistic open and intuitionistic closed.

Definition 3.18. Let (X, T) be an intuitionistic topological space.

(i) An intuitionistic open orbit set is an intuitionistic set which is both intuitionistic open and intuitionistic orbit.
(ii) An intuitionistic closed orbit set is an intuitionistic set which is both intuitionistic closed and intuitionistic orbit.
(iii) An intuitionistic clopen orbit set is an intuitionistic set which is both intuitionistic clopen and intuitionistic orbit.

Definition 3.19. Let (X, T) be an intuitionistic topological space.

(i) An intuitionistic open chaotic set is an intuitionistic set which is both intuitionistic open and intuitionistic chaotic.
(ii) An intuitionistic closed chaotic set is an intuitionistic set which is both intuitionistic closed and intuitionistic chaotic.
(iii) An intuitionistic clopen chaotic set is an intuitionistic set which is both intuitionistic clopen and intuitionistic chaotic.

Definition 3.20. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. A function $f : (X, T) \to (X, S)$ is said to be ***intuitionistic chaotic continuous*** if for each periodic point $x \in X$ and each intuitionistic clopen chaotic set $F = \langle x, F^1, F^2\rangle$ of $f(x)$ there exists an intuitionistic open orbit set $IO_f(x)$ of the periodic point x such that $f(IO_f(x)) \subseteq F$.

Example 3.21. Let $X = \{a, b, c, d\}$. Then the intuitionistic sets A, B, C, D, E, F, G and H are defined by $A = \langle x, \{a, b\}, \{c, d\}\rangle$, $B = \langle x, \{a\}, \{b, c, d\}\rangle$, $C = \langle x, \{c, d\}, \{a, b\}\rangle$, $D = \langle x, \{a, c, d\}, \{b\}\rangle$, $E = \langle x, \{b, d\}, \{a, c\}\rangle$, $F = \langle x, \{b\}, \{a, c, d\}\rangle$, $G = \langle x, \{a, c\}, \{b, d\}\rangle$ and $H = \langle x, \{a, b, c\}, \{d\}\rangle$. Let $T = \{X_\sim, \phi_\sim, A, B, C, D\}$ and $S = \{X_\sim, \phi_\sim, E, F, G, H\}$ be an intuitionistic topologies on X. Clearly (X, T) and (X, S) be any two intuitionistic chaos spaces. If $n = 1, 3$ then the function $f : (X, T) \to (X, S)$ is defined by $f(a) = b, f(b) = d, f(c) = c$ and $f(d) = a$. Now, the function f is called intuitionistic chaotic continuous.

Proposition 3.22. *Let (X, T) and (X, S) be any two intuitionistic chaos spaces. Let $f : (X, T) \to (X, S)$ be a function. Then the following statements are equivalent:*

(i) *f is intuitionistic chaotic continuous.*

(ii) *Inverse image of every intuitionistic clopen chaotic set of (X, S) is an intuitionistic open orbit set of (X, T).*

(iii) *Inverse image of every intuitionistic clopen chaotic set of (X, S) is an intuitionistic clopen orbit set of (X, T).*

Proof. **(i) $\Rightarrow$ (ii)** Let $F = \langle x, F^1, F^2\rangle$ be an intuitionistic clopen chaotic set of (X, S) and the periodic point $x \in f^{-1}(F)$. Then $f(x) \in F$. Since f is intuitionistic chaotic continuous, there exists an intuitionistic open orbit set $IO_f(x)$ of (X, T) such that $x \in IO_f(x)$, $f(IO_f(x)) \subseteq F$.

That is, $x \in IO_f(x) \subseteq f^{-1}(F)$. Now, $f^{-1}(F) = \cup\{IO_f(x) : x \in f^{-1}(F)\}$.

Since $f^{-1}(F)$ is union of intuitionistic open orbit sets. Therefore, $f^{-1}(F)$ is an intuitionistic open orbit set.

(ii) $\Rightarrow$ (iii) Let F be an intuitionistic clopen chaotic set of (X, S). Then $X - F$ is also an intuitionistic clopen chaotic set, By (b) $f^{-1}(Y - F)$ is intuitionistic open orbit in (X, T). So $X - f^{-1}(F)$ is an intuitionistic open orbit set in (X, T). Hence, $f^{-1}(F)$ is intuitionistic closed orbit in (X, T). By (ii), $f^{-1}(F)$ is an intuitionistic open orbit set of (X, T). Therefore, $f^{-1}(F)$ is both intuitionistic open orbit and intuitionistic closed orbit in (X, T). Hence, $f^{-1}(F)$ is an intuitionistic clopen orbit set of (X, T).

(iii) $\Rightarrow$ (i) Let x be a periodic point, $x \in X$ and F be an intuitionistic clopen chaotic set containing $f(x)$ then $f^{-1}(F)$ is an intuitionistic open orbit set of (X, T) containing x and $f(f^{-1}(F)) \subseteq F$. Hence, f is intuitionistic chaotic continuous. □

Definition 3.23. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. A function $f : (X, T) \to (X, S)$ is said to be ***intuitionistic chaotic* continuous*** if for each periodic point $x \in X$ and each intuitionistic closed chaotic set F containing $f(x)$, there exist intuitionistic open orbit set $IO_f(x)$ containing x such that $f(Icl(IO_f(x))) \subseteq F$.

