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Pareto-optimal solutions in multi-objective linear programming with fuzzy numbers

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Abstract. Supply chain has attracted many interests in operations research area. The supplier selection problem is one of the most interesting problem in supply chain subject and its solution process is related to a multi-objective programming model. In this paper, we consider a Multi-Objective Linear Programming with Fuzzy Numbers (FNMOLP) problem. In fact, we try to reduce the multi-objective linear programming with fuzzy numbers to the classical multi-objective linear programming using a linear ranking function. Finally we solve the obtained Multi-Objective Linear Programming (MOLP) by max-min operator and weighted method. We finally illustrate the mentioned approach with presenting a case study which is formulated from an automobile industrial.

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1. Introduction

One of the most important study discussions in supply chain is the supplier selection. The issue of assigning purchase orders to suppliers that act differently in terms of quality, cost, services, etc criteria is one of the significate concerns of purchase managers in supply chain. To adopt optimal decision, in this regard is basically related to a multi-objective problem that the objectives are contradicting each other and have different importance and priority depending on the location. In practice, Decision Maker (DM) faces to a kind of ambiguity and compiexity of the information related to decision criteria and constraints. In this regard, the emergence of fuzzy set theory as a tool to describe such conditions besides presenting question model realistically, unfortunately few original works have been done in this field. For the first time in a fuzzy supplier selection problem which is considered by Amid, Ghodsypour and O'Brien [2], they developed an asymmetric fuzzy multi-objective linear model that enable the decision maker to assign different weights to various criteria in the problem. In addition, Amid Ghodsypour and O'Brien [3] presented a fuzzy weighted additive and mixed integer linear programming method in supply chain. As a result, in this paper in addition to presenting a new multi-objective fuzzy model being modeled based on assigning purchase order to suppliers in a supply chain a solution method is introduced based on using ranking function. To clarify the solution process modeling and description, a case study is included which is related to automobile parts and accessories company.

This paper is organized as follows: in Section 2, we give some necessary preliminaries about fuzzy numbers, fuzzy arithmetic operators and ranking functions. Section 3, introduces the multi-objective supplier selection model with three objective functions. Section 4 defines a fuzzy number multi-objective linear programming and gives some other useful definitions and theorems. In Section 5, we use the ranking functions, max-min approach and weighted method to get the solution. A case study is also provided in Section 6 to illustrate this study.

2. Fuzzy arithmetic operators and ranking

The fundamental fuzzy concepts and ranking of fuzzy numbers is briefly given in this section and more details can be found in [14, 15, 16, 19].

2.1. **Arithmetic on fuzzy numbers.** A fuzzy set $\tilde{a}$ in universe set X is a set $\tilde{a} = \{(\mu_{\tilde{a}}(x), x) | x \in X\}$ where $\mu_{\tilde{a}}(x)$ is a real number in interval $[0,1]$, called a membership degree from point x to $\tilde{a}$. $\mu_{\tilde{a}}(x)$ is called a membership function in fuzzy set $\tilde{a}$ and

$$\begin{aligned} \mu_{\tilde{a}} \quad : \quad & X \to [0,1] \\ & x \mapsto \mu_{\tilde{a}}(x) \end{aligned}$$

A fuzzy number is a convex normalized fuzzy set of the real line $\mathbb{R}$; whose membership function is piecewise continuous. Denote the set of fuzzy numbers on $\mathbb{R}$ by $F(\mathbb{R})$.

Definition 2.1. Let $\tilde{a}$ be a fuzzy number whose membership function can generally be defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} \mu^L_{\tilde{a}}(x) & a^L - a^\alpha \leq x \leq a^L, \\ 1 & a^L \leq x \leq a^U, \\ \mu^R_{\tilde{a}}(x) & a^U \leq x \leq a^U + a^\beta, \\ 0 & \text{otherwise,} \end{cases}$$

where $\mu^L_{\tilde{a}}(x) : [a^L - a^\alpha, a^L] \to [0,1]$ and $\mu^R_{\tilde{a}}(x) : [a^U, a^U + a^\beta] \to [0,1]$ are strictly monotonic and continuous mappings. Then it is consider as a left-right fuzzy number. If the membership function $\mu_{\tilde{a}}(x)$ is piecewise linear, then it is referred to as a trapezoidal fuzzy number and is denoted by $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$, where $(a^L - \alpha, a^U + \beta)$ is the support of $\tilde{a}$ and $[a^L, a^U]$ is its core. If $a^L = a^U$, then the trapezoidal fuzzy number is turned into a triangular fuzzy number $\tilde{a} = (a^L - a^\alpha, a^L, a^L + a^\beta)$.
Let $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ be two trapezoidal fuzzy numbers.

