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On intutionistic fuzzy α-supra continuous maps

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Abstract. In this paper, we introduce and investigate a new class of sets and functions between supra topological spaces called intuitionistic fuzzy α-supra open set and intuitionistic fuzzy α-supra continuous functions respectively.

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1. Introduction and preliminaries

The concept of intuitionistic fuzzy set is deifined by Atanassov as a generalization of the concept of fuzzy set given by Zadeh [9]. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces.In 1983, A.S. Mashhour et al. [5] introduced the supra topological spaces and studied s-continuous functions and s*-continuous functions. In 1987, M. E. Abd El-Monsef et al. [1] introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions and obtained some properties and characterizations. In 1996, Keun Min [6] introduced fuzzy s-continuous, fuzzy s-open and fuzzy s-closed maps and established number of characterizations. In 2008, R.Devi et al [4] introduced the concept of $s\alpha$-open set, $s\alpha$-continuous functions and studied some of the basic properties for this class of functions. In 1999, Necla Turanl [7] introduced the concept of intuitionistic fuzzy supra topological space. In this paper, we are going to study the basic properties of intuitionistic fuzzy α-supra open sets and introduce the notion of intuitionistic fuzzy α-supra continuous functions.

Throughout this paper, by (X, τ) or simply by X we will denote the intuitionistic fuzzy supra topological space (briefly, IFTS). For a subset A of a space (X, τ), $cl(A)$, $int(A)$ and $\overline{A}$ denote the closure of A, the interior of A and the complement of A respectively. Each intuitionistic fuzzy supra set (briefly,IFS) which belongs to (X, τ) is called an intuitionitic fuzzy supra open set (briefly, IFSOS) in X. The complement

$\overline{A}$ of an IFSOS A in X is called an intuitionistic fuzzy supra closed set (IFSCS) in X.

We introduce some basic notions and results that are used in the sequel.

Definition 1.1 ([2])**.** Let X be a non-empty fixed set and I be the closed interval $[0, 1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the mappings $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}.$$

Definition 1.2 ([2])**.** Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
(ii) $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$;
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$;
(v) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(vi) $[]A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle, x \in X\}$;
(vii) $\langle\rangle A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle, x \in X\}$;
(viii) $1_\sim = \{\langle x, 1, 0\rangle, x \in X\}$ and $0_\sim = \{\langle x, 0, 1\rangle, x \in X\}$.

We will use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 1.3 ([7])**.** A family τ of IFSs on X is called an intuitionistic fuzzy supratopology (IFST for short) on X if $0_\sim \in \tau$, $1_\sim \in \tau$ and τ is closed under arbitrary suprema. Then we call the pair (X, τ) an intuitionistic fuzzy supratopological space (IFSTS for short).

Each member of τ is called an intuitionistic fuzzy supraopen set and the complement of an intuitionistic fuzzy supraopen set is called an intuitionistic fuzzy supraclosed set. The intuitionistic fuzzy supraclosure of an IFS A is denoted by $s\text{-}cl(A)$. Here $s\text{-}cl(A)$ is the intersection of all intuitionistic fuzzy supraclosed sets containing A. The intuitionistic fuzzy suprainterior of A will be denoted by $s\text{-}int(A)$. Here, $s\text{-}int(A)$ is the union of all intuitionistic fuzzy supraopen sets contained in A.

Definition 1.4 ([8])**.** Let (X, τ) be an intuitionistic fuzzy supratopological space. An IFS $A \in IF(X)$ is called

(a) intuitionistic fuzzy semi-supraopen iff $A \subseteq s\text{-}cl(s\text{-}int(A))$,
(b) intuitionistic fuzzy α-supraopen iff $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$,
(c) intuitionistic fuzzy pre-supraopen iff $A \subseteq s\text{-}int(s\text{-}cl(A))$.

Let f be a mapping from an ordinary set X into an ordinary set Y, If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle : y \in Y\}$ is an IFST in Y , then the inverse image of B under f is an IFST defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle : x \in X\}$$

The image of IFST $A = \{\langle y, \mu_A(y), \nu_A(y)\rangle : y \in Y\}$ under f is an IFST defined by $f(A) = \{\langle y, f(\mu_A)(y), f(\nu_A)(y)\rangle : y \in Y\}$.

2. Intuitionistic fuzzy α-supra open set

Definition 2.1. Let (X, τ) be an intuitionistic fuzzy supra topological space. An intuitionistic fuzzy set A is called an intuitionistic fuzzy α-supra open set (briefly, IFαSOS) [8] if $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. The complement of an intuitionistic fuzzy α-supra open set is called an intuitionistic fuzzy α-supra closed set.

