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On soft separation axioms

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Abstract. We showed that if a soft topological space (X, τ, E) is a soft T_0 space, then topological space (X, τ_α) is a T_0 space for which $\alpha \in E$ and we showed that if a soft topological space (X, τ, E) is a soft T_1 space, then topological space (X, τ_α) is a T_1 space for which $\alpha \in E$. Won Keun Min (2011) [7, Theorem 3.21] indicated that if a soft topological space (X, τ, E) is soft T_3 space, then (x, E) is soft closed set for each $x \in X$. We developed this result and we showed that if a soft topological space (X, τ, E) is a soft T_2 space, then (x, E) is soft closed set for each $x \in X$.

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1. Introduction

None mathematical tools can successfully deal with the several kinds of uncertainties in complicated problems in engineerig, economics, environment, sociology, medical science, etc, so Molodtsov [12] introduced the concept of a soft set in order to solve these problems in 1999. However, there are some theories such as theory of probability, theory of fuzzy sets [13], theory of intuitionistic fuzzy sets [1], theory of vague sets [3], theory of interval mathematics [5] and the theory of rough sets [9], which can be taken into account as mathematical tools for dealing with uncertainties. But these theories have their own difficulties. Maji et al. [6] introduced a few operators for soft set theory and made a more detailed theoretical study of the soft set theory. Recently, study on the soft set theory and its applications in different fields has been making progress rapidly [11, 4, 10]. Shabir and Naz [12] introduced the concept of soft topological spaces which are defined over an inital universe with fixed set of parameter. They indicated that a soft topological space gives a parameterized family of topological spaces and introduced the concept of soft open sets, soft closed sets, soft interior point, soft closure and soft seperation axioms. They indicated that if a soft topological space (X, τ, E) is a soft T_2 spaces, then topological space (X, τ_e) is T_2 space for all $e \in E$ ([12]).

In the present paper, firstly we show that if a soft topological space (X, τ, E) is a soft T_0 space, then topological space (X, τ_α) is a T_0 space for which $\alpha \in E$ (Theorem 3.2). Secondly, we show that if a soft topological space (X, τ, E) is a soft T_1 space then, topological space (X, τ_α) is a T_1 space for which $\alpha \in E$ (Theorem 3.5). Finally, in [12]it was indicated that if (x, E) is soft closed set for each $x \in X$ in a soft topological space (X, τ, E), then (X, τ, E) is a soft T_1 space and in [7, Theorem 3.21] it was indicated that if a soft topological space (X, τ, E) is soft T_3 space, then (x, E) is soft closed for each $x \in X$. In this paper, we develop Won Keun Min's this theorem, then we show that if a soft topological space (X, τ, E) is a soft T_2 space, then (x, E) is soft closed set for each $x \in X$ (Theorem 3.7).

2. Preliminaries

Definition 2.1 ([8])**.** Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For $e \in A, F(e)$ may be considered as the set of $e-$approximate elements of the soft set (F, A). Clearly, a soft set is not a set

Definition 2.2 ([6])**.** For two soft sets (F, A) and (G, B) over a common universe U, (F, A) is a soft subset of (G, B), denoted by $(F, A)\tilde{\subseteq}(G, B)$, if $A \subset B$ and $e \in A$, $F(e) \subseteq G(e)$. (F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A), $(F, A)\tilde{\supseteq}(G, B)$.

Definition 2.3 ([6])**.** Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4 ([6])**.** A soft set (F, A) over U is said to be a NULL soft set denoted by $\tilde{\varnothing}$ if for all $e \in A$, $F(e) = \varnothing$ (null set).