Now define

$$x \cdot \tilde{a} = (xa^L, xa^U, xa^\alpha, xa^\beta); \quad x \geq 0, x \in \mathbb{R},$$
$$x \cdot \tilde{a} = (xa^U, xa^L, -xa^\beta, -xa^\alpha); \quad x < 0, x \in \mathbb{R},$$
$$\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, a^\alpha + b^\alpha, a^\beta + b^\beta).$$

2.2. **Ranking of fuzzy numbers.** Ranking of fuzzy numbers is an important issue in the study of fuzzy set theory where fuzzy mathematical programming is one of the useful application of this approach [13, 17, 18]. There are numerous methods proposed in the literature for the ranking of fuzzy numbers. Dubois and Prade used maximizing sets to order fuzzy numbers and proposed the ranking of fuzzy numbers in the setting of possibility theory. Efstathiou and Tang in 1982 [7] used a decision theoretic approach for ranking of fuzzy sets. Nowadays, many researchers have developed methods to compare and to rank fuzzy numbers, e.g., ranking fuzzy numbers with an area method using radius of gyration [4], ranking based on deviation degree [20] and so on. Here we describe only three simple methods for the ordering of fuzzy numbers.

An effective approach for ordering the elements of $F(\mathbb{R})$ is to define a ranking function $R : F(\mathbb{R}) \to \mathbb{R}$ which maps each fuzzy number into the real line, where a natural order exists. We define orders on $F(\mathbb{R})$ as follows:

$$\tilde{a} \succeq \tilde{b} \iff R(\tilde{a}) \geq R(\tilde{b}),$$
$$\tilde{a} \succ \tilde{b} \iff R(\tilde{a}) > R(\tilde{b}),$$
$$\tilde{a} \simeq \tilde{b} \iff R(\tilde{a}) = R(\tilde{b}),$$

where $\tilde{a}$ and $\tilde{b}$ are in $F(\mathbb{R})$. We write $\tilde{a} \preceq \tilde{b}$ if and only if $\tilde{b} \succeq \tilde{a}$.
We restrict our attention to linear ranking function R such that

$$R(k\tilde{a} + \tilde{b}) = kR(\tilde{a}) + R(\tilde{b}), \quad \forall\ \tilde{a}, \tilde{b}\ \in F(\mathbb{R}), \quad k \in \mathbb{R}.$$

Without any loss of generality, we will only calculate the second Yager's ranking function for trapezoidal fuzzy numbers. Yager proposed the following linear raking function (see in [11]):

$$R_Y(\tilde{a}) = \frac{1}{2}\int_0^1 (\inf[\tilde{a}]_\alpha + \sup[\tilde{a}]_\alpha) d\alpha.$$

The second approach called the k-Preference index approach suggested by Adamo [1]. Let $\tilde{a}$ be the given fuzzy number and $k \in [0, 1]$. The k-preference index of $\tilde{a}$ is defined as $F_k(\tilde{a}) = max\,\{x : \mu_{\tilde{a}}(x) \geq k\}$. Now, using this k-preference index, for two fuzzy numbers $\tilde{a}$, $\tilde{b}$, $\tilde{a} \preceq \tilde{b}$ with degree $k \in [0, 1]$ if and only if $F_k(\tilde{a}) \leq F_k(\tilde{b})$.
The third approach for ranking of fuzzy numbers is based on possibility theory. In fact Dubois and Prade [5] studied the ranking of fuzzy numbers in the setting of possibility theory. To develop this, suppose we have two fuzzy number $\tilde{a}$ and $\tilde{b}$. Then in accordance with the extension principle of Zadeh, the crisp inequality $x \leq y$ can be extended to obtain the truth value of the assertion that $\tilde{a}$ is less than or equal to $\tilde{b}$, as follows:

$$T(\tilde{a} \preceq \tilde{b}) = \sup_{x \leq y} \left(min(\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)) \right).$$

