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The countability aspects of fuzzy soft topological spaces

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Abstract. In this paper, we have studied fuzzy soft topological spaces. We introduce some new concepts in fuzzy soft topological spaces such as fuzzy soft first-countable space, fuzzy soft second-countable space, fuzzy soft Lindelöf spaces, and some basic properties of these spaces are explored.

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1. Introduction

Many complicated problems arising in the fields of economics, sociology, engineering, environmental sciences, medical sciences etc. are highly dependent on the task of modelling uncertain data. When the uncertainty is highly complicated and difficult to characterize, classical mathematical approaches are found to be inadequate to derive effective or useful models. Zadeh [12] proposed the concept of fuzzy set theory in 1965, which has become a very important tool to solve such kinds of problems and provides an appropriate framework for representing vague concepts by allowing partial membership to the elements under consideration. Since then many applications of fuzzy set theory have been arisen in various fields of mathematics and computer sciences, such as fuzzy control systems, fuzzy automata, fuzzy logic and so on. Beside this theory, there are many theories like theory of probability, theory of intuitionistic fuzzy sets, theory of rough sets etc., which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their own limitations and difficulties. For example, the theory of probability can deal only with possibility, and in fuzzy set theory, the difficulty of setting the appropriate membership function in each particular case. Intuitionistic fuzzy set theory is more generalized concept than fuzzy set theory, but this theory has the same difficulty.

In 1999, the Russian researcher Molodtsov [7] pointed out that the reason for these difficulties was, possibly, the inadequacy of the parametrization tool of the theory, and he proposed the concept of soft set theory for dealing with uncertainties. Soft set theory is free from the difficulties mentioned above. In his paper, Molodtsov pointed out several directions for the applications of soft sets, such as game theory, Riemann integration, theory of measurement, smoothness of functions and so on. In 2003, Maji et al. [6] studied the theory of soft sets initiated by Molodtsov. They defined and studied several basic notions of soft sets with examples.

In recent times, many researchers have contributed a lot towards fuzzification of Soft Set Theory. Maji et al. [5] introduced the concept of fuzzy soft set. He defined fuzzy soft union, intersection, complement of fuzzy soft set, De Morgan Law etc. Then many researcher have applied this concept on group theory [1], decision making problems [3], relations [9] etc. In 2011, Tanay et al. [10] initially gave the concept of fuzzy soft topology using the fuzzy soft sets and gave the basic notions of it by following Chang [2]. Pazar Varol and Aygun [11] defined fuzzy soft topology in Lowen's sense. Further, many other concepts of fuzzy soft topology such as hausdorffness, compactness, connectedness, separation axioms etc. have been studied by researchers.

In this paper, we introduce some new concepts in fuzzy soft topological spaces such as fuzzy soft first-countable space, fuzzy soft second countable space and fuzzy soft Lindelöf space, and some properties and relations in these spaces are discussed.

2. Preliminaries

Throughout this paper, X refers to an initial universe, E is the set of all parameters for X.

Definition 2.1 ([12]). A fuzzy set A of non-empty set X is characterized by a membership function μ_A which associates each point of X to a real number in the interval $[0, 1]$, with the value $\mu_A(x)$ at x representing the "grade of membership" of x in A.

Definition 2.2 ([12]).
(1) Fuzzy sets A and B are said to be equal, written as $A = B$, if and only if $\mu_A(x) = \mu_B(x)$ for all $x \in X$.
(2) Fuzzy set A is contained in a fuzzy set B if and only if $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$.
(3) The union of two fuzzy sets A and B with respective membership functions μ_A and μ_B is a fuzzy set C, written as $C = A \cup B$ whose membership function is related to those of A and B is given by $\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$.
(4) The intersection of two fuzzy sets A and B with respective membership functions μ_A and μ_B is a fuzzy set C, written as $C = A \cap B$ whose membership function is related to those of A and B is given by $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$.
(5) Let Ω be an index set and $\{f_i : i \in \Omega\}$ be the family of fuzzy sets in X. Then their union $\bigcup_{i\in\Omega} f_i$ and intersection $\bigcap_{i\in\Omega} f_i$ are defined, respectively as follows:
 (i) $(\bigcup_{i\in\Omega} f_i)(x) = sup\{f_i(x) : i \in \Omega\}, \forall x \in X$,

(ii) $(\bigcap_{i\in\Omega} f_i)(x) = inf\{f_i(x) : i \in \Omega\}$, $\forall x \in X$.