Theorem 2.2. *Every intuitionistic fuzzy supra open set is intuitionistic fuzzy α-supra open.*

Proof. Let A be an intuitionistic fuzzy supra open set in (X, τ). Since $A \subseteq s\text{-}cl(A)$, we get $A \subseteq s\text{-}cl(s\text{-}int(A))$. Then $s\text{-}int(A) \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. Hence $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.3. Let $X = \{a, b\}$, $A = \{x, \langle 0.3, 0.4\rangle, \langle 0.4, 0.5\rangle\}$, $B = \{x, \langle 0.4, 0.2\rangle, \langle 0.5, 0.3\rangle\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Let $C = \{x, \langle 0.4, 0.6\rangle, \langle 0.3, 0.4\rangle\}$. Then C is intuitionistic fuzzy α-supra open but not intuitionistic fuzzy supra open.

Theorem 2.4. *Every intuitionistic fuzzy α-supra open set is intuitionistic fuzzy semi-supra open.*

Proof. Let A be an intuitionistic fuzzy α-supra open set in (X, τ). Then, $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. It is obvious that $s\text{-}int(s\text{-}cl(s\text{-}int(A))) \subseteq s\text{-}cl(s\text{-}int(A))$. Hence $A \subseteq s\text{-}cl(s\text{-}int(A))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.5. Let $X = \{a, b\}$, $A = \{x, \langle 0.3, 0.5\rangle, \langle 0.4, 0.5\rangle\}$, $B = \{x, \langle 0.4, 0.3\rangle, \langle 0.5, 0.4\rangle\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Let $C = \{x, \langle 0.4, 0.4\rangle, \langle 0.4, 0.4\rangle\}$. Then C is intuitionistic fuzzy semi-supra open but not intuitionistic fuzzy α-supra open.

Theorem 2.6. *Every intuitionistic fuzzy α-supra open set is intuitionistic fuzzy pre-supra open.*

Proof. Let A be an intuitionistic fuzzy α-supra open set in (X, τ). Then, $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. It is obvious that $A \subseteq s\text{-}int(s\text{-}cl((A)))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.7. In Example 2.5, let $C = \{x, \langle 0.4, 0.5\rangle, \langle 0.5, 0.4\rangle\}$. Here C is intuitionistic fuzzy pre-supra open but not intuitionistic fuzzy α-supra open.

Theorem 2.8. *If (X, τ) is an intuitionistic fuzzy supra topological space, then the following holds:*

(i) *Arbitrary union of intuitionistic fuzzy α-supra open sets is always intuitionistic fuzzy α-supra open set.*
(ii) *Finite intersection of intuitionistic fuzzy α-supra open sets may fail to be intuitionistic fuzzy α-supra open set.*

(iii) $1_\sim$ *is an intuitionistic fuzzy α-supra open set.*

Proof. (i) Let $\{A_\lambda : \lambda \in \Lambda\}$ be a family of intuitionistic fuzzy α-supra open set in a topological space X. Then for any $\lambda \in \Lambda$, we have $A_\lambda \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A_\lambda)))$. Hence

$$\begin{aligned}\cup_{\lambda\in\Lambda} A_\lambda &\subseteq \cup_{\lambda\in\Lambda}(s\text{-}int(s\text{-}cl(s\text{-}int(A_\lambda))))\\ &\subseteq s\text{-}int(\cup_{\lambda\in\Lambda}(s\text{-}cl(s\text{-}int(A_\lambda))))\\ &\subseteq s\text{-}int(s\text{-}cl(s\text{-}int(\cup_{\lambda\in\Lambda}A_\lambda))).\end{aligned}$$

Therefore, $\cup_{\lambda\in\Lambda}A_\lambda$ is an intuitionistic fuzzy α-supra open set.

(ii) Let $X = \{a, b\}$, $A = \{x, \langle 0.3, 0.4\rangle, \langle 0.4, 0.5\rangle\}$, $B = \{x, \langle 0.2, 0.4\rangle, \langle 0.5, 0.3\rangle\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Here A and B are intuitionistic fuzzy α-supra open but $A \cap B$ is not intuitionistic fuzzy α-supra open. □

Theorem 2.9. *If (X, τ) is an intuitionistic fuzzy supra topological space, then the following holds:*

(i) *Arbitrary intersection of intuitionistic fuzzy α-supra closed sets is always intuitionistic fuzzy α-supra closed set.*

(ii) *Finite union of intuitionistic fuzzy α-supra closed sets may fail to be intuitionistic fuzzy α-supra closed set.*

Proof. (i) The proof follows immediately from Theorem 2.8.