This truth value $T(\tilde{a} \preceq \tilde{b})$ is also called the grade of possibility of dominance of $\tilde{b}$ on $\tilde{a}$ and is denoted by $Poss(\tilde{a} \preceq \tilde{b})$. Now define $\tilde{a} \preceq \tilde{b}$ if and only if $Poss(\tilde{a} \preceq \tilde{b}) \leq Poss(\tilde{b} \preceq \tilde{a})$.
Now, since each fuzzy number linear programming problem can be changed to equivalent crisp linear programming problem using linear ranking function, we use here the first class for ranking of fuzzy numbers.

3. Multi-objective linear programming with fuzzy numbers

Let a Multi-Objective Linear Programming (MOLP) problem with k objective functions be as follows:

$$\begin{aligned} \min \quad & Z(x) = (\sum_{j=1}^{n} c_{1j}x_j, \sum_{j=1}^{n} c_{2j}x_j, \ldots, \sum_{j=1}^{n} c_{kj}x_j) \\ s.t. \quad & \sum_{j=1}^{n} a_{ij}x_j \geq b_i, \quad i = 1, \ldots, m \\ & x_j \geq 0, \quad j = 1, \ldots, n, \end{aligned} \tag{3.1}$$

or

$$\begin{aligned} \min \quad & Z(x) = (Z_1(x), Z_2(x), \ldots, Z_k(x)) = (c_1x, c_2x, \ldots, c_kx) \\ s.t. \quad & Ax \geq b, \\ & x \geq 0. \end{aligned}$$

Linear programming problems with fuzzy parameters can be considered as linear programming problems with fuzzy numbers (FNLP) and linear programming problems with fuzzy variables (FVLP) [10, 12]. In this section, we consider a multi-objective linear programming with fuzzy numbers shortly denoted by FNMOLP .

Definition 3.1. A Fuzzy Number Multi-Objective Linear Programming (FNMOLP) is defined as follows:

$$\begin{aligned} \min \quad & \tilde{Z}(x) = \{\tilde{Z}_1(x), \tilde{Z}_2(x), \ldots, \tilde{Z}_k(x)\} \simeq \{\tilde{c}_1x, \tilde{c}_2x, \ldots, \tilde{c}_kx\} \\ s.t. \quad & \tilde{A}x \succeq \tilde{b}, \\ & x \geq 0, \end{aligned} \tag{3.2}$$

where $\tilde{b} \in (F(\mathbb{R}))^m, x \in \mathbb{R}^n, \tilde{A} = (\tilde{a}_{ij})_{m\times n} \in (F(\mathbb{R}))^{m\times n}, \tilde{c}_l^T = (\tilde{c}_{l1}, \ldots, \tilde{c}_{ln})^T \in (F(\mathbb{R}))^n$, for $l = 1, \ldots k$.

Some definitions are given as follows which will be used later in throughout of the paper (see [6, 9]):

Definition 3.2. We say that the real number a corresponds to the fuzzy number $\tilde{a}$, with respect to a given linear ranking function R, if $a = R(\tilde{a})$.

Definition 3.3. We say that a vector $x \in \mathbb{R}^n$ is a feasible solution to the FNMOLP problem (3.2) if and only if x satisfies the constraints of the problem.

Definition 3.4. A feasible solution x_* is a pareto-optimal solution to MOLP problem (3.1) if and only if for all feasible solutions x to (3.1), we have $z_l(x_*) \leq z_l(x)$, for all $l = 1, \ldots, k$ and $z_l(x_*) < z_l(x)$, for at least one t.

The following theorem shows that any FNMOLP can be reduced to the classical multi-objective linear programming.

