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I-statistically pre-Cauchy sequences of fuzzy numbers

Sarita Ojha, P. D. Srivastava

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Abstract. In this paper, we have introduced the concept of I-statistically pre-Cauchy sequences of fuzzy numbers. It is shown that an I-statistically convergent sequence of fuzzy numbers is I-statistically pre-Cauchy. The converse of this result is also true under some condition. We have also derived a necessary and sufficient condition for a sequence of fuzzy numbers to be a pre-Cauchy sequence.

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Corresponding Author: Sarita Ojha (sarita.ojha89@gmail.com)

1. Introduction

The concept of fuzzy set is first introduced by Zadeh [22]. Later on, the sequences of fuzzy numbers is discussed by several mathematicians such as Matloka [11], Nanda [12], Savas [14] and others. They have introduced the sequence spaces of fuzzy numbers. The concepts such as statistical convergence, summability etc. are generalized for fuzzy numbers by Nuray and Savas [13].
As the set of all real numbers can be embedded in the set of all fuzzy numbers, so statistical convergence in reals can be considered as a special case of the fuzzy numbers. However since the set of all fuzzy numbers is partially ordered and does not carry a group structure (In general $X + (-X) \neq \bar{0}$ for the case of fuzzy numbers), most of the results known for the sequences of real numbers may not be valid in fuzzy setting. Therefore, the theory is not a trivial extension of what has been known in real case.

The concept of statistical convergence is introduced by Steinhaus and Fast [8] and later on, is reintroduced by Schoenberg independently. Savas [14] has introduced and studied λ-statistical convergence of sequence of fuzzy real numbers which has been further investigated and generalized by several authors such as Nuray [13], Colak and Mursaleen [1], Srivastava and Ojha [21] etc. The idea of statistical convergence is extended to I-convergence in case of reals by using the notion of ideals of $\mathbb{N}$. More investigations in this direction and more applications of ideals of $\mathbb{N}$ can be found in [2], [4], [3]. In an analogous way, Savas [20, 18, 19, 17, 16], Savas and Das [15] used ideals to introduce the concept of I-statistical convergence for reals which have extended the notion of statistical convergence. These authors have also studied some basic properties of this more general convergence. The notion of I-statistically pre-Cauchy sequences are also introduced by them.

In the present paper, we have extended these results to introduce the concept of I-statistically pre-Cauchy sequences for the sequence of fuzzy numbers.

2. Definition and Preliminaries

Definition 2.1 ([9])**.** A non-empty family $I \in 2^{\mathbb{N}}$ is said to be an ideal of $\mathbb{N}$ (set of all natural numbers) if
(a) $A, B \in I$ imply $A \cup B \in I$, and
(b) $A \in I$, $B \subset A$ imply $B \in I$ hold.

Definition 2.2 ([9])**.** A proper ideal I is said to be admissible if $\{n\} \in I$ for each $n \in \mathbb{N}$.

Note: Throughout this paper, I stands for a proper admissible ideal of $\mathbb{N}$.

Definition 2.3 ([8])**.** A sequence $x = (x_k)$ of real numbers is said to be statistically convergent to a number l if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leq n : |x_k - l| \geq \varepsilon\}| = 0$$

where the vertical bars $|\cdot|$ indicate the cardinality of the enclosed set.

Definition 2.4 ([9])**.** A sequence (x_n) of real numbers is said to be I-convergent to $L \in R$ if for each $\varepsilon > 0$, the set $A(\varepsilon) = \{n \in \mathbb{N} : |x_n - L| \geq \varepsilon\} \in I$.

Definition 2.5 ([15])**.** A sequence (x_n) of real numbers is said to be I-statistically convergent to L or $S(I)$-convergent to L if, for each $\varepsilon > 0$ and $\delta > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : |x_k - L| \geq \varepsilon\}| \geq \delta\right\} \in I$$

and is denoted as $x_k \to L(S(I))$. The class of all I-statistically convergent sequences is denoted by $S(I)$.

