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Completely generalized alpha continuous mappings in intuitionistic fuzzy topological spaces

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Abstract. The purpose of this paper is to introduce and study the concepts of completely generalized alpha continuous mappings in intuitionistic fuzzy topological spaces. We investigate some of their properties. Some of results related to product of mappings and the graph of mappings are obtained. Inter relation between other continuous mappings are established with counter example

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1. Introduction

In 1965 Zadeh [16] introduced the concept of fuzzy sets. Using the concept of fuzzy sets, Chang introduced the concept of fuzzy topological spaces. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. Joen et al [6] obtained some interesting results about the intuitionistic α continuity and intuitionistic fuzzy pre continuity. Continuity plays an important role in the study of Topological spaces. A new weaker form of continuity called as completely continuity was introduced by I.M.Hanafy [5] in intuitionistic fuzzy topological spaces. In this paper we introduce Intuitionistic fuzzy completely generalized alpha continuous mappings and studied some of their properties. We provide some characterizations of intuitionistic fuzzy completely generalized alpha continuous mappings.

2. Preliminaries

Definition 2.1 ([1])**.** Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$ where the functions $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2 ([1])**.** Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle | x \in X\}$ Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $A^c = \{\langle x, \gamma_A(x), \mu_A(x)\rangle | x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle | x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle | x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \gamma_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$

Definition 2.3 ([2])**.** An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$.
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$.
(iii) $\cup G_i \in \tau$ for any family $\{G_i | i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 ([1])**.** Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \gamma_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

(i) $int(A) = \cup\{G | G$ is an IFOS in X and $G \subseteq A\}$,
(ii) $cl(A) = \cap\{K | K$ is an IFCS in X and $A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5 ([7])**.** An IFS $A = \langle x, \mu_A, \gamma_A\rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$,
(ii) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$,
(iii) intuitionistic fuzzy ?-closed set ($IF\alpha CS$ in short) if $cl(int(cl(A)) \subseteq A$,
(iv) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$,

The family of all IFCS (respectively IFSCS, $IF\alpha CS$, IFRCS) of an IFTS (X, τ) is denoted by IFC(X) (respectively IFSC(X), IFαC(X), IFRC(X)).

Definition 2.6 ([7])**.** An IFS $A = \langle x, \mu_A, \gamma_A\rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$,

(ii) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq int(cl(A))$,
(iii) intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq int(cl(int(A)))$,
(iv) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$,

The family of all IFOS (respectively IFSOS, IFαOS, IFROS) of an IFTS (X, τ) is denoted by IFO(X) (respectively IFSO(X), IFαO(X), IFRO(X)).

Definition 2.7 ([7]). Let an IFS A of an IFTS. Then $\alpha int A) = \cup\{K|K$ is an IFαOS in X and $K \subseteq A\}$.
$\alpha cl(A) = \cap\{K|K$ is an IFαCS in X and $A \subseteq K\}$.
Note that for any IFS A in(X, τ) , we have $\alpha cl(A^c) = (\alpha int(A))^c$ and $\alpha int(A^c) = (\alpha cl(A))^c$.

Definition 2.8. An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.[15]
(ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.[14]
(iii) intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in X.[11]
(iv) intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.[10]
(v) intuitionistic fuzzy α generalized closed set (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.
(vi) intuitionistic fuzzy generalized pre closed set (IFGPCS in short) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.[8]

Definition 2.9 ([7]). An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized α closed set (IFGαCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFαOS in X.

Example 2.10 ([7]). Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is an IFT on X, where $G = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ and G. Then the IFS $A = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$ is an IFGαCS in (X, τ).

Result 2.11 ([7]). Every IFCS,IFGCS,IFRCS, IFαCS is an IFGαCS but the converses are not true in general.

Definition 2.12 ([7]). An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized α open set (IFGαOS in short) if the complement A^c is an IFGαCS in X.

Definition 2.13 ([7]). Let an IFS A of an IFTS (X, τ). Then $g\alpha int(A) = \cup\{K|K$ is an IFGαOS in X and $K \subseteq A\}$.
$g\alpha cl(A) = \cap\{K|K$ is an IFGαCS in X and $A \subseteq K\}$.

