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Some properties of intuitionistic volterra structure spaces with reference to t-open sets

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Abstract. The purpose of this paper is to introduce the concept of an intuitionistic Volterra structure space. Characterizations and properties of intuitionistic volterra feebly t-continuous functions are presented by making use of intuitionistic volterra feebly open sets and intuitionistic volterra feebly closed sets.

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1. Introduction

The concept of an intuitionistic set was introduced by D. Coker in [2]. The intuitionistic set is the discrete form of intuitionistic fuzzy set. In this paper we introduce the concepts of an intuitionistic Volterra structure space. Characterizations and properties of intuitionistic volterra feebly t-continuous functions are presented by making use of intuitionistic volterra feebly open sets and intuitionistic volterra feebly closed sets. Some Properties of slightly intuitionistic volterra t-continuous functions, intuitionistic volterra clopen continuous functions are also discussed.

2. Preliminaries

Definition 2.1 ([1]). Let X be a nonempty fixed set.An *intuitionistic set*(IS for short) A is an object having the form $A = \langle x, A^1, A^2\rangle$ for all $x \in X$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A.

Every crisp set A on a non-empty set X is obviously an intuitionistic set having the form $\langle x, A, A^c \rangle$, and one can define several relations and operations between intuitionistic sets as follows:

Definition 2.2 ([1])**.** Let X be a nonempty set, $A = \langle x, A^1, A^2 \rangle$ for all $x \in X$, $B = \langle x, B^1, B^2 \rangle$ for all $x \in X$ be intuitionistic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = \langle x, A_i^1, A_i^2 \rangle$ for all $x \in X$.

(i) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(iii) $\overline{A} = \langle x, A^2, A^1 \rangle$
(iv) $\cup A_i = \langle x, \cup A_i^1, \cap A_i^2 \rangle$
(v) $\cap A_i = \langle x, \cap A_i^1, \cup A_i^2 \rangle$
(vi) $A - B = A \cap \overline{B}$
(vii) $\phi_\sim = \langle x, \phi, X \rangle$ and $X_\sim = \langle x, X, \phi \rangle$.

Definition 2.3 ([1])**.** Let X and Y be two nonempty sets and $f : X \to Y$ a function

(i) If $B = \langle x, B^1, B^2 \rangle$ for all $x \in X$ is an intuitionistic set in Y,then the *preimage* of B under f,denoted by $f^{-1}(B)$,is an intuitionistic set in X defined by $f^{-1}(B) = \langle x, f^{-1}(B^1), f^{-1}(B^2) \rangle$.
(ii) If $A = \langle x, A^1, A^2 \rangle$ for all $x \in X$ is an intuitionistic set in X,then the *image* of A under f,denoted by $f(A)$,is the intuitionistic set in Y defined by $f(A) = \langle y, f(A^1), f_-(A^2) \rangle$ where $f_-(A^2) = (f(A^2)^c)^c$.

Definition 2.4 ([1])**.** An *intuitionistic topology* (IT for short) on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(i) $\phi_\sim$ and $X_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the ordered pair (X, τ) is called an *intuitionistic topological space* (ITS for short) and any intuitionistic set in τ is known as an *intuitionistic open set* (IOS for short) in X. The complement $\overline{A}$ of an intuitionistic open set A is called an *intuitionistic closed set* (ICS for short) in X.

Definition 2.5 ([1])**.** Let (X, τ) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X.Then the intuitionistic interior and intuitionistic closure of A are defined by

$cl(A) = \cap\{K : K$ is an intuitionistic closed set in X and $A \subseteq K\}$,
$int(A) = \cup\{G : G$ is an intuitionistic open set in X and $G \subseteq A\}$.

It can be also shown that $cl(A)$ is an intuitionistic closed set and $int(A)$ is an intuitionistic open set in X and A is an intuitionistic closed set in X if $cl(A) = A$ and A is an intuitionistic open set in X if $int(A) = A$.

