On intuitionistic fuzzy $\alpha$-supra continuous maps

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Received 16 September 2014; Revised 4 November 2014; Accepted 15 November 2014

Abstract. In this paper, we introduce and investigate a new class of sets and functions between supra topological spaces called intuitionistic fuzzy $\alpha$-supra open set and intuitionistic fuzzy $\alpha$-supra continuous functions respectively.

2010 AMS Classification: 03F55, 54A40

Keywords: Supra topological spaces, Intuitionistic fuzzy supra open sets, Intuitionistic fuzzy $\alpha$-supra open sets, Intuitionistic fuzzy $\alpha$-supra continuous functions.

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1. Introduction and preliminaries


Throughout this paper, by $(X, \tau)$ or simply by $X$ we will denote the intuitionistic fuzzy supra topological space (briefly, IFTS). For a subset $A$ of a space $(X, \tau)$, $\text{cl}(A)$, $\text{int}(A)$ and $\overline{A}$ denote the closure of $A$, the interior of $A$ and the complement of $A$ respectively. Each intuitionistic fuzzy supra set (briefly, IFS) which belongs to $(X, \tau)$ is called an intuitionistic fuzzy supra open set (briefly, IFSOS) in $X$. The complement
\( A \) of an IFSOS \( A \) in \( X \) is called an intuitionistic fuzzy supra closed set (IFSCS) in \( X \).

We introduce some basic notions and results that are used in the sequel.

**Definition 1.1** ([2]). Let \( X \) be a non-empty fixed set and \( I \) be the closed interval \([0,1]\). An intuitionistic fuzzy set (IFS) \( A \) is an object of the following form

\[
A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \}
\]

where the mappings \( \mu_A : X \to I \) and \( \nu_A : X \to I \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of nonmembership (namely \( \nu_A(x) \)) for each element \( x \in X \) to the set \( A \), respectively, and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \).

Obviously, every fuzzy set \( A \) on a nonempty set \( X \) is an IFS of the following form

\[
A = \{ (x, \mu_A(x), 1 - \mu_A(x)) : x \in X \}.
\]

**Definition 1.2** ([2]). Let \( A \) and \( B \) be IFSs of the form \( A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \} \) and \( B = \{ (x, \mu_B(x), \nu_B(x)) : x \in X \} \). Then

(i) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \);
(ii) \( \overline{A} = \{ (x, \nu_A(x), \mu_A(x)) : x \in X \} \);
(iii) \( A \cap B = \{ (x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) : x \in X \} \);
(iv) \( A \cup B = \{ (x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) : x \in X \} \);
(v) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \);
(vi) \( \sim A = \{ (x, 1 - \mu_A(x), \nu_A(x)), x \in X \} \);
(vii) \( \delta A = \{ (x, 1 - \nu_A(x), \mu_A(x)) : x \in X \} \);
(viii) \( 1_\sim = \{ (x, 1, 0) : x \in X \} \) and \( 0_\sim = \{ (x, 0, 1) : x \in X \} \).

We will use the notation \( A = (x, \mu_A, \nu_A) \) instead of \( A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \} \).

**Definition 1.3** ([7]). A family \( \tau \) of IFSs on \( X \) is called an intuitionistic fuzzy supratopology (IFST for short) on \( X \) if \( 0_\sim \in \tau \), \( 1_\sim \in \tau \) and \( \tau \) is closed under arbitrary suprema. Then we call the pair \( (X, \tau) \) an intuitionistic fuzzy supratopological space (IFSTS for short).

Each member of \( \tau \) is called an intuitionistic fuzzy supraopen set and the complement of an intuitionistic fuzzy supraopen set is called an intuitionistic fuzzy supraclosed set. The intuitionistic fuzzy supraclosure of an IFS \( A \) is denoted by \( s-cl(A) \). Here \( s-cl(A) \) is the intersection of all intuitionistic fuzzy supraclosed sets containing \( A \). The intuitionistic fuzzy suprainterior of \( A \) will be denoted by \( s-int(A) \). Here, \( s-int(A) \) is the union of all intuitionistic fuzzy supraopen sets contained in \( A \).

