Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 9, No. 5, (May 2015), pp. 801–815
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

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On lower and upper α-continuous intuitionistic fuzzy multifunctions

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Received 9 June 2014; Revised 14 October 2014; Accepted 12 Nov 2014

Abstract. In this paper we introduce and characterize the concepts of upper and lower α-continuous intuitionistic fuzzy multifunctions from a topological space to an intuitionistic fuzzy topological space.

2010 AMS Classification: 54A99, 03E99

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy multifunctions, Lower α-continuous and upper α-continuous Intuitionistic fuzzy multifunctions.

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1. Introduction

The study of multifunctions was initiated by Strother [26]under the name of multivalued functions in the year 1951. He further studied the continuity of multivalued functions [27] in the year 1955. A systematic study of continuity of multifunctions was given by Berge in 1959[5]. Multifunctions are indispensable tools in stability and sensitivity analysis of mathematical problems. Multifunctions implemented in optimal control theory, especially differential inclusions and related subjects as game theory, where the Kae kutani fixed-point theorem for multifunctions has been applied to prove existence of Nash equilibria. This amongst many other properties loosely associated with approximability of upper hemicontinuous multifunctions via continuous functions explains why upper hemicontinuity is more preferred than lower hemicontinuity. Nevertheless, lower hemicontinuous multifunctions usually possess continuous selections as stated in the Michael selection theorem, which provides another characterization of paracompact spaces. Other selection theorems like Bressan-Colombo directional continuous selection, Kuratowski-Ryll-Nardzewski measurable selection, Aumann measurable selection, Fryszkowski selection for decomposable maps are important in optimal control and the theory of differential inclusions. In Physics, multifunctions form the mathematical basis for Dirac's magnetic monopoles, for the theory of defects in crystal and the resulting plasticity of materials, for vortices in superfluids and superconductors, and for phase transitions in these systems, for instance melting and quark confinement. They are the origin of gauge field structures in many branches of physics.

After the introduction of fuzzy sets by Zadeh [33] in 1965 and fuzzy topology by Chang in [7], several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1, 2, 3] as a generalization of fuzzy sets. In the last 27 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [8]introduced the concept of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [10], fuzzy connectedness [32], fuzzy separation axioms [4, 13, 14], fuzzy nets and filters [25], fuzzy metric spaces [31], and fuzzy continuity have been generalized for intuitionistic fuzzy topological spaces [12]. In 1999, Ozbakir and Coker [21] introduced the concept intuitionistic fuzzy multifunctions as a generalization of fuzzy multifunction due to Evert[11] and Papageorgiou[22] and studied their lower and upper intuitionistic fuzzy semi continuity from a topological space to an intuitionistic fuzzy topological space. Recently various forms of lower and upper semi-continuity such as quasi upper and lower continuity [29], strongly upper and lower semi-continuity[30] have been studied by the authors. In the present paper we extend the concepts of lower and upper α-continuous multifunctions due to Popa and Noiri [23] to intuitionistic fuzzy multifunctions and obtain some of their characterizations and properties.

2. Preliminaries

Throughout this paper (X, τ) and (Y, Γ) represents a topological space and an intuitionistic fuzzy topological space respectively.

Definition 2.1 ([13, 19])**.** A subset A of a topological space (X, τ) is called :
(a) Semi open [13] if $A \subset Cl(Int(A))$.
(b) semi closed [13] if its complement is semi open.
(c) α-open [19]if$A \subset Int(Cl(Int(A)))$.
(d) α-closed [19] if its complement is α-open.

Remark 2.2. every open (resp. closed) set is α-open (resp. α-closed) and every α-open (resp. α-closed) set is semi open (resp. semi closed) but the converses may not be true.

The family of all α-open (resp. α-closed) subsets of topological space (X, τ) is denoted by $\alpha O(X)$ (resp. $\alpha C(X)$). The intersection of all α-closed (resp. semi closed) sets of X containing a set A of X is called the α-closure [15] (resp. semi closure[6]) of A. It is denoted by $\alpha Cl(A)$ (resp. $sCl(A)$). The union of all α-open (resp. semi open) sub sets of A of X is called the α-interior [15] (resp. semi interior[6]) of A .It is denoted by $\alpha Int(A)$ (resp. $sInt(A)$) . A subset A of X is α-closed (resp. semi closed) if and only if $A \supset Cl(Int(Cl(A)))$ (resp.$A \supset Int(Cl(A))$). A subset N of a topological space (X, τ) is called a α-neighborhood [15] of a point x of X if there exists a α-open set O of X such that $x \in O \subset N$. A is a α-open in X if and only if it is a α-neighborhood of each of its points. A subset V of X is called a α-neighborhood of a subset A of X if there exists $U \in \alpha O(X)$ such that $A \subset U \subset V$. A mapping f from a topological space (X, τ) to another topological space (X^*, τ^*) is said to be α-continuous [17, 18, 20] if the inverse image of every open set of X^* is α-open in X. Every continuous mapping is α-continuous but the converse may not be true[17]. A multifunction F from a topological space (X, τ) to another topological space (X^*, τ^*) is said to be lower α-continuous [23] (resp. upper α-continuous[23]) at a point $x_0 \in X$ if for every open set V of X^* such that $F(x_0) \cap V \neq \phi$ (resp. $F(x_0) \subset V$) there is an α-open set U of X containing x_0 such that $F(u) \cap V \neq \phi, \forall u \in U$ (resp. $F(U) \subset V$).

Lemma 2.3 ([23]). *The following properties holds for a subset A of a topological space (X, τ):*
(a) *A is α-closed in $X \Leftrightarrow sInt(Cl(A) \subset A$;*
(b) *$sInt(Cl(A)) = Cl(Int(Cl(A)))$;*
(c) *$\alpha Cl(A) = A \cup Cl(Int(Cl(A)))$.*

Lemma 2.4 ([23]). *The following are equivalent for a subset A of a topological space X:*
(a) *$A \in \alpha O(X)$,*
(b) *$U \subset A \subset Int(Cl(U))$ for some open set U.*
(c) *$U \subset A \subset sCl(U)$ for some open set U.*
(d) *$A \subset sCl(Int(A))$.*

Definition 2.5 ([1, 2, 3]). Let Y be a nonempty fixed set. An intuitionistic fuzzy set A in Y is an object having the form

$$\tilde{A} = \{< y, \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y) >: y \in Y\}$$

where the functions $\mu_{\tilde{A}}(y) : Y \to I$ and $\nu_{\tilde{A}}(y) : Y \to I$ where $I = [0, 1]$, denotes the degree of membership (namely $\mu_{\tilde{A}}(y)$) and the degree of non membership (namely $\nu_{\tilde{A}}(y)$) of each element $y \in Y$ to the set $\tilde{A}$ respectively, and $0 \leq \mu_{\tilde{A}}(y) + \mu_{\tilde{A}}(y) \leq 1$ for each $y \in Y$.

