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View on intuitionistic rough paracompactness and intuitionistic rough nearly paracompactness

P. Saranya, M. K. Uma, E. Roja

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Abstract. In this paper the concepts of an intuitionistic rough topological space and intuitionistic rough paracompact spaces are introduced. Also, the concepts of an intuitionistic rough nearly paracompact spaces are introduced. In this connection some interesting properties and characterization are established.

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Corresponding Author: P. Saranya (pssaranyalakshmi@gmail.com)

1. Introduction

The concept of intuitionistic sets in topological spaces was introduced by Çoker in [2]. He studied topology on intuitionistic sets in [1]. Pawlak [5] introduced the concept of rough sets. In this paper the concepts of an intuitionistic rough topological space and intuitionistic rough paracompact spaces are introduced. Also, the concepts of an intuitionistic rough nearly paracompact spaces are introduced. In this connection some interesting properties and characterization are established.

2. Preliminaries

Definition 2.1 ([2])**.** Let X be a non empty set. An intuitionistic set (IS for short) A is an object having the form $A = \langle x, A^1, A^2\rangle$, for all $x \in X$ where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \varnothing$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A. Every crisp set A on a nonempty set X is obviously an intuitionistic set having the form $\langle x, A, A^c\rangle$.

Definition 2.2 ([2]). Let X be a non empty set and let the intuitionistic sets A and B be in the form $A = \langle x, A^1, A^2 \rangle$, $B = \langle x, B^1, B^2 \rangle$, respectively. Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = \langle x, {A_i}^1, {A_i}^2 \rangle$. Then

(i) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $A^2 \supseteq B^2$;
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(iii) $\overline{A} = \langle x, A^2, A^1 \rangle$;
(iv) $\cup A_i = \langle x, \cup {A_i}^1, \cap {A_i}^2 \rangle$;
(v) $\cap A_i = \langle x, \cap {A_i}^1, \cup {A_i}^2 \rangle$;
(vi) $\varnothing_\sim = \langle x, \varnothing, X \rangle$; $X_\sim = \langle x, X, \varnothing \rangle$.

Definition 2.3 ([3]). An intuitionistic topology (IT for short) on a nonempty set X is a family T of intuitionistic sets in X satisfying the following axioms:

(i) $\varnothing_\sim$, $X_\sim \in T$;
(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$;
(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i : i \in J\} \subseteq T$.

In this case the pair (X, T) is called an intuitionistic topological space (ITS for short) and any intuitionistic set in T is called an intuitionistic open set(IOS for short) in X. The complement $\overline{A}$ of an intuitionistic open set A is called an intuitionistic closed set (ICS for short) in X.

Definition 2.4 ([3]). Let (X, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X. Then the intuitionistic closure and intuitionistic interior of A are defined by

$$Icl(A) = \cap\{K : K \text{ is an intuitionistic closed set in } X \text{ and } A \subseteq K\}.$$
$$Iint(A) = \cup\{G : G \text{ is an intuitionistic open set in } X \text{ and } G \subseteq A\}.$$

Definition 2.5 ([3]). Let X and Y be two nonempty sets and $f : X \to Y$ a function, $B = \langle y, B^1, B^2 \rangle$ is an intuitionistic set in Y and $A = \langle x, A^1, A^2 \rangle$ is an intuitionistic set in X. Then the preimage of B under f, denoted by $f^{-1}(B)$, is the intuitionistic set in X defined by $f^{-1}(B) = \langle x, f^{-1}(B^1), f^{-1}(B^2) \rangle$, and the image of A under f, denoted by $f(A)$, is the intuitionistic set in Y defined by $f(A) = \langle x, f(A^1), \underline{f}(A^2) \rangle$ where $\underline{f}(A^2) = Y - (f(X - A^2))$.

Definition 2.6 ([4]). Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x \in U}\{R(x) : R(x) \cap X \neq \varnothing\}$, where $R(x)$ denotes the equivalence class determined by x.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.7 ([4]). Let X be a topological space. A collection $\mathcal{A}$ of subsets of X is said to be *locally finite* in X if every point of X has a neighbourhood that intersects only finitely many elements of A.

Definition 2.8 ([4]). A space X is paracompact if every open covering $\mathcal{A}$ of X has a locally finite open refinement $\mathfrak{B}$ that covers X.