Definition 2.6 ([3])**.** Let (X, τ) be an intuitionistic topological space. Let $b \in X$ and an intuitionistic set $A = \langle x, A^1, A^2 \rangle$ of X is called an *intuitionistic neighborhood* of x if there is an intuitionistic open set $B = \langle x, B^1, B^2 \rangle$ in X such that $x \in B \subseteq A$ where $x \in B^1$ and $x \notin B^2$. The family of all the intuitionistic neighborhood system of x is called the system of intuitionistic neighborhood of x denoted by $IN(x)$.

Definition 2.7 ([2])**.** Let (X, τ) be an intuitionistic topological space on X.

(a) A family $\beta \subseteq \tau$ is called an *intuitionistic base* for (X, τ) if and only if each member of τ can be written as a union of elements of β.
(b) A family $\gamma \subseteq \tau$ is called an *intuitionistic subbase* for (X, τ) if and only if the family of finite intersection of elements in γ forms a base for (X, τ). In this case the intuitionistic topology τ is said to be generated by γ.

Definition 2.8 ([6]). A subset A of a space X is called a G_δ*-set* in X if it equals the intersection of a countable collection of open subsets of X.

Definition 2.9 ([4]). A topological space X is called *Volterra*, if for any G_δ-sets A_1 and A_2 which are dense in X, $A_1 \cap A_2$ is dense in X.

Definition 2.10 ([4]). A subset A of a topological space X is called a clopen set if it is both open and closed.

Definition 2.11 ([7]). Let (X, τ) be a topological space. A subset A of (X, τ) is said to be t-open set if $int(A) = int(cl(A))$.

The complement of t-open set is called a t-closed set. The intersection of all t-closed sets containing A is called t-closure of A and is denoted by $tcl(A)$. Dually , the t-interior of A denoted by $tint(A)$, is defined to be the union of all t-open sets contained in A.

Definition 2.12 ([5]). Let (X, τ) be a topological space. A subset A of (X, τ) is said to be feebly open set if there exists an open set U such that $U \subset A \subset scl(U)$ and A is called feebly closed if there exists a closed set U in X such that $sint(U) \subset A \subset U$.

Intersection of all feebly closed sets containing A is called feebly closure of A and is denoted by $fcl(A)$. Dually, the feebly interior of A denoted by $fint(A)$, and is defined to be the union of all feebly open sets contained in A.

Definition 2.13 ([5]). A function $f : (X, T) \to (Y, V)$ is said to be feebly continuous if the inverse image by f of each open set A of Y is feebly open in X.

3. Charecterizations of Intuitionistic Volterra structure spaces

Definition 3.1. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ of an intuitionistic topological space (X, τ) is called an intuitionistic G_δ*-set* in X if it equals the intersection of a countable collection of intuitionistic open sets of X.

Definition 3.2. Let X be a nonempty set. A family $\mathcal{V}$ of intuitionistic G_δ sets in X is said to be intuitionistic Volterra structure on X if it satisfies the following axioms:
(i) $\phi_\sim, X_\sim \in \mathcal{V}$
(ii) $G_1 \cap G_2 \in \mathcal{V}$ for any $G_1, G_2 \in \mathcal{V}$
(iii) $\cup G_i \in \mathcal{V}$ for arbitrary family $\{G_i \mid i \in Z\} \subseteq \mathcal{V}$
(iv) $Icl(G_i) = X_\sim, i \in Z$ implies $Icl(G_j \cap G_k) = X_\sim$ for any $j, k \in \rho$.

Then the ordered pair $(X, \mathcal{V})$ is called an intuitionistic volterra structure space. Every member of $\mathcal{V}$ is called an intuitionistic volterra open set in $(X, \mathcal{V})$. The complement of an intuitionistic volterra open set is called an intuitionistic volterra closed set in $(X, \mathcal{V})$.

Definition 3.3. Let $(X, \mathcal{V})$ be any intuitionistic volterra structure space.An intuitionistic set $A = \langle x, A^1, A^2\rangle$ is said to be an intuitionistic volterra clopen set if A is both intuitionistic volterra closed and intuitionistic volterra open in X.

