Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 9, No. 4, (April 2015), pp. 675–682
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Soft I-proximity spaces

A. Kandil, O. A. Tantawy, S. A. El-Sheikh, A. Zakaria

Received 27 July 2014; Revised 21 September 2014; Accepted 1 November 2014

Abstract. An ideal I on a nonempty set X is a subfamily of $P(X)$ which is closed under finite unions and subsets. In this paper, we introduce a new approach of soft proximity structure based on the ideal notion. For $I = \{\phi\}$, we have the soft proximity structure [9] and for the other types of I, we have many types of soft proximity structures.

2010 AMS Classification: 54A05, 54A10, and 54H99

Keywords: Ideal, Soft set, $I-$basic proximity space, Soft $I-$basic proximity.

Corresponding Author: Amr Zakaria (amr.zakaria@edu.asu.edu.eg)

1. Introduction

The fundamental concept of Efremovič proximity space has been introduced by Efremovič [4]. In addition to, Leader [16, 17] and Lodato [18, 19] have worked with weaker axioms than those of Efremovič proximity space enabling them to introduce an arbitrary topology on the underlying set. Furthermore, proximity relations are useful in solving problems based on human perception [24].

In 1999, Molodtsov [22] proposed the novel concept of soft set theory, which provides a completely new approach for modeling vagueness and uncertainty. Soft set theory has a rich potential for applications in several directions, few of which were shown by Molodtsov in [22]. After Molodtsov's work, some different applications of soft sets were studied in Chen et al. [3]. Further theoretical aspects of soft sets were explored by Maji et al. [20]. Also the same authors [21] presented the definition of a fuzzy soft set. The algebraic nature of the soft set has been studied by several researchers. Aktas and Cagman [1] initiated soft groups, and Feng [5] defined soft semirings. Sun [25] introduced a basic version of soft module theory, which extends the notion of a module by including some algebraic structures in soft sets.

Recently, research on soft set theory has been progressing rapidly. Zhi Xiao [27] proposed the notion of the exclusive disjunctive soft set and gave an application of exclusive disjunctive soft sets, which shows that it can be applied to attribute reduction of incomplete information system. Ke Gong [11] proposes the concept of the bijective soft set and some of its operations and gives an application of the bijective soft set in decision making problems. Jiang et al. [26] present an adjustable approach to intuitionistic fuzzy soft set based decision making by using level soft sets of intuitionistic fuzzy soft sets. The lattice structures of soft sets were constructed by Qin and Hong [12].

Babitha and Sunil [2] defined soft set relations and functions. Yang and Guo [7] introduced kernels and closures of soft set relations, and soft set relation mappings using soft set relations and functions. Hazra et al. [10] introduced the notion of basic proximity of soft sets. Also, the same authors [9] proposed the notion of soft proximity. Finally, A. Kandil et al. [13, 14, 15] introduced a new approaches of proximity structures [23] based on the ideal and soft set notions. In this paper, we make an attempt through this paper to widen the set theoretical aspect of proximity spaces via ideal and soft set notions. This paper is arranged as follows, Section 2 has a collection of all basic definitions and notions for further study. The purpose of Section 3 is to construct a new approach of the basic proximity via ideal. Furthermore, a $R_o - \check{C}$hech topology on X has been obtained. The essential goal of Section 4 is to exhibit the relation between the topology generated via this new approach of proximity spaces and the soft topological space (X, τ, E).

2. Preliminaries

In this section we will collect the basic definitions and notations as introduced by Molodtsov [22], Maji et al. [20] and Hazra et al. [8].

Definition 2.1. Let X be a nonempty set, E be a set of parameters, and $P(X)$ denotes the power set of X. A pair (F, E) is called a soft set over X , where F is a mapping given by $F : E \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the set X. For a particular $e \in E$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, E), i.e. $F = \{F(e) : e \in E,\ F : E \rightarrow P(X)\}$. The family of all these soft sets on (X, E) denoted by $P(X)^E$.

Definition 2.2. Let $F,\ G \in P(X)^E$. Then F is soft subset of G, denoted by $F \tilde{\subseteq} G$, if $F(e) \subseteq G(e)$, $\forall e \in E$.

Definition 2.3. Two soft subset F and G over a nonempty set X are said to be soft equal if F is a soft subset of G and G is a soft subset of F.

Definition 2.4. The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined by $(F, E)^c = (F^c, E)$, $F^c : E \rightarrow P(X)$ is a mapping given by $F^c(e) = X - F(e)$, $\forall\, e \in E$ and F^c is called the soft complement function of F.

