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Some aspects of fuzzy $\widetilde{e}$-closed set

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Abstract. In this paper, a new class of fuzzy sets called fuzzy $\widetilde{e}$-closed sets is introduced and its properties are studied. Also the concepts of fuzzy $\widetilde{e}$-continuous, fuzzy $\widetilde{e}$-irresolute maps and fuzzy $\widetilde{e}$-homeomorphism are studied and their properties are investigated with the help of fuzzy $\tilde{e}$-open sets.

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1. Introduction

The concept fuzzy has invaded almost all branches of mathematics with the introduction of fuzzy sets by Zadeh[8] of 1965. The theory of fuzzy topological spaces was introduced and developed by Chang[3]. The concept of fuzzy e-open sets and fuzzy e-continuity and separations axioms and their properties were defined by Seenivasan etal[6]. In this paper the notion of fuzzy $\widetilde{e}$-closed sets is introduced and its properties are studied. Also the fuzzy $\widetilde{e}$-continuous, fuzzy $\widetilde{e}$-irresolute maps, fuzzy $\widetilde{e}$-homeomorphism and fuzzy $\widetilde{e}$- connectedness and their properties are investigated with the help of fuzzy $\tilde{e}$-open sets.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, ρ) (or simply X, Y and Z) represent non-empty fuzzy topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let A be subset of a space X. The fuzzy closure of A, fuzzy interior of A, fuzzy δ-closure of A and the fuzzy δ-interior of A are denoted by cl(A), int(A), $cl_\delta(A)$, $\mathrm{int}_\delta(A)$ respectively.

Definition 2.1. A subset A of space X is called fuzzy regular open [2](resp.fuzzy regular closed) if $A = \mathrm{int}(cl(A))$ (resp. $A = cl(\mathrm{int}(A))$).

Definition 2.2. The fuzzy δ- interior of subset A of X is the union of all fuzzy regular open sets contained in A.

Definition 2.3. A subset A is called fuzzy δ -open[7] if $A = \text{int}_\delta(A)$. The complement of fuzzy δ-open set is called fuzzy fuzzy δ -closed (i.e, $A = cl_\delta(A)$).

Definition 2.4. A subset A of a space X is called fuzzy δ-preopen [1](resp. fuzzy δ -semi open[4], fuzzy e-open[6]) if $A \leq int\ (cl_\delta(A))$ (resp. $A \leq cl\ (\text{int}_\delta(A))$, $A \leq cl\ (int_\delta(\ A)) \vee\ int(\ cl_\delta(A))$.

Definition 2.5. The complement of a fuzzy δ-preopen set (resp. fuzzy δ-semiopen set, fuzzy e-open) is called fuzzy δ-preclosed (resp.fuzzy δ-semiclosed, fuzzy e-closed).

Definition 2.6. The intersection of all fuzzy e-closed sets containing A is called fuzzy e-closure of A and is denoted by $ecl(A)$ and the union of all fuzzy e-open sets contained in A is called fuzzy e-interior of A and is denoted by $e\,\text{int}(A)$.

Definition 2.7. A mapping $f : X \to Y$ is said to be a fuzzy e- continuous[6] (resp. fuzzy e-irresolute) if $f^{-1}(\lambda)$ is fuzzy e-open in X for every fuzzy open (resp. fuzzy e-open) set λ in Y.

Definition 2.8. A fuzzy set μ is quasi-coincident[5] with a fuzzy set λ denoted by $\mu q \lambda$ iff there exist $x \in X$ such that $\mu(x) + \lambda(x) > 1$. If μ and λ are not quasi-coincident then we write $\mu \bar{q} \lambda$. Note that $\mu \leq \lambda \Leftrightarrow \mu \bar{q}(1 - \lambda)$.

Definition 2.9. A fuzzy point x_p is quasi-coincident[5] with a fuzzy set λdenoted by $x_p q \lambda$ iff there exist $x \in X$ such that $p + \lambda(x) > 1$.