Definition 2.6 ([1, 2, 3]). Let Y be a nonempty set and the intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ be in the form $\tilde{A} = \{< y, \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y) >: y \in Y\}$, $\tilde{B} = \{< y, \mu_{\tilde{B}}(y), \nu_{\tilde{B}}(y) >: y \in Y\}$ and let $\{\tilde{A}_\alpha : \alpha \in \Lambda\}$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then:
(a) $\tilde{A} \subseteq \tilde{B}$ if $\forall y \in Y$ $[\mu_{\tilde{A}}(y) \leq \mu_{\tilde{B}}(y)$ and $\nu_{\tilde{A}}(y) \geq \nu_{\tilde{B}}(y)]$
(b) $\tilde{A} = \tilde{B}$ if $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$;
(c) $\tilde{A}^c = \{< y, \nu_{\tilde{A}}(y), \mu_{\tilde{A}}(y) >: y \in Y\}$;
(d) $\tilde{0} = \{< y, 0, 1 >: y \in Y\}$ and $\tilde{1} = \{< y, 1, 0 >: y \in Y\}$
(e) $\cap \tilde{A}_\alpha = \{< y, \wedge \mu_{\tilde{A}}(y), \vee \nu_{\tilde{A}}(y) >: y \in Y\}$
(f) $\cup \tilde{A}_\alpha = \{< y, \vee \mu_{\tilde{A}}(y), \wedge \nu_{\tilde{A}}(y) >: y \in Y\}$

Definition 2.7 ([9]). Two Intuitionistic Fuzzy Sets $\tilde{A}$ and $\tilde{B}$ of Y are said to be quasi-coincident ($\tilde{A}q\tilde{B}$ for short) if $\exists y \in Y$ such that
$\mu_{\tilde{A}}(y) > \nu_{\tilde{B}}(y)$ or $\nu_{\tilde{A}}(y) < \mu_{\tilde{B}}(y)$

Lemma 2.8 ([9]). *For any two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ of Y,*
$\rceil(\tilde{A}q\tilde{B}) \Leftrightarrow \tilde{A} \subset \tilde{B}^c$.

Definition 2.9 ([8]). An intuitionistic fuzzy topology on a non empty set Y is a family Γ of intuitionistic fuzzy sets in Y which satisfy the following axioms:

O_1. $\tilde{0}, \tilde{1} \in \Gamma$,

O_2. $\tilde{A}_1 \cap \tilde{A}_2 \in \Gamma$ for any $\tilde{A}_1, \tilde{A}_2 \in \Gamma$,

O_3. $\cup \tilde{A}_\alpha \in \Gamma$ for arbitrary family $\{\tilde{A}_\alpha : \alpha \in \Lambda\} \in \Gamma$.

In this case the pair (Y, Γ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in Γ, is known as an intuitionistic fuzzy open set in Y. The complement $\tilde{B}^c$ of an intuitionistic fuzzy open set $\tilde{B}$ is called an intuitionistic fuzzy closed set in Y.

Definition 2.10 ([9]). Let Y be a nonempty set and $c \in Y$ a fixed element in Y. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha + \beta < 1$ then,

(a) $c(\alpha, \beta) =< y, c_\alpha, c_{1-\beta} >$ is called an intuitionistic fuzzy point(IFP in short) in Y, where α denotes the degree of membership of $c(\alpha, \beta)$, and β denotes the degree of non membership of $c(\alpha, \beta)$.

(b) $c(\beta) =< y, 0, 1 - c_{1-\beta} >$ is called a vanishing intuitionistic fuzzy point (VIFP in short) in Y, where β denotes the degree of non membership of $c(\beta)$.

Definition 2.11 ([8]). Let (Y, Γ) be an intuitionistic fuzzy topological space and $\tilde{A}$ be an intuitionistic fuzzy set in Y. Then the interior and closure of $\tilde{A}$ are defined by:

$cl(\tilde{A})$=$\cap\{\tilde{K} : \tilde{K}$ is an intuitionistic fuzzy closed set in Y and $\tilde{A} \subseteq \tilde{K}\}$,

$Int(\tilde{A})$=$\cup\{\tilde{G} : \tilde{G}$ is an intuitionistic fuzzy open set in Y and $\tilde{G} \subseteq \tilde{A}\}$.

Lemma 2.12 ([7]). *For any intuitionistic fuzzy set $\tilde{A}$ in (Y, Γ) we have:*

(a) *$\tilde{A}$ is an intuitionistic fuzzy closed set in $Y \Leftrightarrow Cl(\tilde{A}) = \tilde{A}$*

(b) *$\tilde{A}$ is an intuitionistic fuzzy open set in $Y \Leftrightarrow Int(\tilde{A}) = \tilde{A}$*

(c) $Cl(\tilde{A}^c) = (Int\tilde{A})^c$

(d) $Int(\tilde{A}^c) = (Cl\tilde{A})^c$

Definition 2.13 ([21]). Let X and Y are two non empty sets. A function $F : (X, \tau) \to (Y, \Gamma)$ is called intuitionistic fuzzy multifunction if $F(x)$ is an intuitionistic fuzzy set in Y, $\forall x \in X$.

Definition 2.14 ([28]). Let $F :(X, \tau) \to (Y, \Gamma)$ is an intuitionistic fuzzy multifunction and A be a subset of X. Then $F(A) = \cup_{x\in A} F(x)$.