Definition 2.5 ([6])**.** A soft set (F, A) over U is said to be an absolute soft set denoted by $\tilde{A}$ if for all $e \in A, F(e) = U$. Clearly $\tilde{A}^c = \tilde{\varnothing}$ and $\tilde{\varnothing}^c = \tilde{A}$

Definition 2.6 ([6])**.** The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.7 ([2])**.** The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A)\tilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.8 ([12])**.** The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\widetilde{\setminus}(G, E)$, is defined as $H(e) = F(e)\setminus G(e)$ for all $e \in E$

Definition 2.9 ([12])**.** Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$, $x \notin (F, E)$,if $x \notin F(\alpha)$ for some $\alpha \in E$

Definition 2.10 ([12])**.** Let Y be a non-empty subset of X, then $\tilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by $\tilde{X}$.

Definition 2.11 ([12])**.** Let $x \in X$, then (x, E) denotes the soft set over X for which $x(e) = \{x\}$, for all $e \in E$.

Definition 2.12 ([12])**.** The relative complement of a soft set (F, A) is denoted by $(F, A)\prime$ and is defined by $(F, A)\prime = (F\prime, A)$ where $F\prime : A \to P(U)$ is a mapping given by $F\prime(e) = U - F(e)$ for all $e \in A$.

Definition 2.13 ([12])**.** Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

(1) $\breve{\varnothing}, \breve{X}$ belong to τ
(2) the union of any number of soft sets in τ belongs to τ
(3) the intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.14 ([12])**.** Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition 2.15 ([12])**.** Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ

Proposition 2.16 ([12])**.** *Let (X, τ, E) be a soft space over X. Then*

(1) $\tilde{\varnothing}$, $\tilde{X}$ are closed soft sets over X
(2) the intersection of any number of soft closed sets is a soft closed set over X
(3) the union of any two soft closed sets is a soft closed set over X.

Proposition 2.17 ([12])**.** *Let (X, τ, E) be a soft space over X. Then the collection $\tau_e = \{F(e)|(F, E) \in \tau\}$ for each $e \in E$, defines a topology on X.*

Definition 2.18 ([12])**.** Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \notin (F, E)$ or $y \in (G, E)$, $x \notin (G, E)$, then (X, τ, E) is called a soft T_0 space.

Definition 2.19 ([12])**.** Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \notin (F, E)$ and $y \in (G, E)$, $x \notin (G, E)$, then (X, τ, E) is called a soft T_1 space.

Definition 2.20 ([12])**.** Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E)\widetilde{\cap}(G, E) = \widetilde{\varnothing}$, then (X, τ, E) is called a soft T_2 space.

Proposition 2.21 ([12]). *Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft T_2 space, then (X, τ_e) is a T_2 space for each $e \in E$.*

Theorem 2.22 ([7]). *Let (X, τ, E) be a soft topological space over X and $x \in X$. If X is a soft T_2 space, then $(x, E) = \widetilde{\cap}(F, E)$ for each soft open set (F, E) with $x \in (F, E)$.*

Corollary 2.23 ([7]). *Let (X, τ, E) be a soft topological space over X and $x \in X$. If X and E are finite, and if X is a soft T_2 space, then (x, E) is a soft open set for $x \in X$.*

3. Main results

Now, we will give the following Remark to establish our one of the main theorems in this section.

Remark 3.1. Let (X, τ, E) be a soft T_0 space. Then there exist soft open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \notin (F, E)$ or $y \in (G, E)$, $x \notin (G, E)$ from Definition 2.18. Also we know that for each $e \in E$, (X, τ_e) is a topological space from Proposition 2.17. Then we can see that clearly, since $x \in (F, E)$, there exists open set $F(e)$ in τ_e such that $x \in F(e)$ for all $e \in E$; and since $y \notin (F, E)$, there exists open set $F(e_i)$ in τ_{e_i} such that $y \notin F(e_i)$ for $e_i \in E$, $i \in I$. Or similarly since $y \in (G, E)$, there exists open set $G(e)$ in τ_e such that $y \in G(e)$ for all $e \in E$; and since $x \notin (G, E)$, there exist open sets $G(e_j)$ in τ_{e_j} such that $x \notin G(e_j)$ for $e_j \in E$, $j \in I$.

