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Clinical monitoring using fuzzy system

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Abstract. The use of fuzzy system in clinical monitoring is studied here. A case of a patient at a particular time is taken into account for the clinical monitoring. The diseased conditions are taken as states and the medicines or other methods of treatment which causes the changes of the diseased condition of the patient are considered as the inputs. The states and inputs are fuzzified according to a standard rule. A rule base formed due to the the medical knowledge is taken into account for the treatment of the patient. The fuzzy production rules are used to infer the next state of a patient. An algorithm for this purpose is also presented in this paper.

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1. Introduction

The pioneering publication of L.A Zadeh in 1965 on fuzzy sets opened a new horizon in the field of vague concepts. Some authors worked on the fuzzy system using this vague concept [17] [18]. Medical science, which is sometimes considered as a borderline between Science and Art, is an excellent field where the concept of vagueness can be applied successfully. Many researches have already done in this field [5] [6]. In the medical science; impreciseness, vagueness, subjectivity, imprecision of measurement and medical intuition of the physicians are intricately combined with a large database. The degree of membership can be used to express, for example, the degree of abnormality in hypertension and/or in blood sugar. The decision making system such as expert system and pattern classification system can be used to deal with the patients' abnormalities of states. Usually the source of data is the patients' record. This data is fuzzified with some suitable scheme. The vagueness which is intuitionally dealt with the physicians and the other attendants can be successfully handled by the fuzzy sets. These fuzzy sets are also capable of the computations and

the conclusion can be made with the help of reasoning method. The algebraic structure of fuzzy set theory by some researchers [15] [25] [7] [11] [1] made a robust base for developing the clinical monitoring by fuzzy system. The closely related application with the clinical monitoring is clinical diagnosis using fuzzy automata. Some authors [8] [21] used the concepts of fuzzy input, fuzzy reasoning and fuzzy classes to study clinical diagnosis. Some authors [2] used hierarchical rule based monitoring and fuzzy logic control for neuromuscular block. A monitoring framework was developed [24] that allows the construction of problem oriented diagnostic monitors. It allows time in the data model. The model detects the trend and tracks the disease history. Some works has been done on the fuzzy logic in medicine [16].This work is mainly focused on lung diseases and this work includes rule based fuzzy systems in medicine. Some authors studied the intuitionistic fuzzy set theory [26] and described the case study of some patients [3]. The max-min method is applied here for it is an intuitive recipe and easy to work. The composition considers the extreme values. The physician's medical knowledge as a fuzzy relation between symptoms and diseases was elaborated by [23]. An intelligent state monitor which makes an abstraction of a patient's current status by fuzzy state transitions is described by [22]. The reasoning as well as the inverse approximate reasoning methodology [12] [13] [14] [19] plays an important role in clinical monitoring. These may be extensively used in dealing with the problems of the clinical monitoring.

This paper contains five sections. Section 1 contains some introductory concepts about the clinical monitoring. Section 2 contains some basic concepts and some definitions to improve the readability of this paper. Some preliminary concepts of the fuzzy sets are described in section 3. In section 4, similarity based approximate reasoning and a fuzzy system as a state monitor is described. Some examples are provided in this section to illustrate the matter. Section 5 concludes the paper. A comprehensive list of reference is provided at last.

2. Basic concepts

The clinical monitoring requires (i) The state of a patient (ii) The inputs required to treat the patient (iii) A set of rules connecting the state with the input and (iv) Conclusion about the state of the patient. Sometimes, defuzzification is done to precisely understand the state of the patient and to select the therapy over the time. The term *disease* is broadly used to indicate the deviation from the normal physical state of the patient. There are some precise definitions of the disease which are important for the monitoring of the patients.

Definition 2.1. A disease is a pathological condition of a part, organ or system of an organism resulting from various causes, such as infection, genetic defect or environmental stress which characterized by an identifiable group of signs or symptoms [29].

Definition 2.2. A disease is a disordered or abnormal condition of an organ or other part of an organism resulting from the effect of genetic or developmental errors, infection, nutritional deficiency, toxicity or unfavorable environmental factors, illness, sickness [28].

There are other medical terminologies which refer to the abnormal condition of the human being. These are *Disorder*, *Morbidity* and *Illness* although they have subtle differences from the medical point of view. For mathematical modelling we need not distinguish all of them. We accept the term *Disease* only which contains all the terminologies in a broad sense, that serves our purpose.Another terminology which is helpful to construct a fuzzy state is *syndrome*.