**Definition 1.4** ([8]). Let \( (X, \tau) \) be an intuitionistic fuzzy supratopological space. An IFS \( A \in IF(X) \) is called

\( (a) \) intuitionistic fuzzy semi-supraopen if \( A \subseteq s-cl(s-int(A)) \),
\( (b) \) intuitionistic fuzzy \( \alpha \)-supraopen if \( A \subseteq s-int(s-cl(s-int(A))) \),
\( (c) \) intuitionistic fuzzy pre-supraopen if \( A \subseteq s-int(s-cl(A)) \).

Let \( f \) be a mapping from an ordinary set \( X \) into an ordinary set \( Y \). If \( B = \{ (y, \mu_B(y), \nu_B(y)) : y \in Y \} \) is an IFST in \( Y \), then the inverse image of \( B \) under \( f \) is an IFST defined by

\[
f^{-1}(B) = \{ (x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) : x \in X \}
\]

The image of IFST \( A = \{ (y, \mu_A(y), \nu_A(y)) : y \in Y \} \) under \( f \) is an IFST defined by

\[
f(A) = \{ (y, f(\mu_A(y)), f(\nu_A(y)) : y \in Y \}.
\]
2. INTUITIONISTIC FUZZY $\alpha$-SUPRA OPEN SET

Definition 2.1. Let $(X, \tau)$ be an intuitionistic fuzzy supra topological space. An intuitionistic fuzzy supra open set $A$ is called an intuitionistic fuzzy $\alpha$-supra open set (briefly, IFoSOS) \[8\] if $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. The complement of an intuitionistic fuzzy $\alpha$-supra open set is called an intuitionistic fuzzy $\alpha$-supra closed set.

Theorem 2.2. Every intuitionistic fuzzy supra open set is intuitionistic fuzzy $\alpha$-supra open.

Proof. Let $A$ be an intuitionistic fuzzy supra open set in $(X, \tau)$. Since $A \subseteq s\text{-}cl(A)$, we get $A \subseteq s\text{-}cl(s\text{-}int(A))$. Then $s\text{-}int(A) \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. Hence $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.3. Let $X = \{a, b\}$, $A = \{(0.3, 0.4), (0.4, 0.5)\}$, $B = \{(0.4, 0.2), (0.5, 0.3)\}$ and $\tau = \{0, 1\}$. Let $C = \{(0.4, 0.6), (0.3, 0.4)\}$. Then $C$ is intuitionistic fuzzy $\alpha$-supra open but not intuitionistic fuzzy supra open.

Theorem 2.4. Every intuitionistic fuzzy $\alpha$-supra open set is intuitionistic fuzzy semi-$\alpha$-supra open.

Proof. Let $A$ be an intuitionistic fuzzy $\alpha$-supra open set in $(X, \tau)$. Then, $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. It is obvious that $s\text{-}int(s\text{-}cl(s\text{-}int(A))) \subseteq s\text{-}cl(s\text{-}int(A))$. Hence $A \subseteq s\text{-}cl(s\text{-}int(A))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.5. Let $X = \{a, b\}$, $A = \{(0.3, 0.5), (0.4, 0.5)\}$, $B = \{(0.4, 0.3), (0.5, 0.4)\}$ and $\tau = \{0, 1\}$. Let $C = \{(0.4, 0.4), (0.4, 0.4)\}$. Then $C$ is intuitionistic fuzzy semi-$\alpha$-supra open but not intuitionistic fuzzy $\alpha$-supra open.

Theorem 2.6. Every intuitionistic fuzzy $\alpha$-supra open set is intuitionistic fuzzy pre-$\alpha$-supra open.

Proof. Let $A$ be an intuitionistic fuzzy $\alpha$-supra open set in $(X, \tau)$. Then, $A \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A)))$. It is obvious that $A \subseteq s\text{-}int(s\text{-}cl(A))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.7. In Example 2.5, let $C = \{(0.4, 0.5), (0.5, 0.4)\}$. Here $C$ is intuitionistic fuzzy pre-$\alpha$-supra open but not intuitionistic fuzzy $\alpha$-supra open.

Theorem 2.8. If $(X, \tau)$ is an intuitionistic fuzzy supra topological space, then the following holds:

(i) Arbitrary union of intuitionistic fuzzy $\alpha$-supra open sets is always intuitionistic fuzzy $\alpha$-supra open set.

(ii) Finite intersection of intuitionistic fuzzy $\alpha$-supra open sets may fail to be intuitionistic fuzzy $\alpha$-supra open set.
(iii) \( L \) is an intuitionistic fuzzy \( \alpha \)-supra open set.

