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Fixed point results for hybrid pairs of occasionally weakly compatible mappings defined on fuzzy metric space

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Abstract. The intent of this paper is to obtain some common fixed point theorems for hybrid pairs of single and multi-valued occasionally weakly compatible mappings using a symmetric δ derived from an ordinary symmetric d in fuzzy metric space.

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1. Introduction

Fuzzy set was defined by Zadeh [8]. Kramosil and Michalek [6] introduced fuzzy metric space, George and Veermani [2] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have obtained common fixed point theorems for mappings satisfying different types of commutativity conditions. The study of fixed point theorems, involving four single-valued maps, began with the assumption that all of the maps are commuted. Sessa [7] weakened the condition of commutativity to that of pairwise weakly commuting. Jungck generalized the notion of weak commutativity to that of pairwise compatible [3] and then pairwise weakly compatible maps [4]. Jungck and Rhoades [5] introduced the concept of occasionally weakly compatible maps.

Abbas and Rhoades [1] generalized the concept of weak compatibility in the setting of single and multi-valued maps by introducing the notion of occasionally weakly compatible (owc).

This paper presents some common fixed point theorems for more general commutative condition i.e. occasionally weakly compatible mappings in fuzzy metric space. The aim of this paper is to obtain some common fixed point theorems for owc maps to hybrid pairs of single and multi-valued maps using a symmetric δ derived from an ordinary symmetric d in fuzzy metric space.

2. Preliminary Notes

Definition 2.1. A fuzzy set A in X is a function with domain X and values in [0,1].

Definition 2.2. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norms if $*$ is satisfying conditions:
(i) $*$ is an commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.3. A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$,
(i) $M(x, y, t) > 0$;
(ii) $M(x, y, t) = 1$ if and only if $x = y$;
(iii) $M(x, y, t) = M(y, x, t)$;
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(v) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.
Then M is called a fuzzy metric on X. Then $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Example 2.4. Let (X, d) be a metric space. Denote $a * b = ab$ for all $a, b \in [0,1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d = \frac{t}{t + d(x, y)}.$$

Then $(X, M_d, *)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric.

Throughout the paper X will represent the fuzzy metric space $(X, M, *)$ and $CB(X)$, the set of all non empty closed and bounded sub-sets of X. For $A, B \in CB(X)$ and for every $t > 0$, denote

$$H(A, B, t) = \sup \{M(a, b, t); a \in A, b \in B\}$$
$$\text{and } \ \delta_M(A, B, t) = \inf \{M(a, b, t); a \in A, b \in B\}.$$

If A consist of a single point a, we write $\delta_M(A, B, t) = \delta_M(a, B, t)$. If B also consists of a single point b, we write $\delta_M(A, B, t) = M(a, b, t)$.

It follows immediately from definition that

$$\delta_M(A, B, t) = \delta_M(B, A, t) \geq 0$$
$$\delta_M(A, B, t) = 1 \Leftrightarrow A = B = \{a\} \text{ for all } A, B \in CB(X).$$

Lemma 2.5. *Let $(X, M, *)$ be a fuzzy metric space. If there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.*

Definition 2.6. A point $x \in X$ is called a coincidence point (resp. fixed point) of $A : X \to X,\ B : X \to CB(X)$ if $Ax \in Bx$ (resp. $x = Ax \in Bx$).

Definition 2.7. Maps $f : X \to X$ and $T : X \to CB(X)$ are said to be weakly compatible if they commute at their coincidence points, that is $fx \in Tx$ for some $x \in X$ then $fTx = Tfx$.

Definition 2.8. Maps $f : X \to X$ and $T : X \to CB(X)$ are said to be occasionally weakly compatible (owc) if and only if there exist some point x in X such that $fx \in Tx$ and $fTx \subseteq Tfx$.