Theorem 3.5. *The multi-objective linear programming problem in (3.1) and the FNMOLP problem in (3.2) are equivalent, where a_{ij}, b_i, c_{lj}, $l = 1, \ldots, k$ in (3.1) are real numbers corresponding to the fuzzy numbers $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_{lj}$, $l = 1, \ldots, k$ in (3.2) with respect to a given linear ranking function R, respectively.*

Proof. It is straightforward. (see for more details [6, 11]) □

Remark 3.6. The equivalence also holds even if the x_j's are unrestricted.

4. Fuzzy multi-objective supplier selection model

A typical multi-objective supplier selection problem which is used in this article can be stated as follows:

$$\begin{aligned}
\tilde{z}_1 &= \sum_{i=1}^{n} \tilde{P}_i x_i \\
\tilde{z}_2 &= \sum_{i=1}^{n} \tilde{F}_i x_i \\
\tilde{z}_3 &= \sum_{i=1}^{n} \tilde{S}_i x_i \\
s.t. \quad & \sum_{i=1}^{n} \tilde{Q}_i x_i \leq \tilde{G}, \\
& \sum_{i=1}^{n} x_i \simeq \tilde{D}, \\
& x_i \leq c_i, \\
& x_i \geq 0 \text{ and integer}, \\
& i = 1, 2, \ldots, n,
\end{aligned}$$

where D is demand over period x_i is the number of units purchased from the ith-supplier, $\tilde{z}_1$ is the objective function of the minimized production time, $\tilde{z}_2$ is the objective function of the minimized waste and $\tilde{z}_3$ is the objective function of the maximized reliability of delivery times. The first constraint is related to the total variable cost, the second constraint is related to the company demand and the other constraints are written based on opinion of supplier and suppliant. Now apply a linear ranking function conclude the following equivalent classical multi-objective linear programming:

$$
\begin{aligned}
z_1 &= \sum_{i=1}^{n} P_i x_i \\
z_2 &= \sum_{i=1}^{n} F_i x_i \\
z_3 &= \sum_{i=1}^{n} S_i x_i \\
s.t. \quad & \sum_{i=1}^{n} Q_i x_i \leq G, \\
& \sum_{i=1}^{n} x_i = D, \\
& x_i \leq c_i, \\
& x_i \geq 0 \text{ and integer}, \\
& i = 1, 2, \ldots, n,
\end{aligned}
$$

where P_i, F_i, S_i, Q_i, G and D are the corresponding real numbers to the fuzzy numbers $\tilde{P}_i, \tilde{F}_i, \tilde{S}_i, \tilde{Q}_i, \tilde{G}$ and $\tilde{D}$, $(i = 1, \ldots, n)$ respectively, which are conclude based on the linear ranking function as well as Yager's ranking function.

Now we are in a position that presents a solution process for solving the main fuzzy multi-objective linear programming using linear ranking functions. Let us indicate that we are going to solve the obtained MOLP problem (3.2) by max-min operator and weighted method. Here, we explain the method which is used by Amid, Ghodsypour and O'Braien and taken from [3]:

For each objective function in (3.2), we define the membership function

$$
\mu_l(Z_l(x)) = \frac{Z_l(x) - Z_l^-}{Z_l^* - Z_l^-}, \quad l = 1, \ldots, k, \tag{4.1}
$$

where Z_l^* and Z_l^- are ideal solution (best solution) and anti-ideal solution (worst solution) of problem (3.1) respectively, and they will be obtained through solving a single objective optimization problem individually under each objective function. (see [9])

Max-min operator, solves the model

$$
\begin{aligned}
\max \quad & \nu \\
s.t. \quad & \mu_l(Z_l(x)) \geq \nu, \quad l = 1, \ldots, k \\
& x \geq 0,
\end{aligned} \tag{4.2}
$$

where $\mu_l(Z_l(x))$ for $l = 1, \ldots, k$ are defined in (4.1).

In the definition of the fuzzy decision, there is no different between the goals. Therefore, depending on some problems, situation in which fuzzy goals have unequal importance to decision maker and other patterns, as the confluence of objective, should be considered. The weighted additive model can handle this problem, where w_j's are the weighting coefficients that present the relative importance among the goals.