Definition 2.3. A syndrome is an association of several medical signs, symptoms and (or) other characteristics that often occur together[30].

We now describe the state of a patient as *Fuzzy Disease Syndrome (FDS)*. A *FDS* describes the overall state of a patient at time t and the change at time $t+\delta t$. The input of the states are the factors which causes change from one state to another. In the case of fuzzy clinical monitoring, the inputs are medicines or other physical entities which causes the state change(Oxygen, Physiotherapy etc). To construct a useful state monitor, the careful choice of the input in the Rule Base of *FDS(t)* is important. There are some inputs which act on more than one diseases, on the contrary, in some diseases more than one input is required. These characteristics should be taken into account for the construction of a Rule Base for a *FDS(t)* if the situation needs. For the sake of simplicity we consider the effect of a single input on a single syndrome. There are some factors like age, fitness, immunity, stage of the disease etc, which are deciding factors for the selection of the doses of the input. The clinical monitoring is based on the max-min compositional rule. So the next state cannot be more specific than the previous state. So, care must be taken to construct the Rule Base.

3. Preliminaries

Definition 3.1. Let U be a non-empty set, to be called a universal set. A fuzzy set F on U is meant a function $F : U \to [0,1]$. F is called the membership function. $F(x)$ is called the membership grade of x. We can write, $F = \{(x, F(x) : x \in U\}$.

Definition 3.2. The similarity index of the pair of fuzzy sets $\{A, B\}$ is denoted by $S(A,B)$ and is defined by,

$$S(A,B) = 1 - \left(\frac{\sum_u \mid \mu_A(u) - \mu_B(u) \mid^q}{n} \right)^{\frac{1}{q}} \tag{3.1}$$

where n is the cardinality of the universe of discourse and q is the family parameter.

Example 3.3. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$.

$$\begin{aligned} A &= 0.2/u_1 + 1.0/u_2 + 0.3/u_3 + 0.1/u_4 + 0.4/u_5; \\ B &= .04/u_1 + 1.0/u_2 + 0.09/u_3 + 0.01/u_4 + 0.16/u_5; \\ C &= 0.45/u_1 + 1.0/u_2 + 0.55/u_3 + 0.32/u_4 + 0.63/u_5. \end{aligned}$$

With $q = 2$,

$$S(A,B) = \underline{0.8355} \ (\textit{approximately}),$$
$$903$$

and

$$S(A,C) \;=\; \underline{0.7873}\ (\textit{approximately}).$$

The inequality $S(A,B) \geq S(A,C)$ will imply that 'B is at least as close to A as C is close to A'. $S(A,B)$ is quite sensitive,because every change in A or B will be reflected in $S(A,B)$.

Definition 3.4. A fuzzy sequential machine without output is a four tuple $S =< S, \Sigma_k, M, a >$ where S is the set of internal states, Σ_k contains the set of finite input alphabets $\{\sigma_0, \sigma_1, ..., \sigma_n\}$. M is a function from $S \times \Sigma_k \times S \rightarrow [0,1]$, called the transition function and a is called the initial state.

Definition 3.5. A fuzzy automaton is a five tuple $S =< S, \Sigma_k, M, a, F >$, where $S =< S, \Sigma_k, M, a >$ is fuzzy sequential machine without output and F is a subset of S called the set of final states or output set.

A peak hold property of the clinical monitor was stated by [22]

Definition 3.6. A fuzzy automaton is said to provide a *peak hold* if there is a transition for every state to itself on every input that leads to that state, i.e, if $\forall q', i, q : M(q', i) = q \rightarrow M(q, i) = q$.

Definition 3.7. Defuzzification is the process by means of which we can have a single real value from a fuzzy set. There are several methods of defuzzification. One of them, which is used here, is Modified Centre of Gravity Method. In case of discrete universe $U = \{u_1, u_2, ..., u_n\}$ the defuzzified value for the given fuzzy set will be given as :

$$u^* = \frac{\sum_{i=1}^{n} u_i . \mu_U(u_i)}{\sum_{i=1}^{n} \mu_U(u_i)} \tag{3.2}$$

and in case of continuous universe U the defuzzified value for the given fuzzy set will be given as :

$$u^* = \frac{\int_U u . \mu_U(u) du}{\int_U \mu_U(u) du} \tag{3.3}$$

The state monitor for the clinical monitoring requires a knowledge base. A knowledge base consists of a data base and a rule base. The data base provides information for the proper functioning of the fuzzification module, the rule base and the defuzzification module. A rule base represents a set of production rules in a structured way. The widely used three major types of rules are (i) Mamdani Type (ii) Takagi-Sugeno-Kang Type (TSK Type) and (iii) Tsukamoto Type. The Mamdani Type rule is,