Theorem 3.2. *Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$ and let $i, j \in I$ such that mentioned in Remark 3.1, $e \in E$. If (X, τ, E) is a soft T_0 space, then at least one of (X, τ_{e_i}) and (X, τ_{e_j}) are T_0 spaces.*

Proof. Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$ and let $i, j \in I$ such that mentioned in Remark 3.1, $e \in E$. Give us (X, τ, E) is a soft T_0 space. We can see that clearly from Remark3.1, there exists open set $F(e_i)$ in τ_{e_i} such that $x \in F(e_i)$, $y \notin F(e_i)$. Or similarly there exists open set $G(e_j)$ in τ_{e_j} such that $y \in G(e_j)$, $x \notin G(e_j)$. As a consuquence, at least one of (X, τ_{e_i}) and (X, τ_{e_j}) are T_0 spaces. □

Example 3.3. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E)\}$ where

$F_1(e_1) = \{x, y\}$, $F_1(e_2) = \{x\}$,
$F_2(e_1) = \{y\}$, $F_2(e_2) = \{x\}$.

Then, (X, τ, E) is a soft topological space over X. We note that, (X, τ, E) is a soft T_0 space because there exist soft open set (F_1, E) such that $x \in (F_1, E)$ and $y \notin (F_1, E)$. We can see that (X, τ_{e_1}) is T_0 space because there exist open set $F_2(e_1)$ such that $y \in F_2(e_1)$ and $x \notin F_2(e_1)$.(see $\tau_{e_1} = \{\varnothing,\ X, \{y\}\}$). Also we can see that τ_{e_2} is a T_0 space because there exist open set $F_1(e_2)$ such that $x \in F_1(e_2)$ and $y \notin F_1(e_2)$.(see $\tau_{e_2} = \{\varnothing, X, \{x\}\}$.

Now, we will give the following Remark to establish our one of the main theorems in this section.

Remark 3.4. Let (X, τ, E) be a soft T_1 space, then there exist soft open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \notin (F, E)$ and $y \in (G, E)$, $x \notin (G, E)$ from Definition 2.19. Also we know that for each $e \in E$, (X, τ_e) is a topological space from Proposition 2.17. Then we can see that clearly, since $x \in (F, E)$, there exists open set $F(e)$ in τ_e such that $x \in F(e)$ for all $e \in E$; and since $y \notin (F, E)$, there exists open set $F(e_i)$ in τ_{e_i} such that $y \notin F(e_i)$ for $e_i \in E$, $i \in I$. And similarly since $y \in (G, E)$, there exists open set $G(e)$ in τ_e such that $y \in G(e)$ for all $e \in E$; and since $x \notin (G, E)$, there exists open set $G(e_j)$ in τ_{e_j} such that $x \notin G(e_j)$ for $e_j \in E$, $j \in I$.

Theorem 3.5. *Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$ and let $i, j \in I$ such that mentioned in Remark 3.4, $e \in E$. Let $k, l \in I$ such that $e_{i_k} = e_{j_l}$. If (X, τ, E) is a soft T_1 space, then $(X, \tau_{e_{i_k}})$ are T_1 spaces.*

Proof. Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$ and let $i, j \in I$ such that mentioned in Remark 3.4, $e \in E$. Give us (X, τ, E) is a soft T_1 space. We can see that clearly from Remark 3.4, there exists open set $F(e_i)$ in τ_{e_i} such that $x \in F(e_i)$, $y \notin F(e_i)$. And similarly there exists open set $G(e_j)$ in τ_{e_j} such that $y \in G(e_j)$, $x \notin G(e_j)$. As a consuquence, there exist open sets $F(e_{i_k})$ and $G(e_{i_k})$ in $\tau_{e_{i_k}}$ such that $x \in F(e_{i_k})$, $y \notin F(e_{i_k})$ and $y \in G(e_{i_k})$, $x \notin G(e_{i_k})$ for $k, l \in I$ such that $e_{i_k} = e_{j_l}$. Hence, $(X, \tau_{e_{i_k}})$ are T_1 spaces. □

