Completely generalized alpha continuous mappings in intuitionistic fuzzy topological spaces

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ABSTRACT. The purpose of this paper is to introduce and study the concepts of completely generalized alpha continuous mappings in intuitionistic fuzzy topological spaces. We investigate some of their properties. Some of results related to product of mappings and the graph of mappings are obtained. Inter relation between other continuous mappings are established with counter example

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1. Introduction

In 1965 Zadeh [16] introduced the concept of fuzzy sets. Using the concept of fuzzy sets, Chang introduced the concept of fuzzy topological spaces. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. Joen et al [6] obtained some interesting results about the intuitionistic $\alpha$ continuity and intuitionistic fuzzy pre continuity. Continuity plays an important role in the study of Topological spaces. A new weaker form of continuity called as completely continuity was introduced by I.M.Hanafy [5] in intuitionistic fuzzy topological spaces. In this paper we introduce Intuitionistic fuzzy completely generalized alpha continuous mappings and studied some of their properties. We provide some characterizations of intuitionistic fuzzy completely generalized alpha continuous mappings.
2. Preliminaries

Definition 2.1 ([1]). Let $X$ be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) $A$ in $X$ is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$ where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $\gamma_A(x) : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in $X$.

Definition 2.2 ([1]). Let $A$ and $B$ be IFSs of the form $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$ and $B = \{(x, \mu_B(x), \gamma_B(x)) | x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $A^c = \{(x, \mu_A(x), \mu_A(x)) | x \in X\}$
(d) $A \cap B = \{(x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)) | x \in X\}$
(e) $A \cup B = \{(x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)) | x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \gamma_A \rangle$ instead of $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$.

Definition 2.3 ([2]). An intuitionistic fuzzy topology (IFT in short) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms.

(i) $\emptyset, X \in \tau$.
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$.
(iii) $\cup G_i \in \tau$ for any family $\{G_i | i \in I\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS in short) in $X$. The complement $A^c$ of an IFOS $A$ in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS in short) in $X$.

Definition 2.4 ([1]). Let $(X, \tau)$ be an IFTS and $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in $X$. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

(i) $\text{int}(A) = \cup\{G | G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$
(ii) $\text{cl}(A) = \cap\{K | K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$.

Note that for any IFS $A$ in $(X, \tau)$, we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5 ([7]). An IFS $A = \langle x, \mu_A, \gamma_A \rangle$ in an IFTS $(X, \tau)$ is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,
(ii) intuitionistic fuzzy pre closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$,
(iii) intuitionistic fuzzy fuzzy pre closed set (IFpCS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,
(iv) intuitionistic fuzzy regular closed set (IFRC in short) if $A = \text{cl}(\text{int}(A))$.

The family of all IFCS (respectively IFSCS, IFpCS, IFRC) of an IFTS $(X, \tau)$ is denoted by IFC(X) (respectively IFSC(X), IFpCS(X), IFRC(X)).

Definition 2.6 ([7]). An IFS $A = \langle x, \mu_A, \gamma_A \rangle$ in an IFTS $(X, \tau)$ is said to be an

(i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq \text{cl}(\text{int}(A))$,
(ii) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq int(cl(A))$,
(iii) intuitionistic fuzzy $\alpha$-open set (IFOS in short) if $A \subseteq \text{int}(cl(int(A)))$,
(iv) intuitionistic fuzzy regular open set (IFROS in short) if $A = \text{int}(cl(A))$,
(v) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $cl(c^cA) \subseteq A$,
(vi) intuitionistic fuzzy $\alpha$-open set (IFGCS in short) if $cl(c^cA) \subseteq A$.

The family of all IFOS (respectively IFOS, IFOS, IFROS) of an IFTS $(X, \tau)$ is denoted by $\text{IFO}(X)$ (respectively $\text{IFSO}(X)$, $\text{IFOSO}(X)$, $\text{IFRO}(X)$).