$R_i : If\ x_1\ is\ A_{1i}\ \&\ x_2\ is\ A_{2i}\ \&\ ...\ \&\ x_m\ is\ A_{mi}\ then\ y\ is\ B_i$,

where R_i is the i^{th} rule in the fuzzy rule base, $(i = 1, 2, ..., n)$. Here, A_{ki} and B_i are fuzzy sets on certain appropriate domains $(k = 1, 2, ..., m)$. & is a commutative and associative logical connective. Such type of FRB is called Multiple Input Single Output $(MISO)$ system.

Definition 3.8. The Triangular membership function is given by ,

$$\begin{aligned} f_{Triangular}(x,a,b,c) &= 0,\ x \le a \\ &= (x-a)/(b-a),\ a < x \le b \\ &= (c-x)/(c-b),\ b \le x < c \\ &= 0,\ x \ge c \end{aligned}$$

Definition 3.9. The Trapezoidal membership function is given by ,

$$\begin{aligned} f_{Trapezoidal}(x,a,b,c,d) &= 0,\ x \le a \\ &= (x-a)/(b-a),\ a < x \le b \\ &= 1,\ b < x \le c \\ &= (d-x)/(d-c),\ c < x \le d \\ &= 0,\ x > d \end{aligned}$$

The concept of the final state is omitted purposefully by [22] . For our purpose, we have incorporated the idea of *Normal State* instead of final state. The difference between them is that, once a state reaches it's final state, it can not be reversed, but in case of human being the normal state is reversible.

4. Similarity based approximate reasoning in clinical monitoring

4.1. The cognitive process of human reasoning is qualitative and hence it can not be dealt with the two valued logic. The infinitely many valued logic, namely fuzzy logic is useful to deal with such imprecise reasoning. This process is named as Approximate Reasoning. The basic principle of approximate reasoning was introduced by Zadeh and since then many researchers have contributed in this area [4] [20]. The composite fuzzy propositions are obtained by joining the atomic propositions by the logical connectives. Many different forms of approximate reasoning are presented by many authors. The similarity of two objects indicates the degree to which properties of one may be inferred from those of the other . After the transition from the conditional statement into a fuzzy relation, the similarity between the fact and the antecedent of the rule is calculated and is used to modify the relation with some suitable scheme. Let U and V be the universe of discourses and X, Y be the variables taking values from U and V respectively. Let A, A',B,B' are descriptions of X and Y, which are approximated by fuzzy sets over their universal sets. Let us construct the Table 1 for rule based approximate reasoning.

The relation between A and B is denoted by $R(A,B)$ and constructed by min-rule for translation. Two schemes for the similarity based approximate reasoning was suggested by [17]

p :	X is A	then	Y is B
q :	X is A'		
r :			Y is B'.

Table 1. Rule-based ordinary approximate reasoning

Scheme I :

(4.1)

$$R(A,B \mid A') = [r'_{u,v}]_{m\times n}\ where \left[\begin{array}{lll} r'_{u,v} & = \min(1, r_{u,v}/S(A,A')) & \text{if } S(A,A')>0 \\ & = 1 & \text{otherwise.} \end{array}\right]$$

Scheme II:

$$\mu_{R(A,B|A')}(u,v) = 1-(1-\mu_{R(A,B)}(u,v)).S(A,A') \tag{4.2}$$

4.2. Important properties for a set of rules for proper functioning of the monitor are, completeness, consistency, continuity and interaction. The *MISO* system although useful in some control system, it has less importance in clinical monitoring, for most of the practical experiences of a clinical case, a database consists of multiple inputs as well as multiple outputs. For a *MIMO* system, the state vector is taken as $s = (s_1, s_2, ...s_n)$, the input vector is taken as $i = (i_1, i_2, ..., i_m)$ and the output vector is taken as $t = (t_1, t_2, ..., t_l)$. A fuzzy model for the clinical monitoring consists the rules of the form,

$$\begin{array}{lll} R_k & = & if\ (s_1\ is\ L(s_1^k))\&(s_2\ is\ L(s_2^k))\&...\&(s_n\ is\ L(s_n^k)) \\ \& & & (i_1\ is\ L(i_1^k))\&(i_2\ is\ L(i_2^k))\&...(i_m\ is\ L(i_m^k)) \\ then & & (t_1\ is\ L(t_1^k))\&(t_2\ is\ L(t_2^k))\&...\&(t_l\ is\ L(t_l^k)) \end{array} \tag{4.3}$$