Lemma 2.15 ([28]). *Let $F :(X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction. Then*

(a) *$A \subseteq B \Rightarrow F(A) \subseteq F(B)$ for any subsets A and B of X.*

(b) *$F(A \cap B) \subseteq F(A) \cap F(B)$ for any subsets A and B of X.*

(c) *$F(\cup_{\alpha\in\Lambda} A_\alpha) = \cup\{F(A_\alpha) : \alpha \in \Lambda\}$ for any family of subsets $\{A_\alpha : \alpha \in \Lambda\}$ in X.*

Definition 2.16 ([21]). Let $F : (X, \tau) \to (Y, \Gamma)$ is an intuitionistic fuzzy multifunction. Then the upper inverse $F^+(\tilde{A})$ and lower inverse $F^-(\tilde{A})$ of an intuitionistic fuzzy set $\tilde{A}$ in Y are defined as follows:

$$F^+(\tilde{A}) = \{x \in X : F(x) \subseteq \tilde{A}\}$$

$$F^-(\tilde{A}) = \{x \in X : F(x) q \tilde{A}\}$$

Lemma 2.17 ([28]). *Let $F : (X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction and $\tilde{A}, \tilde{B}$ be intuitionistic fuzzy sets in Y. Then:*
(a) $F^+(\tilde{1}) = F^-(\tilde{1}) = X$,
(b) $F^+(\tilde{A}) \subseteq F^-(\tilde{A})$,
(e) $[F^-(\tilde{A})]^c = [F^+(\tilde{A})^c]$,
(d) $[F^+(\tilde{A})]^c = [F^-(\tilde{A})^c]$, *If **(e)*** $\tilde{A} \subseteq \tilde{B}$, *then* $F^+(\tilde{A}) \subseteq F^+(\tilde{B})$
(f) *If* $\tilde{A} \subseteq \tilde{B}$, *then* $F^-(\tilde{A}) \subseteq F^-(\tilde{B})$.

Definition 2.18 ([21]). An Intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ is said to be:

(a) Intuitionistic fuzzy upper semi continuous at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0) \subset \tilde{W}$ there exists an open set $U \subset X$ containing x_0 such that $F(U) \subset \tilde{W}$.

(b) Intuitionistic fuzzy lower semi continuous at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0)q\tilde{W}$ there exists an open set $U \subset X$ containing x_0 such that $F(x)q\tilde{W}, \forall x \in U$.

(c) Intuitionistic fuzzy upper semi-continuous (intuitionistic fuzzy lower semi-continuous) if it is intuitionistic fuzzy upper semi-continuous (Intuitionistic fuzzy lower semi-continuous) at each point of X.

Definition 2.19 ([30]). An Intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ is said to be:

(a) Intuitionistic fuzzy strongly upper semi continuous at a point $x \in X$, if for any intuitionistic fuzzy set $\tilde{B}$ of Y such that $F(x) \subseteq \tilde{B}$ $\exists$ a neighborhood U of x such that $U \subseteq F^+(\tilde{B})$.

(b) Intuitionistic fuzzy strongly lower semi continuous at a point $x \in X$, if for any intuitionistic fuzzy set $\tilde{B}$ of Y such that $F(x)q\tilde{B}$ $\exists$ a neighborhood U of x such that $U \subseteq F^-(\tilde{B})$.

(c) Intuitionistic fuzzy strongly upper semi-continuous (intuitionistic fuzzy strongly lower semi-continuous) if it is intuitionistic fuzzy strongly upper semi-continuous (Intuitionistic fuzzy strongly lower semi-continuous) at each point of X.

Remark 2.20 ([30]). Every intuitionistic fuzzy strongly lower semi continuous (resp. intuitionistic fuzzy strongly upper semi continuous) multifunction is intuitionistic fuzzy lower semi continuous (resp. intuitionistic fuzzy upper semi continuous) but the converse may not be true.

Definition 2.21 ([29]). An Intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ is said to be:

(a) Intuitionistic fuzzy lower quasi-continuous at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0)q\tilde{W}$ there exists $U \in SO(X)$ containing x_0 such that $F(x)q\tilde{W}, \forall x \in U$.

(b) Intuitionistic fuzzy upper quasi-continuous at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0) \subseteq \tilde{W}$ there exists $U \in SO(X)$ containing x_0 such that $F(U) \subseteq \tilde{W}, \forall x \in U$.

(c) Intuitionistic fuzzy upper quasi-continuous (intuitionistic fuzzy lower quasi-continuous) if it is intuitionistic fuzzy upper quasi-continuous (Intuitionistic fuzzy lower quasi-continuous) at each point of X.

3. Lower α-continuous intuitionistic fuzzy multifunctions

Definition 3.1. An Intuitionistic fuzzy multifunction $F:(X,\tau)\to(Y,\Gamma)$ is said to be:

(a) Intuitionistic fuzzy lower α-continuous at a point $x_0\in X$, if for any intuitionistic fuzzy open set $\tilde{W}\subset Y$ such that $F(x_0)q\tilde{W}$ there exists $U\in\alpha O(X)$ containing x_0 such that $F(x)q\tilde{W}$, $\forall x\in U$.

(b) Intuitionistic fuzzy lower α-continuous if it is intuitionistic fuzzy lower α-continuous at every point of X.

Remark 3.2. Every intuitionistic fuzzy lower semi-continuous intuitionistic fuzzy multifunction is intuitionistic fuzzy lower α-continuous but the converse may not be true. For,

Example 3.3. Let $X=\{a,b,c\}, Y=[0,1]$ and let $\tau=\{\phi,\{a\},X\}$ and

$$\Gamma=\{\tilde{0},\tilde{1},C_{(1/2,1/2)},C_{(1/3,2/3)}\}$$

are topology and intuitionistic fuzzy topology on X and Y respectively. We use the notion $C_{(\alpha,\beta)}(0\le\alpha\le1, 0\le\beta\le1$ and $0\le\alpha+\beta\le1)$ to denote the constant intuitionistic fuzzy sets such that $C_{(\alpha,\beta)}(y)=\{<y,\alpha,\beta>;\forall y\in Y\}$. Then the intuitionistic fuzzy multifunction $F:(X,\tau)\to(Y,\Gamma)$ defined by $F(a)=C_{(5/6,1/6)}$, $F(b)=C_{(1/2,1/2)}$ and $F(c)=C_{(3/4,1/4)}$ is intuitionistic fuzzy lower α-continuous but not intuitionistic fuzzy lower semi-continuous, because $F^{-}(C_{(1/2,1/2)})=\{a,c\}$ and $F^{-}(C_{(1/3,2/3)})=\{a,c\}$ are α-open but not open in X.