Example 3.6. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where

$F_1(e_1) = \{x, y\}, \quad F_1(e_2) = \{x\},$
$F_2(e_1) = \{y\}, \quad F_2(e_2) = \{y\},$
$F_3(e_1) = \{y\}, \quad F_3(e_2) = \varnothing$

We note that (X, τ, E) is a soft T_1 space because there exist soft open sets (F_1, E) and (F_2, E) such that $x \in (F_1, E)$, $y \notin (F_1, E)$ and $y \in (F_2, E)$, $x \notin (F_2, E)$. We can see that (X, τ_{e_1}) is not T_1 space because of $\tau_{e_1} = \{\varnothing, X, \{y\}\}$. Also we can see that, (X, τ_{e_2}) is a T_1 space because of $\tau_{e_2} = \{\varnothing, X, \{x\}, \{y\}\}$.

Theorem 3.7. *Let (X, τ, E) be a soft topological space over X and $x \in X$. If (X, τ, E) is a soft T_2 space then, (x, E) is a soft closed set in X.*

Proof. Let $x \in X$ and $y \in X - \{x\}$, then, $x \neq y$. Since (X, τ, E) is a soft T_2 space, there exist soft open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \in (G, E)$ and $(F, E) \widetilde{\cap} (G, E) = \widetilde{\varnothing}$. Since $(G, E) \widetilde{\cap} (x, E) = \widetilde{\varnothing}$, $(G, E) \widetilde{\subseteq} (x, E)'$. So, $\underset{y \in X - \{x\}}{\widetilde{\cup}} (G, E) \widetilde{\subseteq} (x, E)'$ (1). In other words, let $\underset{y \in X - \{x\}}{\widetilde{\cup}} (G, E) = (H, E)$ where $H(e) = \underset{y \in X - \{x\}}{\cup} G(e)$ for all $e \in E$. Now, we know that, from Definition 2.11 and Definition 2.12, $(x, E)' = (x', E)$ where $x'(e) = X - \{x\}$ for each $e \in E$. Then, for each $y \in X - \{x\}$ and for each $e \in E$, $x'(e) = X - \{x\} = \underset{y \in X - \{x\}}{\cup} \{y\} = \underset{y \in X - \{x\}}{\cup} y(e) \subset \underset{y \in X - \{x\}}{\cup} G(e) = H(e)$. This implies that $(x, E)' \widetilde{\subseteq} \underset{y \in X - \{x\}}{\widetilde{\cup}} (G, E)$ (2) from Definition 2.2. So $(x, E)^c = \underset{y \in X - \{x\}}{\widetilde{\cup}} (G, E)$ from (1) and (2). Since (G, E) is soft open for each $y \in X - \{x\}$, $(x, E)'$ is soft open. Therefore, (x, E) is soft closed set in X. □

Corollary 3.8. *Let* (X, τ, E) *be a soft topological space over* X *and* $x \in X$. *If* (X, τ, E) *is a soft* T_2 *space, the union of finite number of* (x, E) *is soft closed set in* X.

Theorem 3.9. *Let* (X, τ, E) *be a soft topological space over* X *and* $x \in X$. *And let* X *and* E *are finite. If* (X, τ, E) *is a soft* T_2 *space,* (x, E) *is soft open and soft closed set in* X, *for each* $x \in X$.

Proof. It is obvious that Corollary 2.23 and Corollary 3.8. □

Corollary 3.10. *Let* (X, τ, E) *be a soft topological space over* X *and* $x \in X$. *And let* X *and* E *are finite. If* (X, τ, E) *is a soft* T_2 *space, the union of finite number of* (x, E) *is soft closed and soft open set in* X, *for each* $x \in X$.

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