**Proof.** (i) Let \( \{ A_\lambda : \lambda \in \Lambda \} \) be a family of intuitionistic fuzzy \( \alpha \)-supra open set in a topological space \( X \). Then for any \( \lambda \in \Lambda \), we have \( A_\lambda \subseteq s\text{-}int(s\text{-}cl(s\text{-}int(A_\lambda))) \). Hence

\[
\bigcup_{\lambda \in \Lambda} A_\lambda \subseteq \bigcup_{\lambda \in \Lambda} \left( s\text{-}int(s\text{-}cl(s\text{-}int(A_\lambda))) \right)
\]

\[
\subseteq s\text{-}int\left( \bigcup_{\lambda \in \Lambda} (s\text{-}cl(s\text{-}int(A_\lambda))) \right)
\]

\[
\subseteq s\text{-}int(s\text{-}cl(s\text{-}int(\bigcup_{\lambda \in \Lambda} A_\lambda))).
\]

Therefore, \( \bigcup_{\lambda \in \Lambda} A_\lambda \) is an intuitionistic fuzzy \( \alpha \)-supra open set.

(ii) Let \( X = \{ a, b \} \), \( A = \{ x, (0.3, 0.4), (0.4, 0.5) \} \), \( B = \{ x, (0.2, 0.4), (0.5, 0.3) \} \) and \( \tau = \{ 0_\alpha, 1_\alpha, A, B, A \cup B \} \). Here \( A \) and \( B \) are intuitionistic fuzzy \( \alpha \)-supra open but \( A \cap B \) is not intuitionistic fuzzy \( \alpha \)-supra open. \( \square \)

**Theorem 2.9.** If \((X, \tau)\) is an intuitionistic fuzzy supra topological space, then the following holds:

(i) Arbitrary intersection of intuitionistic fuzzy \( \alpha \)-supra closed sets is always intuitionistic fuzzy \( \alpha \)-supra closed set.

(ii) Finite union of intuitionistic fuzzy \( \alpha \)-supra closed sets may fail to be intuitionistic fuzzy \( \alpha \)-supra closed set.

**Proof.** (i) The proof follows immediately from Theorem 2.8.

(ii) Let \( X = \{ a, b \} \), \( A = \{ x, (0.3, 0.5), (0.6, 0.5) \} \), \( B = \{ x, (0.6, 0.3), (0.3, 0.4) \} \) and \( \tau = \{ 0_\alpha, 1_\alpha, A, B, A \cup B \} \). Let \( C = \{ x, (0.3, 0.5), (0.4, 0.5) \} \) and \( D = \{ x, (0.3, 0.4), (0.6, 0.3) \} \). Here \( C \) and \( D \) are intuitionistic fuzzy \( \alpha \)-supra closed but \( C \cup D \) is not intuitionistic fuzzy \( \alpha \)-supra closed. \( \square \)

**Definition 2.10.** The intuitionistic fuzzy \( \alpha \)-supra-closure of a set \( A \) is denoted by \( \alpha\text{-}cl(A) = \bigcup \{ G : G \text{ is an IF\( \alpha \)SCS in } X \text{ and } G \subseteq A \} \) and the intuitionistic fuzzy \( \alpha \)-supra-interior of a set \( A \) is denoted by \( \alpha\text{-}int(A) = \cap \{ G : G \text{ is an IF\( \alpha \)SOS in } X \text{ and } G \supseteq A \} \)

**Remark 2.11.** It is clear that \( \alpha\text{-}int(A) \) is an intuitionistic fuzzy \( \alpha \)-supra open set and \( \alpha\text{-}cl(A) \) is an intuitionistic fuzzy \( \alpha \)-supra closed set.