Definition 2.5. The difference of two soft sets (F, E) and (G, E) over a nonempty set X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.6. Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7. A soft set (F, E) over X is said to be a null soft set denoted by $\tilde{\phi}$ if for all $e \in E,\ F(e) = \phi$ (null set).

Definition 2.8. A soft set (F, E) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \in E,\ F(e) = X$.

Definition 2.9. The intersection of two soft sets (F, E) and (G, E) over a nonempty set X is the soft set (H, E), where for all $e \in E$, $H(e) = F(e) \cap G(e)$.

Definition 2.10. Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a subfamily of $P(X)^E$.

- **(1):** The union of L is the soft set (H, E), where $H(e) = \cup_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\cup}_{i \in I}(F_i, E) = (H, E)$.
- **(2):** The intersection of L is the soft set (M, E), where $M(e) = \cap_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\cap}_{i \in I}(F_i, E) = (M, E)$.

Definition 2.11. Let τ be a collection of soft sets a nonempty set X with a fixed set of parameters E, then $\tau \subseteq P(X)^E$ is called a soft topology on X if

- **(1):** $\tilde{X}, \tilde{\phi} \in \tau$,
- **(2):** the union of any number of soft sets in τ belongs to τ,
- **(3):** the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are called open soft sets in X. Also, a soft set (F, E) is called closed soft if the complement $(F, A)^c$ belongs to τ . The family of closed soft sets is denoted by τ^c.

Definition 2.12. Let (X, τ, E) be a soft topological space. A soft set (F, E) over X is said to be closed soft set in X, if its complement $(F, A)^c$ is an open soft set.

Definition 2.13. Let (X, τ, E) be a soft topological space and $F \in P(X)^E$. The soft closure of F, denoted by $C(F)$ is the intersection of all closed soft super sets of F i.e

$$C(F) = \tilde{\cap}\{H \in P(X)^E : H \textit{ is closed soft set and } F \tilde{\subseteq} H\}. \tag{2.1}$$

Definition 2.14. The soft set $F \in P(X)^E$ is called a soft point if there exists $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.

Definition 2.15. The soft point x_e is said to be belonging to the soft set $G \in P(X)^E$, denoted by $x_e \tilde{\in} G$, if for $e \in E$, $F(e) \subseteq G(e)$.

Definition 2.16 ([9]). Let E be a set of parameters, X be a nonempty set, and $\mathcal{A}$ be a set of basic proximities on X. The pair (δ, E) is called a basic soft proximity on (X, E) if δ is a mapping given by $\delta : E \to \mathcal{A}$.

Definition 2.17 ([10]). Let C be a Čhech closure operator of soft sets on (X, E). Then C is said to be R_o if for any $x, y \in X$ such that $x \neq y$ and $\forall\ F \in P(X)^E$ with $F(e) = \{x\}$, $y \in C(F)(e) \Rightarrow x \in C(G)(e)\ \forall G \in P(X)^E$ with $G(e) = \{y\}$.

Definition 2.18 ([6]). A nonempty collection I of subsets of a set X is called an ideal on X, if it satisfies the following conditions

(1) $A \in I$ and $B \in I \Rightarrow A \cup B \in I$,
(2) $A \in I$ and $B \subseteq A \Rightarrow B \in I$.

Definition 2.19 ([23]). A binary relation δ on $P(X)$ is called a basic proximity on X if δ satisfies the following conditions:-

(P_1) $A\delta B \Rightarrow B\delta A$,
(P_2) $A\delta(B \cup C) \Leftrightarrow A\delta B$ or $A\delta C$,
(P_3) $A\delta B \Rightarrow A \neq \phi, B \neq \phi$,
(P_4) $A \cap B \neq \phi \Rightarrow A\delta B$.

A basic proximity space is a pair (X, δ) consisting of a set X and a basic proximity relation on X. We shall write $A\delta B$ if the sets $A, B \subseteq X$ are δ-related, otherwise we shall write $A\not\delta B$.

3. I-basic proximity

Definition 3.1. Let I be an ideal on a nonempty set X. A binary relation γ_I on $P(X)$ is called an I-basic proximity on X if γ_I satisfies the following conditions:-

(IP_1) $A\gamma_I B \Rightarrow B\gamma_I A$,
(IP_2) $A\gamma_I(B \cup C) \Leftrightarrow A\gamma_I B$ or $A\gamma_I C$,
(IP_3) $A\not\gamma_I B\ \forall A \in I,\ B \in P(X)$,
(IP_4) $A \cap B \notin I \Rightarrow A\gamma_I B$.