Proposition 3.24. *An intuitionistic chaotic continuous function is an intuitionistic chaotic* continuous function.*

Proof. Since f is an intuitionistic chaotic continuous function, F is an intuitionistic clopen chaotic set containing $f(x)$, there exists an intuitionistic open orbit set $IO_f(x)$ containing x such that $f(IO_f(x)) \subseteq F$. Then $f^{-1}(F)$ is an intuitionistic clopen chaotic set of (X, S). By (iii) of Proposition 3.21., $f^{-1}(F)$ is an intuitionistic clopen orbit set in (X, T). Therefore, F is an intuitionistic closed chaotic set containing

$f(x)$ and $f^{-1}(F)$ is an intuitionistic open orbit set such that $f(f^{-1}(F)) \subseteq F$. Since $f^{-1}(F)$ is intuitionistic closed orbit set, $Icl(f^{-1}(F)) = f^{-1}(F)$. This implies that, $f(Icl(f^{-1}(F))) \subseteq F$. Hence, f is an intuitionistic chaotic* continuous function. $\square$

Remark 3.25. The converse of Proposition 3.23. need not be true as shown in Example 3.25.

Example 3.26. Let $X = \{a, b, c, d\}$. Then the intuitionistic sets A, B, C, D, E and F are defined by $A = \langle x, \{a, b\}, \{c, d\}\rangle$, $B = \langle x, \{a\}, \{b, c, d\}\rangle$, $C = \langle x, \{c\}, \{a, b, d\}\rangle$, $D = \langle x, \{a, b, c\}, \{d\}\rangle$, $E = \langle x, \{a, c\}, \{b, d\}\rangle$ and $F = \langle x, \{a, c\}, \{b, d\}\rangle$. Let $T = \{X_\sim, \phi_\sim, A, B, C, D, E\}$ and $S = \{X_\sim, \phi_\sim, , B, D, E, F\}$ are intuitionistic topologies on X. Clearly (X, T) and (X, S) be any two intuitionistic chaos spaces. The function $f : (X, T) \to (X, S)$ is defined by $f(a) = b, f(b) = d, f(c) = c$ and $f(d) = a$. Now the function f is intuitionistic chaotic* continuous but not intuitionistic chaotic continuous. Hence, **intuitionistic chaotic* continuous function need not be intuitionistic chaotic continuous function.**

Definition 3.27. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. A function $f : (X, T) \to (X, S)$ is said to be ***intuitionistic chaotic** continuous*** if for each periodic point $x \in X$ and each intuitionistic closed chaotic set F of $f(x)$, there exists an intuitionistic open orbit set $IO_f(x)$ of the periodic point x such that $f(IO_f(x)) \subseteq Iint(F)$.

Proposition 3.28. *An intuitionistic chaotic continuous function is an intuitionistic chaotic** continuous function.*

Proof. Since f is an intuitionistic chaotic continuous function, F is an intuitionistic clopen chaotic set containing $f(x)$, there exists an intuitionistic open orbit set $IO_f(x)$ containing x such that $f(IO_f(x)) \subseteq F$. Since F is an intuitionistic open orbit set in (X, S), $F = Iint(F)$. This implies that, $f(IO_f(x)) \subseteq Iint(F)$. Hence, f is an intuitionistic chaotic** continuous function. $\square$

Remark 3.29. The converse of Proposition 3.27 need not be true as shown in the Example 3.29.

Example 3.30. Let $X = \{a, b, c\}$. Then the intuitionistic sets A, B, C, D, E and F are defined by $A = \langle x, \{a\}, \{b\}\rangle$, $B = \langle x, \{a\}, \{b, c\}\rangle$, $C = \langle x, \{b\}, \{a\}\rangle$, $D = \langle x, \{a, b\}, \{c\}\rangle$, $E = \langle x, \{a, b\}, \{\phi\}\rangle$, $F = \langle x, \{b\}, \{a, c\}\rangle$ and $G = \langle x, \{b\}, \{c\}\rangle$. Let $T = \{X_\sim, \phi_\sim, A, B, C, E\}$ and $S = \{X_\sim, \phi_\sim, B, C, E, F, G\}$ are intuitionistic topologies on X. Clearly (X, T) and (X, S) be any two intuitionistic chaos spaces. The function $f : (X, T) \to (X, S)$ is defined by $f(a) = b, f(b) = d, f(c) = c$ and $f(d) = a$. Now the function f is intuitionistic chaotic** continuous but not intuitionistic chaotic continuous. Hence, **intuitionistic chaotic** continuous function need not be intuitionistic chaotic continuous function.**

Definition 3.31. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. A function $f : (X, T) \to (X, S)$ is said to be an ***intuitionistic chaotic*** continuous*** if for each periodic point $x \in X$ and each intuitionistic closed chaotic set F of $f(x)$ there exists an intuitionistic clopen orbit set $IO_f(x)$ of the periodic point x such that $f(Iint(IO_f(x))) \subseteq F$.

Proposition 3.32. *An intuitionistic chaotic continuous function is an intuitionistic chaotic*** continuous function.*

Proof. Since f is an intuitionistic chaotic continuous function, F is an intuitionistic clopen chaotic set containing $f(x)$, there exists an intuitionistic open orbit set $IO_f(x)$ containing x such that $f(IO_f(x)) \subseteq F$. This implies that, $IO_f(x) \subseteq f^{-1}(F)$. Then, $f^{-1}(F)$ is an intuitionistic clopen chaotic set of (X, S). By (iii) of Proposition 3.21, $f^{-1}(F)$ is an intuitionistic clopen orbit set in (X, T). Therefore, F is an intuitionistic closed chaotic set containing $f(x)$ and $f^{-1}(F)$ is an intuitionistic open orbit set such that $f(f^{-1}(F)) \subseteq F$. Since $f^{-1}(F)$ is intuitionistic open orbit set, $Iint(f^{-1}(F)) = f^{-1}(F)$. This implies that, $f(Iint(f^{-1}(F))) \subseteq F$. Hence, f is an intuitionistic chaotic*** continuous function. □

Remark 3.33. The converse of Proposition 4.1.4 need not be true as shown in the Example 4.1.7.