R_k designates the k^{th} rule and $L(s_p^k)$, $(p = 1, 2, ..., n)$; $L(i_q^k)$, $(q = 1, 2, ..., m)$, and $L(t_r^k)$, $(r = 1, 2, ..., l)$ are linguistic values of the state, input and output variables respectively.

In fuzzy model linguistic values are represented by the fuzzy sets $L(\tilde{s_p^k})$,$L(\tilde{i_q^k})$ and $L(\tilde{t_r^k})$. These fuzzy sets are defined on the respective universes of discourse S_p, I_q and T_r respectively. The meaning of such a rule is given as a fuzzy relation $R^{(k)}$ on $S \times I \times T$ where,

$S = S_1 \times S_2 \times ... \times S_n$
$I = I_1 \times I_2 \times ... \times I_m$
$T = T_1 \times T_2 \times ... \times T_l$

As a particular case for Mamdani *MIMO* type implication, let

$$\begin{array}{lll} \tilde{R^k} & = & (L(\tilde{s_1^k}) \times ...L(\tilde{s_n^k})) \\ & \times & (L(\tilde{i_1^k}) \times ... \times L(\tilde{i_m^k})) \\ & \times & L(\tilde{t_1^k}) \times ...L(\tilde{t_l^k}) \end{array} \tag{4.4}$$

For the whole set of rules,

$$\tilde{R} = \cup_k \tilde{R^{(k)}} \tag{4.5}$$

Let

$L(\tilde{s}) = L(\tilde{s_1}) \times ... \times L(\tilde{s_n})$
$L(\tilde{i}) = L(\tilde{i_1}) \times ... \times L(\tilde{i_m})$
$L(\tilde{t}) = L(\tilde{i_1}) \times ... \times L(\tilde{i_l})$

Therefore,

$$\tilde{R^k} = L\tilde{(s)} \times L\tilde{(i)} \times L\tilde{(t)} \tag{4.6}$$

The fuzzy model for clinical monitoring is given in a concise form as

$$L\tilde{(t)} = (L\tilde{(s)} \times L\tilde{(u)}) \odot \tilde{R} \tag{4.7}$$

The *MIMO* system can be decomposed into a *MISO* system.

4.3. **State monitor.** In a fuzzy system, the state, the input and the state transition functions are all fuzzy concepts. A strategy is used here for the next state generation in a fuzzy automata using the concepts of fuzzification, knowledge base, inference, similarity measure and defuzzification.

Although the science of medicine is not related to control engineering directly, the use of control techniques for on line devices plays a major role from simple dosage prescription to highly sophisticated adaptive controllers. The fuzzy set theory is capable of describing inexact medical entities as fuzzy sets. The if-then rules can be successfully applied in selecting the dosage of an input medicine such as, 'if the *FBS* is *High* and the input of medicine is *Medium* then *FBS* is *Normal*'[10].The concept of similarity between the fuzzy states are used to select the rules from the rule base and to be fired for a particular input. Here we propose the strategy for monitoring a patients state. Let us model the process by $if - then$ rules.

if present state is s_1 *and present state is* s_2 *... and present state is* s_n *and present input is* i_1 *then next state is* s'_1 *and next state is* s'_2 *... and next state is* s'_n.

Fuzzification : The data acquired from the patient's data base is needed to be fuzzified for generally the data is very precise, time dependent and crisp. The data is fuzzified, normally in trapezoidal or triangular form, because these are economic and easy to understand.

Knowledge Base : The knowledge base consists of data base and rule base. The data base consists of patients' data in the form of *FDS* and it gives information for the fuzzification. The rule base represents a set of rules generally in tabular form. The rules come from the knowledge and experience from a medical practitioner or expert. Now a days, a data bank is maintained by the health institutions, which can be used for the clinical monitoring.