3. Fuzzy $\widetilde{e}$-closed sets

Definition 3.1. A fuzzy set λ in a fuzzy topological space $(X,\ \tau)$ is called

(a) fuzzy $\widetilde{e}$- closed (briefly f$\widetilde{e}$-closed) iff $ecl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy e-open in X.

(b) fuzzy $\widetilde{e}$- open (briefly f$\widetilde{e}$-open) iff $\mu \leq e\,\text{int}(\lambda)$, whenever $\mu \leq \lambda$ and μ is fuzzy e-closed in X.

Theorem 3.2. *Every fuzzy closed, fuzzy δ-pre closed, fuzzy δ-semi closed is fuzzy $\widetilde{e}$-closed but, the converse may not be true in general.*

Proof. Proof follows immediately from the definition. □

Example 3.3. Let $X = \{a, b\}$ and the fuzzy sets u, v, w, h are defined as follows.

$$u(a) = 0.3,\ v(a) = 0.4,\ w(a) = 0.4,\ h(a) = 0.6.$$
$$u(b) = 0.5,\ v(b) = 0,\ w(b) = 0.6,\ h(b) = 0.5.$$

Let $\tau = \{0, 1, u\}$ and h is e-open of X. Then v is f$\widetilde{e}$-closed set for $v \leq w$, $ecl(v) \leq w$, where w is fuzzy e-open, but not fuzzy δ-pre closed.

Example 3.4. Let $X = \{a, b\}$ and the fuzzy sets u, v, w, h are defined as follows.

$$u(a) = 0.5,\ v(a) = 0.1,\ w(a) = 0.5,\ h(a) = 0.6.$$
$$u(b) = 0.1,\ v(b) = 0.3,\ w(b) = 1.0,\ h(b) = 0.3.$$

Let $\tau = \{0, 1, u\}$ and h is fuzzy e-open of X. Then v is f$\widetilde{e}$-closed set for $v \leq w$, $ecl(v) \leq w$, where w is fuzzy e-open, but not fuzzy δ-semi closed.

Theorem 3.5. *If u is fuzzy e-open and f$\widetilde{e}$-closed in (X, τ), then u is fuzzy e-closed in (X, τ) .*

Proof. Let u be fuzzy e-open and f$\widetilde{e}$-closed set in X. For $u \leq u$, by definition $ecl(u) \leq u$. But $u \leq ecl(u)$, which implies $u = ecl(u)$. Hence u is fuzzy e-closed set in X. □

Theorem 3.6. *Let (X, τ) be a fuzzy topological spaces and u be a fuzzy set of X. Then u is f$\widetilde{e}$-closed if and only if $u\overline{q}v$ implies $ecl(u)\overline{q}v$ for every fuzzy e-closed set v of X.*

Proof. Suppose u be a f$\widetilde{e}$-closed set of X. Let v be a fuzzy e-closed set in X such that $u\overline{q}v$. Then by definition that implies $u \leq 1 - v$ and $1 - v$ is fuzzy e-open set of X. Therefore, $ecl(u) \leq ecl(1 - v) \leq 1 - v$ as u is f$\widetilde{e}$-closed. Hence $ecl(u)\overline{q}v$. Conversely, let d be fuzzy e-open set in X such that $u \leq d$. Then by definition $u\overline{q}(1-d)$ and $1-d$ is fuzzy e-closed set in X. By hypothesis, $ecl(u)\overline{q}(1 - d)$, which implies $ecl(u) \leq d$. Hence u is f$\widetilde{e}$-closed. □

Theorem 3.7. *Let u be f$\widetilde{e}$-closed set in (X, τ) and x_p be a fuzzy point of (X, τ) such that $x_p q(ecl(u))$ then $(ecl(x_p))qu$*

Proof. Let u be f$\widetilde{e}$-closed and x_p be a fuzzy point of X. Suppose $ecl(x_p)\overline{q}u$, then by definition $ecl(x_p) \leq 1-u$ which implies $u \leq 1-(ecl(x_p))$. So $ecl(u) \leq 1-(ecl(x_p) \leq 1 - x_p$, because $1 - (ecl(x_p))$ is fuzzy e-open in X and u is f$\widetilde{e}$-closed in X. Hence $x_p\overline{q}(ecl(u))$, which is a contradiction. □