**Definition 2.7** ([7]). Let an IFS $A$ of an IFTS. Then $a\text{int}A = \bigcup \{K | K \text{ is an IFOS in } X \text{ and } K \subseteq A\}.$

Note that for any IFS $A$ in $(X, \tau)$, we have $\text{oc}(A^c) = (a\text{int}(A))^c$ and $a\text{int}(A^c) = (\text{oc}(A))^c$.

**Definition 2.8.** An IFS $A$ of an IFTS $(X, \tau)$ is an

(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.[15]
(ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFROS in $X$.[14]
(iii) intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if $sd(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.[11]
(iv) intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $sd(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.[10]
(v) intuitionistic fuzzy $\alpha$-generalized closed set (IFGCS in short) if $oc(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.
(vi) intuitionistic fuzzy generalized pre closed set (IFGPCS in short) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.[8]

**Definition 2.9** ([7]). An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized $\alpha$ closed set (IFGCS in short) if $oc(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.

**Example 2.10** ([7]). Let $X = \{a, b\}$ and let $\tau = \{0_{\sim}, G, 1_{\sim}\}$ is an IFT on $X$, where $G = \{x, (0.2, 0.3), (0.8, 0.7)\}$. Here the only $\alpha$ open sets are $0_{\sim}$, $1_{\sim}$ and G. Then the IFS $A = \{x, (0.6, 0.7), (0.4, 0.3)\}$ is an IFGCS in $(X, \tau)$.

**Result 2.11** ([7]). Every IFCS,IFGCS,IFRCS, IFaCS is an IFGCS but the converses are not true in general.

**Definition 2.12** ([7]). An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized $\alpha$ open set (IFGOS in short) if the complement $A^c$ is an IFGCS in $X$.

**Definition 2.13** ([7]). Let an IFS $A$ of an IFTS $(X, \tau)$. Then $g\text{aint}(A) = \bigcup \{K | K \text{ is an IFGOS in } X \text{ and } K \subseteq A\}.$

$g\text{oc}(A) = \bigcap \{K | K \text{ is an IFGCS in } X \text{ and } A \subseteq K\}.$

**Definition 2.14.** Let $f$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is said to be an:

(a) intuitionistic fuzzy continuous(IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$. [4]
(b) intuitionistic fuzzy $\alpha$ continuous (IF$\alpha$ continuous in short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$.\textsuperscript{[6]}

(c) intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in IFGO(X)$ for every $B \in \sigma$.\textsuperscript{[13]}

(d) intuitionistic fuzzy semi generalized continuous (IFSG continuous in short) if $f^{-1}(B) \in IFSGO(X)$ for every $B \in \sigma$.\textsuperscript{[12]}

(e) intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B) \in IFGSO(X)$ for every $B \in \sigma$.\textsuperscript{[10]}

(f) intuitionistic fuzzy generalized $\alpha$ continuous (IFG$\alpha$ continuous in short) if $f^{-1}(B) \in IFG\alpha O(X)$ for every $B \in \sigma$.\textsuperscript{[9]}

\textbf{Definition 2.15 (\cite{5}).} Let $f$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is said to be an Intuitionistic fuzzy completely continuous if $f^{-1}(B) \in IF\alpha CO(X)$ for every $B \in \sigma$.

\textbf{Definition 2.16 (\cite{5}).} Let $X, Y$ be nonempty sets and $A = \{ (x, \mu_A(x), \gamma_A(x)) \}, B = \{ (x, \mu_B(x), \gamma_B(x)) \}$ be IFSs of $X$ and $Y$ respectively. Then $A \times B$ is an IFS of $X \times Y$ defined by $(A \times B)(x, y) = \{ (x, \mu_A(x), \mu_B(y)), (x, \gamma_A(x), \gamma_B(y)) \}$.

\textbf{Definition 2.17 (\cite{5}).} Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2)), \forall (x_1, x_2) \in X_1 \times X_2$.

\textbf{Definition 2.18 (\cite{5}).} Let $f : X \to Y$ be a function. The graph $g : X \to X \times Y$ of $f$ is defined by $g(x) = (x, f(x)), \forall x \in X$.