Inference : The inference is made on the basis of similarity measure. To select a rule for firing, the steps are as follows:

Let $A^i_1, A^i_2, ..., A^i_n$ be the fuzzified value of the present state, $B^i_1, B^i_2, ..., B^i_m$ be the fuzzified value of the present input and $C^i_1, C^i_2, ..., C^i_n$ be the fuzzified value of the next state at time t. Let $D^i_1, D^i_2, ..., D^i_n$ be the fuzzified value of the observed present state and $E^i_1, E^i_2, ..., E^i_m$ be the fuzzified value of the observed present input in the i^{th} rule. Let $\alpha_i = min\{sim(A^i_1, D^i_1), sim(A^i_2, D^i_2), ..., sim(A^i_n, D^i_n), sim(B^i_1, E^i_1)\}$. Let $max_i\alpha_i = \alpha_j$ (say). The j^{th} rule is fired if it exceeds a pre-defined threshold value λ. If more than one α exceeds λ then their union is taken. This completes a single phase for clinical monitoring.

Similarity measure : Let for a sequential machine, each input takes the time δt. After m inputs the time is $m\delta t$. So the acquired state after computation of the first phase appears at time $t + m\delta t$. This state is compared with the normal state of

the patient, which is defined before by the experts, with the help of the similarity measure. If the normalcy level hopefully exceeds a threshold value N_α (say) then the patient has reached the normal state.

Defuzzification : The output state sometimes need defuzzification. In this clinical monitoring scheme, the modified centre of gravity method is used. If the similarity value between the fuzzy output state and the fuzzy normal state lacks behind N_α, then the fuzzification of the defuzzified state is again necessary for the next loop of iteration by the state monitor.

Definition 4.1. A fuzzy state monitor is a fuzzy system whose states are the states of a patient at time t, whose inputs are fuzzy states which enables the state transitions with the help of fuzzy rule base. The state monitor gives the information about the state of the patient at time $t + \delta t$.

Let us now develop an algorithm for the clinical monitoring.

ALGORITHM CM : Clinical Monitoring :

Step I : Input : Fuzzy finite state machine $< S, \Sigma, M, a >$, a set of states (s_1, s_2, ..., s_n), a set of inputs $(i_1, i_2, ..., i_m)$, a set of if-then rules R_k, with the corresponding fuzzy states, where,

$$\begin{aligned} R_k \quad = \quad & if\ (s_1\ is\ L(s_1^k))\&(s_2\ is\ L(s_2^k))\&...\&(s_n\ is\ L(s_n^k)) \\ \& \quad & (i_1\ is\ L(i_1^k))\&(i_2\ is\ L(i_2^k))\&...(i_m\ is\ L(i_m^k)) \\ then \quad & (t_1\ is\ L(t_1^k))\&(t_2\ is\ L(t_2^k))\&...\&(t_n\ is\ L(t_n^k)) \end{aligned}$$

Observed present state $L'(s_1^k), L'(s_2^k), ..., L'(s_n^k)$, Observed input $L'(i_1^k)$.

Step II : Compute the similarity index

$$\alpha_k = min\{sim(L(s_1^k), L'(s_1^k)), sim(L(s_2^k), L'(s_2^k)), ..., sim(L(s_n^k), L'(s_n^k)), sim(L(i_1^k), L'(i_1^k))\}.$$

α_k is taken as matching grade of the rule and the fact.

Step III : Compute $R(L(s_1^k), L(s_2^k), ..., L(s_n^k), L(i_1^k), L(t_1^k), L(t_2^k), ..., L(t_n^k))$ using a rule, say min-rule for translation.

Step IV : Modify the rule with α_k. Obtain the modified conditional relation $R(L(s_1^k), L(s_2^k), ..., L(s_n^k), L(i_1^k), L(t_1^k), L(t_2^k), ..., L(t_n^k) | L'(s_1^k), L'(s_2^k), ..., L'(s_n^k), L'(i_1^k))$

Step V : Use max-projection operation on R to obtain the fuzzy output state $L'(t_0^k), L'(t_1^k), ..., L'(t_n^k)$ as

$max_{(s_1^k, s_2^k, ..., s_n^k, i_1^k)}\{\mu_R(s_1^k, s_2^k, ..., s_n^k, i_1^k, t_1^k, t_2^k, ..., t_n^k)\}$

Step VI : Goto Step I and replace sequentially i_1^k with i_2^k, i_3^k,...,i_m^k.