Definition 2.14. Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be an:

(a) intuitionistic fuzzy continuous(IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.[4]

(b) intuitionistic fuzzy α continuous (IFα continuous in short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$.[6]
(c) intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in IFGO(X)$ for every $B \in \sigma$.[13]
(d) intuitionistic fuzzy semi generalized continuous (IFSG continuous in short) if $f^{-1}(B) \in IFSGO(X)$ for every $B \in \sigma$. [12]
(e) intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B) \in IFGSO(X)$ for every $B \in \sigma$. [10]
(f) intuitionistic fuzzy generalized α continuous (IFGα continuous in short) if $f^{-1}(B) \in IFG\alpha O(X)$ for every $B \in \sigma$. [3] (g) intuitionistic fuzzy α generalized continuous (IFαG continuous in short) if $f^{-1}(B) \in IF\alpha GO(X)$ for every $B \in \sigma$.[9]

Definition 2.15 ([5])**.** Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be an Intuitionistic fuzzy completely continuous if $f^{-1}(B) \in IFRO(X)$ for every $B \in \sigma$.

Definition 2.16 ([5])**.** Let X,Y be nonempty sets and $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle\}$ be IFSs of X and Y respectively. Then $A \times B$ is an IFS of $X \times Y$ defined by

$(A \times B)(x, y) = \{(A \times B)(x, y) = \{\langle(x, y), min(\mu_A(x), \mu_B(y)), max(\gamma_A(x), \gamma_B(y)\}$.

Definition 2.17 ([5])**.** Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2)$, $\forall(x_1, x_2) \in X_1 \times X_2$.

Definition 2.18 ([5])**.** Let $f : X \to Y$ be a function. The graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$,$\forall x \in X$.

Result 2.19 ([13])**.** Every IF continuous mapping is an IFG continuous mapping.

Definition 2.20 ([12])**.** Let X be a non empty set and $c \in X$ a fixed element in X. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha + \beta \leq 1$ then $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta}\rangle$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of $c(\alpha, \beta)$ and β denotes the degree of non membership of $c(\alpha, \beta)$.

Definition 2.21 ([12])**.** Two IFSs A and B in X are said to be q-coincident ($A_qB$in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

Definition 2.22 ([3])**.** An IFTS (X, τ) is said to be an intuitionistic fuzzy $\alpha_kT_{\frac{1}{2}}$ (IF $\alpha_kT_{\frac{1}{2}}$in short) space if every IFGαCS in X is an IFCS in X.

Definition 2.23 ([3])**.** An IFTS (X, τ) is said to be an intuitionistic fuzzy $\alpha_lT_{\frac{1}{2}}$ (IF $\alpha_lT_{\frac{1}{2}}$ in short) space if every IFGαCS in X is an IFαCS in X.

3. Intuitionistic fuzzy completely generalized alpha continuous mappings

In this section we introduce intuitionistic fuzzy completely generalized alpha continuous mappings and studied some of its properties.

Definition 3.1. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is called an intuitionistic fuzzy completely generalized alpha continuous (IF completely Gα continuous in short) if $f^{-1}(B)$ is an IFRCS in (X,τ) for every $IFG\alpha CS$ B of (Y,σ).

Theorem 3.2. *Every IF completely Gα continuous mapping is an IFGα continuous mapping but not conversely.*

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Since every IFRCS is an IFGαCS, $f^{-1}(B)$ is an IFGαCS in X. Hence f is an IFGα continuous mapping. □

Example 3.3. Let us consider $X=\{a,b\},Y=\{u,v\},G_1=\langle x,(0.2,0.3),(0.7,0.6)\rangle$, and $G_2=\langle y,(0.4,0.4),(0.5,0.6)\rangle$. Then $\tau=\{0_\sim,G_1,1_\sim\}$ and $\sigma=\{0_\sim,G_2,1_\sim\}$ are IFTS on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=u,f(b)=v$. Then f is an IFGα continuous mapping but not IF completely Gα continuous mapping. Here G_2 is an IFGαCS in Y but not IFRCS in X since $cl(int(f^{-1}(G_2^c)))=G_1^c\neq f^{-1}(G_2^c)$