Theorem 3.8. *If u is f$\widetilde{e}$-closed set in (X, τ) and $u \leq v \leq ecl(u)$, then v is f$\widetilde{e}$-closed set in (X, τ).*

Proof. Let d be fuzzy e- open set of (X, τ)such that $v \leq d$. Then we get $u \leq d$. Since u is f$\widetilde{e}$-closed, it follows that $ecl(u) \leq d$. Now, $v \leq ecl(u)$ implies $ecl(v) \leq ecl(ecl(u)$ $= ecl(u)$. Thus $ecl(v) \leq d$. This proves that v is also a f$\widetilde{e}$-closed set of (X, τ). □

Theorem 3.9. *If u is f$\widetilde{e}$-open set in (X, τ) and $e\,\text{int}(u) \leq v \leq u$ then v is f$\widetilde{e}$-open set in (X, τ) .*

4. $f\widetilde{e}$-continuous and $f\widetilde{e}$-irresolute mappings

Definition 4.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy $\widetilde{e}$-continuous (briefly, f$\widetilde{e}$-continuous), if $f^{-1}(\lambda)$ is f$\widetilde{e}$-closed set in X, for every fuzzy closed set λ in Y.

Definition 4.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy $\widetilde{e}$-irresolute (briefly f$\widetilde{e}$-irresolute), if $f^{-1}(\lambda)$ is f$\widetilde{e}$-closed set in X, for every f$\widetilde{e}$-closed set λ in Y.

Theorem 4.3. *Every f$\widetilde{e}$-irresolute map is f$\widetilde{e}$-continuous.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be f$\widetilde{e}$-irresolute and let λ be fuzzy closed set in Y. Since every fuzzy closed set is also f$\widetilde{e}$-closed, λ is f$\widetilde{e}$-closed in Y. Since $f : (X, \tau) \to (Y, \sigma)$ is f$\widetilde{e}$-irresolute, we have $f^{-1}(\lambda)$ is f$\widetilde{e}$-closed. Thus inverse image of every fuzzy closed set in Y is f$\widetilde{e}$-closed in X. Therefore the function $f : (X, \tau) \to (Y, \sigma)$ is f$\widetilde{e}$-continuous. □

The converse of above theorem need not be true as shown in the following example.

Example 4.4. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$ and the fuzzy sets u_1, u_2, v, w are defined as follows.

$$u_1(a) = 0.1,\ u_2(a) = 0.7,\ v(x) = 0.2,\ w(x) = 0.3,$$
$$u_1(b) = 0.4,\ u_2(b) = 0.2,\ v(y) = 0.4,\ w(y) = 0.5,$$
$$u_1(c) = 0.6,\ u_2(c) = 0.4,\ v(z) = 0.5,\ w(z) = 0.2$$

Let $\tau = \{0, 1, u_1, u_2, u_1 \vee u_2, u_1 \wedge u_2\}$ and $\sigma = \{0, v, 1\}$ and the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$, $f(c) = z$. Then f is f$\widetilde{e}$-continuous but not f$\widetilde{e}$-irresolute as the fuzzy set w is f$\widetilde{e}$-closed in Y but $f^{-1}(w)$ is not f$\widetilde{e}$-closed set in X.

Theorem 4.5. *Every fuzzy continuous map is fuzzy $\widetilde{e}$-continuous.*

However, converse need not be true as shown in the following example.

Example 4.6. Let $X = \{a, b\}$ and $Y = \{x, y\}$ and the fuzzy sets A, B, C are defined as follows.

$$A(a) = 0.2,\ B(x) = 0.6,\ C(a) = 0.5$$
$$A(b) = 0.4,\ B(y) = 0.4,\ C(b) = 0.3$$

Let $\tau = \{0, 1, A\}$ and $\sigma = \{0, B, 1\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x, f(b) = y$. Then f is f$\widetilde{e}$-continuous as C is fuzzy e-open sets in X but f is not fuzzy continuous in Y, since $B \in \sigma$ and $f^{-1}(B) = B \notin \tau$.