An I-basic proximity space is a pair (X, γ_I) consisting of a set X and an $I-$basic proximity relation on X.

Proposition 3.2. *If $I = \{\phi\}$ in Definition 3.1, then we get the basic proximity relation in Definition 2.19.*

Proof. Straightforward. □

Lemma 3.3. *If $A\gamma_I B$, $A \subseteq C$, and $B \subseteq D$, then $C\gamma_I D$.*

Proof. The result follows immediately from (IP_1) and (IP_2). □

Theorem 3.4. *Let (X, γ_I) be an $I-$ basic proximity space. Then the γ_I- operator*

$$\gamma_I : P(X) \to P(X)$$

defined by:

$$A^{\gamma_I} = \{x \in X : x\gamma_I A\} \tag{3.1}$$

satisfies the following:-

(1) $\phi^{\gamma_I} = \phi$
(2) $A \subseteq B \Rightarrow A^{\gamma_I} \subseteq B^{\gamma_I}$,
(3) $(A \cup B)^{\gamma_I} = A^{\gamma_I} \cup B^{\gamma_I}$,
(4) $(A \cap B)^{\gamma_I} \subseteq A^{\gamma_I} \cap B^{\gamma_I}$.

Proof. (1) If $\exists\, x \in X$ such that $x\gamma_I\phi$, which is contradiction from (IP_3). Thus $\phi^{\gamma_I} = \phi$.

(2) Let $x \in A^{\gamma_I}$. Then Eq. (3.1) implies that $x\gamma_I A$ and Lemma 3.3 implies that $x\gamma_I B$. Hence $x \in B^{\gamma_I}$.

(3) By part (2), we get $A^{\gamma_I} \cup B^{\gamma_I} \subseteq (A \cup B)^{\gamma_I}$. To prove the other inclusion, let $x \in (A \cup B)^{\gamma_I}$. Then $x\gamma_I(A \cup B)$. Hence (IP_2) implies that $x\gamma_I A$ or $x\gamma_I B$, consequently $x \in (A^{\gamma_I} \cup B^{\gamma_I})$. Hence the result.

(4) The result is a direct consequence of part (1).

□

The following example shows that $A \nsubseteq A^{\gamma_I}$, in general.

Example 3.5. Let $X = \{a,\ b,\ c,\ d\}$, $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$, $A = \{b\}$ and γ_I is defined as

$$A\gamma_I B \Leftrightarrow A \cap B \notin I. \tag{3.2}$$

Then $A^{\gamma_I} = \phi$.

Theorem 3.6. *Let (X, γ_I) be an I-basic proximity space. Then the operator*

$$-^{\gamma_I} : P(X) \to P(X)$$

defined by

$$\overline{A}^{\gamma_I} = A \cup A^{\gamma_I} \tag{3.3}$$

is a Čhech closure operator and induces R_o-topology on X called τ_{γ_I} given by

$$\tau_{\gamma_I} = \{A \subseteq X : \overline{A^c}^{\gamma_I} = A^c\}. \tag{3.4}$$

Proof. (1) By Theorem 3.4 part (1) $\phi^{\gamma_I} = \phi$, and hence $\overline{\phi}^{\gamma_I} = \phi$.

(2) Eq. (3.3) implies that $A \subseteq \overline{A}^{\gamma_I}$.

(3) By Theorem 3.4 part (3), we have $\overline{(A \cup B)}^{\gamma_I} = \overline{A}^{\gamma_I} \cup \overline{B}^{\gamma_I}$. Thus $-^{\gamma_I}$ is a Čhech closure operator. Let $x, y \in X$ such that $x \neq y$. Let $x \in \overline{y}^{\gamma_I}$, It follows that $x\gamma_I y$. Then (IP_1) implies that $y\gamma_I x$ and hence $y \in \overline{x}^{\gamma_I}$. Therefore, τ_{γ_I} is R_o topology on X.

□

4. Soft I-proximity

Definition 4.1. Let E be a set of parameters, X be a nonempty set, and $\mathcal{A}$ be a set of I-basic proximities on X. The pair (δ_I, E) is called an I-basic soft proximity on (X, E) if δ_I is a mapping given by $\delta_I : E \to \mathcal{A}$.

The set of all I-basic soft proximities on (X, E) will be denoted by $SP(X, E)$. In addition, if $(\delta_I, E) \in SP(X, E)$, then $((X, E), \delta_I)$ is called an $I-$basic proximity space.