3. Properties and characterizations of intuitionistic roughparacompactness

Definition 3.1. Let X be a non-empty set, R be an equivalence relation on X and (X, R) be an approximation space. Then the *intuitionistic right neighbourhood* of an element $x \in X$ denoted as x_R and defined as $x_R = \langle x, x_R^1, x_R^2 \rangle$ where $x_R^1 = \{y \in X : x_R y\}$ and $x_R^2 = X \setminus \{y \in X : x_R y\}$.

Definition 3.2. Let (X, R) be any approximation space. Let $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X. Then

(i) an *intuitionistic lower approximation of* A is defined and denoted as $\mathcal{L}(A) = \cap_{x \in X}\{x_R : x_R$ is an intuitionistic right neighbourhood set and $x_R \subseteq A\}$.

(ii) an *intuitionistic upper approximation of* A is defined and denoted as $\mathcal{U}(A) = \cup_{x \in X}\{x_R : x_R$ is an intuitionistic right neighbourhood set and $x_R \cap A \neq \varnothing_\sim\}$.

Proposition 3.3. *Let* (X, R) *be any approximation space. Let* $P = (x, P^1, P^2)$ *and* $Q = (x, Q^1, Q^2)$ *be any two intuitionistic sets in* X*. Then the following properties hold:*

(i) $\mathcal{L}(P) \subseteq P \subseteq \mathcal{U}(P)$;
(ii) *If* $P \subseteq Q$ *then* $\mathcal{L}(P) \subseteq \mathcal{L}(Q)$ *and* $\mathcal{U}(P) \subseteq \mathcal{U}(Q)$;
(iii) $\mathcal{U}(P \cup Q) = \mathcal{U}(P) \cup \mathcal{U}(Q)$;
(iv) $\mathcal{U}(P \cap Q) \subseteq \mathcal{U}(P) \cap \mathcal{U}(Q)$;
(v) $\mathcal{L}(P \cup Q) \supseteq \mathcal{L}(P) \cup \mathcal{L}(Q)$;
(vi) $\mathcal{L}(P \cap Q) = \mathcal{L}(P) \cap \mathcal{L}(Q)$;
(vii) $\mathcal{U}(\overline{P}) = \overline{\mathcal{L}(P)}$;
(viii) $\mathcal{L}(\overline{P}) = \overline{\mathcal{U}(P)}$;
(ix) $\mathcal{U}(\mathcal{U}(P)) = \mathcal{U}(P)$;
(x) $\mathcal{L}(\mathcal{L}(P)) = \mathcal{L}(P)$.

Proof. The proof is obvious. □

Definition 3.4. Let X be a non-empty set. An *intuitionistic rough set* A is an object having the form $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$ where $\mathcal{L}(A^1)$, $\mathcal{L}(A^2)$, $\mathcal{U}(A^1)$ and $\mathcal{U}(A^2)$ are subsets of X satisfying $\mathcal{L}(A^1) \cap \mathcal{L}(A^2) = \varnothing$ and $\mathcal{U}(A^1) \cap \mathcal{U}(A^2) = \varnothing$. The set $\mathcal{L}(A^1)$ and $\mathcal{U}(A^1)$ are called the set of members of A, while $\mathcal{L}(A^2)$ and $\mathcal{U}(A^2)$ is called the set of nonmembers of A.

Definition 3.5. Let R_i $(i = 1, 2, ..., n)$ be an equivalence relations. An *intuitionistic rough topology* on a nonempty set X is a family $\mathcal{R}$ of intuitionistic rough sets in X satisfying the following axioms:

(i) $\varnothing_\sim$, $X_\sim \in \mathcal{R}$;
(ii) $G_1 \cap G_2 \in \mathcal{R}$ for any $G_1, G_2 \in \mathcal{R}$;
(iii) $\cup G_i \in \mathcal{R}$ for arbitrary family $\{G_i : i \in J\} \subseteq \mathcal{R}$.

Then the ordered pair $(X, \mathcal{R})$ is called an *intuitionistic rough topological space*. Every member in $\mathcal{R}$ is called an *intuitionistic rough open set* in X. The complement of an intuitionistic rough open set A is an *intuitionistic rough closed set* in X.

Definition 3.6. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. A collection $\mathcal{A}$ of intuitionistic rough sets of a space X is said to *intuitionistic rough cover* X, or to be intuitionistic rough covering of X, if the union of the elements of $\mathcal{A}$ is equal to X.