Remark 3.4. Every intuitionistic fuzzy lower α-continuous intuitionistic fuzzy multifunction is intuitionistic fuzzy lower quasi-continuous but the converse may not be true. For,

Example 3.5. Let $X=\{a,b,c\}, Y=[0,1]$ and let $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$ and

$$\Gamma=\{\tilde{0},\tilde{1},C_{(1/2,1/2)},C_{(1/3,2/3)}\}$$

are topology and intuitionistic fuzzy topology on X and Y respectively. We use the notion $C_{(\alpha,\beta)}(0\le\alpha\le1, 0\le\beta\le1$ and $0\le\alpha+\beta\le1)$ to denote the constant intuitionistic fuzzy sets such that $C_{(\alpha,\beta)}(y)=\{<y,\alpha,\beta>;\forall y\in Y\}$. Then the intuitionistic fuzzy multifunction $F:(X,\tau)\to(Y,\Gamma)$ defined by $F(a)=C_{(5/6,1/6)}$, $F(b)=C_{(1/2,1/2)}$ and $F(c)=C_{(3/4,1/4)}$ is intuitionistic fuzzy lower quasi-continuous but it is not intuitionistic fuzzy lower α-continuous, because $F^{-}(C_{(1/2,1/2)})=\{a,c\}$ and $F^{-}(C_{(1/3,2/3)})=\{a,c\}$ are semi-open but not α-open in X.

Definition 3.6. Let $\tilde{A}$ be an intuitionistic fuzzy set of an intuitionistic fuzzy topological space (Y,Γ). Then $\tilde{V}$ is said to be a neighbourhood of $\tilde{A}$ in Y if there exists an intuitionistic fuzzy open set U of Y such that $\tilde{A}\subset\tilde{U}\subset\tilde{V}$.

Theorem 3.7. *Let $F:(X,\tau)\to(Y,\Gamma))$ be an intuitionistic fuzzy multifunction and let $x\in X$. Then the following statements are equivalent:*

(a) *F is intuitionistic fuzzy lower α-continuous at x.*

(b) *For each intuitionistic fuzzy open set $\tilde{B}$ of Y with $F(x)q\tilde{B}$, implies $x\in sCl(Int(F^{-}(\tilde{B}))$.*

(c) *For any semi-open set U of X containing x and for any intuitionistic fuzzy open set $\tilde{B}$ of Y with $F(x)q\tilde{B}$, there exists a non empty open set $V \subset U$ such that $F(v)q\tilde{B}, \forall v \in V$.*

Proof. $(a) \Rightarrow (b)$. Let $x \in X$ and $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$. Then by (a) $\exists U \in \alpha O(X)$ such that $x \in U$ and $F(v)q\tilde{B}, \forall v \in U$. Thus $x \in U \subset F^-(\tilde{B})$. Now $U \in \alpha O(X)$ implies $U \subset sCl(Int(U))$. Hence $x \in sCl(IntF^-(\tilde{B}))$.

$(b) \Rightarrow (c)$. Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$, then $x \in sCl(IntF^-(\tilde{B}))$. Let U be any semi-open set of X containing x. Then $U \cap Int(F^-(\tilde{B})) \neq \phi$. Put $V = U \cap Int(F^-(\tilde{B}))$, then V is a semi-open set of X, $V \subset U, V \neq \phi$ and $F(v)q\tilde{B}, \forall v \in V$.

$(c) \Rightarrow (a)$. Let $\{U_x\}$ be the system of the semi-open sets in X containing x. For any semi-open set $U \subset X$ such that $x \in U$ and any intuitionistic fuzzy open set $\tilde{B}$ of Y such that $F(x)q\tilde{B}$, there exists a non empty open set $B_U \subset U$ such that $F(v)q\tilde{B}, \forall v \in B_U$. Let $W = \bigcup_{U \in U_x} B_U$, then W is open in X , $x \in sCl(W)$ and $F(v)q\tilde{B}, \forall v \in W$. Put $S = W \cup \{x\}$, then $W \subset S \subset sCl(W)$.Thus $S \in \alpha O(X), x \in S$ and $F(v)q\tilde{B}, \forall v \in S$. Hence F is intuitionistic fuzzy lower α-continuous at x. □

Definition 3.8 ([22]). Let X and Y are two non empty sets. A function $F : X \to Y$ is called fuzzy multifunction if $F(x)$ is a fuzzy set in $Y, \forall x \in X$.

Corollary 3.9 ([24]). *Let F be a fuzzy multifunction from a topological space (X, τ) into a fuzzy topological (Y, σ) and $x \in X$. Then the following statements are equivalent:*

(a) *F is fuzzy lower α-continuous at x.*

(b) *For each fuzzy open set B of Y with $F(x)qB$, implies $x \in sCl(Int(F^-(B))$.*

(c) *For any semi-open set U of X containing x and for any non-empty fuzzy open set B of Y with $F(x)qB$, there exists a non empty open set $V \subset U$ such that $F(v)qB, \forall v \in V$.*

Corollary 3.10 ([23]). *For a multifunction $F : X \to Y$ and point $x \in X$ the following statements are equivalent :*

(a) *F is lower α-continuous at x.*

(b) *For each non-empty open set B of Y with $F(x) \cap B \neq \phi$, implies $x \in sCl(Int(F^-(B))$.*

(c) *For any semi-open set U of X containing x and for any non-empty open set B of Y with $F(x) \cap B \neq \phi$, there exists a non empty open set $V \subset U$ such that $F(v) \cap B \neq \phi, \forall v \in V$.*

Theorem 3.11. *Let $F : (X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) *F is intuitionistic fuzzy lower α-continuous.*

(b) *$F^-(\tilde{G}) \in \alpha O(X)$,for every intuitionistic fuzzy open set $\tilde{G}$ of Y.*

(c) *$F^+(\tilde{V}) \in \alpha C(X)$ for every intuitionistic fuzzy closed set $\tilde{V}$ of Y.*