Example 3.34. Let $X = \{a, b, c, d\}$. Then the intuitionistic sets A, B, C, D, E and F are defined by $A = \langle x, \{a, b\}, \{c, d\}\rangle$, $B = \langle x, \{a\}, \{b, c, d\}\rangle$, $C = \langle x, \{c\}, \{a, b, d\}\rangle$, $D = \langle x, \{a, b, c\}, \{d\}\rangle$, $E = \langle x, \{a, c\}, \{b, d\}\rangle$ and $F = \langle x, \{a, c\}, \{b, d\}\rangle$. Let $T = \{X_\sim, \phi_\sim, A, B, C, D, F\}$, $S = \{X_\sim, \phi_\sim, , A, B, D, F\}$ are intuitionistic topologies on X. Clearly (X, T) and (X, S) be any two intuitionistic chaos spaces. The function $f : (X, T) \to (X, S)$ is defined by $f(a) = b, f(b) = d, f(c) = c$ and $f(d) = a$. Now the function f is intuitionistic chaotic*** continuous but not intuitionistic chaotic continuous. Hence, **intuitionistic chaotic*** continuous function need not be intuitionistic chaotic continuous function.**

Remark 3.35. The interrelation among the functions introduced are given clearly in the following diagram.

4. Properties of intuitionistic chaotic continuous functions

Definition 4.1. An intuitionistic chaos space (X, T) is said to be an ***intuitionistic orbit extremally disconnected space*** if the intuitionistic closure of every intuitionistic open orbit set is intuitionistic open orbit.

Proposition 4.2. *Let (X, T) and (X, S) be any two intuitionistic chaos spaces. If $f : (X, T) \to (X, S)$ is an intuitionistic chaotic continuous function and (X, T) is an intuitionistic orbit extremally disconnected space then f is an intuitionistic chaotic* continuous function.*

Proof. Let x be a periodic point and $x \in X$. Since f is intuitionistic chaotic continuous, $F = \langle x, F^1, F^2\rangle$ is an intuitionistic clopen chaotic set of (X, S), there exists an intuitionistic open orbit set $IO_f(x)$ of (X, T) containing x such that $f(IO_f(x)) \subseteq F$. Therefore, $IO_f(x)$ is an intuitionistic open orbit set $IO_f(x)$ of (X, T). Since (X, T) is intuitionistic orbit extremally disconnected, $Icl(IO_f(x))$ is an intuitionistic open orbit set. Therefore, F is an intuitionistic closed chaotic set containing $f(x)$ there exists an intuitionistic open orbit set $Icl(IO_f(x))$ such that $f(Icl(IOf(x))) \subseteq F$. Hence, f is intuitionistic chaotic* continuous. □

Definition 4.3. An intuitionistic chaos space (X, T) is said to be ***intuitionistic chaotic 0- dimensional*** if it has an intuitionistic base consisting of intuitionistic clopen chaotic sets.

Proposition 4.4. *Let (X, T) and (X, S) be any two intuitionistic chaos spaces. Let $f : (X, T) \to (X, S)$ be an intuitionistic chaotic*** continuous function. If (X, S) is intuitionistic chaotic 0-dimensional then f is an intuitionistic chaotic continuous function.*

Proof. Let the periodic point $x \in X$. Since (X, S) is intuitionistic chaotic 0-dimensional, there exists an intuitionistic clopen chaotic set $F = \langle x, F^1, F^2\rangle$ in (X, S). Since f is an intuitionistic chaotic*** continuous function, there exists an intuitionistic clopen orbit set $IO_f(x)$ such that $f(Iint(IO_f(x))) \subseteq F$. Since $IO_f(x)$ is an intuitionistic open orbit set, $Iint(IO_f(x) = IO_f(x)$. This implies that, $f(IO_f(x)) \subseteq F$. Therefore, f is intuitionistic chaotic continuous. □

Definition 4.5. An intuitionistic chaos space (X, T) is said to be an ***intuitionistic orbit connected space*** if $X_\sim$ cannot be expressed as the union of two intuitionistic open orbit sets $IO_f(x)$ and $IO_f(y)$, $x, y \in X$ of (X, T) with $IO_f(x), IO_f(y) \neq \phi_\sim$.

Definition 4.6. An intuitionistic chaos space (X, T) is said to be an ***intuitionistic chaotic connected space*** if $X_\sim$ cannot be expressed as the union of two intuitionistic open chaotic sets $U = \langle x, U^1, U^2\rangle$ and $V = \langle x, V^1, V^2\rangle$ of (X, T) with $U, V \neq \phi_\sim$.

Proposition 4.7. *An intuitionistic chaotic continuous image of an intuitionistic orbit connected space is an intuitionistic chaotic connected space.*

Proof. Let (X, S) be intuitionistic chaotic disconnected. Let $F_1 = \langle x, F_1^1, F_1^2\rangle$ and $F_2 = \langle x, F_2^1, F_2^2\rangle$ be an intuitionistic chaotic disconnected sets of (X, S). Then $F_1 \neq \phi_\sim$ and $F_2 \neq \phi_\sim$ are intuitionistic clopen chaotic sets in (X, S) and $Y_\sim = F_1 \cup F_2$ where $F_1 \cap F_2 = \phi_\sim$. Now,

$$X_\sim = f^{-1}(Y_\sim) = f^{-1}(F_1 \cup F_2) = f^{-1}(F_1) \cup f^{-1}(F_2).$$

Since f is intuitionistic chaotic continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are intuitionistic open orbit sets in (X, T). Also $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi_\sim$. Therefore, (X, T) is not intuitionistic orbit connected. Which is a contradiction. Hence, (X, S) is intuitionistic chaotic connected. □

Proposition 4.8. *Let (X, T) and (X, S) be any two intuitionistic chaos spaces. If $f : (X, T) \to (X, S)$ is an intuitionistic chaotic continuous function and $IO_f(x)$ is intuitionistic open orbit set then the restriction $f|IO_f(x) : IO_f(x) \to (X, S)$ is intuitionistic chaotic continuous.*