Then we solve the following model

$$\begin{aligned}
\max \quad & \sum_{l=1}^{k} w_l \nu_l \\
s.t. \quad & \mu_l(Z_l(x)) \geq \nu_l, \quad l = 1, \ldots, k \\
& \sum_{l=1}^{k} w_l = 1 \\
& x \geq 0,
\end{aligned} \tag{4.3}$$

where the parameters of the problem are the same as defined in (4.2).
Now, any optimal solution x_{**} to problem (4.3) is a pareto-optimal solution to the MOLP problem (3.2). Since the proof is given in [8], we omit it here.
In the next section, we present a case study to illustrate our results.

5. Case study

In this section, an application of the proposed model in the context of supplier selection as one of the important area in operations research is discussed. The company that we are going to study on it, is one of the active company in the field of spare parts, automobile parts and accessories in Iran and active in polymer and plastic parts. The company provides its products to order of supplier companies in the automobile industrial for using in the assemble. This company to supply of one of its pants, have 10 suppliers and also the aim of company is selection of 4 suppliers from 10, so that the production time and the waste are minimized and also reliability of delivery times is maximized. The company needs for these parts, is not determined exactly. It means that the company needs about 2400 parts per month. Also the company is willing that the total cost not be more than $\widetilde{13000}$. Data of the company for 4 factors including of the production time, variable costs and waste for every part and reliability of delivery times is given in Table 1.

Table 1. Data of the company in the different indices

Company	Production time	Variable cost	Waste	Reliability of delivery time
1	$\widetilde{239.16}$	$\widetilde{6.009}$	$\widetilde{0.333}$	$\widetilde{0.9}$
2	$\widetilde{282.14}$	$\widetilde{5.57}$	$\widetilde{0.166}$	$\widetilde{0.6}$
3	$\widetilde{224}$	$\widetilde{5.853}$	$\widetilde{0.166}$	$\widetilde{0.5}$
4	$\widetilde{221.25}$	$\widetilde{5.152}$	$\widetilde{0.416}$	$\widetilde{0.9}$
5	$\widetilde{231.25}$	$\widetilde{5.985}$	$\widetilde{0.5}$	$\widetilde{1.3}$
6	$\widetilde{339.8}$	$\widetilde{6.4444}$	$\widetilde{0.75}$	$\widetilde{1.1}$
7	$\widetilde{217.5}$	$\widetilde{5.05}$	$\widetilde{0.83}$	$\widetilde{0.4}$
8	$\widetilde{211.66}$	$\widetilde{4.898}$	$\widetilde{0.95}$	$\widetilde{0.6}$
9	$\widetilde{336.25}$	$\widetilde{7.28}$	$\widetilde{0.5}$	$\widetilde{0.9}$
10	$\widetilde{348}$	$\widetilde{7.37}$	$\widetilde{0.333}$	$\widetilde{0.6}$

In this problem, the first objective function is considered for minimizing the production time, the second objective function and the third objective function are respectively considered for minimizing the waste and for maximizing the reliability of delivery times. The first and the second constraints are established for the total variable cost and for the company demand and the other constraints are written for selecting 4 companies from 10. Also these constraints ensure that the order of any supplier will be nonnegative.

Decision variables in this problem are as follows:

y_j: Select or deselect of the j-th company,

x_{j_m}: The measure of order for j-th company if it selected.