(d) *$sInt(Cl(F^+(\tilde{B}))) \subset F^+(Cl(\tilde{B}))$, for each intuitionistic fuzzy set $\tilde{B}$ of Y.*

(e) *$F(sInt(Cl(A))) \subset Cl(F(A))$, for each subset A of X.*

(f) *$F(\alpha Cl(A)) \subset Cl(F(A))$,for each subset A of X,*

(g) *$\alpha Cl(F^+(\tilde{B})) \subset F^+(Cl(\tilde{B}))$, for each Intuitionistic fuzzy set $\tilde{B}$ of Y.*
(h) *$F(Cl(Int(Cl(A)))) \subset Cl(F(A))$ for any subset A of X.*

Proof. $(a) \Rightarrow (b)$. Let $\tilde{G}$ be any intuitionistic fuzzy open set of Y and $x \in F^-(\tilde{G})$, so $F(x)q\tilde{G}$, since F is Intuitionistic Fuzzy lower α-continuous, by Theorem 3.7 it follows that $x \in sCl(IntF^-(\tilde{G}))$. As x is chosen arbitrarily in $F^-(\tilde{G})$, we have $F^-(\tilde{G}) \subset sCl(IntF^-(\tilde{G}))$ and thus $F^-(\tilde{G}) \in \alpha O(X)$.

$(b) \Rightarrow (a)$. Let x be arbitrarily chosen in X and $\tilde{G}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{G}$, so $x \in F^-(\tilde{G})$. By hypothesis $F^-(\tilde{G}) \in \alpha O(X)$, we have $x \in F^-(\tilde{G}) \subset sCl(Int(F^-(\tilde{G})))$ and thus F is intuitionistic fuzzy lower α-continuous at x according to Theorem 3.7. As x was arbitrarily chosen, F is intuitionistic fuzzy lower α-continuous.

$(b) \Leftrightarrow (c)$. Obvious.

$(c) \Rightarrow (d)$. Let $\tilde{B}$ be any arbitrary intuitionistic fuzzy set of Y. Since $Cl(\tilde{B})$ is intuitionistic fuzzy closed set in Y by hypothesis, $F^+(Cl(\tilde{B})) \in \alpha C(X)$. Hence by lemma 2.3, we obtain $F^+(Cl(\tilde{B})) \supset sInt(Cl(F^+(Cl\tilde{B})) \supset sInt(Cl(F^+(\tilde{B})))$.

$(d) \Rightarrow (e)$. Suppose that (d) holds, and let A be an arbitrary subset of X. Let us put $\tilde{B} = F(A)$, then $A \subset F^+(\tilde{B})$. Therefore, by hypothesis, we have

$$sInt(Cl(A)) \subset sInt(Cl(F^+(\tilde{B}))) \subset F^+(Cl(\tilde{B})).$$

Therefore

$$F(sInt(Cl(A))) \subset F(F^+(Cl(\tilde{B}))) \subset Cl(\tilde{B}) = Cl(F(A)).$$

$(e) \Rightarrow (c)$. Suppose that (e) holds, and let $\tilde{B}$ be any intuitionistic fuzzy closed set of Y. Put $A = F^+(\tilde{B})$, then $F(A) \subset \tilde{B}$. Therefore, by hypothesis, we have

$$F(sInt(Cl(A))) \subset Cl(F(A)) \subset Cl(\tilde{B}) = \tilde{B}$$

and

$$F^+(F(sInt(Cl(A)))) \subset F^+(\tilde{B}).$$

Since we always have $F^+(F(sInt(Cl(A)))) \supset sInt(Cl(A))$, we obtain $F^+(\tilde{B}) \supset sInt(Cl(F^+(\tilde{B})))$.Hence by lemma, 2.3, $F^+(\tilde{B}) \in \alpha C(X)$.

$(c) \Rightarrow (f)$. Since $A \subset F^+(F(A))$, we have $A \subset F^+(Cl(F(A)))$. Now $Cl(F(A))$ is an intuitionistic fuzzy closed set in Y and so by hypothesis $F^+(Cl(F(A))) \in \alpha C(X)$. Thus $\alpha Cl(A) \subset F^+(Cl(F(A)))$. Consequently, $F(\alpha Cl(A)) \subset F(F^+(Cl(F(A)))) \subset Cl(F(A))$.

$(f) \Rightarrow (c)$. Let $\tilde{B}$ be any intuitionistic fuzzy closed set of Y. Replacing A by $F^+(\tilde{B})$ we get by (f),

$$F(\alpha Cl(F^+(\tilde{B}))) \subset Cl(F(F^+(\tilde{B}))) \subset Cl(\tilde{B}) = \tilde{B}.$$

Consequently, $\alpha Cl(F^+(\tilde{B})) \subset F^+(\tilde{B})$. But $F^+(\tilde{B}) \subset \alpha Cl(F^+(\tilde{B}))$, and so, $\alpha Cl(F^+(\tilde{B})) = F^+(\tilde{B})$. Thus $F^+(\tilde{B}) \in \alpha C(X)$.

$(f) \Rightarrow (g)$. Let $\tilde{B}$ be any intuitionistic fuzzy set of Y. Replacing A by $F^+(\tilde{B})$ we get by (f),

$$F(\alpha Cl(F^+(\tilde{B}))) \subset Cl(F(F^+(\tilde{B}))) \subset Cl(\tilde{B}).$$

Thus $\alpha Cl(F^+(\tilde{B})) \subset F^+(Cl(\tilde{B}))$.

$(g) \Rightarrow (f)$. Replacing $\tilde{B}$ by $F(A)$, where A is a subset of X, we get by (g),

$$\alpha Cl(A) \subset \alpha Cl(F^+(F(A))) = \alpha Cl(F^+(\tilde{B})) = F^+(Cl(\tilde{B})) = F^+(Cl(F(A))).$$

Thus $F(\alpha Cl(A)) \subset F(F^+(Cl(F(A)))) \subset Cl(F(A))$.

$(e) \Rightarrow (h)$. Follows from by Lemma 2.17.