Step VII : Defuzzify the output state and compute similarity index with the defuzzified normal state. If the similarity does not exceed a threshold value λ (say), goto Step I.

The algorithm CM is based on *MISO* system. Lee [9] has proved that, if the consequences of the rules within the fuzzy logic system are not applied to the antecedent of other rules, then a *MIMO* system can be considered as a collection of *MISO* systems [9].

Case Study : Let the Rule Base for the Systolic Blood Pressure (SBP) be given by Table 2 and the Rule Base for the Fasting Blood Sugar (FBS) be given by Table 3 [27]. Here ϕ is taken as forbidden state.

Let $FDS(t) = \{SBP\ is\ High,\ FBS\ is\ Very\ High\}$

σ_{SBP}	$Medium$	$Large$	$Very\ Large$
$High$	Normal	ϕ	ϕ
$Above\ High$	ϕ	Normal	ϕ
$Very\ High$	ϕ	ϕ	Normal

TABLE 2. Rule Base for SBP

σ_{FBS}	$Medium$	$Large$	$Very\ Large$
$High$	Normal	ϕ	ϕ
$Above\ High$	ϕ	Normal	ϕ
$Very\ High$	ϕ	ϕ	Normal

TABLE 3. Rule Base for FBS

Considering that the patient has two states and two inputs there are exactly nine rules as in 4.8:

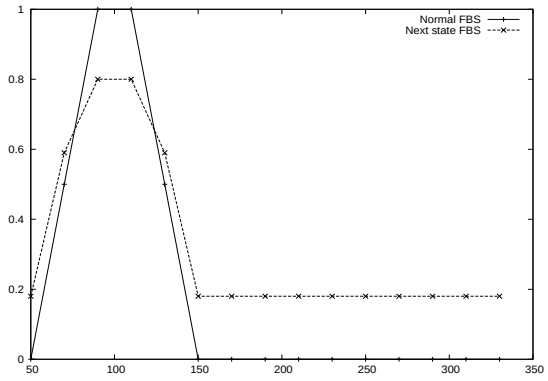

FIGURE 1. Comparison between Normal FBS and Next Output FBS

(4.8)

$$R^{(k)} = SBP\ is\ L^{(k)}(SBP)\ \wedge\ FBS\ is\ L^{(k)}(FBS)\ \wedge\ \sigma_{SBP}\ is\ L^{(k)}(\sigma_{SBP}) \wedge \sigma_{FBS}\ is\ L^{(k)}(\sigma_{FBS}) \rightarrow\ SBP\ is\ L^{(k)}(SBP) \wedge\ FBS\ is\ L^{(k)}(FBS)$$

k=1,2,...9.

Here $L^{(k)}$ indicates the linguistic variable of the concerned fuzzy set. These nine rules can further be decomposed into eighteen $MISO$ rules,

$Similarity(Next\ State\ FBS, Normal\ FBS) = 0.83$

(4.9)

$$R^{(k)} = SBP\ is\ L^{(k)}(SBP)\ \wedge\ FBS\ is\ L^{(k)}(FBS)\ \wedge\ \sigma SBP\ is\ L^{(k)}(\sigma_{SBP})\ \wedge\ \sigma_{FBS}\ is\ L^{(k)}(\sigma_{FBS})\ \rightarrow\ SBP\ is\ L^{(k)}(SBP)$$

k=1,2,...9.