From here onwards, A denotes the subset of the parameters set E.

Definition 2.3 ([7])**.** A pair (F, A) is called a soft set over X if and only if F is a mapping from A into the set of all subsets of the set X i.e., $F : A \to \mathcal{P}(X)$, where $\mathcal{P}(X)$ denotes the power set of X.

Definition 2.4 ([8])**.** Let I^X be the collection of all fuzzy sets in X. A fuzzy soft set f_A over X is a mapping from E to I^X i.e., $f_A : E \to I^X$ such that $f_A(e) \neq 0_X$; if $e \in A \subseteq E$, and $f_A(e) = 0_X$; otherwise, where 0_X denotes the null fuzzy set in X, given by $0_X(x) = 0$, $\forall x \in X$.

Definition 2.5 ([11])**.** For two fuzzy soft sets f_A and g_B over a common universe X, f_A is said to be fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $f_A(e) \subseteq g_B(e)$, $\forall e \in E$.

Definition 2.6 ([11])**.** Two fuzzy soft sets f_A and g_B over a common universe X are said to be fuzzy soft equal if $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$.

Definition 2.7 ([11])**.** The complement of a fuzzy soft set f_A, denoted by f_A^c, is the fuzzy soft set over X, defined by $f_A^c(e) = 1_X - f_A(e)$, $\forall e \in E$, where 1_X denotes the absolute fuzzy set in X given by $1_X(x) = 1$, $\forall x \in X$.

Definition 2.8 ([11])**.** The universal fuzzy soft set 1_E over X is given by $1_E(e) = 1_X$, $\forall e \in E$ and the null fuzzy soft set 0_E over X is given by $0_E(e) = 0_X$, $\forall e \in E$.

Clearly, we have $1_E^c = 0_E$ and $0_E^c = 1_E$.

Definition 2.9 ([11])**.** The fuzzy soft union of f_A and g_B, denoted by $f_A \sqcup g_B$, is the fuzzy soft set over X defined by $(f_A \sqcup g_B)(e) = f_A(e) \cup g_B(e)$, $\forall e \in E$.

Definition 2.10 ([11])**.** The fuzzy soft intersection of f_A and g_B, denoted by $f_A \sqcap g_B$, is the fuzzy soft set over X defined by $(f_A \sqcap g_B)(e) = f_A(e) \cap g_B(e)$, $\forall e \in E$.

Definition 2.11 ([11])**.** Let Ω be an index set and $\{(f_A)_i : i \in \Omega\}$ be a family of fuzzy soft sets over X. Then their union $\sqcup_{i\in\Omega}(f_A)_i$ and intersection $\sqcap_{i\in\Omega}(f_A)_i$ are defined, respectively as follows:

(1) $(\sqcup_{i\in\Omega}(f_A)_i)(e) = \cup_{i\in\Omega}(f_A)_i(e)$, $\forall e \in E$,
(2) $(\sqcap_{i\in\Omega}(f_A)_i)(e) = \cap_{i\in\Omega}(f_A)_i(e)$, $\forall e \in E$.

Definition 2.12 ([11])**.** A fuzzy soft topological space is a pair (X, τ) consisting of a non-empty set X and a family τ of fuzzy soft sets over X satisfying the following conditions:

(1) 0_E, 1_E belong to τ,
(2) The union of any number of fuzzy soft sets in τ belongs to τ,
(3) The intersection of any two fuzzy soft sets in τ belongs to τ.

Then τ is called a fuzzy soft topology over X. Members of τ are called fuzzy soft open sets. A fuzzy soft set f_A over X is called fuzzy soft closed set if $f_A^c \in \tau$.

Definition 2.13 ([8]). A fuzzy soft topological space is called indiscrete if its topology contains only 0_E and 1_E, while the topology of the discrete fuzzy soft topological space consists of all fuzzy soft sets over X.

Definition 2.14 ([8]). A fuzzy soft set g_B over a fuzzy soft topological space (X,τ) is called a fuzzy soft neighbourhood of the fuzzy soft set f_A if there exist a fuzzy soft open set h_C such that $f_A \sqsubseteq h_C \sqsubseteq g_B$.