Theorem 4.7. *If $f : (X, \tau) \to (Y, \sigma)$ is f$\widetilde{e}$-continuous if and only if the inverse image of each fuzzy open set of Y is f$\widetilde{e}$-open set of X.*

Proof. Let u be a f$\widetilde{e}$-open set of Y then $1 - u$ is f$\widetilde{e}$-closed in Y. Since $f : X \to Y$ is f$\widetilde{e}$-continuous $f^{-1}(1-u) = 1-f^{-1}(u)$ is f$\widetilde{e}$-closed set of X. That is $f^{-1}(u)$ is f$\widetilde{e}$-open set of X. The converse is obvious. □

Theorem 4.8. *If $f : (X, \tau) \to (Y, \sigma)$ is f$\widetilde{e}$-continuous then*

(a) *for each fuzzy point x_α of X and each $\lambda \in Y$ such that $f(x_\alpha)q\lambda$, there exists a f$\widetilde{e}$-open set μ of X such that $x_\alpha \in \mu$ and $f(\mu) \le \lambda$.*
(b) *for each fuzzy point x_α of X and each $\lambda \in Y$ such that $f(x_\alpha)q\lambda$, there exists a f$\widetilde{e}$-open set μ of X such that $x_\alpha q\mu$ and $f(\mu) \le \lambda$.*

Proof. (a) Let x_α be a fuzzy point of X. Then $f(x_\alpha)$ is a fuzzy point in Y. Now, let $\lambda \in Y$ be a f$\widetilde{e}$-open set such that $f(x_\alpha)q\lambda$. For $\mu = f^{-1}(\lambda)$ as f is f$\widetilde{e}$-continuous we have μ is f$\widetilde{e}$-open set of X and $x_\alpha \in \mu$. Therefore $f(\mu) = f(f^{-1}(\lambda)) \le \lambda$.
(b) Let x_α be a fuzzy point of X, and let $\lambda \in Y$ such that $f(x_\alpha)q\lambda$. Taking $\mu = f^{-1}(\lambda)$ we get μ is f$\widetilde{e}$-open set of X such that $x_\alpha \in \mu$ and $f(\mu) = f(f^{-1}(\lambda)) \le \lambda$. □

Definition 4.9. A fuzzy topological space X is fuzzy $eT_{1/2}$ space (briefly, $feT_{1/2}$ space) if every f$\widetilde{e}$-closed set in X is fuzzy e-closed set in X.

Theorem 4.10. *A fuzzy topological space X is $feT_{1/2}$ if and only if every fuzzy set in X is both fuzzy e-open and f$\widetilde{e}$-open.*

Proof. Let X be $feT_{1/2}$ and let μ be f$\widetilde{e}$-open in X. Then $1-\mu$ is f$\widetilde{e}$-closed. By hypothesis every f$\widetilde{e}$-closed set is fuzzy e-closed,$1-\mu$ is fuzzy e-closed set and hence μ is fuzzy e-open in X. Conversely, let μ be f$\widetilde{e}$- closed. Then $1-\mu$ is f$\widetilde{e}$-open which implies $1-\mu$ is fuzzy e-open. Hence μ is fuzzy e-closed. Every f$\widetilde{e}$-closed set in X is fuzzy e-closed. Therefore X is $feT_{1/2}$ space. □

Definition 4.11. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy $\widetilde{e}$-open (briefly f$\widetilde{e}$-open) if the image of every fuzzy open set in X, is f$\widetilde{e}$-open in Y.

Definition 4.12. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy $\widetilde{e}$-closed (briefly f$\widetilde{e}$-closed) if the image of every fuzzy closed set in X is f$\widetilde{e}$-closed in Y.