(ii) Let $X = \{a, b\}$, $A = \{x, \langle 0.3, 0.5\rangle, \langle 0.6, 0.5\rangle\}$, $B = \{x, \langle 0.6, 0.3\rangle, \langle 0.3, 0.4\rangle\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A\cup B\}$. Let $C = \{x, \langle 0.3, 0.5\rangle, \langle 0.4, 0.5\rangle\}$ and $D = \{x, \langle 0.3, 0.4\rangle, \langle 0.6, 0.3\rangle\}$. Here C and D are intuitionistic fuzzy α-supra closed but $C \cup D$ is not intuitionistic fuzzy α-supra closed. □

Definition 2.10. The intuitionistic fuzzy α-supra-closure of a set A is denoted by $\alpha s\text{-}cl(A) = \cup\{G : G$ is an IFαSCS in X and $G \subseteq A\}$ and the intuitionistic fuzzy α-supra-interior of a set A is denoted by $\alpha s\text{-}int(A) = \cap\{G : G$ is an IFαSOS in X and $G \supseteq A\}$

Remark 2.11. It is clear that $\alpha s\text{-}int(A)$ is an intuitionistic fuzzy α-supra open set and $\alpha s\text{-}cl(A)$ is an intuitionistic fuzzy α-supra closed set.

Theorem 2.12. *Let X be an intuitionistic fuzzy supra open space. If A and B are two subsets of X, then*

(i) $X - \alpha s\text{-}int(A) = \alpha s\text{-}cl(X - A)$
(ii) $X - \alpha s\text{-}cl(A) = \alpha s\text{-}int(X - A)$
(iii) *If $A \subseteq B$, then $\alpha s\text{-}cl(A) \subseteq \alpha s\text{-}cl(B)$ and $\alpha s\text{-}int(A) \subseteq \alpha s\text{-}int(B)$*

Proof. It is obvious. □

Theorem 2.13. *Let X be an intuitionistic fuzzy supra open space. If A and B are two subsets of X, then*

(i) $\alpha s\text{-}int(A) \cup \alpha s\text{-}int(B) \subseteq \alpha s\text{-}int(A \cup B)$
(ii) $\alpha s\text{-}int(A \cap B) \subseteq \alpha s\text{-}int(A) \cap \alpha s\text{-}int(B)$
(iii) *If $A \subseteq B$, then $\alpha s\text{-}cl(A) \subseteq \alpha s\text{-}cl(B)$ and $\alpha s\text{-}int(A) \subseteq \alpha s\text{-}int(B)$*

Proof. It is obvious. □

Theorem 2.14. *If (X, τ) is an intuitionistic fuzzy supra topological space, then the following holds:*

(i) *The intersection of an intuitionistic fuzzy-supra open set and an intuitionistic fuzzy α-supra open set is intuitionistic fuzzy α-supra open.*

(ii) *The intersection of an intuitionistic fuzzy α-supra open set and an intuitionistic fuzzy pre-supra open set is intuitionistic fuzzy pre-supra open.*

Proof. It is obvious. □

3. Intuitionistic fuzzy α-supra continuous map

Definition 3.1. Let (X, τ) and (Y, σ) be two intuitionistic fuzzy α-supra open sets and μ be an associated supra topology with τ. A map $f : (X, \tau) \to (Y, \sigma)$ is called intuitionistic fuzzy α-supra continuous map if the inverse image of each open set in Y is an intuitionistic fuzzy α-supra open set in X.

Theorem 3.2. *Every intuitionistic fuzzy supra continuous map is intuitionistic fuzzy α-supra continuous map.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy supra continuous map and A is an open set in Y. Then $f^{-1}(A)$ is an open set in X. Since μ is associated with τ, then $\tau \subseteq \mu$. Therefore, $f^{-1}(A)$ is an intuitionistic fuzzy supra open set in X which is an intuitionistic fuzzy supra open set in X. Hence f is an intuitionistic fuzzy α-supra continuous map. □

Remark 3.3. Every intuitionistic fuzzy α-supra continuous map need not be intuitionistic fuzzy supra continuous map.