Definition 2.15 ([8]). Let (X,τ) be a fuzzy soft topological space and let f_A be a fuzzy soft set over X. Then

(1) The fuzzy soft closure of f_A is defined as the intersection of all fuzzy soft closed sets which contain f_A and is denoted by $\overline{f_A}$. We write $\overline{f_A} = \sqcap\{g_B : g_B$ is fuzzy soft closed and $f_A \sqsubseteq g_B\}$,

(2) The fuzzy soft interior of f_A is defined as the union of all fuzzy soft open sets contained in f_A and is denoted by f_E^o. We write $f_E^o = \sqcup\{g_B : g_B$ is fuzzy soft open and $g_B \sqsubseteq f_A\}$.

Definition 2.16 ([4]). A fuzzy soft set f_A is said to be a fuzzy soft point, denoted by $e(f_A)$, if for the element $e \in A$, $f_A(e) \neq 0_X$ and for $e \neq e' \in A$, $f_A(e') = 0_X$.

Theorem 2.17 ([4]). *Let (X,τ) be a fuzzy soft topological space. A fuzzy soft point $e(f_A) \in \overline{g_B}$ if and only if each fuzzy soft neighbourhood of $e(f_A)$ intersects g_B.*

Definition 2.18 ([1]). Let $\mathcal{F}(X,E)$ and $\mathcal{F}(Y,K)$ be the families of fuzzy soft sets over X and Y respectively and E, K be the parameters sets for the universe X and Y respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then the fuzzy soft mapping
$(u,p) : \mathcal{F}(X,E) \to \mathcal{F}(Y,K)$ is defined as:

(1) Let f_A be a fuzzy soft set in $\mathcal{F}(X,E)$. Then the image of f_A under (u,p), written as $(u,p)f_A$, is a fuzzy soft set in $\mathcal{F}(Y,K)$ such that

$$(u,p)f_A(k)(y) = \begin{cases} \underset{x\in p^{-1}(y)}{sup}\ \underset{e\in u^{-1}(k)}{sup}\ f_A(e)(x), & \text{if } p^{-1}(y) \neq \phi, u^{-1}(k) \neq \phi. \\ 0, & \text{otherwise,} \end{cases}$$

$\forall y \in Y$ and $\forall k \in K$.

(2) Let g_B be a fuzzy soft set in $\mathcal{F}(Y,K)$. The inverse image of g_B under (u,p), written as $(u,p)^{-1}(g_B)$ is a fuzzy soft set in $\mathcal{F}(X,E)$ such that $(u,p)^{-1}g_B(e)(x) = g_B(u(e))(p(x))$, $\forall x \in X$ and $\forall e \in E$.

3. Fuzzy soft first and second-countable spaces

Definition 3.1 ([11]). Let (X,τ) be a fuzzy soft topological space and $\mathcal{B}$ be a subfamily of τ. If every element of τ can be written as the arbitrary fuzzy soft union of some elements of $\mathcal{B}$, then $\mathcal{B}$ is called a fuzzy soft basis for the fuzzy soft topology τ.

Definition 3.2 ([11]). Let (X,τ) be a fuzzy soft topological space. Then a subfamily $\mathcal{S}$ of τ is called a subbase for τ if every member of τ is a union of finite intersections of members of $\mathcal{S}$.

Definition 3.3 ([10]). Let (X, τ) is a fuzzy soft topological space and $f_A \in \mathcal{F}(X, E)$, then the topology τ' defined by $\tau' = \{f_A \sqcap g_B : g_B \in \tau\}$ is a fuzzy soft topology on the fuzzy soft subset f_A and called the fuzzy soft subspace topology. With respect to this topology, (f_A, τ') is called fuzzy soft subspace of (X, τ).

Definition 3.4 ([10]). Let (X, τ) be a fuzzy soft topological space, $\mathcal{B}$ be a fuzzy soft basis for (X, τ) and $f_A \in \mathcal{F}(X, E)$. Then the collection $\mathcal{B}' = \{f_A \sqcap g_B : g_B \in \mathcal{B}\}$ is a fuzzy soft basis for the fuzzy soft subspace topology (f_A, τ').

Definition 3.5. Let (X, τ) be a fuzzy soft topological space, and let $\mathcal{U}$ be a family of fuzzy soft neighbourhoods of some fuzzy soft point $e(f_A)$ in X. If for each fuzzy soft neighbourhood g_B of $e(f_A)$, there exist h_C in $\mathcal{U}$ such that $e(f_A) \in h_C \sqsubseteq g_B$ then we say that $\mathcal{U}$ is a fuzzy soft neighbourhood base at $e(f_A)$.