Definition 4.13. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy $\widetilde{e}^*$-open (briefly f$\widetilde{e}^*$-open) if the image of every f$\widetilde{e}$-open set in X is f$\widetilde{e}$-open in Y.

Definition 4.14. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy $\widetilde{e}^*$-closed (briefly f$\widetilde{e}^*$-closed) if the image of every f$\widetilde{e}$-closed set in X is f$\widetilde{e}$-closed.

Theorem 4.15. *If λ is f$\widetilde{e}$-closed in X and $f:X\to Y$ is bijective, fe-irresolute and f$\widetilde{e}$-closed, then $f(\lambda)$ is f$\widetilde{e}$-closed in Y.*

Proof. Let $f(\lambda)\leq\mu$ where μ is fuzzy e-open in Y. Since f is fuzzy e -irresolute, $f^{-1}(\mu)$ is fuzzy e-open containing λ. Hence $ecl(\lambda)\leq f^{-1}(\mu)$ as λ is f$\widetilde{e}$ -closed. Since f is f$\widetilde{e}$-closed, $f(ecl(\lambda))$ is f$\widetilde{e}$-closed set contained in the fuzzy e-open set μ, which implies that $ecl(f(ecl(\lambda)))\leq\mu$ and hence $eclf(\lambda)\leq\mu$. So $f(\lambda)$ is f$\widetilde{e}$-closed in Y. □

Definition 4.16. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy e-open(resp. fuzzy e*-open) if the image of every fuzzy open (resp. fuzzy e-open)set in X is fuzzy e-open set in Y.

Theorem 4.17. *Let $f:(X,\tau)\to(Y,\sigma)$ be onto f$\widetilde{e}$-irresolute and fuzzy e*-closed. If X is $feT_{1/2}$ space, then (Y,σ) is $feT_{1/2}$ -space.*

Proof. Let μ be a f$\widetilde{e}$-closed set in Y. Since $f:X\to Y$ is f$\widetilde{e}$-irresolute, $f^{-1}(\mu)$ is f$\widetilde{e}$-closed set in X. As X is $feT_{1/2}$ -space, $f^{-1}(\mu)$ is fuzzy e-closed set in X. Also $f:X\to Y$ is fuzzy e^*-closed, $f(f^{-1}(\mu))$ is fuzzy e-closed in Y. Since $f:X\to Y$is onto, $f(f^{-1}(\mu))\leq\mu$. Thus μ is fuzzy e-closed in Y. Hence (Y,σ) is also $feT_{1/2}$ -space. □

Theorem 4.18. *If the bijective map $f:(X,\tau)\to(Y,\sigma)$ is fuzzy e*-open and fuzzy e-irresolute, then $f:(X,\tau)\to(Y,\sigma)$ is f$\widetilde{e}$-irresolute.*

Proof. Let λ be a f$\widetilde{e}$-closed set in Y and let $f^{-1}(\lambda)\leq\mu$ where μ is a fuzzy e-open set in X. Clearly, $\lambda\leq f(\mu)$. Since $f:X\to Y$ is fe*-open map, $f(\mu)$ is fuzzy e-open in Y and λ is f$\widetilde{e}$-closed set in Y. Then $ecl(\lambda)\leq f(\mu)$, and thus $f^{-1}(ecl(\lambda))\leq\mu$. Also $f:X\to Y$is fuzzy e- irresolute and $ecl(\lambda)$ is a fuzzy e-closed set in Y, then $f^{-1}(ecl(\lambda))$ is fuzzy e-closed set in X. Thus $ecl(f^{-1}(\lambda))\leq ecl(f^{-1}(ecl(\lambda))\leq\mu$. So $f^{-1}(\lambda)$ is f$\widetilde{e}$-closed set in X. Hence $f:X\to Y$is f$\widetilde{e}$-irresolute map. □

Theorem 4.19. *Let $f:(X,\tau)\to(Y,\sigma)$. Then the following statements are equivalent.*