Example 2.9. Let $(X, F, *)$ be a fuzzy metric space, where $X = [0, \infty)$ with $a * b = \min\{a, b\}$ and

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

Let $A : X \to X$ & $B : X \to CB(X)$ be single valued and set-valued maps defined by

$$A(x) = \begin{cases} 0, & \text{if } x = 0; \\ x^2, & \text{if } x \in (0,\infty). \end{cases} \qquad B(X) = \begin{cases} \{0\}, & \text{if } x = 0; \\ \{3x\}, & \text{if } x \in (0,\infty). \end{cases}$$

Here, 0 and 3 are two coincidence points of A and B. That is $A0 = \{0\} \in B(0), A(3) = \{9\} \in B(3)$, but $AB(0) = \{0\} = BA(0), AB(3)eqBA(3)$. Thus A and B are owc but not weakly compatible.

3. Main results

Theorem 3.1. *Let $(X, M, *)$ be a fuzzy metric space with $t * t = t$ for all $t \in [0, 1]$. Let $A, B : X \to X$ and $S, T : X \to CB(X)$ be single valued and multi valued mappings respectively such that $\{A, S\}$ and $\{B, T\}$ are owc. If there exist $q \in (0, 1)$ such that*

$$\begin{aligned} \delta_M(Sx, Ty, qt) \geq \min\{&M(Ax, By, t), H(Ax, Sx, t), H(By, Ty, t), \\ &H(Ax, Ty, t), H(By, Sx, t)\} \end{aligned} \tag{3.1}$$

for all $x, y \in X$. Then A, B, S and T have a unique common fixed point.

Proof. Since the pairs $\{A, S\}$ & $\{B, T\}$ are owc, therefore, there exist two elements $u, v \in X$ such that $Au \in Su,\ ASu \subseteq SAu$ and $Bv \in Tv,\ BTv \subseteq TBv$.
First we prove that $Au = Bv$.
As $Au \in Su$ so $AAu \subset ASu \subset SAu,\ Bv \in Tv$ so $BBv \subset BTv \subset TBv$ and hence $M(A^2u, B^2v, t) \geq \delta_M(SAu, TBv, t)$ and if $\delta_M(SAu, TBv, t) < 1$. Using (3.1) for

$x = Au, y = Bv$

$$\begin{aligned}\delta_M(SAu, TBv, qt) &\geq \min\{M(AAu, BBv, t), H(A^2u, SAu, t), H(BBv, TBv, t),\\ &\qquad H(A^2u, TBv, t), H(BBv, SAu, t)\}\\ &\geq \min\{M(AAu, BBv, t), M(A^2u, SAu, t), M(BBv, TBv, t),\\ &\qquad M(A^2u, TBv, t), M(BBv, SAu, t)\}\\ &\geq \min\{M(A^2u, B^2v, t), 1, 1, M(A^2u, TBv, t), M(BBv, SAu, t)\}\\ &\geq \min\{\delta_M(SAu, TBv, t), 1, 1, \delta_M(SAu, TBv, t), \delta_M(SAu, TBv, t)\}\\ &= \delta_M(SAu, TBv, t), \text{ a contradiction.}\end{aligned}$$

Hence $Au = Bv$.

$$\text{Also,} \qquad \begin{aligned} M(A^2u, Bu, t) &\geq \delta_M(SAu, Tu, t)\\ M(A^2u, Tu, t) &\geq \delta_M(SAu, Tu, t).\end{aligned}$$

Now we claim that $Au = u$. If not, then $\delta_M(SAu, Tu, t) < 1$.

Considering (3.1) for $Au = x,\ y = u$

$$\begin{aligned}\delta_M(SAu, Tu, qt) &\geq \min\{M(AAu, Bu, t), H(A^2u, SAu, t), H(Bu, Tu, t),\\ &\qquad H(A^2u, Tu, t), H(Bu, SAu, t)\}\\ &\geq \min\{M(AAu, Bu, t), M(A^2u, SAu, t), M(Bu, Tu, t),\\ &\qquad M(A^2u, Tu, t), M(Bu, SAu, t)\}\\ &\geq \min\{\delta_M(SAu, Tu, t), 1, 1, \delta_M(SAu, Tu, t), \delta_M(SAu, Tu, t)\}\\ &= \delta_M(SAu, Tu, t), \text{ which is again a contradiction and hence Au=u.}\end{aligned}$$