Definition 3.6. Let (X, τ) be a fuzzy soft topological space, and let $e(f_A)$ be a fuzzy soft point in X. If $e(f_A)$ has a countable fuzzy soft neighbourhood base, then we say that (X, τ) is fuzzy soft first-countable at the fuzzy soft point $e(f_A)$. If (X, τ) is fuzzy soft first-countable at each of its fuzzy soft points, then we say that (X, τ) is fuzzy soft first-countable.

Even nicer than first-countable fuzzy soft spaces are the second-countable fuzzy soft spaces, in which we can describe all the fuzzy soft open sets in terms of a countable subcollection of fuzzy soft sets.

Definition 3.7. A fuzzy soft topological space (X, τ) is fuzzy soft second-countable if it has a countable fuzzy soft base $\mathcal{B}$ for its topology τ, say $\mathcal{B} = \{(f_A)_1, (f_A)_2, (f_A)_3, ...\}$. That is, given any open set f_A and point $e(g_B) \in f_A$, there is $(f_A)_n \in \mathcal{B}$ such that $(f_A)_n \sqsubseteq f_A$ with $e(g_B) \in (f_A)_n$.

Proposition 3.8. *Each fuzzy soft second-countable space is fuzzy soft first-countable.*

Proof. Let (X, τ) be a fuzzy soft second-countable space and suppose $\mathcal{B} = \{(f_A)_1, (f_A)_2, (f_A)_3, ...\}$ be the countable fuzzy soft base. We can take for a fuzzy soft basis at $e(f_A)$ the sequence of all $(f_A)_n$ which contain $e(f_A)$, call this collection as $\mathcal{B}'$. Then $\mathcal{B}'$ is countable as it is fuzzy soft subset of the countable fuzzy soft basis $\mathcal{B}$, and since $\mathcal{B}$ is fuzzy soft basis, for any fuzzy soft neighbourhood g_B of $e(f_A)$, there exists $(f_A)_n \in \mathcal{B}'$ such that $e(f_A) \in (f_A)_n \sqsubseteq g_B$. This implies that (X, τ) is fuzzy soft first-countable. □

Proposition 3.9. *A subspace of fuzzy soft first-countable space is fuzzy soft first-countable and same holds for second-countability.*

Proof. We prove for second countability and first follows from second. Let (X, τ) is a fuzzy soft second-countable space and (f_A, τ') be a fuzzy soft subspace of (X, τ). Suppose $\mathcal{B} = \{(f_A)_1, (f_A)_2, (f_A)_3, ...\}$ is a countable fuzzy soft basis for the space (X, τ). Then take the basis for subspase (f_A, τ') as $\mathcal{B}' = \{f_A \sqcap g_B : g_B \in \mathcal{B}\}$, which is countable. Therefore (f_A, τ') is fuzzy soft second-countable. □

Remark 3.10. There exists a fuzzy soft space which is not fuzzy soft first-countable.

Remark 3.11. There exists a fuzzy soft first-countable space which is not fuzzy soft second-countable space.

Example 3.12. (1) Let X be an uncountable set, and $A = \{e\}$. Let $\tau =\{f_A : f_A(e) = 1_X - g_A(e)$, $g_A(e)$ is finite fuzzy set in $X\}$, then (X, τ) is a fuzzy soft topological space, which is not fuzzy soft first-countable.

(2) If we take the discrete fuzzy soft topology τ^1 on X, over the parameter set E, then each fuzzy soft set in X is open with respect to discrete fuzzy soft topology. Take $\mathcal{B}_{e(f_A)} = \{\{e(f_A)\}\}$ as fuzzy soft neighborhood base at each fuzzy soft point $e(f_A)$. Then $\mathcal{B}_{e(f_A)}$ is countable and for each fuzzy soft neighborhood g_B of $e(f_A)$, there is always $\{e(f_A)\} \in \mathcal{B}_{e(f_A)}$ such that $e(f_A) \in \{e(f_A)\} \sqsubseteq g_B$. Therefore (X, τ^1) is fuzzy soft first-countable space but it is not fuzzy soft second-countable space.

Theorem 3.13.