(a) *f is f$\widetilde{e}$-irresolute.*

(b) *for every $\widetilde{fe}$-closed set λ in Y , $f^{-1}(\lambda)$is $\widetilde{fe}$-closed in X.*
(c) *for every fuzzy point x_p of X and every $\widetilde{fe}$-open λ of Y such that $f(x_p) \in \lambda$, there exist a $\widetilde{fe}$-open set such that $x_p \in \mu$ and $f(\mu) \leq \lambda$.*

Proof. $(a) \Rightarrow (b)$ Obvious.
$(b) \Rightarrow (c)$ Let λ be $\widetilde{fe}$-open in Y which implies $1-\lambda$ is $\widetilde{fe}$-closed in Y. By (ii), $f^{-1}(\lambda)$is $\widetilde{fe}$- closed in X, $f^{-1}(1-\lambda)$ is $\widetilde{fe}$- closed in X. Let x_p be a fuzzy point of X such that $f(x_p) \in\lambda$ implies that $x_p \in f^{-1}(\lambda)$ is $\widetilde{fe}$-open in X. Let $\mu = f^{-1}(\lambda)$ which implies that$f(\mu) = f(f^{-1}(\lambda)) \leq \lambda$.
$(c) \Rightarrow (a)$Let λ be a $\widetilde{fe}$-open set in Y and $x_p \in f^{-1}(\lambda)$which implies $f(x_p) \in \lambda$. Then there exist a $\widetilde{fe}$-open set μ in X such that $x_p \in \mu$ and $f(\mu) \leq \lambda$. Hence $x_p \in \mu \leq f^{-1}(\lambda)$. Hence $f^{-1}(\lambda)$is $\widetilde{fe}$-open in X. Hence f is $\widetilde{fe}$-irresolute. □

Theorem 4.20. *Let $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \rho)$ be two maps such that $g \circ f : (X, \tau) \to (Z, \rho)$ is $\widetilde{fe}$-closed.*

(a) *If f is fuzzy continuous and surjective, then g is $\widetilde{fe}$-closed.*
(b) *If g is $\widetilde{fe}$-irresolute and injective, then f is $\widetilde{fe}$-closed.*

Proof. (a) Let U be fuzzy closed in Y . Then $f^{-1}(U)$ is fuzzy closed in X, as f is fuzzy continuous. Since $g \circ f$ is $\widetilde{fe}$-closed map and f is surjective, $(g \circ f)f^{-1}(U) = g(U)$ is $\widetilde{fe}$-closed in Z. Hence $g : Y \to Z$ is $\widetilde{fe}$-closed.
(b) Let U be a fuzzy closed in X. Then $(g \circ f)(U)$ is feg-closed in Z. Since g is $\widetilde{fe}$-irresolute and injective $g^{-1}(g \circ f)(U) = f(U)$ is $\widetilde{fe}$-closed in Y. Hence f is a $\widetilde{fe}$-closed. □

Theorem 4.21. *Let $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \rho)$ be two maps such that $g \circ f : (X, \tau) \to (Z, \rho)$ is $\widetilde{fe}^*$-closed.*

(a) *If f is fuzzy continuous and surjective, then g is $\widetilde{fe}$-closed.*
(b) *If g is $\widetilde{fe}$-irresolute and injective, then f is $\widetilde{fe}^*$-closed.*

Theorem 4.22. *For the functions $f : X \to Y$ and $g : Y \to Z$ the following relations hold:*

(a) *If $f : X \to Y$ is $\widetilde{fe}$-continuous and $g : Y \to Z$ is fuzzy continuous then $g \circ f : X \to Z$ is $\widetilde{fe}$-continuous.*
(b) *If $f : X \to Y$ and $g : Y \to Z$ are $\widetilde{fe}$-irresolute then $g \circ f : X \to Z$ is $\widetilde{fe}$-irresolute.*
(c) *If $f : X \to Y$ is $\widetilde{fe}$-irresolute and $g : Y \to Z$ is $\widetilde{fe}$-continuous then $g \circ f : X \to Z$ is $\widetilde{fe}$-continuous.*