Notation 3.1. *Let $(X, \mathcal{V})$ be any intuitionistic volterra structure space. Then*

(i) *$O(X)$ denotes the family of all intuitionistic volterra open sets in $(X, \mathcal{V})$.*
(ii) *$C(X)$ denotes the family of all intuitionistic volterra closed sets in $(X, \mathcal{V})$.*

Definition 3.4. Let $(X, \mathcal{V})$ be any intuitionistic volterra structure space. Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set in X. Then

(i) the intuitionistic volterra interior of A is defined and denoted as $I\mathcal{V}int(A) = \cup\{B = \langle x, B^1, B^2\rangle \mid B \in O(X) \text{ and } B \subseteq A\}$.
(ii) the intuitionistic volterra closure of A is defined and denoted as $I\mathcal{V}cl(A) = \cap\{B = \langle x, B^1, B^2\rangle \mid B \in C(X) \text{ and } A \subseteq B\}$.

Remark 3.5. Let $(X, \mathcal{V})$ be any intuitionistic volterra structure space. Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set in X. Then the following statements hold:

(i) $I\mathcal{V}cl(A) = A$ if and only if A is an intuitionistic volterra closed set.
(ii) $I\mathcal{V}int(A) = A$ if and only if A is an intuitionistic volterra open set.
(iii) $I\mathcal{V}int(A) \subseteq A \subseteq I\mathcal{V}cl(A)$.
(iv) $I\mathcal{V}int(X_\sim) = X_\sim$ and $I\mathcal{V}int(\phi_\sim) = \phi_\sim$.
(v) $I\mathcal{V}cl(X_\sim) = X_\sim$ and $I\mathcal{V}cl(\phi_\sim) = \phi_\sim$.
(vi) $I\mathcal{V}cl(\overline{A}) = \overline{I\mathcal{V}int(A)}$ and $I\mathcal{V}int(\overline{A}) = \overline{I\mathcal{V}cl(A)}$.

Proof. The proof is simple. □

Definition 3.6. Let $(X, \mathcal{V})$ be an intuitionistic volterra structure space. An intuitionistic set A of $(X, \mathcal{V})$ is said to be intuitionistic volterra t-open set if $I\mathcal{V}int(A) = I\mathcal{V}int(I\mathcal{V}cl(A))$.

The complement of intuitionistic volterra t-open set is called intuitionistic volterra t-closed set. The intersection of all intuitionistic volterra t-closed sets containing A is called intuitionistic volterra t-closure of A and is denoted by $I\mathcal{V}tcl(A)$. Dually , the intuitionistic volterra t-interior of A denoted by $I\mathcal{V}tint(A)$, is defined to be the union of all intuitionistic volterra t-open sets contained in A.

Definition 3.7. Let $(X, \mathcal{V})$ be an intuitionistic volterra structure space. An intuitionistic set A of $(X, \mathcal{V})$ is said to be intuitionistic volterra t-clopen set if it is both intuitionistic volterra t-open and intuitionistic volterra t-closed.

Definition 3.8. Let $(X, \mathcal{V})$ be any intuitionistic volterra structure space.An intuitionistic set $A = \langle x, A^1, A^2\rangle$ is said to be an intuitionistic volterra feebly open set if there exists an intuitionistic volterra open set U such that $U \subset A \subset I\mathcal{V}tcl(U)$ and A is called intuitionistic volterra feebly closed if there exists an intuitionistic volterra closed set U in X such that $I\mathcal{V}tint(U) \subset A \subset U$.

Intersection of all intuitionistic volterra feebly closed sets containing A is called intuitionistic volterra feebly closure of A and is denoted by $I\mathcal{V}fcl(A)$. Dually, the intuitionistic volterra feebly interior of A denoted by $I\mathcal{V}fint(A)$, and is defined to be the union of all intuitionistic volterra feebly open sets contained in A.