Theorem 3.4. *Every IF completely Gα continuous mapping is an IF continuous mapping but not conversely.*

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Since every IFRCS is an IFCS, $f^{-1}(B)$ is an IFCS in X. Hence is an IF continuous mapping. □

Example 3.5. Let us consider $X=\{a,b\},Y=\{u,v\},G_1=\langle x,(0.2,0.2),(0.3,0.7)\rangle$, and $G_2=\langle x,(0.5,0.4),(0.4,0.6)\rangle$ and $G_3=\langle y,(0.5,0.4),(0.4,0.6)\rangle$. Then $\tau=\{0_\sim,G_1,G_2,1_\sim\}$ and $\sigma=\{0_\sim,G_3,1_\sim\}$ are IFTS on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=u,f(b)=v$. Then f is an IF continuous mapping but not IF completely Gα continuous mapping. Here G_3 is an IFGαCS in Y but not an IFRCS in X since $cl(int(f^{-1}(G_3^c)))=G_1^c\neq f^{-1}(G_3^c)$.

Theorem 3.6. *Every IF completely Gα continuous mapping is an IF α continuous mapping but not conversely.*

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Since every IFRCS is an IFαCS, $f^{-1}(B)$ is an IFαCS in X. Hence f is an IFα continuous mapping. □

Example 3.7. Let us consider $X=\{a,b\},Y=\{u,v\},G_1=\langle x,(0.2,0.4),(0.8,0.6)\rangle$, and $G_2=\langle y,(0.2,0.4),(0.8,0.6)\rangle$. Then $\tau=\{0_\sim,G_1,1_\sim\}$ and $\sigma=\{0_\sim,G_2,1_\sim\}$ are IFTS on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=u,f(b)=v$. Then f is an IF α continuous mapping but not IF completely Gα continuous mapping. Since $A=\langle y,(0.1,0.2),(0.9,0.8)\rangle$ is an IFGαCS in Y but not IFRCS in X, $cl(int(f^{-1}(A^c)))=G_1^c=\neq A^c$.

Theorem 3.8. *Every IF completely Gα continuous mapping is an IFαG continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Since every IFRCS is an IFαGCS, $f^{-1}(B)$ is an IFαGCS in X. Hence f is an IFαG continuous mapping. □

Example 3.9. Let us consider $X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.8, 0.8), (0.2, 0.1)\rangle$, and $G_2 = \langle y, (0.1, 0.3), (0.9, 0.7)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u, f(b) = v$. Then f is an IFαG continuous mapping but not an IF completely Gα continuous mapping. Here G_2 is an IFGαCS in Y but not IFRCS in X since $cl(int(f^{-1}(G_2^c)) = 0 \neq f^{-1}(G_2^c)$.

Theorem 3.10. *Every IF completely Gα continuous mapping is an IF generalized semi continuous but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Since every IFRCS is an IFGSCS, $f^{-1}(B)$ is an IFGSCS in X. Hence f is an IF generalized semi continuous mapping. □

Example 3.11. Let us consider $X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.2, 0.2), (0.4, 0.5)\rangle$, and $G_2 = \langle y, (0, 0.1), (0.9, 0.8)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u, f(b) = v$. Clearly f is an intuitionistic fuzzy generalized semi- continuous mapping, but not an intuitionistic fuzzy completely Gα continuous mapping. Here G_2^c is an IFGαCS in Y but not IFRCS in X since $cl(int(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$.

Theorem 3.12. *Every IF completely Gα continuous mapping is an IF semi generalized continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Since every IFRCS is an IFSGCS, $f^{-1}(B)$ is an IFSGCS in X. Hence f is an IF semi generalized continuous mapping. □

Example 3.13. Let us consider $X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.3, 0.3), (0.4, 0.4)\rangle$, and $G_2 = \langle y, (0.1, 0.1), (0.5, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u, f(b) = v$. Then f is an intuitionistic fuzzy semi generalized continuous mapping, but not an intuitionistic fuzzy completely Gα continuous mapping. Here G_2^c is an IFGαCS in Y but not an IFRCS in X since $cl(int(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$..