Similarly, we can get $Bv = v$.
Thus A, B, S & T have a common fixed point.
For uniqueness let $u \neq u'$ be another fixed point of A, B, S & T, then (3.1) gives

$$\begin{aligned}\delta_M(Su, Tu', qt) &\geq \min\{M(Au, Bu', t), H(Au, Su, t), H(Bu', Tu', t), H(Au, Tu', t),\\ &\qquad H(Bu', Su, t)\}\\ &\geq \min\{M(Au, Bu', t), M(Au, Su, t), M(Bu', Tu', t), M(Au, Tu', t),\\ &\qquad M(Bu', Su, t)\}\\ &\geq \min\{\delta_M(Su, Tu', t), 1, 1, \delta_M(Su, Tu', t), \delta_M(Su, Tu', t)\}\\ &= \delta_M(Su, Tu', t), \text{ a contradiction.}\end{aligned}$$

Hence $Su = Tu'$. i.e., $u = u'$.
Thus, A, B, S & T have a unique common fixed point. □

tExample 3.1.1. Let $X = [0,4]$ with the metric d defined by $d(x,y) =\mid x-y \mid$ and for each $t \in [0,1]$, define

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0 \end{cases}$$

for all $x, y \in X$. Clearly $(X, F, *)$ be a fuzzy metric space, where $X = [0,\infty)$ with $a * b = \min\{a,b\}$. Define the single-valued maps $S, T : X \to X$ and set-valued maps $A, B : X \to CB(X)$ defined by

$$A(x) = \begin{cases} \{2\}, & \text{if } 0 \leq x \leq 2; \\ \{0\}, & \text{if } 2 \leq x \leq 4. \end{cases} \qquad S(X) = \begin{cases} x, & \text{if } 0 \leq x \leq 2; \\ 3, & \text{if } 2 \leq x \leq 4. \end{cases}$$

$$B(x) = \begin{cases} \{2\}, & \text{if } 0 \leq x \leq 2; \\ \{4\}, & \text{if } 2 \leq x \leq 4. \end{cases} \qquad T(X) = \begin{cases} 2, & \text{if } 0 \leq x \leq 2; \\ \frac{x}{4}, & \text{if } 2 \leq x \leq 4. \end{cases}$$

Clearly all the conditions of the above theorem are satisfied. That is,

$$S(2) = \{2\} \in A(2) \text{ and } SA(2) = \{2\} = AS(2),$$
$$T(2) = \{2\} \in B(2) \text{ and } TB(2) = \{2\} = BT(2),$$

So, A and S as well as B and T are owc maps. Also 2 is the unique common fixed point of A, B, S and T.

Theorem 3.2. *Let $(X, M, *)$ be a fuzzy metric space with $t * t = t$ for all $t \in [0,1]$. Let $A, B : X \to X$ and $S, T : X \to CB(X)$ be single valued and multi valued mappings respectively such that $\{A, S\}$ and $\{B, T\}$ are owc. If there exist $q \in (0,1)$ such that*

$$\delta_M(Sx, Ty, qt) \geq \min\Big\{M(Ax, By, t), H(Ax, Sx, t), H(By, Ty, t), \frac{H(Ax, Ty, t) + H(By, Sx, t)}{2}\Big\} \tag{3.2}$$

for all $x, y \in X$. Then A, B, S and T have a unique common fixed point.