Definition 3.7. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and let $\mathcal{U}$ and $\mathcal{V}$ be an intuitionistic rough covers of X. We say that $\mathcal{U}$ is *intuitionistic rough refinement* of $\mathcal{V}$, or $\mathcal{U}$ refines $\mathcal{V}$, and write $\mathcal{U} < \mathcal{V}$, provided that for each $U \in \mathcal{U}$ there exists $V \in \mathcal{V}$ such that $U \subseteq V$.

Definition 3.8. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and let $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$ be an intuitionistic rough set of X, and let $\mathcal{U}$ be an intuitionistic rough cover of X. The *intuitionistic rough star of A with respect to $\mathcal{U}$*, denoted by $I\mathcal{R}St(A,\mathcal{U})$, is $\cup\{U \in \mathcal{U} : A \cap U \neq \varnothing_\sim\}$. If $x \in X$, we write $I\mathcal{R}St(x,\mathcal{U})$ to mean $I\mathcal{R}St(\{x\},\mathcal{U})$.

Definition 3.9. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and let $\mathcal{U}$ and $\mathcal{V}$ be an intuitionistic rough covers of X. Then $\mathcal{U}$ is an *intuitionistic rough star refinement* of $\mathcal{V}$, or $\mathcal{U}$ *intuitionistic rough star refines* $\mathcal{V}$, denoted by $\mathcal{U}^* < \mathcal{V}$, provided that for each $U \in \mathcal{U}$ there exists $V \in \mathcal{V}$ such that $I\mathcal{R}St(U,\mathcal{U}) \subseteq V$.

Definition 3.10. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and let $\mathcal{U}$ and $\mathcal{V}$ be an intuitionistic rough covers of X. Then $\mathcal{U}$ is an *intuitionistic rough barycentric refinement* of $\mathcal{V}$, denoted by $\mathcal{U}\Delta\mathcal{V}$, provided $\{I\mathcal{R}St(x,\mathcal{U}) : x \in X\}$ is an intuitionistic rough refinement of $\mathcal{V}$.

Notation 3.11. *Let (X, R) be any intuitionistic rough topological space. Let $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$ be an intuitionistic rough set in X. For each point $x \in A$ that means $x \in \mathcal{L}(A^1)$, $x \in \mathcal{U}(A^1)$, $x \notin \mathcal{L}(A^2)$ and $x \notin \mathcal{U}(A^2)$.*

Proposition 3.12. *Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and let $\mathcal{U}$ and $\mathcal{V}$ be an intuitionistic rough covers of a set X such that $\mathcal{U}^* < \mathcal{V}$. Then $\mathcal{U}\Delta\mathcal{V}$.*

Proof. Let $x \in X$. Since $\mathcal{U}$ intuitionistic rough covers X, there exists $U \in \mathcal{U}$ where $U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle)$ such that $x \in U$. Since $\mathcal{U}^* < \mathcal{V}$, there exists $V \in \mathcal{V}$ where $V = (\langle x, \mathcal{L}(V^1), \mathcal{L}(V^2)\rangle, \langle x, \mathcal{U}(V^1), \mathcal{U}(V^2)\rangle)$ such that $I\mathcal{R}St(U,\mathcal{U}) \subseteq V$. Since $x \in U$, $I\mathcal{R}St(x,\mathcal{U}) \subseteq I\mathcal{R}St(U,\mathcal{U})$. Hence $I\mathcal{R}St(x,\mathcal{U}) \subseteq V$, and so $\mathcal{U}\Delta\mathcal{V}$. □

Definition 3.13. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and $x \in X$. An intuitionistic rough set $V = (\langle x, \mathcal{L}(V^1), \mathcal{L}(V^2)\rangle, \langle x, \mathcal{U}(V^1), \mathcal{U}(V^2)\rangle)$ of X is called an *intuitionistic rough neighbourhood* of x if there exists an intuitionistic rough open set $U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle)$ of X such that $x \in U \subseteq V$.

Definition 3.14. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. A collection $\mathcal{U}$ of intuitionistic rough sets of an intuitionistic rough topological space $(X, \mathcal{R})$ is said to be *intuitionistic rough locally finite* provided each $x \in X$ has an intuitionistic rough neighbourhood that intersects only a finite number of members of $\mathcal{U}$.