Theorem 3.14. *Every IF completely Gα continuous mapping is an IF generalized pre continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Since every IFRCS is an IFGPCS, $f^{-1}(B)$ is an IFGPCS in X. Hence f is an IF generalized pre continuous mapping. □

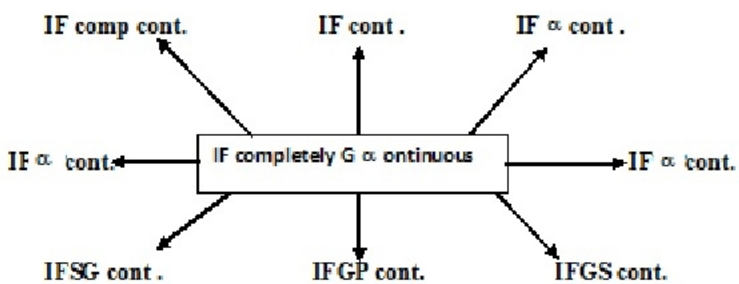
Example 3.15. Let us consider $X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$, and $G_2 = \langle y, (0.3, 0.3), (0.7, 0.7)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u, f(b) = v$. Then f is an intuitionistic fuzzy generalized pre continuous mapping but not an intuitionistic fuzzy completely Gα continuous mapping. Here G_2^c is an IFGαCS in Y but not an IFRCS in X since $cl(int(f^{-1}(G_2^c))) = G_1^c \neq f^{-1}(G_2^c)$.

Theorem 3.16. *Every IF completely Gα continuous mapping is an IF completely continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely Gα continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFGαCS, B is an IFGαCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. Then every IFCS in Y is an IFRCS in X. Hence f is an IF completely continuous mapping. □

Example 3.17. Let us consider $X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.2, 0.3), (0.5, 0.7)\rangle$, and $G_2 = \langle y, (0.2, 0.3), (0.5, 0.7)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u, f(b) = v$. Clearly f is an intuitionistic fuzzy completely continuous mapping but not an IF completely Gα continuous mapping. But $A = \langle y, (0.4, 0.3), (0.6, 0.7)\rangle$ is an IFGαCS in Y but not an IFRCS in X, since $cl(int(f^{-1}(A^c))) = G_1^c \neq f^{-1}(A^c)$.

The following diagram implications are true:



IF comp cont. means Intuitionistic fuzzy completely continuous mapping

Theorem 3.18. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF completely Gα continuous mapping if for every IFP $c(\alpha, \beta) \in X$ and for every IFN A of $f(c(\alpha, \beta))$, there exists an IFROS $B \subseteq X$ such that $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.*

Proof. Let $c(\alpha, \beta) \in X$ and let A be an IFN of $f(c(\alpha, \beta))$.Then there exists an IFOS U in Y such that $f(c(\alpha, \beta)) \in U \subseteq A$. Since every IFOS is an IFGαOS, U is an IFGαOS in Y. Hence by hypothesis $f^{-1}(U)$ is an IFROS in X and $c(\alpha, \beta) \in f^{-1}(U)$. Let $B = f^{-1}(U)$. Therefore $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$. □

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Theorem 3.19. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF completely Gα continuous mapping then $cl(int(cl(f^{-1}(B))) \subseteq f^{-1}(B)$ for every IFS B in Y.*

Proof. Let B be an IFS in Y. Then $cl(B)$ is an IFCS in Y. Then $cl(B)$ is an IFGαCS in Y. By hypothesis $f^{-1}(cl(B))$ is an IFRCS in X. Hence $cl(int(cl(f^{-1}(B))) \subseteq f^{-1}(cl(B)) = f^{-1}(B)$. □

Theorem 3.20. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF completely Gα continuous mapping then the following are equivalent.*

(i) *for any IFGαOS A in Y and for any IFP $c(\alpha, \beta) \in X$ if $f(c(\alpha, \beta))_q A$ then $c(\alpha, \beta)_q int(f^{-1}(A))$*
(ii) *for any IFGαOS A in Y and for any $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta))_q A$ then there exist an IFOS B in X such that $c(\alpha, \beta)_q B$ and $f(B) \subseteq A$*