Proposition 4.2. *If $I = \{\phi\}$ in Definition 4.1, then we get the basic soft proximity relation in Definition 2.16.*

Proof. Straightforward. □

Example 4.3. Let I be an ideal on a nonempty set X, E be a set of parameters, and $\delta_I = \{(F(e), G(e)) \in P(X) \times P(X) : F(e) \cap G(e) \notin I\}$. Then one easily sees that (δ_I, E) is an I-basic soft proximity on (X, E).

Example 4.4. Let I be an ideal on a nonempty set X, E be a set of parameters, and $\delta_I = \{(F(e), G(e)) \in P(X) \times P(X) : F(e) \notin I$ and $G(e) \notin I\}$. Then it follows directly from the definition that (δ_I, E) is an I-basic soft proximity on (X, E).

Theorem 4.5. *Let $((X, E), \delta_I) \in SP(X, E)$. Then the $\tilde{\delta_I}$- operator*

$$\tilde{\delta_I} : P(X)^E \to P(X)^E$$

defined by:

$$F^{\tilde{\delta_I}}(e) = \{x \in X : (x, F(e)) \in \delta_I\} \tag{4.1}$$

satisfies the following:-

(1) $(\tilde{\phi})^{\tilde{\delta_I}} = \tilde{\phi}$,
(2) $F \tilde{\subseteq} G \Rightarrow F^{\tilde{\delta_I}} \tilde{\subseteq} G^{\tilde{\delta_I}}$,
(3) $(F \tilde{\cup} G)^{\tilde{\delta_I}} = F^{\tilde{\delta_I}} \tilde{\cup} G^{\tilde{\delta_I}}$,
(4) $(F \tilde{\cap} G)^{\tilde{\delta_I}} \tilde{\subseteq} F^{\tilde{\delta_I}} \tilde{\cap} G^{\tilde{\delta_I}}$.

Proof. (1) If $\exists\ x \in X$ and $e \in E$ such that $x \delta_I \tilde{\phi}(e)$, i.e. $x \delta_I \phi$, which is contradiction from (IP_3). So, $(\tilde{\phi})^{\tilde{\delta_I}} = \tilde{\phi}$.
(2) Let $x \in F^{\tilde{\delta_I}}(e)$. Then Eq. (4.1) implies that $x \delta_I F(e)$ and Lemma 3.3 implies that $x \delta_I G(e)$. Hence $x \in G^{\tilde{\delta_I}}(e)$.
(3) By part (2), we get $F^{\tilde{\delta_I}} \tilde{\cup} G^{\tilde{\delta_I}} \tilde{\subseteq} (F \tilde{\cup} G)^{\tilde{\delta_I}}$. To prove the other inclusion, let $x \in (F \tilde{\cup} G)^{\tilde{\delta_I}}(e)$. Then $x \delta_I (F(e) \cup G(e))$. Hence (IP_2) implies that $x \delta_I F(e)$ or $x \delta_I G(e)$. Therefore, $x \in F^{\tilde{\delta_I}}(e)$ or $x \in G^{\tilde{\delta_I}}(e)$. Hence $x \in (F^{\tilde{\delta_I}}(e) \cup G^{\tilde{\delta_I}}(e))$. Then $(F \tilde{\cup} G)^{\tilde{\delta_I}} \tilde{\subseteq} F^{\tilde{\delta_I}} \tilde{\cup} G^{\tilde{\delta_I}}$. Consequently, $(F \tilde{\cup} G)^{\tilde{\delta_I}} = F^{\tilde{\delta_I}} \tilde{\cup} G^{\tilde{\delta_I}}$.
(4) The result is a direct consequence of part (2).

□

The following example shows that $F \tilde{\nsubseteq} F^{\tilde{\delta_I}}$, in general.

Example 4.6. Let $((X, E), \delta_I) \in SP(X, E)$ and $I = I_f$. Then for each $F \in P(X)^E$, $F^{\tilde{\delta_I}}(e) = \phi$.

Theorem 4.7. *Let $((X, E), \delta_I) \in SP(X, E)$. Then the $C^{\tilde{\delta_I}}$- operator*

$$C^{\tilde{\delta_I}} : P(X)^E \to P(X)^E$$

defined by:

$$C^{\tilde{\delta_I}}(F)(e) = F(e) \cup F^{\tilde{\delta_I}}(e) \tag{4.2}$$

is a $R_o -$ Čhech closure operator of soft sets.