$(h) \Rightarrow (a)$. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set in Y such that $F(x)q\tilde{V}$. Then $x \in F^-(\tilde{V})$. We shall show that $F^-(\tilde{V}) \in \alpha(X)$. By the hypothesis, we have

$$F(Cl(Int(Cl(F^+(\tilde{V}^c))))) \subset Cl(F(F^+(\tilde{V}^c))) \subset (\tilde{V}^c),$$

which implies that

$$Cl(Int(Cl(F^+(\tilde{V}^c))))) \subset F^+(\tilde{V}^c) \subset (F^-(\tilde{V}))^c.$$

Therefore, we obtain $F^-(\tilde{V}) \subset Int(Cl(Int(F^-(\tilde{V}))))$. Hence $F^-(\tilde{V}) \in \alpha(X)$. Put $U = F^-(\tilde{V})$. Then $x \in U \in \alpha O(X)$ and $F(u)q\tilde{V}$ for every $u \in U$ thus F is intuitionistic fuzzy lower α-continuous. □

Corollary 3.12 ([24]). *Let $F : (X, \tau) \to (Y, \sigma)$ be a fuzzy multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) *F is fuzzy lower α-continuous.*
(b) *$F^-(G) \in \alpha O(X)$,for every fuzzy open set G of Y.*
(c) *$F^+(V) \in \alpha C(X)$ for every fuzzy closed set V of Y.*
(d) *$sInt(Cl(F^+(B))) \subset F^+(Cl(B))$, for each fuzzy set B of Y.*
(e) *$F(sInt(Cl(A))) \subset Cl(F(A))$, for each subset A of X.*
(f) *$F(\alpha Cl(A)) \subset Cl(F(A))$,for each subset A of X,*
(g) *$\alpha Cl(F^+(B)) \subset F^+(Cl(B))$, for each fuzzy set B of Y.*
(h) *$F(Cl(Int(Cl(A)))) \subset Cl(F(A))$ for any subset A of X.*

Corollary 3.13 ([23]). *Let $F : X \to Y$ be a multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) *F is lower α-continuous.*
(b) *$F^-(G) \in \alpha O(X)$,for every open set G of Y.*
(c) *$F^+(V) \in \alpha C(X)$ for every closed set V of Y.*
(d) *$sInt(Cl(F^+(B))) \subset F^+(Cl(B))$, for each set B of Y.*
(e) *$F(sInt(Cl(A))) \subset Cl(F(A))$, for each subset A of X.*
(f) *$F(\alpha Cl(A)) \subset Cl(F(A))$,for each subset A of X,*
(g) *$\alpha Cl(F^+(B)) \subset F^+(Cl(B))$, for each set B of Y.*
(h) *$F(Cl(Int(Cl(A)))) \subset Cl(F(A))$ for any subset A of X.*

4. Upper α-continuous intuitionistic fuzzy multifunctions

Definition 4.1. An Intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ is said to be:

(a) Intuitionistic fuzzy upper α-continuous at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0) \subseteq \tilde{W}$ there exists $U \in \alpha O(X)$ containing x_0 such that $F(U) \subseteq \tilde{W}$.

(b) Intuitionistic fuzzy upper α-continuous if it has this property at every point of X.

Remark 4.2. Every intuitionistic fuzzy upper semi-continuous intuitionistic fuzzy multifunction is intuitionistic fuzzy upper α-continuous but the converse may not be true. For,

Example 4.3. Let $X = \{a, b, c\}, Y = [0, 1]$ and let $\tau = \{\phi, \{b\}, X\}$ and

$$\Gamma = \{\tilde{0}, \tilde{1}, C_{(5/6,1/6)}, C_{(1/3,2/3)}\}$$

are topology and intuitionistic fuzzy topology on X and Y respectively. We use the notion $C_{(\alpha,\beta)}(0 \leq \alpha \leq 1, 0 \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1)$ to denote the constant intuitionistic fuzzy sets such that $C_{(\alpha,\beta)}(y) = \{< y, \alpha, \beta >; \forall y \in Y\}$. Then the intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ defined by $F(a) = C_{(5/6,1/6)}$, $F(b) = C_{(1/2,1/2)}$ and $F(c) = C_{(6/7,1/7)}$ is intuitionistic fuzzy upper α-continuous but not intuitionistic fuzzy upper semi-continuous,because $F^+(C_{(5/6,1/6)}) = \{a, b\}$ is α-open but not open in X.

Remark 4.4. Every intuitionistic fuzzy upper α-continuous intuitionistic fuzzy multifunction is intuitionistic fuzzy upper quasi-continuous but the converse may not be true. For,

Example 4.5. Let $X = \{a, b, c\}, Y = [0, 1]$ and let $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and

$$\Gamma = \{\tilde{0}, \tilde{1}, C_{(1/3,2/3)}, C_{(5/6,1/6)}\}$$

are topology and intuitionistic fuzzy topology on X and Y respectively. We use the notion $C_{(\alpha,\beta)}(0 \leq \alpha \leq 1, 0 \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1)$ to denote the constant intuitionistic fuzzy sets such that $C_{(\alpha,\beta)}(y) = \{< y, \alpha, \beta >; \forall y \in Y\}$. Then the intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ defined by $F(a) = C_{(5/6,1/6)}$, $F(b) = C_{(1/2,1/2)}$ and $F(c) = C_{(6/7,1/7)}$ is intuitionistic fuzzy upper quasi-continuous but it is not intuitionistic fuzzy upper α-continuous, because $F^+(C_{(5/6,1/6)}) = \{a, b\}$ is semi-open but not α-open in X.

Theorem 4.6. *For an intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ and let $x \in X$, the following statements are equivalent:*

(a) *F is intuitionistic fuzzy upper α-continuous at x.*

(b) *For each intuitionistic fuzzy open set $\tilde{G}$ of Y with $F(x) \subset \tilde{G}$, there results the relation $x \in sCl(Int(F^+(\tilde{G}))$.*

(c) *For any semi-open set $U \subset X$ containing x and for any intuitionistic fuzzy open set $\tilde{G}$ of Y, $F(x) \subset \tilde{G}$, there exists a non empty open set $V \subset U$ such that $F(V) \subset \tilde{G}$.*

Proof. **(a)⇒(b)**. Let $x \in X$ and $\tilde{G}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{G}$, there is a $U \in \alpha O(X)$ such that $x \in U$ and $F(u) \subset \tilde{G}, \forall u \in U$. Thus $x \in U \subset F^+(\tilde{G})$. Since $U \in \alpha O(X), U \subset sCl(Int(U)) \subset sCl(Int(F^+(\tilde{G})))$. Hence , $x \in sCl(Int(F^+(\tilde{G})))$.