Definition 3.15. Let $(X, \mathcal{R})$ be an intuitionistic rough topological space and $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$ be an intuitionistic rough set in X. Then the *intuitionistic rough closure* of A is defined and denoted by

$$I\mathcal{R}cl(A) = \cap\{K : K = (\langle x, \mathcal{L}(K^1), \mathcal{L}(K^2)\rangle, \langle x, \mathcal{U}(K^1), \mathcal{U}(K^2)\rangle) \text{ is an intuitionistic rough closed set in } X \text{ and } A \subseteq K\}.$$

Notation 3.16. *Let (X, R) be any intuitionistic rough topological space. Let $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$ be an intuitionistic rough set in X. Then $\overline{A} = (x, \langle x, \mathcal{L}(A^2), \mathcal{L}(A^1)\rangle, \langle x, \mathcal{U}(A^2), \mathcal{U}(A^1)\rangle)$.*

Proposition 3.17. *Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. Let $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$ be an intuitionistic rough set in X. Then $x \in I\mathcal{R}cl(A)$ if and only if every intuitionistic rough open set U containing x intersects A.*

Proof. Let us assume that $x \notin I\mathcal{R}cl(A)$ if and only if every intuitionistic rough open set $U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle)$ containing x that does not intersects A. If x is not in $I\mathcal{R}cl(A)$, the intuitionistic set $U = \overline{I\mathcal{R}cl(A)}$ is an intuitionistic rough open set containing x that does not intersect A.

Conversely, if there exists an intuitionistic rough open U containing x which does not intersect A, then $\overline{U}$ is an intuitionistic rough closed set containing A. By the definition of the intuitionistic rough closure $I\mathcal{R}cl(A)$, the intuitionistic set $\overline{U}$ must contain $I\mathcal{R}cl(A)$; therefore, x cannot be in $I\mathcal{R}cl(A)$. □

Proposition 3.18. *Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. Let $\mathcal{U}$ be a intuitionistic rough locally finite collection of intuitionistic rough sets of an intuitionistic rough topological space $(X, \mathcal{R})$. Then*

$$\{I\mathcal{R}cl(U) : U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle) \in \mathcal{U}\}$$

is intuitionistic rough locally finite.

Proof. Let $x \in X$. Then there exists an intuitionistic rough neighbourhood $V = (\langle x, \mathcal{L}(V^1), \mathcal{L}(V^2)\rangle, \langle x, \mathcal{U}(V^1), \mathcal{U}(V^2)\rangle)$ of x that intersects only a finite number of members of $\mathcal{U}$. Suppose $U \in \mathcal{U}$ and $V \cap U = \varnothing_\sim$. Then no member of V belongs to $I\mathcal{R}cl(U)$, so $V \cap I\mathcal{R}cl(U) = \varnothing_\sim$. Therefore V intersects only a finite number of members of $\mathcal{U}$. □

Proposition 3.19. *Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. Let $\{A_\alpha : \alpha \in \Lambda\}$ be an intuitionistic rough locally finite collection of intuitionistic rough sets of an intuitionistic rough topological space. Then $\cup_{\alpha\in\Lambda} I\mathcal{R}cl(A_\alpha) = I\mathcal{R}cl(\cup_{\alpha\in\Lambda} A_\alpha)$.*

Proof. First we prove that $\cup_{\alpha\in\Lambda} I\mathcal{R}cl(A_\alpha) \subseteq I\mathcal{R}cl(\cup_{\alpha\in\Lambda} A_\alpha)$. Let $x \in \cup_{\alpha\in\Lambda} I\mathcal{R}cl(A_\alpha)$. Then there exists $\beta \in \Lambda$ such that $x \in I\mathcal{R}cl(A_\beta)$. Thus every intuitionistic rough neighbourhood of x intersects A_β, and hence every intuitionistic rough neighbourhood of x intersects $\cup_{\alpha\in\Lambda} A_\alpha$. Therefore $x \in I\mathcal{R}cl(\cup_{\alpha\in\Lambda} A_\alpha)$ and so $\cup_{\alpha\in\Lambda} I\mathcal{R}cl(A_\alpha) = I\mathcal{R}cl(\cup_{\alpha\in\Lambda} A_\alpha)$.