Proof. (1) Theorem 4.5 part (1) implies that $C^{\tilde{\delta_I}}(\tilde{\phi}) = \tilde{\phi}$.
(2) Eq. (4.2) implies that $F \tilde{\subseteq} C^{\tilde{\delta_I}}(F)$.
(3) By Theorem 4.5 part (3), we have $C^{\tilde{\delta_I}}(F \tilde{\cup} G) = C^{\tilde{\delta_I}}(F) \tilde{\cup} C^{\tilde{\delta_I}}(G)$. Thus $C^{\tilde{\delta_I}}$ is a Čhech closure operator of soft sets on (X, E). Let $x, y \in X$ such that $x \neq y$. Let $F \in P(X)^E$ such that $F(e) = \{x\}$. Then $y \in C^{\tilde{\delta_I}}(F)(e) \Leftrightarrow (\{y\}, \{x\}) \in \delta_I \Leftrightarrow (\{x\}, \{y\}) \in \delta_I \Leftrightarrow (\{x\}, G(e)) \in \delta_I\ \forall\ G \in P(X)^E$ such that $G(e) = \{y\} \Leftrightarrow \{x\} \in C^{\tilde{\delta_I}}(G)(e)\ \forall\ G \in P(X)^E$ such that $G(e) = \{y\}$. Consequently, $C^{\tilde{\delta_I}}$ is a $R_o -$ Čhech closure operator of soft sets on (X, E).

□

Example 4.8. Let I be an ideal on a nonempty set X, C be a Čhech closure operator of soft sets on (X, E), and δ_I be a binary relation on $P(X)$ defined as:
(4.3)
$\delta_I = \{(F(e), G(e)) : (C(F)(e) \cap G(e)) \cup (F(e) \cap C(G)(e)) \notin I \text{ and } F(e), G(e) \notin I\}$.

Then (δ_I, E) is an I-basic soft proximity on (X, E). It's clear that δ_I satisfies (IP_1). To prove that δ_I satisfies (IP_2). $\forall F, G, H \in P(X)^E$, $(F(e), G(e) \cup H(e)) \in \delta_I \Leftrightarrow \{C(F)(e) \cap (G(e) \cup H(e))\} \cup \{(F(e) \cap C(G(e) \cup H(e))\} \notin I$ *and* $F(e), G(e) \cup H(e) \notin I \Leftrightarrow \{C(F)(e) \cap (G(e) \cup H(e))\} \cup \{(F(e) \cap (C(G)(e) \cup C(H)(e))\} \notin I$and $F(e) \notin I$ and $G(e)$ or $H(e) \notin I \Leftrightarrow \{C(F)(e) \cap (G(e)\} \cup \{C(F)(e) \cap H(e)\} \cup \{(F(e) \cap C(G)(e)\} \cup \{F(e) \cap C(H)(e)\} \notin I, F(e) \notin I$ and $G(e)$ or $H(e) \notin I \Leftrightarrow \{C(F)(e) \cap (G(e)\} \cup \{(F(e) \cap C(G)(e)\} \notin I$ and $F(e), G(e) \notin I$ or $\{C(F)(e) \cap H(e)\} \cup \{F(e) \cap C(H)(e)\} \notin I$ and $F(e), H(e) \notin I \Leftrightarrow (F(e), G(e)) \in \delta_I$ or $(F(e), H(e)) \in \delta_I$. (IP_3) is a direct consequence from the definition of δ_I. For (IP_4), Let $F, G \in P(X)^E$ such that $F(e) \cap G(e) \notin I$. Then Definition 2.18 part (2) implies that $\{C(F)(e) \cap (G(e)\} \cup \{(F(e) \cap C(G)(e)\} \notin I$. Therefor $(F(e), G(e)) \in \delta_I$.

The following theorem shows that the topology generated by the formula (4.2) is finer than the soft topology generated by the formula (2.1).

Theorem 4.9. *Let C be a Čhech closure operator of soft sets on (X, E), and δ_I is the formula (4.3). Then $\forall\, F \in P(X)^E$, $C(F) \tilde{\subseteq} C^{\tilde{\delta_I}}(F)$.*

Proof. Let $F \in P(X)^E$, $e \in E$, and $x \in C^{\tilde{\delta_I}}(F)(e)$. It follows that $x_e \delta_I F$, and hence $\exists\, e_1 \in E$ such that $(C(x_e)(e_1) \cap F(e)) \cup (x_e(e_1) \cap C(F)(e_1)) \notin I$ and $x_e, F(e) \notin I$. □

5. Conclusions

This paper has presented an extension of soft proximity spaces introduced by Hazra et. al [9] based on the ideal notion. The significance of this extension is that for $I = \{\phi\}$, we have the soft proximity structure [9] and for the other types of I, we have many types of soft proximity structures.