Proof. Let $F = \langle x, F^1, F^2\rangle$ be an intuitionistic clopen chaotic set in (X, S). Then, $(f|IO_f(x))^{-1}(F) = f^{-1}(F) \cap IO_f(x)$. Since f is intuitionistic chaotic continuous, $f^{-1}(F)$ is intuitionistic open orbit in (X, T) and $IO_f(x)$ is an intuitionistic open orbit set. This implies that, $f^{-1}(F) \cap IO_f(x)$ is an intuitionistic open orbit set. Therefore, $(f|IO_f(x))^{-1}(F)$ is intuitionistic open orbit in (X, T). Hence, $f|IO_f(x)$ is intuitionistic chaotic continuous. □

Definition 4.9. Let (X, T) be an intuitionistic chaos space. If a family $\{IO_f(x_i) : i \in J\}$ of intuitionistic open orbit set in (X, T) satisfies the condition $\cup IO_f(x_i) = X_\sim$, then it is called an ***intuitionistic open orbit cover*** of (X, T).

Proposition 4.10. *Let $\{IO_f(x)_\gamma : \gamma \in \Gamma\}$ be any intuitionistic open orbit cover of an intuitionistic chaos space (X, T). A function $f : (X, T) \to (X, S)$ is an intuitionistic chaotic continuous function if and only if the restriction $f|IO_f(x)_\gamma : IO_f(x)_\gamma \to (X, S)$ is intuitionistic chaotic continuous for each $\gamma \in \Gamma$.*

Proof. Let γ be an arbitrarily fixed index and $IO_f(x)_\gamma$ be an intuitionistic open orbit set of (X, T). Let the periodic point $x \in IO_f(x)_\gamma$ and $F = \langle x, F^1, F^2\rangle$ is intuitionistic clopen chaotic set containing $(f|IO_f(x)_\gamma)(x) = f(x)$. Since f is intuitionistic chaotic continuous there exists an intuitionistic open orbit set $IO_f(x)$ containing x such that $f(IO_f(x)) \subseteq F$. Since $(IO_f(x)_\gamma)$ is intuitionistic open orbit cover in (X, T), $x \in IO_f(x) \cap IO_f(x)_\gamma$ and

$$\begin{aligned}(f|IO_f(x)_\gamma)(IO_f(x) \cap (IO_f(x)_\gamma) &= f(IO_f(x) \cap (IO_f(x)_\gamma))\\ &\subset f(IO_f(x)\\ &\subset F.\end{aligned}$$

Hence $f|IO_f(x)_\gamma$ is an intuitionistic chaotic continuous function.

Conversely, let the periodic point $x \in X$ and F be an intuitionistic chaotic set containing $f(x)$. There exists an $\gamma \in \Gamma$ such that $x \in IO_f(x)_\gamma$.

Since $(f|IO_f(x)_\gamma) : IO_f(x)_\gamma \to (X, S)$ is intuitionistic chaotic continuous, there exists an $IO_f(x) \in IO_f(x)_\gamma$ containing x such that $(f|IO_f(x)_\gamma)(IO_f(x)) \subseteq F$. Since $IO_f(x)$ is intuitionistic open orbit in (X, T), $f(IO_f(x)) \subseteq F$. Hence, f is intuitionistic chaotic continuous. □

Proposition 4.11. *If a function $f : (X, T) \to \prod(X, S)_\lambda$ is intuitionistic chaotic continuous then $P_\lambda \circ f : (X, T) \to (X, S)_\lambda$ is intuitionistic chaotic continuous for each $\lambda \in \Lambda$, where P_λ is the projection of $\prod(X, S)_\lambda$ onto $(X, S)_\lambda$.*

Proof. Let $F_\lambda = \langle x, F^1_\lambda, F^2_\lambda\rangle$ be any intuitionistic clopen chaotic set of $(X, S)_\lambda$. Then $P^{-1}_\lambda(F_\lambda)$ is an intuitionistic clopen chaotic set in $\prod(X, S)_\lambda$ and hence $(P_\lambda \circ f)^{-1}(F_\lambda)$ $= f^{-1}(P^{-1}_\lambda(F_\lambda))$ is an intuitionistic open orbit set in (X, T). Therefore, $P_\lambda \circ f$ is intuitionistic chaotic continuous. □

Proposition 4.12. *If a function $f : \prod(X, T)_\lambda \to \prod(X, S)_\lambda$ is intuitionistic chaotic continuous then $f_\lambda : (X, T)_\lambda \to (X, S)_\lambda$ is an intuitionistic chaotic continuous function for each $\lambda \in \Lambda$.*

Proof. Let $F_\lambda = \langle x, F^1_\lambda, F^2_\lambda\rangle$ be any intuitionistic clopen chaotic set of $(X, S)_\lambda$. Then $P^{-1}_\lambda(F_\lambda)$ is intuitionistic clopen chaotic in $\prod(X, S)_\lambda$ and $f^{-1}(P^{-1}_\lambda(F_\lambda)) = f^{-1}_\lambda(F_\lambda) \times \prod\{(X, T)_\alpha : \alpha \in \Lambda - \{\lambda\}\}$.

Since f is intuitionistic chaotic continuous, $f^{-1}(P^{-1}_\lambda(F_\lambda))$ is an intuitionistic open orbit set in $\prod(X, T)_\lambda$. Since the projection P_λ of $\prod(X, T)_\lambda$ onto $(X, T)_\lambda$ is an intuitionistic open function, $f^{-1}_\lambda(F_\lambda)$ is intuitionistic open orbit in $(X, T)_\lambda$. Hence, f_λ is intuitionistic chaotic continuous. □

Definition 4.13. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. A function $f : (X, T) \to (X, S)$ is said to be ***intuitionistic chaotic irresolute*** if for each intuitionistic clopen chaotic set $F = \langle x, F^1, F^2 \rangle$ in (X, S), $f^{-1}(A)$ is an intuitionistic clopen chaotic set of (X, T).