\textbf{Result 2.19 (\cite{13}).} Every IF continuous mapping is an IFG continuous mapping.

\textbf{Definition 2.20 (\cite{12}).} Let $X$ be a non empty set and $c \in X$ a fixed element in $X$. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha + \beta \leq 1$ then $c(\alpha, \beta) = (x, c_\alpha, c_{1-\beta})$ is called an intuitionistic fuzzy point in $X$, where $\alpha$ denotes the degree of membership of $c(\alpha, \beta)$ and $\beta$ denotes the degree of non membership of $c(\alpha, \beta)$.

\textbf{Definition 2.21 (\cite{12}).} Two IFSs $A$ and $B$ in $X$ are said to be $q$-coincident ($A \equiv B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

\textbf{Definition 2.22 (\cite{3}).} An IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $\alpha_1 T \frac{1}{2}$ (IF $\alpha_1 T \frac{1}{2}$ in short ) space if every IFG$\alpha$CS in $X$ is an IFCS in $X$.

\textbf{Definition 2.23 (\cite{3}).} An IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $\alpha_1 T \frac{1}{2}$ (IF $\alpha_1 T \frac{1}{2}$ in short ) space if every IFG$\alpha$CS in $X$ is an IF$\alpha$CS in $X$.

3. **Intuitionistic Fuzzy Completely Generalized Alpha Continuous Mappings**

In this section we introduce intuitionistic fuzzy completely generalized alpha continuous mappings and studied some of its properties.
Definition 3.1. A mapping \( f : (X, \tau) \to (Y, \sigma) \) is called an intuitionistic fuzzy completely generalized alpha continuous (IF completely Go continuous in short) if \( f^{-1}(B) \) is an IFRCS in \( (X, \tau) \) for every IFGoCS \( B \) of \( (Y, \sigma) \).

Theorem 3.2. Every IF completely Go continuous mapping is an IFGo continuous mapping but not conversely.

Proof. Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF completely Go continuous mapping. Let \( B \) be an IFCS in \( Y \). Since every IFCS is an IFGoCS, \( B \) is an IFGoCS in \( Y \). Then \( f^{-1}(B) \) is an IFRCS in \( X \). Since every IFRCS is an IFGoCS, \( f^{-1}(B) \) is an IFGoCS in \( X \). Hence \( f \) is an IFGo continuous mapping.

\( \square \)

Example 3.3. Let us consider \( X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.2, 0.3), (0.7, 0.6) \rangle \), and \( G_2 = \langle y, (0.4, 0.4), (0.5, 0.6) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTS on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Then \( f \) is an IFGo continuous mapping but not IF completely Go continuous mapping. Here \( G_2 \) is an IFGoCS in \( Y \) but not IFRCS in \( X \) since \( c(\text{int}(f^{-1}(G_2))) = G_1^c \neq f^{-1}(G_2^c) \).

Theorem 3.4. Every IF completely Go continuous mapping is an IF continuous mapping but not conversely.

Proof. Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF completely Go continuous mapping. Let \( B \) be an IFCS in \( Y \). Since every IFCS is an IFGoCS, \( B \) is an IFGoCS in \( Y \). Then \( f^{-1}(B) \) is an IFRCS in \( X \). Since every IFRCS is an IFGoCS, \( f^{-1}(B) \) is an IFGoCS in \( X \). Hence \( f \) is an IF continuous mapping.

\( \square \)

Example 3.5. Let us consider \( X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.2, 0.2), (0.3, 0.7) \rangle \), and \( G_2 = \langle x, (0.5, 0.4), (0.4, 0.6) \rangle \) and \( G_3 = \langle y, (0.5, 0.4), (0.4, 0.6) \rangle \). Then \( \tau = \{0, 1\} \) and \( \sigma = \{0, G_3, 1\} \) are IFTS on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Then \( f \) is an IF continuous mapping but not IF completely Go continuous mapping. Here \( G_3 \) is an IFGoCS in \( Y \) but not an IFRCS in \( X \) since \( c(\text{int}(f^{-1}(G_3))) = G_1^c \neq f^{-1}(G_3^c) \).