Definition 2.6 ([4])**.** A sequence (x_k) of real numbers is said to be I-statistically pre-Cauchy if, for any $\varepsilon > 0$ and $\delta > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{n^2}|\{(j,k) : |x_j - x_k| \geq \varepsilon\}| \geq \delta, j, k \leq n\right\} \in I.$$

Definition 2.7 ([4])**.** Let I be an admissible ideal of $\mathbb{N}$ and let $x = (x_n)$ be a real sequence. Let, $A_x = \{a \in \mathbb{R} : \{k : x_k < a\} \notin I\}$. Then the I-limit inferior of x is given by,

$$I - \liminf x = \begin{cases} \inf A_x, \text{ if } A_x \neq \varnothing \\ \infty, \text{ if } A_x = \varnothing. \end{cases}$$

It is known that [7] $I - \liminf x = \alpha$ (finite) iff for arbitrary $\varepsilon > 0$,

$$\{k : x_k < \alpha + \varepsilon\} \notin I \text{ and } \{k : x_k < \alpha - \varepsilon\} \in I$$

Sequences of fuzzy numbers and their properties are first developed by Matloka [11]. Later on, Nanda [12] introduced l_p^F spaces of sequences of fuzzy numbers.
A fuzzy real number $X : \mathbb{R} \to [0, 1]$ is a fuzzy set which satisfies the following conditions:
(i) normal i.e. $X(t) = 1$ for some $t \in \mathbb{R}$.
(ii) fuzzy convex i.e. $X(t) \geq \min\{X(s), X(r)\}$ where $s < t < r$.
(iii) X is upper semicontinuous.
(iv) $X^0 = \overline{\{t \in \mathbb{R} : X(t) > 0\}}$ is compact.
The set of all fuzzy numbers satisfying (i)-(iv) is denoted by $L(R)$. Clearly, $\mathbb{R}$ is embedded in $L(R)$ in this way: for each $r \in \mathbb{R}$, $\overline{r} \in L(R)$ is defined as,

$$\overline{r}(t) = \begin{cases} 1, & t = r \\ 0, & t \neq r \end{cases}$$

For, $0 < \alpha \leq 1$, α-cut of a fuzzy number X is defined by $X^{(\alpha)} = \{t \in \mathbb{R} : X(t) \geq \alpha\}$.
Now for any two fuzzy numbers X, Y, Matloka [11] has introduced,

$$\overline{d}(X^{(\alpha)}, Y^{(\alpha)}) = \max\{|\underline{X}^{(\alpha)} - \underline{Y}^{(\alpha)}|, |\overline{X}^{(\alpha)} - \overline{Y}^{(\alpha)}|\}$$

where $\underline{X}^{(\alpha)}$ and $\overline{X}^{(\alpha)}$ are the lower and upper bound of the α-cut and

$$d(X, Y) = \sup_{0 \leq \alpha \leq 1} \overline{d}(X^{(\alpha)}, Y^{(\alpha)}) \tag{2.1}$$

and showed that d is a metric on the set $L(R)$.

Definition 2.8 ([10])**.** A sequence $X = (X_k)$ of fuzzy numbers is said to be I-convergent to X_0 or S_I^F-convergent if for each $\varepsilon > 0$,

$$\Big\{k \in \mathbb{N} : d(X_k, X_0) \geq \varepsilon\Big\} \in I.$$

Definition 2.9 ([5, 6])**.** A sequence $X = (X_k)$ of fuzzy numbers is said to be I-statistically convergent if for any $\varepsilon > 0$ and $\delta > 0$,

$$\Big\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \varepsilon\}| \geq \delta\Big\} \in I.$$

Note: Throughout the paper, small and capital alphabets will be used to denote real and fuzzy numbers respectively and d is the metric defined on the set of all fuzzy numbers $L(R)$ by equation (2.1).

3. Main results

We now define the concept of I-statistical pre-Cauchy sequence of fuzzy numbers as follows.

Definition 3.1. A sequence $X = (X_k)$ of fuzzy numbers is said to be I-statistically pre-Cauchy if for any $\varepsilon > 0$ and $\delta > 0$,

$$\Big\{n \in \mathbb{N} : \frac{1}{n^2}|\{(j, k) : d(X_k, X_j) \geq \varepsilon, j, k \leq n\}| \geq \delta\Big\} \in I.$$

Theorem 3.2. *If a sequence $X = (X_k)$ of fuzzy numbers is I-statistically convergent, then it is I-statistically pre-Cauchy.*

Proof. Let (X_k) is I-statistically convergent to X_0. So, for any $\varepsilon, \delta > 0$, let,

$$A = \Big\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \varepsilon/2\}| \geq \delta\Big\} \in I.$$