Remark 3.9. Let $(X, \mathcal{V})$ be any intuitionistic volterra structure space. Let $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, B^1, B^2\rangle$ be intuitionistic sets in X. Then the following statements hold:

(i) $I\mathcal{V}fcl(A) = A$ if and only if A is an intuitionistic volterra feebly closed set.

(ii) $I\mathcal{V}fint(A) = A$ if and only if A is an intuitionistic volterra feebly open set.
(iii) $I\mathcal{V}fint(A) \subseteq A \subseteq I\mathcal{V}fcl(A)$.
(iv) $I\mathcal{V}fint(X_\sim) = X_\sim$ and $I\mathcal{V}fint(\phi_\sim) = \phi_\sim$.
(v) $I\mathcal{V}fcl(X_\sim) = X_\sim$ and $I\mathcal{V}fcl(\phi_\sim) = \phi_\sim$.
(vi) $I\mathcal{V}fcl(\overline{A}) = \overline{I\mathcal{V}fint(A)}$ and $I\mathcal{V}fint(\overline{A}) = \overline{I\mathcal{V}fcl(A)}$.
(vii) If $A \subset B$ then $I\mathcal{V}fcl(A) \subset I\mathcal{V}fcl(B)$.

Proof. The proof is simple. □

Definition 3.10. Let $(X_1, \mathcal{V}_1)$ and $(X_2, \mathcal{V}_2)$ be any two intuitionistic volterra structure spaces. Let $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ be a function. Then f is said to be an

(i) intuitionistic volterra t-continuous function if $f^{-1}(A)$ is an intuitionistic volterra t-open set in $(X_1, \mathcal{V}_1)$, for every intuitionistic volterra open set A in $(X_2, \mathcal{V}_2)$.
(ii) slightly intuitionistic volterra continuous function if for each point $x \in X_1$ and each intuitionistic volterra clopen set V containing $f(x)$ there exists an intuitionistic volterra open set U containing x such that $f(U) \subset V$.
(iii) slightly intuitionistic volterra t-continuous function if for each point $x \in X_1$ and each intuitionistic volterra clopen set V containing $f(x)$ there exists an intuitionistic volterra t-open set U containing x such that $f(U) \subset V$.
(iv) intuitionistic volterra clopen continuous if for each point x of X and each intuitionistic volterra open set V containing $f(x)$, there exists an intuitionistic volterra clopen set U containing x such that $f(U) \subset V$.
(v) intuitionistic volterra feebly t-continuous if the inverse image of each intuitionistic volterra t-open set A of Y is intuitionistic volterra feebly open in X.

Proposition 3.11. *Let $(X_1, \mathcal{V}_1)$ and $(X_2, \mathcal{V}_2)$ be any two intuitionistic volterra structure spaces. Let $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ be a function. Then the following statements are equivalent:*

(i) *f is an intuitionistic volterra t-continuous function.*
(ii) *$f^{-1}(B)$ is an intuitionistic volterra t-closed set in $(X_1, \mathcal{V}_1)$, for every intuitionistic volterra closed set A in $(X_2, \mathcal{V}_2)$.*
(iii) *$I\mathcal{V}tcl(f^{-1}(A)) \subseteq f^{-1}(I\mathcal{V}tcl(A))$, for each intuitionistic set A in $(X_2, \mathcal{V}_2)$.*
(iv) *$f^{-1}(I\mathcal{V}tint(A)) \subseteq I\mathcal{V}tint(f^{-1}(A))$, for each intuitionistic set A in $(X_2, \mathcal{V}_2)$.*

Proof. The proof is simple. □

Proposition 3.12. *For a function $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ the following statements are equivalent:*

(i) *f is slightly intuitionistic volterra t-continuous function.*
(ii) *inverse image of every intuitionistic volterra clopen set of X_2 is intuitionistic volterra t-open set of X_1.*
(iii) *inverse image of every intuitionistic volterra clopen set of X_2 is intuitionistic volterra t-clopen set of X_1.*

Proof. (i) ⇒ (ii). Let U be an intuitionistic volterra clopen set of X_2 and let $x \in f^{-1}(U)$. Then $f(x) \in U$. Since f is slightly intuitionistic volterra t-continuous function, there exists an intuitionistic volterra t-open set V_x in X_1 such that $x \in$

$V_x, f(V_x) \subseteq U$. That is $x \in V_x \subseteq f^{-1}(U)$. Now $f^{-1}(U) = \cup\{V_x : x \in f^{-1}(U)\}$. Since $f^{-1}(U)$ is union of intuitionistic volterra t-open sets, $f^{-1}(U)$ is intuitionistic volterra t-open.