Proposition 4.14. *Let (X, T) and (X, S) be any two intuitionistic chaos spaces. If $f : (X, T) \to (X, S)$ is an intuitionistic chaotic continuous function and $g : (X, S) \to (X, U)$ is an intuitionistic chaotic irresolute function, then $g \circ f : (X, T) \to (X, U)$ is intuitionistic chaotic continuous.*

Proof. Let $F = \langle x, F^1, F^2 \rangle$ be an intuitionistic clopen set of (X, U). Since g is intuitionistic chaotic irresolute, $g^{-1}(F)$ is intuitionistic clopen chaotic set of (X, S). Since f is intuitionistic chaotic continuous, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is an intuitionistic open orbit set of (X, T) such that $f^{-1}(g^{-1}(F)) \subseteq F$. Hence $g \circ f$ is intuitionistic chaotic continuous. □

Definition 4.15. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. A function $f : (X, T) \to (X, S)$ is said to be ***intuitionistic orbit irresolute*** if for each intuitionistic open orbit set $IO_f(x)$ in (X, S), $f^{-1}(IO_f(x))$ is an intuitionistic open orbit set of (X, T).

Definition 4.16. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. Let $f : (X, T) \to (X, S)$ be a function. Then f is said to be an ***intuitionistic open orbit function*** if the image of every intuitionistic open orbit set in (X, T) is intuitionistic open orbit in (X, S).

Proposition 4.17. *Let $f : (X, T) \to (X, S)$ be intuitionistic orbit irresolute, surjective and intuitionistic open orbit function. Then $g \circ f : (X, T) \to (X, U)$ is intuitionistic chaotic continuous iff $g : (X, S) \to (X, U)$ is intuitionistic chaotic continuous.*

Proof. Let $F = \langle x, F^1_\lambda, F^2_\lambda \rangle$ be an intuitionistic clopen chaotic set of (X, U). Since g is intuitionistic chaotic continuous, $g^{-1}(F)$ is intuitionistic open orbit in (X, S). Since f is intuitionistic orbit irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is intuitionistic open orbit in (X, T). Hence $g \circ f$ is intuitionistic chaotic continuous.

Conversely, let $g \circ f : (X, T) \to (X, U)$ be an intuitionistic chaotic continuous function. Let F be an intuitionistic clopen chaotic set of (Z, U), then $(g \circ f)^{-1}(F)$ is an intuitionistic open orbit set of (X, T). Since f is intuitionistic open orbit and surjective, $f(f^{-1}(g^{-1}(F))$ is an intuitionistic open orbit set of (X, S). Therefore, $g^{-1}(F)$ is an intuitionistic open orbit set in (X, S). Hence, g is intuitionistic chaotic continuous. □

5. Applications of intuitionistic chaotic continuous functions

Definition 5.1. An intuitionistic chaos space (X, T) is said to be ***intuitionistic chaotic locally indiscrete*** if every intuitionistic open orbit set of (X, T) is an intuitionistic open chaotic set in (X, T).

Definition 5.2. Let (X, T) be an intuitionistic chaos space. If a family $\{\langle x, F_i^1, F_i^2 \rangle : i \in J\}$ of intuitionistic open chaotic sets in (X, T) satisfies the condition $\cup\{\langle x, F_i^1, F_i^2 \rangle : i \in J\} = X_\sim$, then it is called an ***intuitionistic open chaotic cover*** of (X, T).

A finite subfamily of an intuitionistic open chaotic cover $\{\langle x, F_i^1, F_i^2\rangle : i \in J\}$ of X, which is also an intuitionistic open chaotic cover of X. Then the intuitionistic open chaotic cover is called a ***finite intuitionistic open chaotic subcover*** of $\{\langle x, F_i^1, F_i^2\rangle : i \in J\}$.

Definition 5.3. An intuitionistic chaos space (X, T) is said to be ***intuitionistic chaotic compact*** if every intuitionistic open chaotic cover has a finite intuitionistic open chaotic subcover.

Definition 5.4. Let (X, T) be an intuitionistic chaos space. If a family $\{IO_f(x_i) : i \in J\}$ of intuitionistic open orbit sets in (X, T) satisfies the condition $\cup\{IO_f(x_i) : i \in J\} = X_\sim$, then it is called an ***intuitionistic open orbit cover*** of (X, T). A finite subfamily of an intuitionistic open orbit cover $\{IO_f(x_i) : i \in J\}$ of X, which is also an intuitionistic open orbit cover of X. Then the intuitionistic open orbit cover is called a ***finite intuitionistic open orbit subcover*** of $\{IO_f(x_i) : i \in J\}$.

Definition 5.5. An intuitionistic chaos space (X, T) is said to be ***intuitionistic orbit compact*** if every intuitionistic open orbit cover has a finite intuitionistic open orbit subcover.

Definition 5.6. Let (X, T) and (X, S) be any two intuitionistic chaos spaces. Let $f : (X, T) \to (X, S)$ be a function. Then f is said to be an ***intuitionistic open chaotic function*** if the image of every intuitionistic open chaotic set in (X, T) is intuitionistic open chaotic in (X, S).