Proof. Omitted. □

Theorem 4.23. *If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy e-irresolute and $g : (Y, \sigma) \to (Z, \rho)$ is $\widetilde{fe}$-continuous then $g \circ f : (X, \tau) \to (Z, \rho)$ is $\widetilde{fe}$-continuous if Y is $feT_{1/2}$ -space.*

Proof. Suppose μ is fuzzy e-closed subset of Z. Since $g : Y \to Z$ is $\widetilde{fe}$-continuous $g^{-1}(\mu)$ is $\widetilde{fe}$-closed subset of Y. Now since Y is $feT_{1/2}$ -space, $g^{-1}(\mu)$ is fuzzy e-closed subset of Y. Also since $f : X \to Y$ is fuzzy e- irresolute $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)$ is fuzzy e-closed. Thus $g \circ f : X \to Z$ is $\widetilde{fe}$-continuous. □

Theorem 4.24. *Let $g \circ f : (X, \tau) \to (Z, \rho)$ be $f\widetilde{e}$-continuous. Then f is fe-continuous if X is $feT_{1/2}$ -space.*

Proof. Let μ be fuzzy closed set in Y. Since $f : X \to Y$ is $f\widetilde{e}$-continuous, $f^{-1}(\mu)$ is $f\widetilde{e}$-closed subset in X. Since X is $feT_{1/2}$ -space, by hypothesis every $f\widetilde{e}$-closed set is fuzzy eclosed. Hence $f^{-1}(\mu)$ is fuzzy e-closed subset in X. Therefore $f : X \to Y$ is fuzzy e-continuous. □

5. $f\widetilde{e}$- homeomorphism and $f\widetilde{e}^*$-homeomorphism

Definition 5.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy $\widetilde{e}$-homeomorphism (briefly $f\widetilde{e}$-homeomorphism) if f and f^{-1} are $f\widetilde{e}$-continuous.

Definition 5.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy $\widetilde{e}^*$-homeomorphism (briefly $\widetilde{e}^*$-homeomorphism) if f and f^{-1} are $f\widetilde{e}$- irresolute.

Theorem 5.3. *Every fuzzy homeomorphism is $f\widetilde{e}$- homeomorphism.*

The converse of the above theorem need not be true as seen from the following example.

Example 5.4. Let $X = Y = \{a, b, c\}$ and the fuzzy sets u, v be defined as, $u(a) = 0.1$, $u(b) = 0.4$, $v(a) = 0.2$, $v(b) = 0.5$. Let $\tau = \{0, 1, u\}$ and $\sigma = \{0, 1, v\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = a$, $f(b) = b$ with $A(a) = 0$, $A(b) = 0.3$ is fuzzy e-open in (X, τ) and $B(a) = 0.1$, $B(b) = 0.3$ is fuzzy e-open (Y, τ) is $f\widetilde{e}$-homeomorphism but not fuzzy homeomorphism as u is open in X, $f(u) = u$ is not open in Y. Hence $f^{-1} : Y \to X$ is not fuzzy continuous.

Theorem 5.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then the following are equivalent:*

(a) *f is $f\widetilde{e}$-homeomorphism.*
(b) *f is $f\widetilde{e}$-continuous and $f\widetilde{e}$-open map.*
(c) *f is $f\widetilde{e}$-continuous and $f\widetilde{e}$-closed map.*