(ii) $\Rightarrow$ (iii). Let A be an intuitionistic volterra clopen set of X_2. Then $\overline{A}$ is also an intuitionistic volterra clopen set and $f^{-1}(\overline{A})$ is intuitionistic volterra t-open in X_1 by (ii). So $X - f^{-1}(A)$ is intuitionistic volterra t-open and hence $f^{-1}(A)$ is intuitionistic volterra t-closed in X_1 and by (ii), $f^{-1}(A)$ is intuitionistic volterra clopen set of X_1.

(iii) $\Rightarrow$ (i). Let $x \in X_1$ and let V be an intuitionistic volterra clopen set of X_2 containing $f(x)$, then $f^{-1}(V)$ is an intuitionistic volterra t-open set of X_1 containing x and $f(f^{-1}(V)) \subseteq V$. □

Proposition 3.13. *If $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ is a slightly intuitionistic volterra t-continuous then f is intuitionistic volterra clopen continuous.*

Proof. Let $x \in X_1$ and V be an intuitionistic volterra t-open set of X_2 containing $f(x)$. There exists an intuitionistic volterra clopen set W in X_2 such that $f(x) \subset W \subset V$. Since f is slightly intuitionistic volterra t-continuous there exists an intuitionistic volterra clopen set U containing x such that $f(U) \subset W$ and hence $f(U) \subset V$. This shows that $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ is intuitionistic volterra clopen continuous. □

Proposition 3.14. *If $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ is a slightly intuitionistic volterra t-continuous and $g : (X_2, \mathcal{V}_2) \to (X_3, \mathcal{V}_3)$ is a slightly intuitionistic volterra continuous function, then $g_o f : (X_1, \mathcal{V}_1) \to (X_3, \mathcal{V}_3)$ is slightly intuitionistic volterra t-continuous.*

Proof. Let F be an intuitionistic volterra clopen set of X_3. Since g is slightly intuitionistic volterra continuous, $g^{-1}(F)$ is intuitionistic volterra clopen set of X_2. Since f slightly intuitionistic volterra t-continuous, $f^{-1}(g^{-1}(F)) = (g_o f)^{-1}(F)$ is an intuitionistic volterra t-clopen set of X_1. So by the above Theorem, $g_o f$ is slightly intuitionistic volterra t-continuous. □

Remark 3.15. (i) Every intuitionistic volterra t-open set is intuitionistic volterra feebly open.
(ii) Every intuitionistic volterra t-closed set is intuitionistic volterra feebly closed.

Proposition 3.16. *Every intuitionistic volterra t-continuous function is intuitionistic volterra feebly t-continuous function*

Proof. Let $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ be intuitionistic volterra t-continuous and let A be any intuitionistic volterra clopen set of X_2. Since f is intuitionistic voloterra t-continuous then $f^{-1}(A)$ is intuitionistic volterra t-open in X, then $f^{-1}(A)$ is intuitionistic volterra feebly open [By using "Every intuitionistic volterra t-open set is intuitionistic volterra feebly open"]. Hence f is intuitionistic volterra feebly t-continuous function. □

Proposition 3.17. *A function $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ is intuitionistic volterra feebly t-continuous if and only if the inverse image of every intuitionistic volterra t-closed set of X_2 is intuitionistic volterra feebly closed in X_1.*

Proof. Let f be intuitionistic volterra feebly t-continuous and let H be intuitionistic volterra t-closed set in X_2, then $\overline{H}$ is intuitionistic volterra t-open, $f^{-1}(\overline{H}) = \overline{(f^{-1}(H))}$ is intuitionistic volterra feebly open in X_1, then $f^{-1}(H)$ is intuitionistic volterra feebly closed.