Theorem 3.6. Every IF completely Go continuous mapping is an IF \( \alpha \) continuous mapping but not conversely.

Proof. Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF completely Go continuous mapping. Let \( B \) be an IFCS in \( Y \). Since every IFCS is an IFGoCS, \( B \) is an IFGoCS in \( Y \). Then \( f^{-1}(B) \) is an IFRCS in \( X \). Since every IFRCS is an IFGoCS, \( f^{-1}(B) \) is an IFGoCS in \( X \). Hence \( f \) is an IFGo continuous mapping.

\( \square \)

Example 3.7. Let us consider \( X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.2, 0.4), (0.8, 0.6) \rangle \), and \( G_2 = \langle y, (0.2, 0.4), (0.8, 0.6) \rangle \). Then \( \tau = \{0, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTS on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Then \( f \) is an IF \( \alpha \) continuous mapping but not IF completely Go continuous mapping. Since \( A = \langle y, (0.1, 0.2), (0.9, 0.8) \rangle \) is an IFGoCS in \( Y \) but not IFRCS in \( X \), \( c(\text{int}(f^{-1}(A^c))) = G_1^c \neq A^c \).

Theorem 3.8. Every IF completely Go continuous mapping is an IF\( \alpha \)G continuous mapping but not conversely.

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Proof. Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF completely \( G_a \) continuous mapping. Let \( B \) be an IFCS in \( Y \). Since every IFCS is an IFGCS, \( B \) is an IFGCS in \( Y \). Then \( f^{-1}(B) \) is an IFRCS in \( X \). Since every IFRCS is an IFGCS, \( f^{-1}(B) \) is an IFGCS in \( X \). Hence \( f \) is an IFG continuous mapping. \( \square \)

Example 3.9. Let us consider \( X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.8, 0.8), (0.2, 0.1) \rangle \), and \( G_2 = \langle y, (0.1, 0.3), (0.9, 0.7) \rangle \). Then \( \tau = \{0_-, G_1, 1_-\} \) and \( \sigma = \{0_-, G_2, 1_-\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Then \( f \) is an IFG continuous mapping but not an IF completely \( G_a \) continuous mapping. Here \( G_2 \) is an IFGCS in \( Y \) but not IFRCS in \( X \) since \( cl(int(f^{-1}(G_2^1))) = 0 \neq f^{-1}(G_2^1) \).

Theorem 3.10. Every IF completely \( G_a \) continuous mapping is an IF generalized semi continuous but not conversely.

Proof. Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF completely \( G_a \) continuous mapping. Let \( B \) be an IFCS in \( Y \). Since every IFCS is an IFGCS, \( B \) is an IFGCS in \( Y \). Then \( f^{-1}(B) \) is an IFRCS in \( X \). Since every IFRCS is an IFGCS, \( f^{-1}(B) \) is an IFGCS in \( X \). Hence \( f \) is an IF generalized semi continuous mapping. \( \square \)

Example 3.11. Let us consider \( X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.2, 0.2), (0.4, 0.5) \rangle \), and \( G_2 = \langle y, (0.1, 0.1), (0.9, 0.8) \rangle \). Then \( \tau = \{0_-, G_1, 1_-\} \) and \( \sigma = \{0_-, G_2, 1_-\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly \( f \) is an intuitionistic fuzzy generalized semi-continuous mapping, but not an intuitionistic fuzzy completely \( G_a \) continuous mapping. Here \( G_2^1 \) is an IFGCS in \( Y \) but not IFRCS in \( X \) since \( cl(int(f^{-1}(G_2^1))) = G_1^1 \neq f^{-1}(G_2^1) \).

Theorem 3.12. Every IF completely \( G_a \) continuous mapping is an IF semi generalized continuous mapping but not conversely.