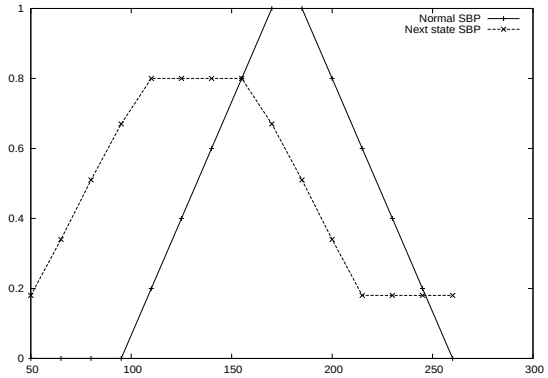

FIGURE 2. Comparison between Normal SBP and Next Output SBP

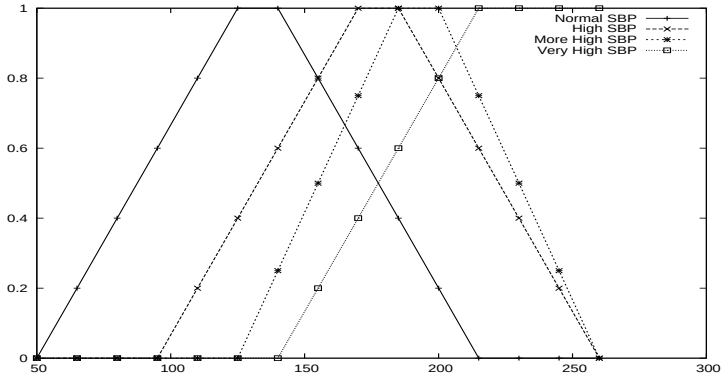

FIGURE 3. Fuzzy sets of SBP

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0	0.18	s_5	0.80	s_{10}	0.34
s_1	0.34	s_6	0.80	s_{11}	0.18
s_2	0.51	s_7	0.80	s_{12}	0.18
s_3	0.67	s_8	0.67	s_{13}	0.18
s_4	0.80	s_9	0.51	s_{14}	0.18

TABLE 4. Next output SBP

and

(4.10)
$$R^{(k)} = SBP \text{ is } L^{(k)}(SBP) \wedge FBS \text{ is } L^{(k)}(FBS) \wedge \sigma_{SBP} \text{ is } L^{(k)}(\sigma_{SBP}) \wedge \sigma_{FBS} \text{ is } L^{(k)}(\sigma_{FBS}) \rightarrow FBS \text{ is } L^{(k)}(FBS)$$

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0	0.18	s_5	0.18	s_{10}	0.18
s_1	0.59	s_6	0.18	s_{11}	0.18
s_2	0.80	s_7	0.18	s_{12}	0.18
s_3	0.80	s_8	0.18	s_{13}	0.18
s_4	0.59	s_9	0.18	s_{14}	0.18

TABLE 5. Next output FBS

k=1,2,...9. The trapezoidal method is taken here for the fuzzification of the acquired data.

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5	1.0	s_{10}	0.2
s_1	0.2	s_6	1.0	s_{11}	
s_2	0.4	s_7	0.8	s_{12}	
s_3	0.6	s_8	0.6	s_{13}	
s_4	0.8	s_9	0.4	s_{14}	

TABLE 6. SBP is *Normal* (Defuzzified value 132.5)

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5	0.4	s_{10}	0.8
s_1		s_6	0.6	s_{11}	0.6
s_2		s_7	0.8	s_{12}	0.4
s_3		s_8	1.0	s_{13}	0.2
s_4	0.2	s_9	1.0	s_{14}	0.0

TABLE 7. SBP is *High* (Defuzzified value 168.2)

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5		s_{10}	1.0
s_1		s_6	0.25	s_{11}	0.75
s_2		s_7	0.50	s_{12}	0.50
s_3		s_8	0.75	s_{13}	0.25
s_4		s_9	1.0	s_{14}	

TABLE 8. SBP is *Above High* (Defuzzified value 192.5)

The observed SBP, observed FBS, observed input for SBP and observed input for FBS are given by the tables Table 17, Table 18, Table 19, and Table 20 respectively.

The next output states of FBS and SBP are respectively given as in the tables Table 4 and Table 5.

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5		s_{10}	0.8
s_1		s_6		s_{11}	1.0
s_2		s_7	0.2	s_{12}	1.0
s_3		s_8	0.4	s_{13}	1.0
s_4	0.2	s_9	0.6	s_{14}	1.0

TABLE 9. SBP is $Very\ High$ (Defuzzified value 220)

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5		s_{10}	
s_1	0.5	s_6		s_{11}	
s_2	1.0	s_7		s_{12}	
s_3	1.0	s_8		s_{13}	
s_4	0.5	s_9		s_{14}	

TABLE 10. FBS is $Normal$ (Defuzzified value 100)

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5	1.0	s_{10}	0.4
s_1	0.2	s_6	1.0	s_{11}	0.2
s_2	0.4	s_7	1.0	s_{12}	
s_3	0.6	s_8	0.8	s_{13}	
s_4	0.8	s_9	0.6	s_{14}	

TABLE 11. FBS is $High$ (Defuzzified value 170)

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5	0.4	s_{10}	0.8
s_1	0.5	s_6	0.6	s_{11}	0.6
s_2		s_7	0.8	s_{12}	0.4
s_3		s_8	1.0	s_{13}	0.2
s_4	0.2	s_9	1.0	s_{14}	