Proof. Since the pairs $\{A, S\}$ & $\{B, T\}$ are owc, therefore, there exist two elements $u, v \in X$ such that $Au \in Su$, $ASu \subseteq SAu$ and $Bv \in Tv$, $BTv \subseteq TBv$.
First we prove that $Au = Bv$.
As $Au \in Su$ so $AAu \subset ASu \subset SAu$, $Bv \in Tv$ so $BBv \subset BTv \subset TBv$ and hence $M(A^2u, B^2v, t) \geq \delta_M(SAu, TBv, t)$ and if $\delta_M(SAu, TBv, t) < 1$. Using (3.2) for

$x = Au,\ y = Bv$

$$\begin{aligned}\delta_M(SAu, TBv, qt) &\geq \min\Big\{M(AAu, BBv, t), H(A^2u, SAu, t), H(BBv, TBv, t),\\ &\qquad \frac{H(A^2u, TBv, t) + H(BBv, SAu, t)}{2}\Big\}\\ &\geq \min\Big\{M(AAu, BBv, t), M(A^2u, SAu, t), M(BBv, TBv, t),\\ &\qquad \frac{M(A^2u, TBv, t) + M(BBv, SAu, t)}{2}\Big\}\\ &\geq \min\Big\{M(A^2u, B^2v, t), 1, 1, \frac{M(A^2u, TBv, t) + M(BBv, SAu, t)}{2}\Big\}\\ &\geq \min\Big\{\delta_M(SAu, TBv, t), 1, 1, \frac{\delta_M(SAu, TBv, t) + \delta_M(SAu, TBv, t)}{2}\Big\}\\ &= \delta_M(SAu, TBv, t), \quad \text{a contradiction.}\end{aligned}$$

Hence $Au = Bv$.

Also,
$$M(A^2u, Bu, t) \geq \delta_M(SAu, Tu, t)$$
$$M(A^2u, Tu, t) \geq \delta_M(SAu, Tu, t).$$

Now we claim that $Au = u$. If not, then $\delta_M(SAu, Tu, t) < 1$.

Considering (3.2) for $Au = x,\ y = u$

$$\begin{aligned}\delta_M(SAu, Tu, qt) &\geq \min\Big\{M(AAu, Bu, t), H(A^2u, SAu, t), H(Bu, Tu, t),\\ &\qquad \frac{H(A^2u, Tu, t) + H(Bu, SAu, t)}{2}\Big\}\\ &\geq \min\Big\{M(AAu, Bu, t), M(A^2u, SAu, t), M(Bu, Tu, t),\\ &\qquad \frac{M(A^2u, Tu, t) + M(Bu, SAu, t)}{2}\Big\}\\ &\geq \min\Big\{\delta_M(SAu, Tu, t), 1, 1, \frac{\delta_M(SAu, Tu, t) + \delta_M(SAu, Tu, t)}{2}\Big\}\\ &= \delta_M(SAu, Tu, t),\end{aligned}$$

which is again a contradiction and hence $Au = u$.

Similarly, we can get $Bv = v$. Thus A, B, S & T have a common fixed point. For uniqueness let $uequ'$ be another fixed point of A, B, S & T, then (3.2) gives

$$\begin{aligned}\delta_M(Su,Tu^{'},qt) &\geq \min\Big\{M(Au,Bu^{'},t),H(Au,Su,t),H(Bu^{'},Tu^{'},t),\\ &\qquad \frac{H(Au,Tu^{'},t)+H(Bu^{'},Su,t)}{2}\Big\}\\ &\geq \min\Big\{M(Au,Bu^{'},t),M(Au,Su,t),M(Bu^{'},Tu^{'},t),\\ &\qquad \frac{M(Au,Tu^{'},t)+M(Bu^{'},Su,t)}{2}\Big\}\\ &\geq \min\Big\{\delta_M(Su,Tu^{'},t),1,1,\frac{\delta_M(Su,Tu^{'},t)+\delta_M(Su,Tu^{'},t)}{2}\Big\}\\ &= \delta_M(Su,Tu^{'},t), \text{ a contradiction.}\end{aligned}$$

Hence $Su = Tu'$. i.e., $u = u'$.
Thus, A, B, S & T have a unique common fixed point. □

Corollary 3.3. *Let $(X, M, *)$ be a fuzzy metric space with $t * t = t$ for all $t \in [0, 1]$. Let $A, B : X \to X$ and $S, T : X \to CB(X)$ be single valued and multi valued mappings respectively such that $\{A, S\}$ and $\{B, T\}$ are owc. If there exist $q \in (0, 1)$ such that*

$$\delta_M(Sx,Ty,qt) \geq \min\Big\{M(Ax,By,t),\frac{H(Ax,Sx,t)+H(By,Ty,t)}{2},\frac{H(Ax,Ty,t)+H(By,Sx,t)}{2}\Big\} \tag{3.3}$$

for all $x, y \in X$. Then A, B, S and T have a unique common fixed point.