Proof. (a)⇒ (b) Let f be $f\widetilde{e}$-homeomorphism. Then f and f^{-1} are $f\widetilde{e}$-continuous. To prove that f is $f\widetilde{e}$-open map, let λ be a fuzzy open set in X. Since $f^{-1} : Y \to X$ is $f\widetilde{e}$-continuous, $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is $f\widetilde{e}$-open in Y. Therefore $f(\lambda)$ is $f\widetilde{e}$-open in Y. Hence f is $f\widetilde{e}$-open.
(b)⇒ (c) Let f be $f\widetilde{e}$-continuous and $f\widetilde{e}$-open map. To prove that$f$ is $f\widetilde{e}$-closed map. Let μ be a fuzzy closed set in X. Then $1 - \mu$ is fuzzy open set in X. Since f is $f\widetilde{e}$-open map, $f(1 - \mu)$ is $f\widetilde{e}$-open set in Y. Now $f(1 - \mu) = 1 - f(\mu)$. Therefore $f(\mu)$ is $f\widetilde{e}$-closed in Y. Hence f is a $f\widetilde{e}$-closed.
(c)⇒ (a) Let f be $f\widetilde{e}$-continuous and $f\widetilde{e}$-closed map. To prove that f^{-1} is $f\widetilde{e}$-continuous. Let λ be a fuzzy open set in X. Then $1-\lambda$ is a fuzzy closed set in X. Since f is $f\widetilde{e}$-closed map, $f(1-\lambda)$ is $f\widetilde{e}$-closed in Y. Now $(f^{-1})^{-1}(1-\lambda) = f(1-\lambda) = 1-f(\lambda)$ is $f\widetilde{e}$- open set in Y. Therefore $f^{-1} : Y \to X$ is $f\widetilde{e}$-continuous. Hence f is $f\widetilde{e}$-homeomorphism. □

Theorem 5.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following are equivalent:*

(a) *f is $f\widetilde{e}^*$-homeomorphism.*
(b) *f is $f\widetilde{e}$-irresolute and $f\widetilde{e}^*$-open.*
(c) *f is $f\widetilde{e}$-irresolute and $f\widetilde{e}^*$-closed.*

Proof. Proof follows from the above theorem. □

6. Fuzzy $\widetilde{e}$-connectedness

Definition 6.1. A fuzzy set v in a fuzzy topological spaces (X, τ) is said to be fuzzy $\widetilde{e}$-connected if and only if v cannot be expressed as the union of two fuzzy $\widetilde{e}$-open sets.

Theorem 6.2. *Every fuzzy $\widetilde{e}$-connected set is fuzzy connected.*

Proof. Let X be fuzzy $\widetilde{e}$-connected and X is not fuzzy connected. Then there exists fuzzy open sets u and v in X such that $1_x = u \vee v$. Since X is fuzzy $\widetilde{e}$-connected set, which implies that u and v is fuzzy $\widetilde{e}$-open set. Clearly, X is not fuzzy $\widetilde{e}$-connected which is a contradiction. □

The converse of the above theorem is not true in general.

Example 6.3. Let $X = \{a, b, c\}$ and $\tau = \{0, 1, A\}$ and the fuzzy set $A(a) = 0$, $A(b) = 1$, $A(c) = 0$. Then (X, τ) is fuzzy connected but not fuzzy $\widetilde{e}$-connected.

Theorem 6.4. *Let $f : (X, \tau) \to (Y, \sigma)$be a fuzzy $\widetilde{e}$-continuous surjective mapping. If v is a fuzzy $\widetilde{e}$ -connected subset in X, then f (v) is fuzzy connected in Y.*

Proof. Suppose that H is not fuzzy connected in Y. Then, there exist fuzzy open sets u and v in Y such that $H = u \vee v$. Since f is fuzzy $\widetilde{e}$-continuous surjective mapping, $f^{-1}(u)$ and $f^{-1}(v)$ are fuzzy $\widetilde{e}$-open set in X and $H = f^{-1}(u \vee v) = f^{-1}(u) \vee f^{-1}(v)$. It is clear that $f^{-1}(u)$ and $f^{-1}(v)$ are fuzzy $\widetilde{e}$-open set in X. Therefore, H is not fuzzy $\widetilde{e}$-connected in X, which is a contradiction. Hence, Y is fuzzy connected. □

7. conclusion

It is interesting to work on $\widetilde{e}$-compactness, $f\widetilde{e}$-continuous and $f\widetilde{e}$-irresolute mappings, fuzzy $\widetilde{e}$-connectedness and various properties of these things. Compositions of mappings can be tried with other forms of $f\widetilde{e}$-irresolute mappings.

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