Now let $x \in I\mathcal{R}cl(\cup_{\alpha\in\Lambda} A_\alpha)$. Since $\{A_\alpha : \alpha \in \Lambda\}$ is intuitionistic rough locally finite, there exists an intuitionistic rough neighbourhood $U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle)$ of x that intersects only a finite number of members $A_{\alpha_1}, A_{\alpha_2}, .., A_{\alpha_n}$ of $\{A_\alpha : \alpha \in \Lambda\}$. Suppose there exists an intuitionistic rough neighbourhood $V = (\langle x, \mathcal{L}(V^1), \mathcal{L}(V^2)\rangle, \langle x, \mathcal{U}(V^1), \mathcal{U}(V^2)\rangle)$ of x that does not intersect $\cup_{i=1}^n A_{\alpha_i}$. Then $U \cap V$ is an intuitionistic rough neighbourhood of x that does not intersect $\cup_{\alpha\in\Lambda} A_\alpha$. This is a contradiction, since $x \in I\mathcal{R}cl(\cup_{\alpha\in\Lambda} A_\alpha)$. Therefore every intuitionistic rough neighbourhood of x intersects $\cup_{i=1}^n A_{\alpha_i}$. Hence $x \in I\mathcal{R}cl(\cup_{i=1}^n A_{\alpha_i})$. But $I\mathcal{R}cl(\cup_{i=1}^n A_{\alpha_i}) = \cup_{i=1}^n I\mathcal{R}cl(A_{\alpha_i})$, so $x \in I\mathcal{R}cl(A_{\alpha_i})$ for some $i = 1, 2, ...n$. Thus $x \in \cup_{\alpha\in\Lambda} I\mathcal{R}cl(A_\alpha)$, and so $I\mathcal{R}cl(\cup_{\alpha\in\Lambda} A_\alpha) \subseteq \cup_{\alpha\in\Lambda} I\mathcal{R}cl(A_\alpha)$. □

Definition 3.20. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. If a family $\{G_i = (\langle x, \mathcal{L}(G_i^1), \mathcal{L}(G_i^2)\rangle, \langle x, \mathcal{U}(G_i^1), \mathcal{U}(G_i^2)\rangle) : i \in J\}$ of an intuitionistic rough open sets in $(X, \mathcal{R})$ satisfies the condition $\cup\{G_i : i \in J\} = X_\sim$, then it is called an *intuitionistic rough open cover* of $(X, \mathcal{R})$.

Definition 3.21. An intuitionistic rough topological space $(X, \mathcal{R})$ is said to be an *intuitionistic rough paracompact* if every intuitionistic rough open cover of X has an intuitionistic rough locally finite open refinement.

Definition 3.22. An intuitionistic rough topological space $(X, \mathcal{R})$ is said to be an *intuitionistic rough regular space* if for each intuitionistic rough closed set $H = (\langle x, \mathcal{L}(H^1), \mathcal{L}(H^2)\rangle, \langle x, \mathcal{U}(H^1), \mathcal{U}(H^2)\rangle)$ and $x \notin H$, there exist two disjoint intuitionistic rough open sets $U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle)$ and $V = (\langle x, \mathcal{L}(V^1), \mathcal{L}(V^2)\rangle, \langle x, \mathcal{U}(V^1), \mathcal{U}(V^2)\rangle)$ such that $H \subseteq U$ and $x \in V$.

Definition 3.23. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. A collection $\mathscr{V}$ of an intuitionistic rough open sets of $(X, \mathcal{R})$ is said to be an *intuitionistic rough open refinement* of $\mathscr{U}$ if for each element $V = (\langle x, \mathcal{L}(V^1), \mathcal{L}(V^2)\rangle, \langle x, \mathcal{U}(V^1), \mathcal{U}(V^2)\rangle)$ of $\mathscr{V}$, there is an element $U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle)$ of $\mathscr{U}$ containing V.

Definition 3.24. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. A collection $\mathscr{V}$ of an intuitionistic rough closed sets of $(X, \mathcal{R})$ is said to be an *intuitionistic rough closed refinement* of $\mathscr{U}$ if for each element $V = (\langle x, \mathcal{L}(V^1), \mathcal{L}(V^2)\rangle, \langle x, \mathcal{U}(V^1), \mathcal{U}(V^2)\rangle)$ of $\mathscr{V}$, there is an element $U = (\langle x, \mathcal{L}(U^1), \mathcal{L}(U^2)\rangle, \langle x, \mathcal{U}(U^1), \mathcal{U}(U^2)\rangle)$ of $\mathscr{U}$ containing V.

Definition 3.25. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space. A collection $\mathcal{U}$ of intuitionistic rough sets of an intuitionistic rough topological space $(X, \mathcal{R})$ is *σ-intuitionistic rough locally finite* provided $\mathcal{U} = \cup_{n\in\mathbb{N}} \mathcal{U}_n$, where each $\mathcal{U}_n$ is an intuitionistic rough locally finite collection of intuitionistic rough sets of X.