Conversely, let H be an intuitionistic volterra t-open set in X_2, $\overline{H}$ is intuitionistic volterra t-closed, then by hypothesis $f^{-1}(\overline{H})$ is intuitionistic volterra feebly closed in X_1, then $f^{-1}(H)$ is intuitionistic volterra feebly open set in X_1. Thus f is an intuitionistic volterra feebly t-continuous function. □

Proposition 3.18. *A function $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ is intuitionistic volterra feebly t-continuous if and only if $f(I\mathcal{V}fcl(A)) \subset I\mathcal{V}tcl(f(A))$ for every $A \subset X_1$.*

Proof. Let f be intuitionistic volterra feebly t-continuous. Since $f(I\mathcal{V}tcl(A))$ is intuitionistic volterra t-closed set in X_2, by the above theorem, we have $f^{-1}(I\mathcal{V}tcl(f(A)))$ is intuitionistic volterra feebly closed in X_1 and therefore

$$I\mathcal{V}fcl(f^{-1}(I\mathcal{V}tcl(f(A)))) = f^{-1}(I\mathcal{V}tcl(f(A))).$$

Now $f(A) \subset I\mathcal{V}tcl(f(A))$ implies $A \subset f^{-1}(I\mathcal{V}tcl(f(A)))$, thus

$$I\mathcal{V}fcl(A) \subset I\mathcal{V}fcl(f^{-1}(I\mathcal{V}tcl(f(A)))).$$

Now we have, $I\mathcal{V}fcl(A) \subset f^{-1}(I\mathcal{V}tcl(f(A)))$ then, $f(I\mathcal{V}fcl(A) \subset I\mathcal{V}tcl(f(A))$.
Conversely, let $f(IVfclA) \subset IVtcl(f(A))$ for every $A \subset X$. Let H is intuitionistic volterra t-closed set in X_2, then $IVtclH = H$, let $f^{-1}(H) \subset X_1$, then by hypothesis, $f(IVfcl(f^{-1}(H))) \subset IVtcl(f(f^{-1}(H))) = IVtcl(H) = H$, thus $IVfcl(f^{-1}(H)) \subset f^{-1}(H)$ but $f^{-1}(H) \subset IVfcl(f^{-1}(H))$, hence $IVfcl(f^{-1}(H)) = f^{-1}(H)$, therefore by Remark (3.2(i)), $f^{-1}(H)$ is intuitionistic volterra feebly closed in X_1, hence by above proportion f is intuitionistic volterra feebly t-continuous. □

Proposition 3.19. *A function $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ is intuitionistic volterra feebly t-continuous if and only if $I\mathcal{V}fcl(f^{-1}(B)) \subset f^{-1}(I\mathcal{V}tcl(B))$ for every $B \subset X_2$.*

Proof. Let f be intuitionistic volterra feebly t-continuous. Since $I\mathcal{V}tcl(B)$ is intuitionistic volterra t-closed set in X_2, by the proportion (3.6), we have $f^{-1}(I\mathcal{V}tcl(B))$ is intuitionistic volterra feebly closed in X_1 and by Remark(3.2)

$$I\mathcal{V}fcl(f^{-1}(I\mathcal{V}tcl(B))) = f^{-1}(I\mathcal{V}tcl(B)).$$

Now $B \subset I\mathcal{V}tcl(B)$ implies $f^{-1}(B) \subset f^{-1}(I\mathcal{V}tcl(B))$, then by Remark-3.2(vii)

$$I\mathcal{V}fcl(f^{-1}(B)) \subset I\mathcal{V}fcl(f^{-1}(I\mathcal{V}tcl(B))).$$

Now we have, $I\mathcal{V}fcl(f^{-1}(B)) \subset f^{-1}(I\mathcal{V}tcl(B))$.