TABLE 12. FBS is *Above High* (Defuzzified value 220)

Also, $Similarity(Next\ State\ SBP, Normal\ SBP) = 0.85$, $Similarity(Next\ State\ FBS, Normal\ FBS) = 0.83$ and when defuzzified, $defuzzy_{\mu(s)>0.5}(SBP) = 132.5$ and $defuzzy_{\mu(t)>0.5}(FBS) = 100$, $\forall\ s \in \{\tilde{SBP}\}$, and $\forall\ t \in \{\tilde{FBS}\}$ which are within normal ranges. Hence $FDS(t+\delta t) = \{SBP\ is\ Normal, FBS\ is\ Normal\}$. It is to be noted that the *Peak Hold* property [22] is satisfied with this type of clinical monitoring.

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5		s_{10}	1.0
s_1		s_6	0.2	s_{11}	1.0
s_2		s_7	0.4	s_{12}	1.0
s_3		s_8	0.6	s_{13}	1.0
s_4		s_9	0.8	s_{14}	1.0

TABLE 13. FBS is *Very High* (Defuzzified value 267.14)

i	$\mu(i)$	i	$\mu(i)$	i	$\mu(i)$
i_0		i_5	0.8	i_{10}	0.6
i_1		i_6	1.0	i_{11}	0.4
i_2	0.2	i_7	1.0	i_{12}	0.2
i_3	0.4	i_8	1.0	i_{13}	
i_4	0.6	i_9	0.8	i_{14}	

TABLE 14. Input is *Medium*

i	$\mu(i)$	i	$\mu(i)$	i	$\mu(i)$
i_0		i_5	0.64	i_{10}	0.36
i_1		i_6	1.0	i_{11}	0.16
i_2	0.04	i_7	1.0	i_{12}	0.04
i_3	0.16	i_8	1.0	i_{13}	
i_4	0.36	i_9	0.64	i_{14}	

TABLE 15. Input is *Large*

i	$\mu(i)$	i	$\mu(i)$	i	$\mu(i)$
i_0		i_5	0.512	i_{10}	0.216
i_1		i_6	1.0	i_{11}	0.064
i_2	0.008	i_7	1.0	i_{12}	0.008
i_3	0.064	i_8	1.0	i_{13}	
i_4	0.216	i_9	0.512	i_{14}	

TABLE 16. Input is *Very Large*

5. Conclusion

A case study of a patient is done here. The case of a patient with High Blood Pressure and Very High Blood Sugar is studied with the help of the fuzzy clinical monitoring. Properties of approximate reasoning are used here to observe the next state. The result obtained here is the normal state of the patient. However, it may happen that the state of the patient is more critical and it requires a continuous monitoring over the time, hopefully until the patient's state becomes normal. Such cases may also be dealt with the proposed fuzzy clinical monitoring system.

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5	0.4	s_{10}	0.7
s_1	0.5	s_6	0.7	s_{11}	0.5
s_2		s_7	0.8	s_{12}	0.1
s_3	0.1	s_8	0.8	s_{13}	
s_4	0.3	s_9	0.8	s_{14}	

TABLE 17. Observed SBP

S	$\mu(S)$	S	$\mu(S)$	S	$\mu(S)$
s_0		s_5	0.15	s_{10}	0.8
s_1	0.5	s_6	0.3	s_{11}	0.8
s_2		s_7	0.4	s_{12}	0.8
s_3		s_8	0.7	s_{13}	0.8
s_4	0.1	s_9	0.8	s_{14}	0.8

TABLE 18. Observed FBS

i	$\mu(i)$	i	$\mu(i)$	i	$\mu(i)$
i_0		i_5	1.0	i_{10}	0.6
i_1		i_6	1.0	i_{11}	0.4
i_2	0.2	i_7	1.0	i_{12}	0.2
i_3	0.4	i_8	1.0	i_{13}	
i_4	0.6	i_9	1.0	i_{14}	

TABLE 19. Observed Input for SBP

i	$\mu(i)$	i	$\mu(i)$	i	$\mu(i)$
i_0		i_5	1.0	i_{10}	0.216
i_1		i_6	1.0	i_{11}	0.064
i_2	0.008	i_7	1.0	i_{12}	0.008
i_3	0.064	i_8	1.0	i_{13}	
i_4	0.216	i_9	1.0	i_{14}	

TABLE 20. Observed Input for FBS

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