Proof. $(i) \Rightarrow (ii)$ Let $A \subseteq Y$ be an IFGαOS and let $c(\alpha, \beta) \in X$. Let $f(c(\alpha, \beta))_q A$. Then $c(\alpha, \beta)_q f^{-1}(A)$. (i) implies that $c(\alpha, \beta)_q int(f^{-1}(A))$, where $int(f^{-1}(A))$, is an IFOS in X. Let $B = int(f^{-1}(A))$ since $int(f^{-1}(A)) \subseteq f^{-1}(A)$,$B \subseteq f^{-1}(A)$. Then $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

$(ii) \Rightarrow (i)$ Let $A \subseteq Y$ be an IFGαOS and let $c(\alpha, \beta) \in X$. Suppose $f(c(\alpha, \beta))_q A$, then by (ii) there exists an IFOS B in X such that $c(\alpha, \beta)_q B$ and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. That is $int(B) \subseteq int(f^{-1}(A))$. Therefore $c(\alpha, \beta)_q B$ implies $c(\alpha, \beta)_q int(f^{-1}(A))$. □

Theorem 3.21. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $g : X \to X \times X$ be the graph of the function f . Then f is completely Gα intuitionistic fuzzy continuous if is so.*

Proof. Let us consider $B \in \sigma$, then $f^{-1}(B) = f^{-1}(1 \times B) = 1 \cap f^{-1}(B) = g^{-1}(1 \times B)$. Since B is an IFOS in Y, and every IFOS is an IFGαOS , $1 \times B$ is an IFGαOS in $X \times Y$. Also the fact that f is an completely IFGα continuous implies that $g^{-1}(1 \times B)$ is an IFROS in X. Hence $f^{-1}(B)$ is an IFROS in X. Hence f is completely Gα intuitionistic fuzzy continuous mapping. □

Theorem 3.22. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following are equivalent.*

(i) *f is an IF completely Gα continuous mapping*
(ii) *$f^{-1}(B)$ is an IFROS in X for every for every IFGαOS B in Y.*
(iii) *for every IFP $c(\alpha, \beta) \in X$ and for every IFGαOS B in Y such that if $f(c(\alpha, \beta)) \in B$ there exists an IFROS A in X such that $c(\alpha, \beta) \in A$ and $f(A) \subseteq B$.*

Proof. $(i) \Rightarrow (ii)$ is obvious.
$(ii) \Rightarrow (iii)$ Let $c(\alpha, \beta) \in X$. Let B be an IFGαOS in Y and $f^{-1}(B)$ is an IFROS in X. Let $f(c(\alpha, \beta)) \in B$ and let $A = f^{-1}(B)$. Then $c(\alpha, \beta) \in f^{-1}(f(c(\alpha, \beta))) \in f^{-1}(B) = A$. Therefore $c(\alpha, \beta) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$. This implies $f(A) \subseteq B$.
$(iii) \Rightarrow (i)$ Let B be an IFGαOS in Y and let $c(\alpha, \beta) \in X$ and $f(c(\alpha, \beta))) \in B$.Then by hypothesis there exists an IFROS G in X such that $c(\alpha, \beta) \in G$ and $f(G) \subseteq B$. Now $c(\alpha, \beta) \in f^{-1}(B)$. But $G \subseteq f^{-1}(B)$, $c(\alpha, \beta) \in G$ and $f(G) \subseteq B$. This implies $G \subseteq f^{-1}(f(G)) \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \bigcup_{c(\alpha,\beta) \in f^{-1}(B)} G \subseteq f^{-1}(B)$. This

implies $f^{-1}(B) = \bigcup_{c(\alpha,\beta)\in f^{-1}(B)} G$ where G is an IFROS and hence $f^{-1}(B)$ is an IFROS in X. Hence is an IF completely Gα continuous mapping. □

Theorem 3.23. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy completely Gα- continuous if and only if $f^{-1}(A)$ is an IFROS in X for every IFGαOS A in Y.*

Proof. Necessity: Let A be an IFGαOS in Y. This implies A^c is an IFGαCS in Y. Since f is an intuitionistic fuzzy completely Gα-continuous mapping, $f^{-1}(A^c)$ is an IFRCS in X. Hence $f^{-1}(A^c) = \overline{f^{-1}(A)}$, $f^{-1}(A)$ is an IFROS in X.