Proof. Clearly the result immediately follows from Theorem 3.2. □

Corollary 3.4. *Let $(X, M, *)$ be a fuzzy metric space with $t * t = t$ for all $t \in [0, 1]$. Let $A, B : X \to X$ and $S, T : X \to CB(X)$ be single valued and multi valued mappings respectively such that $\{A, S\}$ and $\{B, T\}$ are owc. If there exist $q \in (0, 1)$ such that*

$$\delta_M(Sx,Ty,qt) \geq h\min\Big\{M(Ax,By,t),H(Ax,Sx,t),H(By,Ty,t),\frac{H(Ax,Ty,t)+H(By,Sx,t)}{2}\Big\} \tag{3.4}$$

for all $x, y \in X$, $h \in [0, 1)$ and $t > 0$. Then A, B, S and T have a unique common fixed point.

Proof. Clearly the result immediately follows from Theorem 3.2. □

Theorem 3.5. *Let $(X, M, *)$ be a fuzzy metric space with $t * t = t$ for all $t \in [0, 1]$. Let $A, B : X \to X$ and $S, T : X \to CB(X)$ be single valued and multi valued mappings respectively such that $\{A, S\}$ and $\{B, T\}$ are owc. If there exist $q \in (0, 1)$ such that*

$$\begin{aligned}\delta_M(Sx,Ty,qt) &\geq \alpha M(Ax,By,t)+\beta\min\big(M(Ax,By,t),H(Ax,Sx,t),H(By,Ty,t)\big)\\ &\quad +\gamma\min\big(M(Ax,By,t),H(Ax,Ty,t),H(By,Sx,t)\big)\}\end{aligned} \tag{3.5}$$

for all $x, y \in X$ and $t > 0$, where $\alpha, \beta, \gamma > 0$ and $(\alpha + \beta + \gamma) = 1$. Then A, B, S and T have a unique common fixed point.

Proof. Since the pairs $\{A, S\}$ and $\{B, T\}$ are owc, therefore, there exist two elements $u, v \in X$ such that $Au \in Su,\ ASu \subseteq SAu$ and $Bv \in Tv,\ BTv \subseteq TBv$.
First we prove that $Au = Bv$.
As $Au \in Su$ so $AAu \subset ASu \subset SAu,\ Bv \in Tv$ so $BBv \subset BTv \subset TBv$ and hence $M(A^2u, B^2v, t) \geq \delta_M(SAu, TBv, t)$ and if $\delta_M(SAu, TBv, t) < 1$. Using (3.5) for $x = Au,\ y = Bv$

$$\begin{aligned}
\delta_M(SAu, TBv, qt) &\geq \alpha M(AAu, BBv, t) + \beta \min\big(M(AAu, BBv, t), H(A^2u, SAu, t),\\
&\quad H(BBv, TBv, t)\big) + \gamma \min\big(M(AAu, BBv, t), H(A^2u, TBv, t),\\
&\quad H(BBv, SAu, t)\big)\}\\
&\geq \alpha M(AAu, BBv, t) + \beta \min\big(M(AAu, BBv, t), M(A^2u, SAu, t),\\
&\quad M(BBv, TBv, t)\big) + \gamma \min\big(M(AAu, BBv, t), M(A^2u, TBv, t),\\
&\quad M(BBv, SAu, t)\big)\}\\
&\geq \alpha M(A^2u, B^2v, t) + \beta \min\big(M(A^2u, B^2v, t), 1, 1\big)\\
&+ \gamma \min\big(M(A^2u, B^2v, t), M(A^2u, TBv, t), M(BBv, SAu, t)\big)\}\\
&\geq \alpha \delta_M(SAu, TBv, t) + \beta \min\big(\delta_M(SAu, TBv, t), 1, 1\big)\\
&+ \gamma \min\big(\delta_M(SAu, TBv, t), \delta_M(SAu, TBv, t), \delta_M(SAu, TBv, t)\big)\}\\
&= (\alpha + \beta + \gamma)\delta_M(SAu, TBv, t),
\end{aligned}$$