(1) Product of countably many fuzzy soft-first countable spaces is fuzzy soft first-countable space,
(2) Product of countably many fuzzy soft-second countable spaces is fuzzy soft second-countable space.

Proof. We prove only for second-countability and first follows from the second. For each $n \in \mathbb{N}$, let (X_n, τ_n) be the countable fuzzy soft second-countable space over the parameter set E_n. And let $\mathcal{B}_n$ be the countable fuzzy soft base for the fuzzy soft topological space X_n. Put $\mathcal{B} = \Big\{ \prod\limits_{n\in\mathbb{N}} (f_A)_n : (f_A)_n \in \mathcal{B}_n$ and $(f_A)_n$ equals 1_{E_n} except for finitely many values of $n\Big\}$. Then $\mathcal{B}$ is countable and a fuzzy soft base for $\prod\limits_{n\in\mathbb{N}} (X_n, \tau_n)$. □

Theorem 3.14. *The image of fuzzy soft first-countable spaces under a fuzzy soft open continuous map are fuzzy soft first-countable.*

Proof. Let (X, τ_1) and (Y, τ_2) be two fuzzy soft spaces over the parameter set E, and suppose (X, τ_1) is fuzzy soft first-countable, and let (u, p) be an onto fuzzy soft continuous open mapping from (X, τ_1) to (Y, τ_2). Since (u, p) is onto, for any fuzzy soft point $e(g_B)$ in Y, there exists a fuzzy soft point $e(f_A)$ in X such that $(u, p)(e(f_A)) = e(g_B)$. Since (X, τ_1) is fuzzy soft first-countable, there exists a countable fuzzy soft neighborhood base $\{(f_A)_n\}_{n\in\mathbb{N}}$ at $e(f_A)$. Then it is easy to see that $\{(u, p)((f_A)_n)\}_{n\in\mathbb{N}}$ is fuzzy soft neighbourhood base at $e(g_B)$. □

Definition 3.15. A fuzzy soft space (X, τ) is fuzzy soft Lindelöf if each fuzzy soft open covering $\mathcal{A}$ of X has a countable subcover.

Proposition 3.16. *Each fuzzy soft second-countable space is fuzzy soft Lindelöf.*

Proof. Let (X, τ) be a fuzzy soft second-countable space, and let $\mathcal{B}$ be a countable fuzzy soft open base of X. Let $\mathcal{C}$ be an arbitrary fuzzy soft open covering of X. Put $\mathcal{B}' = \{f_A \in \mathcal{B}$: there is $g_B \in \mathcal{C}$ such that $f_A \sqsubseteq g_B\}$. Then $\mathcal{B}'$ is countable. Denote $\mathcal{B}'$ by $\{(f_A)_n : n \in \mathbb{N}\}$. For each $n \in \mathbb{N}$, there exists an $(h_A)_n \in \mathcal{C}$ such that $(f_A)_n \sqsubseteq (h_A)_n$. Then $\{(h_A)_n : n \in \mathbb{N}\}$ is countable fuzzy soft subfamily of $\mathcal{C}$. Next we shall prove that $\{(h_A)_n : n \in \mathbb{N}\}$ is a fuzzy soft cover of (X, τ).
Take an arbitrary fuzzy soft point $e(f_A)$ in $\mathcal{F}(X, E)$. Since $\mathcal{C}$ is a fuzzy soft covering of X, there exists a $C \in \mathcal{C}$ such that $e(f_A) \in C$. Since C is open fuzzy soft set, so

there is a $B \in \mathcal{B}$ such that $e(f_A) \in B \sqsubseteq C$ because $\mathcal{B}$ is a fuzzy soft base of X. Hence $B \in \mathcal{B}'$, and therefore, there is an $n \in \mathbb{N}$ such that $B = (f_A)_n$. Thus $e(f_A) \in (f_A)_n$. Hence $\{(h_A)_n : n \in \mathbb{N}\}$ is a fuzzy soft cover of (X, τ). □

Definition 3.17. Let (X, τ) be a fuzzy soft topological space over the set of parameter E, and let g_B be a fuzzy soft closed set over (X, τ) and a fuzzy soft point $e(h_C)$ such that $e(h_C) \notin g_B$. If there exist fuzzy soft open sets $(f_A)_1$ and $(f_A)_2$ such that $e(h_C) \in (f_A)_1$, $g_B \sqsubseteq (f_A)_2$ and $(f_A)_1 \sqcap (f_A)_2 = \phi$, then (X, τ) is called a fuzzy soft regular space.