Proof. Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF completely \( G_a \) continuous mapping. Let \( B \) be an IFCS in \( Y \). Since every IFCS is an IFGCS, \( B \) is an IFGCS in \( Y \). Then \( f^{-1}(B) \) is an IFRCS in \( X \). Since every IFRCS is an IFGCS, \( f^{-1}(B) \) is an IFGCS in \( X \). Hence \( f \) is an IF semi generalized continuous mapping. \( \square \)

Example 3.13. Let us consider \( X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.3, 0.3), (0.4, 0.4) \rangle \), and \( G_2 = \langle y, (0.1, 0.1), (0.5, 0.6) \rangle \). Then \( \tau = \{0_-, G_1, 1_-\} \) and \( \sigma = \{0_-, G_2, 1_-\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Then \( f \) is an intuitionistic fuzzy semi generalized continuous mapping, but not an intuitionistic fuzzy completely \( G_a \) continuous mapping. Here \( G_2^1 \) is an IFGCS in \( Y \) but not an IFRCS in \( X \) since \( cl(int(f^{-1}(G_2^1))) = G_1^1 \neq f^{-1}(G_2^1) \).

Theorem 3.14. Every IF completely \( G_a \) continuous mapping is an IF generalized pre continuous mapping but not conversely.

Proof. Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IF completely \( G_a \) continuous mapping. Let \( B \) be an IFCS in \( Y \). Since every IFCS is an IFGCS, \( B \) is an IFGCS in \( Y \). Then \( f^{-1}(B) \) is an IFRCS in \( X \). Since every IFRCS is an IFGCS, \( f^{-1}(B) \) is an IFGCS in \( X \). Hence \( f \) is an IF generalized pre continuous mapping. \( \square \)
Example 3.15. Let us consider $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$, and $G_2 = \langle y, (0.3, 0.3), (0.7, 0.7) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(u) = u, f(b) = v$. Then $f$ is an intuitionistic fuzzy generalized pre continuous mapping but not an intuitionistic fuzzy completely $G_a$ continuous mapping. Here $G_2^0$ is an IFGoCS in $Y$ but not an IFRCS in $X$ since $\text{cl}(\text{int}(f^{-1}(G_2^0))) = G_1^0 \neq f^{-1}(G_2^0)$.

Theorem 3.16. Every IF completely $G_a$ continuous mapping is an IF completely continuous mapping but not conversely.

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely $G_a$ continuous mapping. Let $B$ be an IFCS in $Y$. Since every IFCS is an IFGoCS, $B$ is an IFGoCS in $Y$. Then $f^{-1}(B)$ is an IFRCS in $X$. Then every IFCS in $Y$ is an IFRCS in $X$. Hence $f$ is an IF completely continuous mapping.

Example 3.17. Let us consider $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.2, 0.3), (0.5, 0.7) \rangle$, and $G_2 = \langle y, (0.2, 0.3), (0.5, 0.7) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u, f(b) = v$. Clearly $f$ is an intuitionistic fuzzy completely continuous mapping but not an IF completely $G_a$ continuous mapping. But $A = \langle y, (0.4, 0.3), (0.6, 0.7) \rangle$ is an IFGoCS in $Y$ but not an IFRCS in $X$, since $\text{cl}(\text{int}(f^{-1}(A^c))) = G_1^c \neq f^{-1}(A^c)$.

The following diagram implications are true:

IF comp cont. means Intuitionistic fuzzy completely continuous mapping

Theorem 3.18. A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF completely $G_a$ continuous mapping if for every IFP $c(\alpha, \beta) \in X$ and for every IFN $A$ of $f(c(\alpha, \beta))$, there exists an IFROS $B \subseteq X$ such that $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.