Sufficiency : Let A be an IFGαCS in Y. This implies A^c is an IFGαOS in Y. By hypothesis $f^{-1}(A^c)$ is an IFROS in X. Since $f^{-1}(A^c) = \overline{f^{-1}(A)}$, $f^{-1}(A)$ is an IFRCS in X. Hence f is an intuitionistic fuzzy completely Gα- continuous mapping. □

Theorem 3.24. *For any two intuitionistic fuzzy completely Gα continuous mappings $f_1, f_2 : (X, \tau) \to (Y, \sigma)$, the mapping $(f_1, f_2) : (X, \tau) \to (Y \times Y, \sigma \times \sigma)$ is also an IF completely Gα continuous mapping, where $(f_1, f_2)(x) = (f_1(x), f_2(x)), \forall x \in X$.*

Proof. Let $A \times B$ be an IFGαOS in $Y \times Y$. Then

$$(f_1, f_2)^{-1}(A \times B)(x) = (A \times B)(f_1(x), f_2(x))$$
$$= \langle x, min(\mu_A(f_1(x)), \mu_B(f_2(x))), max(\gamma_A(f_1((x)), \gamma_B(f_2(x)))\rangle$$
$$= \langle x, min(f_1^{-1}(\mu_A(x)), f_2^{-1}(\mu_B(x))), max(f_1^{-1}(\gamma_A((x)), f_2^{-1}(\gamma_B(x)))\rangle$$
$$= (f_1^{-1}(A) \cap f_2^{-1}(B))(x).$$

Since f_1 and f_2 are IF completely Gα continuous mappings, $f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X. Since intersection of IFROS is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1, f_2) is an intuitionistic fuzzy Gα- continuous mapping. □

Theorem 3.25. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two mappings where (Z, δ) is an IF$\alpha_k T_{\frac{1}{2}}$ space. Then the following statements hold.*

(i) *f be an IF completely Gα continuous mapping and g be an IF continuous mapping. Then their composition $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF completely Gα continuous mapping.*
(ii) *f be an IF completely Gα continuous mapping and g be an IFα continuous mapping. Then their composition $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF completely Gα continuous mapping.*
(iii) *f be an IF completely Gα continuous mapping and g be an IF completely continuous mapping. Then their composition $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF completely Gαcontinuous mapping.*
(iv) *f be an IF completely Gα continuous mapping and g be an IF continuous mapping. Then their composition $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF completely Gαcontinuous mapping.*

Proof. (i) Let A be an IFGαCS in Z. Since Z is an IF$\alpha_k T_{\frac{1}{2}}$ space, A is IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since every IFCS is an IFGαCS, $g^{-1}(A)$ is an IFGαCS in Y. Therefore $f^{-1}(g^{-1}(A))$ is an IFRCS in X. Hence $g \circ f$

is an IF completely Gαcontinuous mapping.
The proof of (ii), (iii), (iv) is similar to (i). □

Theorem 3.26. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IFGα continuous mapping. Then the following statements hold.*

(i) *$f(g\alpha(cl(A))) \subseteq cl(f(A))$, for every IFS A in X.*
(ii) *$g\alpha cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in X.*

Proof. (i) Let $A \subseteq X$. Then $cl(f(A))$ is an IFCS in Y. Since f is an IF completely Gα continuous mapping, $f^{-1}(cl(f(A)))$ is an IFGαCS in X . Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A))$ and $f^{-1}(cl(f(A)))$ is an IFGα- closed, implies $g\alpha cl(A) \subseteq f^{-1}(cl(f(A)))$. Hence $f(g\alpha(cl(A))) \subseteq cl(f(A))$.
(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(g\alpha cl(f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Hence $g\alpha cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for every IFS B in Y. □

Theorem 3.27. *If $f : (X, \tau) \to (Y, \sigma)$ be an IF completely Gα - continuous mapping then f is an IF α continuous mapping.*

Proof. Let A be an IFCS in Y. Since every closed set is an IFGαCS, A is an IFGαCS in Y. By hypothesis,$f^{-1}(A)$ is an IFRCS and hence $f^{-1}(A)$ is an IFαCS in X. Hence f is IF α continuous mapping. □

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