Then for all $n \in A^c$ (complement of A), we have

$$\frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \varepsilon/2\}| < \delta$$

$$\text{which implies } \frac{1}{n}|\{k \leq n : d(X_k, X_0) < \varepsilon/2\}| > 1 - \delta.$$

Assume $A_n = \{k \leq n : d(X_k, X_0) < \varepsilon/2\}$. Then,

$$\frac{1}{n}|A_n| > 1 - \delta \tag{3.1}$$

So, for all $j, k \in A_n$

$$\begin{aligned}
&d(X_k, X_j) \leq d(X_k, X_0) + d(X_j, X_0) < \varepsilon \\
&\Rightarrow A_n \times A_n \subseteq \{(j,k) : d(X_k, X_j) < \varepsilon, j, k \leq n\} \\
&\Rightarrow \frac{|A_n \times A_n|}{n^2} = \frac{[|A_n|]^2}{n^2} \leq \frac{1}{n^2}|\{(j,k) : d(X_k, X_j) < \varepsilon, j, k \leq n\}| \\
&\Rightarrow \frac{1}{n^2}|\{(j,k) : d(X_k, X_j) < \varepsilon, j, k \leq n\}| \geq \Big[\frac{|A_n|}{n^2}\Big]^2 > (1-\delta)^2 \text{ [from 3.1]}
\end{aligned}$$

So, we have

$$\frac{1}{n^2}|\{(j,k) : d(X_k, X_j) \geq \varepsilon, j, k \leq n\}| < 1 - (1-\delta)^2.$$

For any given $\delta_1 > 0$, we choose $\delta > 0$ in such a way that $1 - (1-\delta)^2 < \delta_1$. Now, for all $n \in A^c$,

$$\begin{aligned}
&\frac{1}{n^2}|\{(j,k) : d(X_k, X_j) \geq \varepsilon, j, k \leq n\}| < \delta_1 \\
\therefore \quad &\Big\{n \in \mathbb{N} : \frac{1}{n^2}|\{(j,k) : d(X_k, X_j) \geq \varepsilon, j, k \leq n\}| \geq \delta_1\Big\} \subset A \in I
\end{aligned}$$

$\Rightarrow (X_k)$ is I-statistically pre-Cauchy. □

The following theorem gives a necessary and sufficient condition for a bounded sequence of fuzzy numbers to be I-statistically pre-Cauchy. The boundedness of a sequence of fuzzy numbers (X_k) we mean, there exists a real number $M > 0$ such that $d(X_k, \bar{0}) \leq M \ \forall \ k \in \mathbb{N}$.

Theorem 3.3. *Let $X = (X_k)$ be a bounded sequence of fuzzy numbers. Then (X_k) is I-statistically pre-Cauchy iff*

$$I - \lim \frac{1}{n^2} \sum_{j,k \leq n} d(X_k, X_j) = 0 \tag{3.2}$$

Proof. (Necessary condition) Suppose (3.2) holds. For any $\varepsilon > 0, n \in \mathbb{N}$, we have,

$$\begin{aligned}\frac{1}{n^2}\sum_{j,k\leq n} d(X_k,X_j) &= \frac{1}{n^2}\sum_{\substack{j,k\leq n\\ d(X_k,X_j)<\varepsilon}} d(X_k,X_j) + \frac{1}{n^2}\sum_{\substack{j,k\leq n\\ d(X_k,X_j)\geq\varepsilon}} d(X_k,X_j)\\ &\geq \varepsilon\Big(\frac{1}{n^2}|\{(j,k): d(X_k,X_j)\geq\varepsilon, j,k\leq n\}|\Big).\end{aligned}$$

Then for any $\delta > 0$, the set

$$\begin{aligned}&\Big\{n\in\mathbb{N} : \frac{1}{n^2}|\{(j,k): d(X_k,X_j)\geq\varepsilon, j,k\leq n\}|\geq\delta\Big\}\\ &\subset \Big\{n\in\mathbb{N} : \frac{1}{n^2}\sum_{j,k\leq n} d(X_k,X_j)\geq\delta\varepsilon\Big\}\in I\end{aligned}$$

because of (3.2). Therefore (X_k) is I-statistically pre-Cauchy.
(Sufficient condition) Let (X_k) is I-statistically pre-Cauchy. Since (X_k) is bounded, so $\exists\ M > 0$ such that $d(X_k, \bar{0}) \leq M$ for all $k \in \mathbb{N}$. Then for any $\varepsilon > 0$ and $n \in \mathbb{N}$,