(b)⇒(c). Let $\tilde{G}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{G}$, then $x \in sCl(Int(F^+(\tilde{G})))$. Let $U \subset X$ be any semi-open set such that $x \in U$, then $U \cap Int(F^+(\tilde{G})) \neq \phi$. Put $V = U \cap Int(F^+(\tilde{G}))$, then V is an semi-open set in X, $V \subset U, V \neq \phi$ and $F(V) \subset \tilde{G}$.

(c)⇒(a). Let $\{U_x\}$ be the system of the semi-open sets in X containing x. For any semi-open set $U \subset X$ such that $x \in U$ and $\tilde{G}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{G}$, there exists a non empty open set $G_U \subset U$ such that $F(G_U) \subset \tilde{G}$. Let $W = \bigcup_{U \in U_x} G_U$, then W is open , $x \in sCl(W)$ and $F(w) \subset \tilde{G}, \forall w \in W$. Put $S = W \cup \{x\}$, then $W \subset S \subset sCl(W)$.Thus $S \in \alpha O(X), x \in S$ and $F(w) \subset \tilde{G}, \forall w \in S$. Hence F is intuitionistic fuzzy upper α-continuous at x.. □

Corollary 4.7 ([24]). *Let $F : (X, \tau) \to (Y, \sigma)$ be a fuzzy multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) *F is fuzzy upper α-continuous at x.*

(b) *For each fuzzy open set G of Y with $F(x) \subset G$, there results the relation $x \in sCl(Int(F^+(G))$.*

(c) *For any semi-open set $U \subset X$ containing x and for any fuzzy open set G of Y with $F(x) \subset G$, there exists a non empty open set $V \subset U$ such that $F(V) \subset G$.*

Corollary 4.8 ([23]). *Let $F : X \to Y$ be a multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) *F is upper α-continuous at x.*

(b) *For each open set G of Y with $F(x) \subset G$, there results the relation $x \in sCl(Int(F^+(G))$.*

(c) *For any semi-open set $U \subset X$ containing x and for any open set G of Y with $F(x) \subset G$, there exists a non empty open set $V \subset U$ such that $F(V) \subset G$.*

Theorem 4.9. *For an intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ and let $x \in X$, the following statements are equivalent:*

(a) *F is intuitionistic fuzzy upper α-continuous.*

(b) *$F^+(\tilde{G}) \in \alpha O(X)$, for every intuitionistic fuzzy open set $\tilde{G}$ of Y.*

(c) *$F^-(\tilde{B}) \in \alpha C(X)$ for each intuitionistic fuzzy closed set $\tilde{B}$ of Y.*

(d) *For each point $x \in X$ and for each neighborhood $\tilde{V}$ of $F(x)$ in $Y, F^+(\tilde{V})$ is a α-neighborhood of x.*

(e) *For each point $x \in X$ and for each neighborhood $\tilde{V}$ of $F(x)$ in Y, there is an α-neighborhood U of x such that $F(U) \subset \tilde{V}$.*

(f) *$\alpha Cl(F^-(\tilde{B})) \subset F^-(Cl(\tilde{B}))$ for each intuitionistic fuzzy set $\tilde{B}$ of Y.*

(g) *$sInt(Cl(F^-(\tilde{B}))) \subset F^-(Cl(\tilde{B}))$ for any intuitionistic fuzzy set $\tilde{B}$ of Y.*

Proof. **(a)⇒(b)**. Let $\tilde{V}$ be any intuitionistic fuzzy open set of Y and $x \in F^+(\tilde{V})$. By Theorem 4.6, $x \in sCl(IntF^+(\tilde{V}))$. Therefore, we obtain $F^+(\tilde{V}) \subset sCl(IntF^+(\tilde{V}))$. Hence by Lemma 2.4,$F^+(\tilde{V}) \in \alpha O(X)$.

(b)⇒(a). Let x be arbitrarily chosen in X and $\tilde{G}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{G}$, so $x \in F^+(\tilde{G})$. By hypothesis $F^+(\tilde{G}) \in \alpha O(X)$, we have $x \in F^+(\tilde{G}) \subset sCl(Int(F^+(\tilde{G})))$ and thus F is intuitionistic fuzzy upper α-continuous at x according to Theorem 4.6. As x is arbitrarily chosen, F is intuitionistic fuzzy upper α-continuous.

(b)⇔(c). It follows from the fact that $[F^-(\tilde{A})]^c = F^+(\tilde{A}^c)$ for every intuitionistic fuzzy set $\tilde{A}$ of Y and compliment of every open set is always closed.

(c)⇒(f). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y .Then by (c), $F^-(Cl(\tilde{B}))$ is an α-closed set in X. Thus by Lemma 2.3 we have

$$\begin{aligned} F^-(Cl(\tilde{B})) &\supset sInt(Cl(F^-(Cl(\tilde{B})))) \supset sInt(Cl(F^-(\tilde{B}))) \\ &\supset F^-(\tilde{B}) \cup sInt(Cl(F^-(\tilde{B}))) \\ &\supset \alpha Cl(F^-(\tilde{B})). \end{aligned}$$

(f)⇒(g). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y. By Lemma 2.3, we have

$$\alpha Cl(F^-(\tilde{B})) = F^-(\tilde{B}) \cup sInt(Cl(F^-(\tilde{B}))) \subset F^-(Cl(\tilde{B})).$$

(g)⇒(c). Let $\tilde{B}$ be any intuitionistic fuzzy closed set of Y. Then by (g) we have ,

$$sInt(Cl(F^-(\tilde{B}))) \subset F^-(\tilde{B}) \cup sInt(Cl(F^-(\tilde{B}))) \subset F^-(Cl(\tilde{B})) = F^-(\tilde{B}).$$

Hence By Lemma 2.3, $F^-(\tilde{B}) \in \alpha C(X)$.