Conversely, let $f(IVfcl(f^{-1}(B))) \subset f^{-1}(IVtcl(B))$ for every $B \subset X_2$. Let H is intuitionistic volterra t-closed set in X_2, then $IVtclH = H$, let $f^{-1}(H) \subset X_1$, then by hypothesis, $IVfcl(f^{-1}(H)) \subset f^{-1}(IVtcl(H)) = f^{-1}(H) = H$, thus $IVfcl(f^{-1}(H)) \subset f^{-1}(H)$ but $f^{-1}(H) \subset IVfcl(f^{-1}(H))$, hence $IVfcl(f^{-1}(H)) = f^{-1}(H)$, therefore by Remark (3.2(i)), $f^{-1}(H)$ is intuitionistic feebly closed in X_1, hence by proportion(3.6) f is intuitionistic volterra feebly t-continuous. □

Proposition 3.20. *Let $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ be a function. If $f(IVfcl(A)) \subset IVfcl(f(A))$ for every $A \subset X_1$ then f is intuitionistic volterra feebly t-continuous.*

Proof. Let H be any intuitionistic volterra t-closed set in X_2, then by Remark-3.3(ii), H is intuitionistic volterra feebly closed. So by Remark-3.2(i), $IVfcl(H) = H$, $f^{-1}(H) \subset X_1$ so that by hypothesis,

$$f(IVfcl(f^{-1}(H))) \subset IVfcl(f(f^{-1}(H))) \subset IVfcl(H) = H,$$

therefore, $IVfcl(f^{-1}(H)) \subset f^{-1}(H)$, but $f^{-1}(H) \subset IVfcl(f^{-1}(H))$. Hence

$$IVfcl(f^{-1}(H)) = f^{-1}(H),$$

then by Remark-3.2(i), $f^{-1}(H)$ is intuitionistic volterra feebly closed in X_1, therefore by proportion(3.6), f is intuitionistic volterra feebly t-continuous. □

Proposition 3.21. *Let $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ be a function. If $IVfcl(f^{-1}(B)) \subset f^{-1}(IVfcl(B))$ for every $B \subset X_2$ then f is intuitionistic volterra feebly t-continuous.*

Proof. Let H be any intuitionistic volterra t-closed set in X_2, then by Remark-3.3(ii), H is intuitionistic volterra feebly closed. So that by Remark-3.2(i), $IVfcl(H) = H$, $f^{-1}(H) \subset X_1$ so that by hypothesis, $IVfcl(f^{-1}(H)) \subset f^{-1}(IVfcl(H)) = f^{-1}(H) = H$, therefore, $IVfcl(f^{-1}(H)) \subset f^{-1}(H)$, but $f^{-1}(H) \subset IVfcl(f^{-1}(H))$. Hence $IVfcl(f^{-1}(H)) = f^{-1}(H)$, then by Remark-3.2(i), $f^{-1}(H)$ is intuitionistic volterra feebly closed in X_1, therefore by proportion(3.6), f is intuitionistic volterra feebly t-continuous. □

Proposition 3.22. *A function $f : (X_1, \mathcal{V}_1) \to (X_2, \mathcal{V}_2)$ is intuitionistic volterra feebly t-continuous if $f^{-1}(IVfint(A)) \subset IVfint(f^{-1}(A))$ for all $A \subset X_2$.*

Proof. Let $f^{-1}(IVfint(A)) \subset IVfint(f^{-1}(A))$ for all $A \subset X_2$. Let H be any intuitionistic volterra t-open set in X_2, then by Remark-3.3(i), H is intuitionistic volterra feebly open. So that by Remark-3.2(ii), $IVfint(H) = H$, since

$$f^{-1}(H) = f^{-1}(IVfint(H)) \subset IVfint(f^{-1}(H)),$$

then, $f^{-1}(H) \subset IVfint(f^{-1}(H))$. Hence Remark-3.2(ii), $f^{-1}(H)$ is intuitionistic volterra feebly open in X_1, therefore by proportion(3.6), f is intuitionistic volterra feebly t-continuous. □

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