$$\begin{aligned}\frac{1}{n^2}\sum_{j,k\leq n} d(X_k,X_j) &= \frac{1}{n^2}\sum_{\substack{j,k\leq n\\ d(X_k,X_j)<\varepsilon/2}} d(X_k,X_j) + \frac{1}{n^2}\sum_{\substack{j,k\leq n\\ d(X_k,X_j)\geq\varepsilon/2}} d(X_k,X_j)\\ &\leq \frac{1}{n^2}\cdot n^2\cdot\frac{\varepsilon}{2} + 2M\cdot\frac{1}{n^2}|\{(j,k): d(X_k,X_j)\geq\varepsilon/2, j,k\leq n\}|\end{aligned}$$

Since (X_k) is I-statistically pre-Cauchy, for $\delta > 0$,

$$A = \Big\{n\in\mathbb{N} : \frac{1}{n^2}|\{(j,k): d(X_k,X_j)\geq\varepsilon/2, j,k\leq n\}|\geq\delta\Big\}\in I$$

Then for all $n \in A^c$,

$$\begin{aligned}&\frac{1}{n^2}|\{(j,k): d(X_k,X_j)\geq\varepsilon/2, j,k\leq n\}| < \delta\\ &\therefore\ \frac{1}{n^2}\sum_{j,k\leq n} d(X_k,X_j)\leq\frac{\varepsilon}{2}+2M\delta\end{aligned}$$

Let $\delta_1 > 0$ be chosen such that $\frac{\varepsilon}{2} + 2M\delta < \delta_1$. Then for all $n \in A^c$,

$$\begin{aligned}&\frac{1}{n^2}\sum_{j,k\leq n} d(X_k,X_j) < \delta_1\\ \therefore\ &\Big\{n\in\mathbb{N} : \frac{1}{n^2}\sum_{j,k\leq n} d(X_k,X_j)\geq\delta_1\Big\}\subset A\in I\end{aligned}$$

This completes the proof. □

Theorem 3.4. *Let a sequence of fuzzy numbers $X = (X_k)$ is I-statistically pre-Cauchy. If X has a subsequence (X_{p_k}) which converges to X_0 and*

$$0 < I - \liminf \frac{1}{n}|\{p_k \leq n : k \in \mathbb{N}\}| < \infty,$$

then X is I-statistically convergent to X_0.

Proof. Let $\varepsilon > 0$. Since $X_{p_k} \to X_0$, we can choose $n_0 \in \mathbb{N}$ such that $d(X_{p_k}, X_0) < \varepsilon/2 \;\forall\; p_k > n_0$.
Let $A = \{p_k : p_k > n_0, k \in \mathbb{N}\}$ and $A(\varepsilon) = \{k : d(X_k, X_0) \geq \varepsilon\}$. Now,

$$\frac{1}{n^2}|\{(j,k) : d(X_k, X_j) \geq \varepsilon/2, j, k \leq n\}| \geq \frac{1}{n^2}\sum_{j,k\leq n} \chi_{A(\varepsilon)\times A}(j,k)$$

$$= \frac{1}{n}|\{p_k \leq n : p_k \in A\}| \cdot \frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \varepsilon\}|$$

Since X is I-statistically pre-Cauchy, so for $\delta > 0$,

$$B = \left\{n \in \mathbb{N} : \frac{1}{n^2}|\{(j,k) : d(X_k, X_j) \geq \varepsilon/2, j, k \leq n\}| \geq \delta\right\} \in I.$$

Thus for all $n \in B^c$,

$$\frac{1}{n^2}|\{(j,k) : d(X_k, X_j) \geq \varepsilon/2, j, k \leq n\}| < \delta$$

Again, since, $I - \liminf \frac{1}{n}|\{p_k \leq n : k \in \mathbb{N}\}| = p > 0$ (say), then the set

$$C = \left\{n \in \mathbb{N} : \frac{1}{n}|\{p_k \leq n : k \in \mathbb{N}\}| < \frac{p}{2}\right\} \in I$$

and so, for all $n \in C^c$, we have

$$\frac{1}{n}|\{p_k \leq n : k \in \mathbb{N}\}| \geq \frac{p}{2}$$

$\therefore$ For all $n \in B^c \cap C^c = (B \cup C)^c$, we get

$$\frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \varepsilon\}| < \frac{2\delta}{p}$$