**Theorem 2.12.** Let \( X \) be an intuitionistic fuzzy supra open space. If \( A \) and \( B \) are two subsets of \( X \), then

(i) \( X - \alpha\text{-}int(A) = \alpha\text{-}cl(X - A) \)

(ii) \( X - \alpha\text{-}cl(A) = \alpha\text{-}int(X - A) \)

(iii) If \( A \subseteq B \), then \( \alpha\text{-}cl(A) \subseteq \alpha\text{-}cl(B) \) and \( \alpha\text{-}int(A) \subseteq \alpha\text{-}int(B) \)

**Proof.** It is obvious. \( \square \)

**Theorem 2.13.** Let \( X \) be an intuitionistic fuzzy supra open space. If \( A \) and \( B \) are two subsets of \( X \), then

(i) \( \alpha\text{-}int(A) \cup \alpha\text{-}int(B) \subseteq \alpha\text{-}int(A \cup B) \)

(ii) \( \alpha\text{-}int(A \cap B) \subseteq \alpha\text{-}int(A) \cap \alpha\text{-}int(B) \)

(iii) If \( A \subseteq B \), then \( \alpha\text{-}cl(A) \subseteq \alpha\text{-}cl(B) \) and \( \alpha\text{-}int(A) \subseteq \alpha\text{-}int(B) \)
Proof. It is obvious.

\begin{theorem}
If \((X, \tau)\) is an intuitionistic fuzzy supra topological space, then the following holds:

(i) The intersection of an intuitionistic fuzzy supra open set and an intuitionistic fuzzy \(\alpha\)-supra open set is intuitionistic fuzzy \(\alpha\)-supra open.

(ii) The intersection of an intuitionistic fuzzy \(\alpha\)-supra open set and an intuitionistic fuzzy pre-supra open set is intuitionistic fuzzy pre-supra open.

\end{theorem}

Proof. It is obvious.

3. INTUITIONISTIC FUZZY \(\alpha\)-SUPRA CONTINUOUS MAP

\begin{definition}
Let \((X, \tau)\) and \((Y, \sigma)\) be two intuitionistic fuzzy \(\alpha\)-supra open sets and \(\mu\) be an associated supra topology with \(\tau\). A map \(f : (X, \tau) \rightarrow (Y, \sigma)\) is called intuitionistic fuzzy \(\alpha\)-supra continuous map if the inverse image of each open set in \(Y\) is an intuitionistic fuzzy \(\alpha\)-supra open set in \(X\).

\end{definition}

\begin{theorem}
Every intuitionistic fuzzy supra continuous map is intuitionistic fuzzy \(\alpha\)-supra continuous map.

\end{theorem}

Proof. Let \(f : (X, \tau) \rightarrow (Y, \sigma)\) be an intuitionistic fuzzy supra continuous map and \(A\) is an open set in \(Y\). Then \(f^{-1}(A)\) is an open set in \(X\). Since \(\mu\) is associated with \(\tau\), then \(\tau \subseteq \mu\). Therefore, \(f^{-1}(A)\) is an intuitionistic fuzzy supra open set in \(X\) which is an intuitionistic fuzzy supra open set in \(X\). Hence \(f\) is an intuitionistic fuzzy \(\alpha\)-supra continuous map.

\begin{remark}
Every intuitionistic fuzzy \(\alpha\)-supra continuous map need not be intuitionistic fuzzy supra continuous map.

\end{remark}

\begin{theorem}
Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \(\mu\) be an associated supra topology with \(\tau\). Let \(f\) be a map from \(X\) into \(Y\). Then the following are equivalent:

(i) \(f\) is an intuitionistic fuzzy supra \(\alpha\)-continuous map

(ii) the inverse image of a closed set in \(Y\) is an intuitionistic fuzzy supra \(\alpha\)-closed set in \(X\)

(iii) \(\alpha\)-cl\((f^{-1}(A))\) \(\subseteq\) \(f^{-1}(\text{cl}(A))\) for every set \(A\) in \(X\).

(iv) \(f(\alpha\text{-cl}(A))\) \(\subseteq\) \(\text{cl}(f(A))\) for every set \(A\) in \(X\).

(v) \(f^{-1}(\text{int}(B))\) \(\subseteq\) \(\alpha\text{-int}(f^{-1}(B))\) for every set \(B\) in \(Y\).

\end{theorem}

Proof. (i) \(\Rightarrow\) (ii): Let \(A\) be a closed set in \(Y\), then \(Y - A\) is open in \(Y\). Thus, \(f^{-1}(X - A) = X - f^{-1}(A)\) is \(\alpha\)-open in \(X\). It follows that \(f^{-1}(A)\) is a \(\alpha\)-closed set of \(X\).

(ii) \(\Rightarrow\) (iii): Let \(A\) be any subset of \(X\). Since \(\text{cl}(A)\) is closed in \(Y\), then it follows that \(f^{-1}(\text{cl}(A))\) is \(\alpha\)-closed in \(X\). Therefore, \(f^{-1}(\text{cl}(A)) = \alpha\text{-cl}(f^{-1}(\text{cl}(A))) \supseteq \alpha\text{-cl}(f^{-1}(A))\).