a contradiction as $\alpha + \beta + \gamma = 1$. Hence $Au = Bv$.

Also, $$\begin{aligned} M(A^2u, Bu, t) &\geq \delta_M(SAu, Tu, t)\\ M(A^2u, Tu, t) &\geq \delta_M(SAu, Tu, t). \end{aligned}$$

Now we claim that $Au = u$. If not, then $\delta_M(SAu, Tu, t) < 1$.
Considering (3.5) for $Au = x,\ y = u$

$$\begin{aligned}
\delta_M(SAu, Tu, qt) &\geq \alpha M(AAu, Bu, t) + \beta \min\big(M(AAu, Bu, t), H(A^2u, SAu, t),\\
&\quad H(Bu, Tu, t)\big) + \gamma \min\big(M(AAu, Bu, t), H(A^2u, Tu, t),\\
&\quad H(Bu, SAu, t)\big)\}\\
&\geq \alpha M(AAu, Bu, t) + \beta \min\big(M(AAu, Bu, t), M(A^2u, SAu, t),\\
&\quad M(Bu, Tu, t)\big) + \gamma \min\big(M(AAu, Bu, t), M(A^2u, Tu, t),\\
&\quad M(Bu, SAu, t)\big)\}\\
&\geq \alpha \delta_M(SAu, Tu, t) + \beta \min\big(\delta_M(SAu, Tu, t), 1, 1\big)\\
&+ \gamma \min\big(\delta_M(SAu, Tu, t), \delta_M(SAu, Tu, t), \delta_M(SAu, Tu, t)\big)\}\\
&= (\alpha + \beta + \gamma)\delta_M(SAu, Tu, t),
\end{aligned}$$

which is again a contradiction as $(\alpha + \beta + \gamma) = 1$ and hence $Au = u$.
Similarly, we can get $Bv = v$.

Thus A, B, S and T have a common fixed point.
For uniqueness let $uequ'$ be another fixed point of A, B, S and T, then (3.5) gives

$$\begin{aligned}
\delta_M(Su, Tu^{'}, qt) &\geq \alpha M(Au, Bu^{'}, t) \\
&+ \beta \min\big(M(Au, Bu^{'}, t), H(Au, Su, t), H(Bu^{'}, Tu^{'}, t)\big) \\
&+ \gamma \min\big(M(Au, Bu^{'}, t), H(Au, Tu^{'}, t), H(Bu^{'}, Su, t)\big)\} \\
&\geq \alpha M(Au, Bu^{'}, t) \\
&+ \beta \min\big(M(Au, Bu^{'}, t), M(Au, Su, t), M(Bu^{'}, Tu^{'}, t)\big) \\
&+ \gamma \min\big(M(Au, Bu^{'}, t), (Au, Tu^{'}, t), M(Bu^{'}, Su, t)\big)\} \\
&\geq \alpha \delta_M(Su, Tu^{'}, t) + \beta \min\big(\delta_M(Su, Tu^{'}, t), 1, 1\big) \\
&+ \gamma \min\big(\delta_M(Su, Tu^{'}, t), \delta_M(Su, Tu^{'}, t), \delta_M(Su, Tu^{'}, t)\big)\} \\
&= (\alpha + \beta + \gamma)\delta_M(Su, Tu^{'}, t), \text{ a contradiction as } (\alpha + \beta + \gamma) = 1.
\end{aligned}$$

Hence $Su = Tu'$. i.e., $u = u'$. Thus, A, B, S and T have a unique common fixed point. □

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