Choose $\delta_1 > 0$ such that $\frac{2\delta}{p} < \delta_1$. Now, for all $n \in (B \cup C)^c$, we have

$$\frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \varepsilon\}| < \delta_1$$

which implies, $$\left\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \varepsilon\}| \geq \delta_1\right\} \subset B \cup C \in I.$$

Therefore X is I-statistically convergent to X_0. □

Theorem 3.5. *Let $X = (X_k)$ be a sequence of fuzzy numbers and $X_k^{(\alpha)} = [\underline{X_k}^{(\alpha)}, \overline{X_k}^{(\alpha)}]$. Let (p, q) is an open interval of real numbers such that for all k, $\underline{X_k}^{(\alpha)}$, $\overline{X_k}^{(\alpha)} \notin (p, q)$ for all $\alpha \in (0, 1]$. Write $A_\alpha = \{k : \underline{X_k}^{(\alpha)} \leq p\}$ and $B_\alpha = \{k : \overline{X_k}^{(\alpha)} \geq q\}$ and further assume that,*

$$\limsup D_n(A_\alpha) - \liminf D_n(B_\alpha) \leq \eta \tag{3.3}$$

for some $0 \leq \eta \leq 1$. Then if X is I-statistically pre-Cauchy, either

$$I - \lim_n D_n(A_\alpha) = 0 \text{ or } I - \lim_n D_n(B_\alpha) = 0$$

for each $\alpha \in (0, 1]$ where $D_n(A_\alpha) = \frac{1}{n}|\{k \leq n : k \in A_\alpha\}|$.

Proof. Clearly, $B_\alpha = \mathbb{N} - A_\alpha$, therefore $D_n(A_\alpha) = 1 - D_n(B_\alpha)\ \forall\ n \in \mathbb{N}$. So, it is left to prove $I - \lim_n D_n(A_\alpha) = 0$ or 1. Let $r = |p - q|$. Now,

$$A_\alpha \times B_\alpha \subset \{(j,k) : d(X_k, X_j) \geq r\}$$

$$\begin{aligned}\text{So, } I - \lim_n \frac{1}{n^2}|\{(j,k) : d(X_k, X_j) \geq r, j,k \leq n\}| &= I - \lim_n D_n(A_\alpha)D_n(\mathbb{N} - A_\alpha)\\ &= I - \lim_n D_n(A_\alpha)[1 - D_n(\ A_\alpha)]\end{aligned}$$

Since X is I-statistically pre-Cauchy, so L.H.S=0. Therefore,

$$I - \lim_n D_n(A_\alpha)[1 - D_n(\ A_\alpha)] = 0 \tag{3.4}$$

Using the definition of I-convergence and from the relation (3.4), we get,

$$\Big\{n \in \mathbb{N} : D_n(A_\alpha)[1 - D_n(A_\alpha)] \geq \frac{1}{9}\Big\} \in I$$

$$\text{i.e. } \Big\{n \in \mathbb{N} : D_n(A_\alpha)[1 - D_n(A_\alpha)] < \frac{1}{9}\Big\} \in S \text{ (say)} \in F(I)$$

This shows that for all $n \in S$, either $D_n(A_\alpha) < \frac{1}{3}$ or $D_n(A_\alpha) > \frac{2}{3}$.
If $D_n(A_\alpha) < \frac{1}{3}\ \forall\ n \in S_0 \subset S$ for some $S_0 \in F(I)$, then for any $\varepsilon, 0 < \varepsilon < \frac{1}{3}$, from the definition of I-convergence, we get,

$$\Big\{n \in \mathbb{N} : D_n(A_\alpha)[1 - D_n(A_\alpha)] < \varepsilon^2\Big\} \in S_1 \text{ (say)} \in F(I)$$

Taking $S_2 = S_0 \cap S_1$, then $S_2 \in F(I)$ and $D_n(A_\alpha) < \varepsilon$ for all $n \in S_2$.
Similarly, if $D_n(A_\alpha) > \frac{2}{3}$ for all $n \in S_3 \subset S$ for some $S_3 \in F(I)$, then using the same arguments, we can show that $I - \lim_n D_n(A_\alpha) = 1$. If neither of them holds then we can find an increasing sequence of integers $\{n_1 < n_2 < n_3 < \cdots\}$ from S such that