Definition 3.18. Let (X, τ) be a fuzzy soft topological space over X, and let $(g_B)_1$ and $(g_B)_2$ be two disjoint fuzzy soft closed sets over X. If there exist fuzzy soft open sets $(f_A)_1$ and $(f_A)_2$ such that $(g_B)_1 \sqsubseteq (f_A)_1$, $(g_B)_2 \sqsubseteq (f_A)_2$ and $(f_A)_1 \sqcap (f_A)_2 = \phi$, then (X, τ) is called a fuzzy soft normal space.

Theorem 3.19. *Each fuzzy soft regular and fuzzy soft Lindelöf space is fuzzy soft normal.*

Proof. Let (X, τ) be a fuzzy soft regular and fuzzy soft Lindelöf space. Let $(f_A)_1$ and $(f_A)_2$ be two disjoint fuzzy soft closed sets over X. For each fuzzy soft point $e(f_A) \in (f_A)_1 \sqsubseteq (f_A)_2^c$, and since (X, τ) is fuzzy soft regular, there exists a fuzzy soft open neighbourhood g_B of $e(f_A)$ such that $e(f_A) \in \overline{g_B} \sqsubseteq (f_A)_2^c$, that is, $\overline{g_B} \sqcap (f_A)_2 = \phi$. Let $\mathcal{G} = \{g_B : e(f_A) \in (f_A)_1\}$ i.e., $\mathcal{G}$ is the collection of fuzzy soft neighbourhoods of $e(f_A)$ such that $e(f_A) \in (f_A)_1$, then $\mathcal{G} \sqcup (f_A)_1^c$ is a fuzzy soft open cover of (X, τ). Since (X, τ) is a fuzzy soft Lindelöf, there exists a countable subcover $\{(g_B)_n : n \in \mathbb{N}\} \sqcup (f_A)_1^c$. Put $G_n = (g_B)_n$ for each $n \in \mathbb{N}$, then $(f_A)_1 \sqsubseteq \bigsqcup_{n \in \mathbb{N}} G_n$ and each $G_n \sqcap (f_A)_2 = \phi$. Similarly, there exist countably many fuzzy soft open sets $\{F_n : n \in \mathbb{N}\}$ such that $(f_A)_2 \sqsubseteq \bigsqcup_{n \in \mathbb{N}} F_n$ and each $F_n \sqcap (f_A)_1 = \phi$.
For each $n \in \mathbb{N}$, put

$$G'_n = G_n \sqcap (\sqcup_{i=1}^n \overline{F_i})^c \quad F'_n = F_n \sqcap (\sqcup_{i=1}^n \overline{G_i})^c.$$

Then for $m, n \in \mathbb{N}$, we have $G'_n \sqcap F'_m = \phi$. Put

$$G = \bigsqcup_{n \in \mathbb{N}} G'_n, \quad F = \bigsqcup_{n \in \mathbb{N}} F'_n$$

Then we have $(f_A)_1 \sqsubseteq G$, $(f_A)_2 \sqsubseteq F$, and $G \sqcap F = \phi$. Therefore, (X, τ) is fuzzy soft normal. □

Corollary 3.20. *Each fuzzy soft regular and fuzzy soft second-countable space is fuzzy soft normal.*

Definition 3.21 ([11])**.** Let (X, τ) be a fuzzy soft topological space, where $\tau = \{(f_A)_\lambda : \lambda \in \delta\}$. Then the collection $\tau_{e_i} = \{(f_A)_\lambda(e_i) : (f_A)_\lambda \in \tau\}$, for each $e_i \in E$, defines a fuzzy topology on X. This fuzzy topology is called e_i-parameter topology.

Proposition 3.22. *Let (X, τ) be a fuzzy soft topological space over the parameter set E. If we take $E = \{e\}$. Then (X, τ) is fuzzy soft Lindelöf if and only if the fuzzy topological space (X, τ_e) is fuzzy Lindelöf.*

4. Conclusions

In the present work, we have presented some concepts such as fuzzy soft first-countable space, fuzzy soft second-countable space and fuzzy soft Lindelöf space. Moreover, we have established several useful results and its fundamental properties in detail with the help of some examples. We hope that the concepts and results discussed in this manuscript will certainly help researcher to enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life.

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