Proposition 5.7. *Let (X, T) and (X, S) be any two intuitionistic chaos spaces. Let $f : (X, T) \to (X, S)$ be an intuitionistic chaotic continuous function, intuitionistic closed and surjection. If (X, T) is intuitionistic orbit compact and intuitionistic locally indiscrete then (X, S) is intuitionistic chaotic compact.*

Proof. Since f is intuitionistic chaotic continuous, $\{F_\alpha = \langle x, F_\alpha^1, F_\alpha^2\rangle : \alpha \in I\}$ is an intuitionistic clopen chaotic cover of (X, S) there exist an intuitionistic open orbit cover $\{f^{-1}(F_\alpha) : \alpha \in I\}$ of (X, T). Since (X, T) is intuitionistic orbit compact, there exists a finite subset I_0 of I such that $X_\sim = \cup\{f^{-1}(F_\alpha) : \alpha \in I_0\}$. Now, $\{f^{-1}(F_\alpha) : \alpha \in I_0\}$ is a finite intuitionistic open orbit subcover of $\{f^{-1}(F_\alpha) : \alpha \in I\}$ of (X, T). Since (X, T) is intuitionistic locally indiscrete $\{f^{-1}(F_\alpha) : \alpha \in I_0\}$ is intuitionistic open chaotic subcover of (X, T). Since f is surjective and intuitionistic open chaotic, $\{f(f^{-1}(F_\alpha)) : \alpha \in I_0\} = \{F_\alpha : \alpha \in I_0\}$ is intuitionistic open chaotic subcover of (X, S). Hence, (X, S) is intuitionistic chaotic compact. □

Definition 5.8. Let (X, T) be an intuitionistic chaos space. If a family $\{\langle x, F_i^1, F_i^2\rangle : i \in J\}$ of intuitionistic closed chaotic sets in (X, T) satisfies the condition $\cup\{\langle x, F_i^1, F_i^2\rangle : i \in J\} = X_\sim$, then it is called an ***intuitionistic closed chaotic cover*** of (X, T). A finite subfamily of an intuitionistic closed chaotic cover $\{\langle x, F_i^1, F_i^2\rangle : i \in J\}$ of X, which is also an intuitionistic closed chaotic cover of X. Then the intuitionistic closed chaotic cover is called a ***finite intuitionistic closed chaotic subcover*** of $\{\langle x, F_i^1, F_i^2\rangle : i \in J\}$.

Definition 5.9. Let (X, T) be an intuitionistic chaos space. An intuitionistic cover is said to be ***intuitionistic clopen chaotic cover*** if it is both intuitionistic open chaotic cover and intuitionistic closed chaotic cover.

Definition 5.10. An intuitionistic chaos space (X,T) is said to be ***intuitionistic mildly orbit compact*** if every intuitionistic clopen chaotic cover has a finite intuitionistic open orbit subcover.

Proposition 5.11. *Let $f:(X,T)\to(X,S)$ be an intuitionistic chaotic continuous function, surjective and intuitionistic open orbit function. If (X,T) is intuitionistic orbit compact, then (X,S) is intuitionistic mildly chaotic compact.*

Proof. Since f is intuitionistic chaotic continuous, $\{F_\alpha=\langle x,F_\alpha^1,F_\alpha^2\rangle:\alpha\in I\}$ is an intuitionistic clopen chaotic cover of (X,S) there exists an intuitionistic open orbit cover $\{f^{-1}(F_\alpha):\alpha\in I\}$ of (X,T). Since (X,T) is intuitionistic orbit compact, there exists a finite subset I_0 of I such that $X_\sim=\cup\{f^{-1}(F_\alpha):\alpha\in I_0\}$ of (X,T). Now $\{f^{-1}(F_\alpha):\alpha\in I_0\}$ is a finite intuitionistic open orbit subcover of $\{f^{-1}(F_\alpha):\alpha\in I\}$ of (X,T). Since f is surjective and intuitionistic open orbit, $\{f(f^{-1}(F_\alpha)):\alpha\in I_0\}=\{F_\alpha:\alpha\in I\}$ is intuitionistic open orbit subcover of (X,S). Hence, (X,S) is intuitionistic mildly chaotic compact. □

Definition 5.12. An intuitionistic chaos space (X,T) is said to be ***countably intuitionistic orbit compact*** if every countable intuitionistic open orbit cover has a finite intuitionistic open orbit subcover.

Definition 5.13. An intuitionistic chaos space (X,T) is said to be ***countably intuitionistic mildly chaotic compact*** if every countable intuitionistic clopen chaotic cover has a finite intuitionistic open orbit subcover.

Proposition 5.14. *If $f:(X,T)\to(X,S)$ intuitionistic chaotic continuous, intuitionistic open orbit and surjection. If (X,T) is countably intuitionistic orbit compact then (Y,T) is countably intuitionistic mildly chaotic compact.*

Proof. The Proof is similar to that of Proposition 5.11. □

Definition 5.15. An intuitionistic chaos space (X,T) is said to be ***intuitionistic closed orbit compact*** if every intuitionistic closed orbit cover has a finite intuitionistic open orbit subcover.

Proposition 5.16. *If $f:(X,T)\to(X,S)$ intuitionistic chaotic*** continuous, intuitionistic open orbit and surjection. If (X,T) is intuitionistic closed orbit compact then (X,S) is intuitionistic closed orbit compact.*

Proof. Since f is intuitionistic chaotic*** continuous, $\{F_\alpha=\langle x,F_\alpha^1,F_\alpha^2\rangle:\alpha\in\Delta\}$ is intuitionistic closed chaotic cover of (X,S) there exists an intuitionistic clopen orbit cover $\{f^{-1}(F_\alpha):\alpha\in\Delta\}$ of (X,T). Therefore, $f^{-1}(F_\alpha)$ is an intuitionistic closed orbit cover of (X,T). Since (X,T) is intuitionistic closed orbit compact, there exists a finite subset Δ_0 of Δ such that $X_\sim=\cup\{f^{-1}(F_\alpha):\alpha\in\Delta_0\}$. Now, $\{f^{-1}(F_\alpha):\alpha\in I_0\}$ is a finite intuitionistic open orbit subcover of $\{f^{-1}(F_\alpha):\alpha\in I\}$ of (X,T). Since f is surjective and intuitionistic open orbit, $\{f(f^{-1}(F_\alpha)):\alpha\in I_0\}=\{F_\alpha:\alpha\in I_0\}$ is intuitionistic open orbit subcover of (X,S). Hence, (X,S) is intuitionistic closed orbit compact. □

Definition 5.17. An intuitionistic chaos space (X,T) is said to be ***countably intuitionistic closed orbit compact*** if every countable intuitionistic closed orbit cover has a finite intuitionistic open orbit subcover.