(b)⇒(d). Let $x \in X$ and $\tilde{V}$ be a neighborhood of $F(x)$ in Y. Then there is an intuitionistic fuzzy open set $\tilde{G}$ of Y such that $F(x) \subset \tilde{G} \subset \tilde{V}$. Hence, $x \in F^+(\tilde{G}) \subset F^+(\tilde{V})$. Now by hypothesis $F^+(\tilde{G}) \in \alpha O(X)$, and thus $F^+(\tilde{V})$ is an α-neighborhood of x.

(d)⇒(e). Let $x \in X$ and $\tilde{V}$ be a neighborhood of $F(x)$ in Y. Put $U = F^+(\tilde{V})$. Then U is an α-neighborhood of x and $F(U) \subset \tilde{V}$.

(e)⇒(a). Let $x \in X$ and $\tilde{V}$ be an intuitionistic fuzzy set in Y such that $F(x) \subset \tilde{V}$. $\tilde{V}$, being an intuitionistic fuzzy open set in Y , is a neighborhood of $F(x)$ and according to the hypothesis there is a α-neighborhood U of x such that $F(U) \subset \tilde{V}$. Therefore there is $A \in \alpha O(X)$ such that $x \in A \subset U$ and hence $F(A) \subset F(U) \subset \tilde{V}$. □

Corollary 4.10 ([24]). *Let $F : (X, \tau) \to (Y, \sigma)$ be a fuzzy multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) *F is fuzzy upper α-continuous.*

(b) *$F^+(G) \in \alpha O(X)$, for every fuzzy open set G of Y.*

(c) *$F^-(B) \in \alpha C(X)$ for each fuzzy closed set B of Y.*

(d) *For $\forall x \in X$ and for each neighborhood V of $F(x)$ in $Y, F^+(V)$ is a α-neighborhood of x.*

(e) *For $\forall x \in X$ and for each neighborhood V of $F(x)$ in Y, there is an α-neighborhood U of x such that $F(U) \subset V$.*

(f) *$\alpha Cl(F^-(B)) \subset F^-(Cl(B))$ for each fuzzy set B of Y.*

(g) *$sInt(Cl(F^-(B))) \subset F^-(Cl(B))$ for any fuzzy set B of Y.*

Corollary 4.11 ([23]). *Let $F : X \to Y$ be a multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) *F is upper α-continuous.*

(b) *$F^+(G) \in \alpha O(X)$, for every open set G of Y.*

(c) *$F^-(B) \in \alpha C(X)$ for each closed set B of Y.*

(d) *For each point $x \in X$ and for each neighborhood V of $F(x)$ in $Y, F^+(V)$ is a α-neighborhood of x.*

(e) *For each point $x \in X$ and for each neighborhood V of $F(x)$ in Y, there is an α-neighborhood U of x such that $F(U) \subset V$.*
(f) *$\alpha Cl(F^-(B)) \subset F^-(Cl(B))$ for each fuzzy set B of Y.*
(g) *$sInt(Cl(F^-(B))) \subset F^-(Cl(B))$ for any fuzzy set B of Y.*

Corollary 4.12 ([17])**.** *For a single-valued mapping $f : (X, \tau) \to (Y, \tau^*)$ the following conditions are equivalent:*
(a) *f is α-continuous.*
(b) *$f^{-1}(B) \in \alpha O(X), \forall$ open sets B of Y.*
(c) *For any open set V of Y and $\forall x \in X$ with $f(x) \in V$, there is $U \in \alpha O(X)$ such that $x \in U$ and $f(U) \subset V$.*
(d) *For every point $x \in X$ and for each neighborhood V of $f(x), f^{-1}(V)$ is a α-neighborhood of x.*
(e) *For every point $x \in X$ and for each neighborhood V of $f(x)$, there is a α-neighborhood U of x such that $f(U) \subset V$.*
(f) *For each closed set V of $Y, f^{-1}(V) \in \alpha C(X)$.*
(g) *For each subset $A \subset X, f(\alpha Cl(A))) \subset Cl(f(A))$.*
(h) *For each subset $B \subset Y, \alpha Cl(f^{-1}(B)) \subset f^{-1}(Cl(B))$.*

5. Application on α-compact spaces

In 1985 Maheshwari and Thakur [16] introduced the concept of α-compact spaces. A topological space is called α-compact if every α-open cover of X has a finite subcover. On the other hand Intuitionistic fuzzy compactness was studied by Coker and Hayder [10] in 1995. An intuitionistic fuzzy topological space Y is called intuitionistic fuzzy compact if every intuitionistic fuzzy open cover of Y has a finite subcover.

Theorem 5.1. *Let $F : (X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy upper α-continuous surjective multifunction such that $F(x)$ is compact for each $x \in X$ if X is α-compact , then Y is intuitionistic fuzzy Compact.*

Proof. Let $\{V_\alpha \mid \alpha \in \Lambda\}$ be an intuitionistic fuzzy open cover of Y and $x \in X$. Then by hypothesis $F(x)$ is compact, therefore there exists a finite subset $\Lambda(x)$ of Λ such that $F(x) \subset \bigcup\{V_\alpha \mid \alpha \in \Lambda(x)\}$. Put $V(x) = \cup\{V_\alpha \mid \alpha \in \Lambda(x)\}$.Then $V(x)$ is an intuitionistic fuzzy open set in Y. Since F is intuitionistic fuzzy upper α-continuous $\exists$ an α-open set $U(x)$ of X containing x such that $F(U(x)) \subset V(x)$. The family $\{U(x) \mid x \in X\}$ is an α-open cover of X and there exists a finite number of points say $x_1, x_2, x_3........x_n$ in X such that $X = \bigcup\{U(x_i) \mid 1 \leq i \leq n\}$. Therefore , we have

$$Y = F(X) = F(\bigcup_{i=1}^{n} U(x_i)) = \bigcup_{i=1}^{n} F(U(x_i)) \subset \bigcup_{i=1}^{n} V(x_i) = \bigcup_{i=1}^{n} \bigcup_{\alpha \in \Lambda(x_i)} V_\alpha,$$

This shows that Y is Intuitionistic fuzzy compact. □

Future and scope of newly defined multifunctions

The theory of intuitionistic fuzzy multifunctions can be applied in functional analysis and Fixed point theory, the classes of intuitionistic fuzzy multifunctions studied in this paper can be used to study α-separation axioms ,α-regular and α-convergence of Nets and Filters in topological and intuitionistic fuzzy topological spaces.

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