$$
\begin{aligned}
\min \quad & \tilde{z}_1 \simeq (238, 241, 1, 2)x_{1m} + (280, 285, 2, 3)x_{2m} \\
& +(220, 225, 4, 1)x_{3m} + (220, 222, 1, 1)x_{4m} \\
& +(230, 236, 1, 5)x_{5m} + (337, 340, 2, 1)x_{6m} \\
& +(215, 219, 2, 2)x_{7m} + (207, 212, 4, 1)x_{8m} \\
& +(334, 338, 2, 2)x_{9m} + (347, 349, 1, 1)x_{10m} \\
\min \quad & \tilde{z}_2 \simeq (0.3, 0.383, 0.033, 0.05)x_{1m} + (0.1, 0.19, 0.066, 0.036)x_{2m} \\
& +(0.120, 0.2, 0.046, 0.034)x_{3m} + (0.4, 0.45, 0.016, 0.034)x_{4m} \\
& +(0.4, 0.6, 0.1, 0.1)x_{5m} + (0.69, 0.80, 0.6, 0.5)x_{6m} \\
& +(0.80, 0.85, 0.03, 0.02)x_{7m} + (0.9, 0.97, 0.05, 0.02)x_{8m} \\
& +(0.45, 0.6, 0.05, 0.1)x_{9m} + (0.3, 0.343, 0.033, 0.013)x_{10m}
\end{aligned}
$$

$$
\begin{aligned}
\max \quad & \tilde{z}_3 \simeq (0.85, 0.93, 0.05, 0.03)x_{1m} + (0.5, 0.7, 0.1, 0.1)x_{2m} \\
& +(0.46, 0.51, 0.04, 0.01)x_{3m} + (0.84, 0.95, 0.06, 0.05)x_{4m} \\
& +(1.1, 1.5, 0.2, 0.2)x_{5m} + (1, 1.2, 0.1, 0.1)x_{6m} \\
& +(0.38, 0.43, 0.02, 0.03)x_{7m} + (0.55, 0.63, 0.05, 0.03)x_{8m} \\
& +(0.8, 0.93, 0.1, 0.03)x_{9m} + (0.56, 0.65, 0.04, 0.05)x_{10m} \\
\\
s.t. \quad & (5.85, 6.119, 0.159, 0.11)x_{1m} + (5.46, 5.83, 0.11, 0.26)x_{2m} \\
& +(4.012, 5.043, 0.886, 0.145)x_{3m} + (5, 5.352, 0.152, 0.2)x_{4m} \\
& +(5.781, 6.001, 0.204, 0.016)x_{5m} + (6.143, 6.743, 0.301, 0.299)x_{6m} \\
& +(5, 5.08, 0.05, 0.03)x_{7m} + (5.151, 5.989, 0.702, 0.136)x_{8m} \\
& +(7.10, 7.42, 0.18, 0.14)x_{9m} + (7.17, 7.57, 0.2, 0.2)x_{10m} \\
& \leq (12000, 14000, 1000, 1000)
\end{aligned}
$$

$$
\begin{aligned}
& x_{1m}+x_{2m}+x_{3m}+x_{4m}+x_{5m}+x_{6m}+x_{7m}+x_{8m}+x_{9m}+x_{10m} \simeq \\
& (2300, 2500, 1000, 1000) \\
& x_{jm} \geq 100\ y_j \\
& x_{jm} \leq 2400\ y_j \\
& y_1+y_2+y_3+\ldots+y_{10}=4 \\
& x_{jm} \geq 0 \quad \textit{and integer} \\
& y_1, y_2, y_3, \ldots, y_{10}=0 \ \textit{or}\ 1 \\
& j=1,2,\ldots,10
\end{aligned}
$$

Now using Yager's ranking function R_Y to the fuzzy numbers in the problem, we get the following problem:

$$
\begin{aligned}
\min \quad & z_1 = 239.75x_{1m}+282.75x_{2m}+221.75x_{3m}+221x_{4m}+234x_{5m} \\
& +338.25x_{6m}+217x_{7m}+208.75x_{8m}+336x_{9m}+348x_{10m} \\
\min \quad & z_2 = 0.34575x_{1m}+0.1375x_{2m}+0.157x_{3m}+0.4295x_{4m}+0.5x_{5m} \\
& +0.72x_{6m}+0.8225x_{7m}+0.9275x_{8m}+0.5375x_{9m}+0.3165x_{10m} \\
\max \quad & z_3 = 0.885x_{1m}+0.6x_{2m}+0.4775x_{3m}+0.87x_{4m}+1.3x_{5m} \\
& +1.1x_{6m}+0.4075x_{7m}+0.585x_{8m}+0.8475x_{9m}+0.6075x_{10m} \\
\\
s.t. \quad & 5.97225x_{1m}+5.6825x_{2m}+4.34225x_{3m}+5.188x_{4m}+5.844x_{5m} \\
& +6.4425x_{6m}+5.035x_{7m}+5.4285x_{8m}+7.25x_{9m}+7.37x_{10m} \leq 13000 \\
& x_{1m}+x_{2m}+x_{3m}+x_{4m}+x_{5m}+x_{6m}+x_{7m}+x_{8m}+x_{9m}+x_{10m} \\
& = 2400 \\
& x_{jm} \geq 100\ y_j \\
& x_{jm} \leq 2400\ y_j \\
& y_1+y_2+y_3+\ldots+y_{10}=4 \\
& x_{jm} \geq 0 \quad \textit{and integer} \\
& y_1, y_2, y_3, \ldots, y_{10}=0 \ \textit{or}\ 1 \\
& j=1,2,\ldots,10
\end{aligned}
$$