Proposition 5.18. *If $f : (X,T) \to (X,S)$ intuitionistic chaotic continuous, intuitionistic open orbit function and surjection. If (X,T) is countably intuitionistic closed orbit compact then (X,S) countably intuitionistic closed orbit compact.*

Proof. The Proof is follows from Proposition 5.16. □

Definition 5.19. An intuitionistic chaos space (X,T) is said to be ***intuitionistic orbit Lindelof*** if every intuitionistic open orbit cover has a countable intuitionistic open orbit subcover.

Definition 5.20. An intuitionistic chaos space (X,T) is said to be ***intuitionistic mildly chaotic Lindelof*** if every intuitionistic clopen chaotic cover has a countable intuitionistic open orbit subcover.

Proposition 5.21. *Let $f : (X,T) \to (X,S)$ be an intuitionistic chaotic continuous function, intuitionistic open orbit and surjection. If (X,T) is intuitionistic orbit Lindelof then (X,S) intuitionistic mildly chaotic Lindelof.*

Proof. Since f is intuitionistic chaotic continuous, $\{F_\alpha = \langle x, F_\alpha^1, F_\alpha^2\rangle : \alpha \in I\}$ is intuitionistic clopen chaotic cover of (X,S) there exists an intuitionistic open orbit cover $\{f^{-1}(F_\alpha) : \alpha \in I\}$. Since (X,T) is intuitionistic orbit Lindelof, there exists a countable subset I_0 of I such that $X_\sim = \cup\{f^{-1}(F_\alpha) : \alpha \in I_0\}$. Now, $\{f^{-1}(F_\alpha) : \alpha \in I_0\}$ is a countable intuitionistic open orbit subcover of $\{f^{-1}(F_\alpha) : \alpha \in I\}$ of (X,T). Since f is surjective and intuitionistic open orbit, $\{f(f^{-1}(F_\alpha)) : \alpha \in I_0\} = \{F_\alpha : \alpha \in I_0\}$ is a countable intuitionistic open orbit subcover of $\{F_\alpha : \alpha \in I\}$. Hence, (X,S) is intuitionistic mildly chaotic Lindelof. □

Definition 5.22. An intuitionistic chaos space (X,T) is said to be ***intuitionistic closed orbit Lindelof*** if every intuitionistic closed orbit cover has a countable intuitionistic open orbit subcover.

Proposition 5.23. *If $f : (X,T) \to (X,S)$ intuitionistic chaotic continuous, intuitionistic closed and surjection. If (X,T) is intuitionistic closed orbit Lindelof then (X,S) is intuitionistic mildly chaotic Lindelof.*

Proof. The Proof is similar to that of Proposition 5.21. □

Definition 5.24. An intuitionistic chaos space (X,T) is said to be ***intuitionistic orbit Hausdorff*** if for every two distinct periodic points of X, there exist intuitionistic open orbit sets $IO_f(x_1)$ and $IO_f(x_2)$ such that $IO_f(x_1) \cap IO_f(x_2) = \phi_\sim$.

Definition 5.25. An intuitionistic chaos space (X,T) is said to be ***intuitionistic chaotic Hausdorff*** if for every two distinct periodic points of X, there exist intuitionistic clopen chaotic sets F_1 and F_2 such that $F_1 \cap F_2 = \phi_\sim$.

Proposition 5.26. *Let $f : (X,T) \to (X,S)$ be intuitionistic chaotic continuous. If (X,S) is intuitionistic chaotic Hausdorff then (X,T) is intuitionistic orbit Hausdorff.*

Proof. Let x_1, x_2 be two distinct periodic points of X. Since (X,S) is intuitionistic chaotic Hausdorff, there exist intuitionistic clopen chaotic sets $F_1 = \langle x, F_1^1, F_2^2\rangle$ and $F_2 = \langle x, F_2^1, F_2^2\rangle$ such that $f(x_1) \in F_1$, $f(x_2) \in F_2$ and $F_1 \cap F_2 = \phi_\sim$. Since f is an intuitonistic chaotic continuous function, $x_i \in f^{-1}(F_i)$ is intuitionistic open orbit in (X,T) for $i=1,2$ and $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi_\sim$. Hence, (X,T) is intuitionistic orbit Hausdorff. □

Definition 5.27. An intuitionistic chaos space (X,T) is said to be ***intuitionistic orbit regular*** if for each intuitionistic closed orbit set $IO_f(x_1)$, $x_1 \in X$ in (X,T) and each periodic points $x \notin IO_f(x_1)$ there exist intuitionistic open orbit sets $IO_f(x_2)$ and $IO_f(x_3)$, $x_2, x_3 \in X$ with $IO_f(x_2), IO_f(x_3) \neq \phi_\sim$ such that $IO_f(x_1) \subseteq IO_f(x_2)$ and $x \in IO_f(x_3)$.

Definition 5.28. An intuitionistic chaos space (X,T) is said to be ***intuitionistic chaotic regular*** if for each intuitionistic closed chaotic set $F = \langle x, F^1, F^2 \rangle$ in (X,T) and each periodic points $x \notin F$ there exist intuitionistic open chaotic sets $U = \langle x, U^1, U^2 \rangle$ and $V = \langle x, V^1, V^2 \rangle$ with $U, V \neq \phi_\sim$ such that $F \subseteq U$ and $x \in V$.