Proof. Let $c(\alpha, \beta) \in X$ and let $A$ be an IFN of $f(c(\alpha, \beta))$. Then there exists an IFOS $U$ in $Y$ such that $f(c(\alpha, \beta)) \in U \subseteq A$. Since every IFOS is an IFGoOS, $U$ is an IFGoOS in $Y$. Hence by hypothesis $f^{-1}(U)$ is an IFROS in $X$ and $c(\alpha, \beta) \in f^{-1}(U)$. Let $B = f^{-1}(U)$. Therefore $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$. 

Theorem 3.19. A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF completely $G_a$ continuous mapping then $\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(B)$ for every IFS $B$ in $Y$. 

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Let $B$ be an IFS in $Y$. Then $cl(B)$ is an IFCS in $Y$. By hypothesis $f^{-1}(cl(B))$ is an IFRCS in $X$. Hence $cl(int(cl(f^{-1}(B)))) \subseteq f^{-1}(cl(B)) = f^{-1}(B)$. 

**Theorem 3.20.** A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF completely $G_{\alpha}$ continuous mapping then the following are equivalent.

(i) for any IFGoOS $A$ in $Y$ and for any IFP $c(\alpha, \beta) \in X$ if $f(c(\alpha, \beta)) q A$ then $c(\alpha, \beta) q int(f^{-1}(A))$

(ii) for any IFGoOS $A$ in $Y$ and for any $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta)) q A$ then there exist an IFOS $B$ in $X$ such that $c(\alpha, \beta) q B$ and $f(B) \subseteq A$

**Proof.** (i) $\Rightarrow$ (ii) Let $A \subseteq Y$ be an IFGoOS and let $c(\alpha, \beta) \in X$. Let $f(c(\alpha, \beta)) q A$. Then $c(\alpha, \beta) q f^{-1}(A)$. (ii) implies that $c(\alpha, \beta) q int(f^{-1}(A))$, where $int(f^{-1}(A))$, is an IFOS in $X$. Let $B = int(f^{-1}(A))$ since $int(f^{-1}(A)) \subseteq f^{-1}(A), B \subseteq f^{-1}(A)$. Then $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(ii) $\Rightarrow$ (i) Let $A \subseteq Y$ be an IFGoOS and let $c(\alpha, \beta) \in X$. Suppose $f(c(\alpha, \beta)) q A$, then by (ii) there exists an IFOS $B$ in $X$ such that $c(\alpha, \beta) q B$ and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$, That is $int(B) \subseteq int(f^{-1}(A))$. Therefore $c(\alpha, \beta) q B$ implies $c(\alpha, \beta) q int(f^{-1}(A))$.

**Theorem 3.21.** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function and $g : X \rightarrow X \times X$ be the graph of the function $f$. Then $f$ is completely $G_{\alpha}$ intuitionistic fuzzy continuous if is so.

**Proof.** Let us consider $B \in \sigma$, then $f^{-1}(B) = f^{-1}(1 \times B) = 1 \cap f^{-1}(B) = g^{-1}(1 \times B)$. Since $B$ is an IFOS in $Y$, and every IFOS is an IFGoOS, $1 \times B$ is an IFGoOS in $X \times Y$. Also the fact that $f$ is an completely IFGo continuous implies that $g^{-1}(1 \times B)$ is an IFROS in $X$. Hence $f^{-1}(B)$ is an IFROS in $X$. Hence $f^{-1}(B)$ is an IFROS in $X$. Hence $f$ is completely $G_{\alpha}$ intuitionistic fuzzy continuous mapping.

**Theorem 3.22.** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then the following are equivalent.

(i) $f$ is an IF completely $G_{\alpha}$ continuous mapping

(ii) $f^{-1}(B)$ is an IFROS in $X$ for every for every IFGoOS $B$ in $Y$.

(iii) for every IFP $c(\alpha, \beta) \in X$ and for every IFGoOS $B$ in $Y$ such that if $f(c(\alpha, \beta)) \in B$ there exists an IFROS $A$ in $X$ such that $c(\alpha, \beta) \in A$ and $f(A) \subseteq B$.