$$D_{n_i}(A_\alpha) < \frac{1}{3} \text{ when } i \text{ is odd}$$

$$\text{and } D_{n_i}(A_\alpha) > \frac{2}{3} \text{ when } i \text{ is even.}$$

which implies that,

$$\limsup D_n(A_\alpha) - \liminf D_n(B_\alpha) > \frac{1}{3}$$

Similarly, if we take,

$$\Big\{n \in \mathbb{N} : D_n(A_\alpha)[1 - D_n(A_\alpha)] < \frac{1}{16}\Big\} \in S \text{ (say)} \in F(I)$$

then also we have either $I - \lim_n D_n(A_\alpha) = 0$ or 1 or neither holds then

$$\limsup D_n(A_\alpha) - \liminf D_n(B_\alpha) > \frac{2}{4}$$

Proceeding in this way, we can see that, this will stop only when we get either $I - \lim_n D_n(A_\alpha) = 0$ or 1. Otherwise, after a finite number of steps, we will have,

$$\limsup D_n(A_\alpha) - \liminf D_n(B_\alpha) > \lim_n \frac{n-2}{n} = 1$$

which violates the condition (3.3). This completes the proof. □

Remark 3.6. An I-statistically pre-cauchy sequence of fuzzy numbers need not be I-statistically convergent.

Before giving a counter example in support of remark 3.6, we will first prove the following proposition.

Proposition 3.7. *If a sequence of fuzzy numbers $X = (X_k)$ is I-statistically convergent, then it must have a convergent subsequence in usual sense.*

Proof. Let (X_k) be I-statistically convergent to X_0. Then taking $\varepsilon = \delta = 1$ in the definition 2.9, we have,

$$A = \Big\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq 1\}| \geq 1\Big\} \in I.$$

Since I is non-trivial, so $A \neq \mathbb{N}$. Choose $n_1 \in \mathbb{N} - A$ such that

$$\frac{1}{n_1}|\{k \leq n_1 : d(X_k, X_0) \geq 1\}| < 1$$
$$\Rightarrow \frac{1}{n_1}|\{k \leq n_1 : d(X_k, X_0) < 1\}| > 0$$

Therefore, $\exists\ k_1 \leq n_1$ such that, $d(X_{k_1}, X_0) < 1$. Now choose, $\varepsilon = \delta = \frac{1}{2}$, we have from definition 2.9,

$$B = \Big\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : d(X_k, X_0) \geq \frac{1}{2}\}| \geq \frac{1}{2}\Big\} \in I.$$

Since I is admissible, so, $B \cup \{1, 2, ..., 3n_1\} \in I$. Again $N \notin I$, choose $n_2 \in \mathbb{N}$ such that $n_2 \notin B$ and $n_2 > 3n_1$. Then

$$\frac{1}{n_2}|\{k \leq n_2 : d(X_k, X_0) \geq \frac{1}{2}\}| < \frac{1}{2}$$
$$\Rightarrow \frac{1}{n_2}|\{k \leq n_2 : d(X_k, X_0) < \frac{1}{2}\}| > \frac{1}{2}$$

Now, if $d(X_k, X_0) \geq \frac{1}{2}$ for all k, where $n_1 < k \leq n_2$, then

$$\frac{1}{n_2}|\{k \leq n_2 : d(X_k, X_0) < \frac{1}{2}\}| \leq \frac{n_1}{n_2} < \frac{1}{3}$$

$\therefore\ \exists\ k, n_1 < k \leq n_2$ such that $d(X_k, X_0) < \frac{1}{2}$. We denote it as k_2. So, $k_2 > k_1$. Proceeding like this, we get an increasing sequence of natural numbers $\{k_1 < k_2 < k_3 < \cdots\}$ such that $d(X_{k_j}, X_0) < \frac{1}{j}$. So, (X_{k_j}) is convergent to X_0. □

Justification of remark 3.6: Now, we define the sequence (X_k) as follows,

$$X_k = \overline{A_m},\ A_m = \sum_{i=1}^{m} i^{-1},\ \text{where } (m-1)! < k \leq m!$$

Then for each $0 < \alpha \leq 1$, the α-cut of (X_k) is given by, $X_k^\alpha = A_m$. Now, proceeding in the same way as in [2], we can prove (X_k) is I-statistically pre-Cauchy, but not I-statistically convergent.

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Sarita Ojha (sarita.ojha89@gmail.com)
Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur-721302, India

P. D. Srivastava (pds@maths.iitkgp.ernet.in)
Professor, Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur-721302, India