Proposition 5.29. *Let $f : (X,T) \to (X,S)$ be an intuitionistic chaotic continuous function. If (X,S) is intuitionistic chaotic regular and f is intuitionistic open orbit or intuitionistic closed orbit then (X,T) is intuitionistic orbit regular.*

Proof. Suppose that f is intuitionistic open orbit. Let the periodic point $x \in X$ and $IO_f(x)$ be an intuitionistic open orbit set containing x. Then $f(IO_f(x))$ is an intuitionistic open orbit set of (X,S) containing $f(x)$. Since (X,S) is intuitionistic chaotic regular, there exists an intuitionistic clopen chaotic set F such that $f(x) \in F \subset f(IO_f(x))$. Then $x \in f^{-1}(F) \subset IO_f(x)$ is intuitionistic clopen chaotic set in (X,S). By (iii) of Proposition 4.1.1., $f^{-1}(F)$ is intuitionistic clopen orbit in (X,T). Therefore, (X,T) is intuitionistic orbit regular.

Suppose that f is intuitionistic closed. Let the periodic point $x \in X$ and $IO_f(x)$ be any intuitionistic closed orbit set of (X,T) not containing x. Since f is injective and intuitionistic closed orbit, $f(x) \notin f(IO_f(x))$ and $f(F)$ is intuitionistic closed orbit in (X,S). By the intuitionistic chaotic regularity of (X,S), there exists an intuitionistic clopen chaotic set F such that $f(x) \in F \subset Y - f(IO_f(x))$. Therefore, $x \in f^{-1}(IO_f(x))$ and $F \subset X - f^{-1}(F)$. By Proposition 4.1.1., $f^{-1}(F)$ is intuitionistic clopen orbit set in (X,T). Thus, (X,T) is intuitionistic orbit regular. □

Definition 5.30. An intuitionistic chaos space (X,T) is said to be ***intuitionistic orbit normal*** if for any pair of intuitionistic closed orbit sets $IO_f(x_1)$ and $IO_f(x_2)$, $x_1, x_2 \in X$ of (X,T) there exist intuitionistic open orbit sets $IO_f(x_3)$ and $IO_f(x_4)$, $x_3, x_4 \in X$ such that $IO_f(x_1) \subseteq IO_f(x_3)$ and $IO_f(x_2) \subseteq IO_f(x_4)$.

Definition 5.31. An intuitionistic chaos space (X,T) is said to be ***intuitionistic chaotic normal*** if for any pair of intuitionistic closed chaotic sets $A = \langle x, A^1, A^2 \rangle$ and $B = \langle x, B^1, B^2 \rangle$ of (X,T) there exist intuitionistic open chaotic sets $U = \langle x, U^1, U^2 \rangle$ and $V = \langle x, V^1, V^2 \rangle$ such that $A \subseteq U$ and $B \subseteq V$.

Proposition 5.32. *Let $f : (X,T) \to (X,S)$ be an intuitionistic chaotic continuous. If (X,S) is intuitionistic chaotic normal and f is intuitionistic closed orbit then (X,T) is intuitionistic orbit normal.*

Proof. Let $IO_f(x_1), IO_f(x_2)$ be any two intuitionistic closed orbit sets of (X,T) with $IO_f(x_1), IO_f(x_2) \neq \phi_\sim$. Since f is intuitionistic closed orbit and injective, $f(IO_f(x_1))$ and $f(IO_f(x_2))$ are intuitionistic closed orbit sets of (X,S) with

$$f(IO_f(x_1)), f(IO_f(x_2)) \neq \phi_\sim.$$

Since (X,S) is intuitionistic chaotic normal, there exist intuitionistic clopen chaotic sets $V_1 = \langle x, V_1^1, V_2^2 \rangle$ and $V_2 = \langle x, V_2^1, V_2^2 \rangle$ such that $F_1 \subset f^{-1}(V_1)$ and $F_2 \subset f^{-1}(V_2)$. Since f is intuitionistic chaotic continuous, $f^{-1}(V_i)$ is intuitionistic open orbit. Moreover, $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi_\sim$. Hence, (X,T) is intuitionistic orbit normal. □

Definition 5.33. An intuitionistic chaos space (X,T) is said to be ***intuitionistic orbit*** T_1 if for each pair of distinct periodic points x and y in X, there exist intuitionistic open orbit sets $IO_f(x)$ and $IO_f(y)$ such that $IO_f(x) \cap IO_f(y) = \phi_\sim$.

Definition 5.34. An intuitionistic chaos space (X,T) is said to be ***intuitionistic chaotic*** T_1 if for each pair of distinct periodic points x and y in X, there exist intuitionistic clopen chaotic sets $F_1 = \langle x, F_1^1, F_1^2 \rangle$ and $F_2 = \langle x, F_2^1, F_2^2 \rangle$ such that $F_1 \cap F_2 = \phi_\sim$.

Proposition 5.35. *Let $f : (X,T) \to (X,S)$ be intuitionistic chaotic continuous. If (X,S) is intuitionistic chaotic T_1 then (X,T) is intuitionistic orbit T_1.*

Proof. Suppose that (X,S) is intuitionistic chaotic T_1. Then for any distinct periodic points x and y in X, there exist intuitionistic clopen chaotic sets F_1 and F_2 containing $f(x)$ and $f(y)$, respectively, such that $f(y) \notin F_1$ and $f(x) \notin F_2$.

Since f is intuitionistic chaotic continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are intuitionistic open orbit sets of (X,T) such that $x \in f^{-1}(F_1), y \notin f^{-1}(F_1), x \notin f^{-1}(F_2)$ and $y \in f^{-1}(F_2)$. Hence, (X,T) is intuitionistic orbit T_1. □

Acknowledgements. The authors would like to thank the referee for reading the paper carefully and giving very valuable suggestions and remarks.

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