(iii) \(\Rightarrow\) (iv): Let \(A\) be any subset of \(X\). By (iii) we obtain, \(f^{-1}(\text{cl}(f(A))) \supseteq \alpha\text{-cl}(f^{-1}(f(A))) \supseteq \alpha\text{-cl}(f(A))\) and hence \(f(\alpha\text{-cl}(A)) \subseteq \text{cl}(f(A))\).

(iv) \(\Rightarrow\) (v): Let \(f(\alpha\text{-cl}(A)) \subseteq \text{cl}(f(A))\) for every set \(A\) in \(X\). Then
\[ \alpha\text{-cl}(A) \subseteq f^{-1}(\text{cl}(f(A))), \ X - \alpha\text{-cl}(A) \supseteq X - f^{-1}(\text{cl}(f(A))) \]

and \( \alpha\text{-int}(X - A) \supseteq f^{-1}(\text{int}(Y - f(A))) \). Then \( \alpha\text{-int}(f^{-1}(B)) \supseteq f^{-1}(\text{int}(B)) \).

Therefore \( f^{-1}(\text{int}(B)) \subseteq \alpha\text{-int}(f^{-1}(B)) \), for every \( B \) in \( Y \).

\((v) \Rightarrow (i)\) : Let \( A \) be an open set in \( Y \). Therefore, \( f^{-1}(\text{int}(A)) \subseteq \alpha\text{-int}(f^{-1}(A)) \), hence \( f^{-1}(A) \subseteq \alpha\text{-int}(f^{-1}(A)) \). But by other hand, we know that,

\[ \alpha\text{-int}(f^{-1}(A)) \subseteq f^{-1}(A). \]

Then \( f^{-1}(A) = \alpha\text{-int}(f^{-1}(A)) \). Therefore, \( f^{-1}(A) \) is a \( \alpha \)-open set. \( \square \)

**Theorem 3.5.** If a map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is a \( \alpha \)-continuous and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) is continuous, then \( (g \circ f) \) is \( \alpha \)-continuous.

**Proof.** Obvious. \( \square \)

**Theorem 3.6.** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an intuitionistic fuzzy \( \alpha \)-continuous map, if one of the following holds:

1. \( f^{-1}(\alpha\text{-int}(A)) \subseteq \text{int}(f^{-1}(A)) \) for every set \( A \) in \( Y \).
2. \( \text{cl}(f^{-1}(A)) \subseteq f^{-1}(\alpha\text{-cl}(A)) \) for every set \( A \) in \( Y \).
3. \( \text{cl}(f(B)) \subseteq \alpha\text{-cl}(f(B)) \) for every set \( B \) in \( X \).

**Proof.** Let \( A \) be any open set of \( Y \), if condition (i) is satisfied, then \( f^{-1}(\alpha\text{-int}(A)) \subseteq \text{int}(f^{-1}(A)) \). We get, \( f^{-1}(A) \subseteq \text{int}(f^{-1}(A)) \). Therefore \( f^{-1}(A) \) is an intuitionistic fuzzy supra open set. Every intuitionistic fuzzy supra open set is intuitionistic fuzzy supra \( \alpha \)-open set. Hence \( f \) is an intuitionistic fuzzy \( \alpha \)-continuous function.

If condition (ii) is satisfied, then we can easily prove that \( f \) is an intuitionistic fuzzy \( \alpha \)-continuous function. If condition (iii) is satisfied, and \( A \) is any open set of \( Y \). Then \( f^{-1}(A) \) is a set in \( X \) and \( f(\text{cl}(f^{-1}(A))) \subseteq \alpha\text{-cl}(f(f^{-1}(A))) \). This implies \( f(\text{cl}(f^{-1}(A))) \subseteq \alpha\text{-cl}(A) \). This is nothing but condition (ii). Hence \( f \) is an intuitionistic fuzzy \( \alpha \)-continuous function. \( \square \)

4. **Conclusion**

We introduced and investigated a new class of set and its function called intuitionistic fuzzy \( \alpha \)-supra open set and intuitionistic fuzzy \( \alpha \)-supra continuous functions respectively. Also we studied some of its properties and relation with the existing sets and functions.

**References**


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