$w_j (j=1,2,3)$ are the associated weights with the j-th objective. In this case study, the assumed decision maker's relative importance or the weights of the fuzzy goals are given as:

$$w_1 = 0.25, \quad w_2 = 0.25, \quad w_3 = 0.50$$

Based on the convex fuzzy decision-making (4.3) and the weights which are given by decision maker, the crisp single objective formulation for the case study is as follows:

$$
\begin{aligned}
\max \quad & 0.25v_1 + 0.25v_2 + 0.50v_3 \\
s.t. \quad & v_1 \leq \frac{636966.3975 - z_1(x)}{132616.3975} \\
& v_2 \leq \frac{2074.52 - z_2}{1699.438} \\
& v_3 \leq \frac{z_3 - 14.4}{1367.26} \\
& 5.97225x_{1m} + 5.6825x_{2m} + 4.34225x_{3m} + 5.188x_{4m} + 5.844x_{5m} \\
& +6.4425x_{6m} + 5.035x_{7m} + 5.4285x_{8m} + 7.25x_{9m} + 7.37x_{10m} \leq 13000 \\
& x_{1m} + x_{2m} + x_{3m} + x_{4m} + x_{5m} + x_{6m} + x_{7m} + x_{8m} + x_{9m} + x_{10m} \\
& = 2400 \\
& x_{jm} \geq 100\ y_j \\
& x_{jm} \leq 2400\ y_j \\
& y_1 + y_2 + y_3 + \cdots + y_{10} = 4 \\
& x_{jm} \geq 0 \quad \textit{and integer} \\
& y_1, y_2, y_3, \ldots, y_{10} = 0 \ \textit{or}\ 1 \\
& j = 1, 2, \ldots, 10 \\
& 0 \leq v_1 \leq 1, \quad 0 \leq v_2 \leq 1, \quad 0 \leq v_3 \leq 1
\end{aligned}
$$

Finally, using the software of LINGO, we can solve this problem. Then, the pareto optimal solutions for the above formulation is obtained as follows:

$$
\begin{aligned}
x_{1m} &= 100, & x_{2m} &= x_{6m} = 0, & v_1 &= 0.6037047, \\
x_{3m} &= 323, & x_{7m} &= x_{8m} = 0, & v_2 &= 0.5930516, \\
x_{4m} &= 101, & x_{9m} &= x_{10m} = 0, & v_3 &= 1, \\
x_{5m} &= 1876.
\end{aligned}
$$

Now it is observed that the major part of purchase has been made from the fifth supplier which has the highest level of reliability, as the weight of third objective function (reliability) is more than the other ones.

6. Conclusion

In a multi-objective linear programming problem, it is unlikely that all objective functions simultaneously achieve their optimal values. In particular, if it includes fuzziness in the coefficients and the right-hand-sides of the constraints, in practice it is difficult for the decision maker to choose a satisfying solution. In this paper, we first reduced the original problem to the one without fuzzy parameters and then solved it by the solution process as well as discussed in Section 4. We emphasize that the same process can be used when a real problem can be formulated as the model which is discussed in the paper. We also understood that the mentioned approach is very simple to solve the original model as well as crisp environment.

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