**Proof.** (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii) Let $c(\alpha, \beta) \in X$. Let $B$ be an IFGoOS in $Y$ and $f^{-1}(B)$ is an IFROS in $X$. Let $f(c(\alpha, \beta)) \in B$ and let $A = f^{-1}(B)$. Then $c(\alpha, \beta) \in f^{-1}(f(c(\alpha, \beta))) \subseteq f^{-1}(B) = A$. Therefore $c(\alpha, \beta) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$. This implies $f(A) \subseteq B$.

(iii) $\Rightarrow$ (i) Let $B$ be an IFGoOS in $Y$ and let $c(\alpha, \beta) \in X$ and $f(c(\alpha, \beta)) \in B$.Then by hypothesis there exists an IFROS $B$ in $X$ such that $c(\alpha, \beta) \in G$ and $f(G) \subseteq B$. Now $c(\alpha, \beta) \in f^{-1}(B)$. But $G \subseteq f^{-1}(B), c(\alpha, \beta) \in G$ and $f(G) \subseteq B$. This implies $G \subseteq f^{-1}(f(G)) \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \bigcup_{c(\alpha, \beta) \in f^{-1}(B)} G \subseteq f^{-1}(B)$. This
A mapping \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy completely \( G_\alpha \)-continuous if and only if \( f^{-1}(A) \) is an IFROS in \( X \) for every IF\( G_\alpha \)OS \( A \) in \( Y \).

**Proof.** Necessity: Let \( A \) be an IF\( G_\alpha \)OS in \( Y \). This implies \( A^c \) is an IF\( G_\alpha \)CS in \( Y \). Since \( f \) is an intuitionistic fuzzy completely \( G_\alpha \)-continuous mapping, \( f^{-1}(A^c) \) is an IF\( G_\alpha \)CS in \( X \). Hence \( f^{-1}(A^c) = f^{-1}(A) \), \( f^{-1}(A) \) is an IFROS in \( X \).

Sufficiency: Let \( A \) be an IF\( G_\alpha \)CS in \( Y \). This implies \( A^c \) is an IF\( G_\alpha \)OS in \( Y \). By hypothesis \( f^{-1}(A^c) \) is an IFROS in \( X \). Since \( f^{-1}(A^c) = f^{-1}(A) \), \( f^{-1}(A) \) is an IF\( G_\alpha \)CS in \( X \). Hence \( f \) is an intuitionistic fuzzy completely \( G_\alpha \)-continuous mapping. \( \square \)

**Theorem 3.24.** For any two intuitionistic fuzzy completely \( G_\alpha \) continuous mappings \( f_1, f_2 : (X, \tau) \to (Y, \sigma) \), the mapping \( (f_1, f_2) : (X, \tau) \to (Y \times Y, \sigma \times \sigma) \) is also an IF completely \( G_\alpha \) continuous mapping, where \( (f_1, f_2)(x) = (f_1(x), f_2(x)), \forall x \in X \).

**Proof.** Let \( A \times B \) be an IF\( G_\alpha \)OS in \( Y \times Y \). Then
\[
(f_1, f_2)^{-1}(A \times B)(x) = (A \times B)(f_1(x), f_2(x))
\]

\[
= (x, \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max(\gamma_A(f_1(x)), \gamma_B(f_2(x))))
\]

\[
= (x, \min(f_1^{-1}(\mu_A(x)), f_2^{-1}(\mu_B(x))), \max(f_1^{-1}(\gamma_A(x)), f_2^{-1}(\gamma_B(x))))
\]

\[
= (f_1^{-1}(A) \cap f_2^{-1}(B))(x).
\]

Since \( f_1 \) and \( f_2 \) are IF completely \( G_\alpha \) continuous mappings, \( f_1^{-1}(A) \) and \( f_2^{-1}(B) \) are IFROS\( s \) in \( X \). Since intersection of IFROS is an IFROS, \( f_1^{-1}(A) \cap f_2^{-1}(B) \) is an IFROS in \( X \). Hence \( (f_1, f_2) \) is an intuitionistic fuzzy \( G_\alpha \)-continuous mapping. \( \square \)

**Theorem 3.25.** Let \( f : (X, \tau) \to (Y, \sigma) \) and \( g : (Y, \sigma) \to (Z, \delta) \) be any two mappings where \( (Z, \delta) \) is an IF\( G_\alpha T_{\frac{1}{2}} \) space. Then the following statements hold.