Acknowledgements. The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

References

[1] H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.
[2] K. V. Babitha and J. J. Sunil, Soft set relations and functions, Comput. Math. Appl. 60 (2010) 1840–1849.
[3] D. Chen, The parametrization reduction of soft sets and its applications, Comput. Math. Appl. 49 (2005) 757–763.
[4] V. A. Efremvich, The geometry of proximity, Mat. Sbornik N. S. 31(73) (1952) 189–200.
[5] F. Feng, Y. B. Jun and X. Z. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008) 2621–2628.
[6] D. Jankovic and T. R. Hamlet, New topologies from old via ideals, Amer. Math. Monthly 97(4) (1990) 295–310.
[7] Hai Long Yang and Zhi Lian Guo, Kernels and closures of soft set relations, and soft set relation mappings, Comput. Math. Appl. 61 (2011) 651–652.
[8] H. Hazra, P. Majumdar and S. K. Samanta, Soft topology, Fuzzy Inf. Eng. 4(1) (2012) 105–115.
[9] H. Hazra, P. Majumdar and S. K. Samanta, Soft proximity, Ann. Fuzzy Math. Inform. 7 (2014) 867–877.

[10] H. Hazra, P. Majumdar and S. K. Samanta, Proximity of soft sets, Ann. Fuzzy Math. Inform. 8 (2014) 113–123.
[11] Ke Gong, Zhi Xiao and Xia Zhang, The bijective soft set with its operations, Comput. Math. Appl. 60 (2010) 2270–2278.
[12] Keyun Qin and Zhiyong Hong, On soft equality, J. Comput. Appl. Math. 234 (2010) 1347–1355.
[13] A. Kandil, O. A. Tantawy, S. A. El-Sheikh and A. Zakaria, I-proximity spaces, Jokull Journal 63 (2013) 237–245.
[14] A. Kandil, O. A. Tantawy, S. A. El-Sheikh and A. Zakaria, New structures of proximity spaces, Information Sciences Letters 3 (2014) 85–89.
[15] A. Kandil, O. A. Tantawy, S. A. El-Sheikh and A. Zakaria, Generalized I-proximity spaces, Mathematical Sciences Letters 3 (2014) 173–178.
[16] S. Leader, Local proximity spaces, Math. Ann. 169 (1957) 275–281.
[17] S. Leader, Extension based on proximity boundedness, Math. Z. 108 (1969) 137–144.
[18] M. W. Lodato, On topologically induced generalized proximity relations I, Proc. Amer. Math. Soc. 15 (1964) 417–422.
[19] M. W. Lodato, On topologically induced generalized proximity relations II, Pacific J. Math. 17 (1966) 131–135.
[20] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[21] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft set theory, J. Fuzzy Math. 3 (2001) 589–602.
[22] D. A. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37 (1999) 19–31.
[23] S. A. Naimpally and B. D. Warrack, Proximity Spaces, Cambridge Tract. 1970.
[24] J. F. Peters and P. Wasilewski, Foundations of near sets, Inform. Sci. 179 (2009) 3091–3109.
[25] Qiu-Mei Sun, Zi-Long Zhang and Jing Liu, Soft sets and soft modules, Rough Sets and Knowledge Technology 5009 (2008) 403–409.
[26] Yuncheng Jiang, Yong Tang and Qimai Chen, An adjustable approach to intuitionistic fuzzy soft sets based decision making, Appl. Math. Model. 35 (2011) 824–836.
[27] Zhi Xiao, Ke Gong, Sisi Xia and Yan Zou, Exclusive disjunctive soft sets, Comput. Math. Appl. 59 (2010) 2128–2137.

A. Kandil (dr.ali_kandil@yahoo.com)
Department of Mathematics, Faculty of Science, Helwan University, Egypt

O. A. Tantawy (drosamat@yahoo.com)
Department of Mathematics, Faculty of Science, Zagazig University, Egypt

S. A. El-Sheikh (elsheikh33@hotmail.com)
Department of Mathematics, Faculty of Education, Ain Shams University, Egypt

A. Zakaria (amr.zakaria@edu.asu.edu.eg)
Department of Mathematics, Faculty of Education, Ain Shams University, Egypt