Theorem 3.4. *Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. Let f be a map from X into Y. Then the following are equivalent:*

(i) *f is an intuitionistic fuzzy supra α-continuous map*
(ii) *the inverse image of a closed set in Y is an intuitionistic fuzzy supra α-closed set in X*
(iii) *$\alpha s\text{-}cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every set A in Y.*
(iv) *$f(\alpha s\text{-}cl(A)) \subseteq cl(f(A))$ for every set A in X.*
(v) *$f^{-1}(int(B)) \subseteq \alpha s\text{-}int(f^{-1}(B))$ for every set B in Y.*

Proof. *(i)* $\Rightarrow$ *(ii)* : Let A be a closed set in Y, then $Y - A$ is open in Y. Thus, $f^{-1}(X - A) = X - f^{-1}(A)$ is αs-open in X. It follows that $f^{-1}(A)$ is a αs-closed set of X.

(ii) $\Rightarrow$ *(iii)*: Let A be any subset of X. Since $cl(A)$ is closed in Y, then it follows that $f^{-1}(cl(A))$ is αs-closed in X. Therefore, $f^{-1}(cl(A)) = \alpha s\text{-}cl(f^{-1}(cl(A))) \supseteq \alpha s\text{-}cl(f^{-1}(A))$.

(iii) $\Rightarrow$ *(iv)* : Let A be any subset of X. By (iii) we obtain, $f^{-1}(cl(f(A))) \supseteq \alpha s\text{-}cl(f^{-1}(f(A))) \supseteq \alpha s\text{-}cl(A)$ and hence $f(\alpha s\text{-}cl(A)) \subseteq cl(f(A))$.

(iv) $\Rightarrow$ *(v)* : Let $f(\alpha s\text{-}cl(A)) \subseteq cl(f(A))$ for every set A in X. Then

$$\alpha s\text{-}cl(A) \subseteq f^{-1}(cl(f(A))),\ X - \alpha s\text{-}cl(A) \supseteq X - f^{-1}(cl(f(A)))$$

and $\alpha s\text{-}int(X - A) \supseteq f^{-1}(int(Y - f(A)))$. Then $\alpha s\text{-}int(f^{-1}(B)) \supseteq f^{-1}(int(B))$. Therefore $f^{-1}(int(B)) \subseteq s\text{-}int(f^{-1}(B))$, for every B in Y.

$(v) \Rightarrow (i)$: Let A be a open set in Y. Therefore, $f^{-1}(int(A)) \subseteq \alpha s\text{-}int(f^{-1}(A))$, hence $f^{-1}(A) \subseteq \alpha s\text{-}int(f^{-1}(A))$. But by other hand, we know that,

$$\alpha s\text{-}int(f^{-1}(A)) \subseteq f^{-1}(A).$$

Then $f^{-1}(A) = \alpha s\text{-}int(f^{-1}(A))$. Therefore, $f^{-1}(A)$ is a αs-open set. □

Theorem 3.5. *If a map $f : (X, \tau) \to (Y, \sigma)$ is a αs-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is continuous, then $(g \circ f)$ is αs-continuous.*

Proof. Obvious. □

Theorem 3.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy αs-continuous map, if one of the following holds:*

(i) *$f^{-1}(\alpha s\text{-}int(A)) \subseteq int(f^{-1}(A))$ for every set A in Y.*
(ii) *$cl(f^{-1}(A)) \subseteq f^{-1}(\alpha s\text{-}cl(A))$ for every set A in Y.*
(iii) *$f(cl(B)) \subseteq \alpha s\text{-}cl(f(B))$ for every set B in X.*

Proof. Let A be any open set of Y, if condition (i) is satisfied, then $f^{-1}(\alpha s\text{-}int(A)) \subseteq int(f^{-1}(A))$. We get, $f^{-1}(A) \subseteq int(f^{-1}(A))$. Therefore $f^{-1}(A)$ is an intuitionistic fuzzy supra open set. Every intuitionistic fuzzy supra open set is intuitionistic fuzzy supra α-open set. Hence f is an intuitionistic fuzzy αs-continuous function. If condition (ii) is satisfied, then we can easily prove that f is an intuitionistic fuzzy αs-continuous function. If condition (iii) is satisfied, and A is any open set of Y. Then $f^{-1}(A)$ is a set in X and $f(cl(f^{-1}(A))) \subseteq \alpha s\text{-}cl(f(f^{-1}(A)))$. This implies $f(cl(f^{-1}(A))) \subseteq \alpha s\text{-}cl(A)$. This is nothing but condition (ii). Hence f is an intuitionistic fuzzy αs-continuous function. □

4. Conclision

We introduced and investigated a new class of set and its function called intuitionistic fuzzy α-supra open set and intuitionistic fuzzy α-supra continuous functions respectively. Also we studied some of its properties and relation with the existing sets and functions.

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