(i) \( f \) be an IF completely \( G_\alpha \) continuous mapping and \( g \) be an IF continuous mapping. Then their composition \( g \circ f : (X, \tau) \to (Z, \delta) \) is an IF completely \( G_\alpha \) continuous mapping.

(ii) \( f \) be an IF completely \( G_\alpha \) continuous mapping and \( g \) be an IF\( G_\alpha \) continuous mapping. Then their composition \( g \circ f : (X, \tau) \to (Z, \delta) \) is an IF completely \( G_\alpha \) continuous mapping.

(iii) \( f \) be an IF completely \( G_\alpha \) continuous mapping and \( g \) be an IF completely continuous mapping. Then their composition \( g \circ f : (X, \tau) \to (Z, \delta) \) is an IF completely \( G_\alpha \) continuous mapping.

(iv) \( f \) be an IF completely \( G_\alpha \) continuous mapping and \( g \) be an IF continuous mapping. Then their composition \( g \circ f : (X, \tau) \to (Z, \delta) \) is an IF completely \( G_\alpha \) continuous mapping.

**Proof.**

(i) Let \( A \) be an IF\( G_\alpha \)CS in \( Z \). Since \( Z \) is an IF\( G_\alpha T_{\frac{1}{2}} \) space, \( A \) is IFCS in \( Z \). Then \( g^{-1}(A) \) is an IFCS in \( Y \), by hypothesis. Since every IFCS is an IF\( G_\alpha \)CS, \( g^{-1}(A) \) is an IF\( G_\alpha \)CS in \( Y \). Therefore \( f^{-1}(g^{-1}(A)) \) is an IF\( G_\alpha \)CS in \( X \). Hence \( g \circ f \)
is an IF completely Gα continuous mapping. The proof of (ii), (iii), (iv) is similar to (i). □

**Theorem 3.26.** Let \( f : (X, \tau) \to (Y, \sigma) \) be an IFGα continuous mapping. Then the following statements hold.

(i) \( f(g(a)(\text{cl}(A))) \subseteq \text{cl}(f(A)) \), for every IFS \( A \) in \( X \).

(ii) \( g(a)(\text{cl}(f^{-1}(B))) \subseteq f^{-1}(\text{cl}(B)) \), for every IFS \( B \) in \( X \).

**Proof.**

(i) Let \( A \subseteq X \). Then \( \text{cl}(f(A)) \) is an IFCS in \( Y \). Since \( f \) is an IF completely Gα continuous mapping, \( f^{-1}(\text{cl}(f(A))) \) is an IFGαCS in \( X \). Since \( A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\text{cl}(f(A))) \) and \( f^{-1}(\text{cl}(f(A))) \) is an IFGα closed, implies \( g(a)(\text{cl}(A)) \subseteq \text{cl}(f(A)) \). Hence \( g(a)(\text{cl}(A)) \subseteq \text{cl}(f(A)) \).

(ii) Replacing \( A \) by \( f^{-1}(B) \) in (i), we get \( f(g(a)(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B) \). Hence \( g(a)(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B)) \) for every IFS \( B \) in \( Y \). □

**Theorem 3.27.** If \( f : (X, \tau) \to (Y, \sigma) \) be an IF completely Gα - continuous mapping then \( f \) is an IF α continuous mapping.

**Proof.** Let \( A \) be an IFCS in \( Y \). Since every closed set is an IFGαCS, \( A \) is an IFGαCS in \( Y \). By hypothesis, \( f^{-1}(A) \) is an IFRCS and hence \( f^{-1}(A) \) is an IFαCS in \( X \). Hence \( f \) is IF α continuous mapping. □

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