Proposition 3.26. *Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and $\mathcal{A}$ be an intuitionistic rough locally finite collection of intuitionistic rough sets of X. Then any subcollection of $\mathcal{A}$ is intuitionistic rough locally finite.*

Proof. The proof is obvious. □

Definition 3.27. Let $(X,\mathcal{R})$ be an intuitionistic rough topological space and $A = (\langle x,\mathcal{L}(A^1),\mathcal{L}(A^2)\rangle,\langle x,\mathcal{U}(A^1),\mathcal{U}(A^2)\rangle)$ be an intuitionistic rough set in X. Then

(i) $\overline{\cup A_i} = \cap \overline{A_i}$.

(ii) $\overline{\cap A_i} = \cup \overline{A_i}$.

Proposition 3.28. *Let $(X,\mathcal{R})$ be any intuitionistic rough regular space. Then the following are equivalent:*

(i) *$(X,\mathcal{R})$ is intuitionistic rough paracompact.*

(ii) *Every intuitionistic rough open cover of $(X,\mathcal{R})$ has σ-intuitionistic rough locally finite open refinement.*

(iii) *Every intuitionistic rough open cover of $(X,\mathcal{R})$ has an intuitionistic rough locally finite refinement.*

(vi) *Every intuitionistic rough open cover of $(X,\mathcal{R})$ has an intuitionistic rough locally finite, intuitionistic rough closed refinement.*

Proof. (i) ⇒ (ii) Let $\mathcal{U}$ be an intuitionistic rough open cover of $(X,\mathcal{R})$. Then there exists an intuitionistic rough locally finite refinement $\mathcal{V}$ of $\mathcal{U}$. By Proposition 3.26 $\mathcal{V}$ is an σ-intuitionistic rough locally finite open refinement, and the proof is complete.

(ii)⇒ (iii) Let $\mathcal{U}$ be an intuitionistic rough open cover of $(X,\mathcal{R})$. Then there exists an σ-intuitionistic rough locally finite open refinement $\mathcal{V}$ of $\mathcal{U}$. So $\mathcal{V} = \cup_{n\in\mathbb{N}}\mathcal{V}_n$, where each $\mathcal{V}_n$ is intuitionistic rough locally finite. For each $n \in \mathbb{N}$, let $W_n = \cup\{V : V \in \mathcal{V}_n\}$ where $V = (\langle x,\mathcal{L}(V^1),\mathcal{L}(V^2)\rangle,\langle x,\mathcal{U}(V^1),\mathcal{U}(V^2)\rangle)$. Then $\{W_n : n \in \mathbb{N}\}$ is an intuitionistic rough open cover of X. For each $n \in \mathbb{N}$, let $A_n = W_n \cap \overline{\cup_{i=1}^{n-1} W_i}$. It is clear that $\{A_n : n \in \mathbb{N}\}$ is an intuitionistic rough refinement of $\{W_n : n \in \mathbb{N}\}$. Let $x \in X$, and let n_x be the smallest member of $\{n \in \mathbb{N} : x \in W_n\}$. Then $x \in A_{n_x}$,and hence $\{A_n : n \in \mathbb{N}\}$ covers X. Also W_{n_x} is an intuitionistic rough neighbourhood of x that does not intersects A_n for any $n > n_x$, and so $\{A_n : n \in \mathbb{N}\}$ is intuitionistic rough locally finite. Let $\mathcal{A} = \{A_n \cap V : n \in \mathbb{N}$ and $V \in \mathcal{V}_n\}$. Since $\mathcal{V}$ is an intuitionistic rough refinement of $\mathcal{U}$, $\mathcal{A}$ intuitionistic rough refines $\mathcal{U}$. Let $x \in X$. Since $\{A_n : n \in \mathbb{N}\}$ is intuitionistic rough locally finite, there exists an intuitionistic rough neighbourhood $M = (\langle x,\mathcal{L}(M^1),\mathcal{L}(M^2)\rangle,\langle x,\mathcal{U}(M^1),\mathcal{U}(M^2)\rangle)$ of x that intersects only a finite number of members of $A_{n_1}, A_{n_2}, ..., A_{n_k}$ of $\{A_n : n \in \mathbb{N}\}$. For each $i = 1, 2, ..., k$, there exists an intuitionistic rough neighbourhood $P_{n_i} = (\langle x,\mathcal{L}(P^1_{n_i}),\mathcal{L}(P^2_{n_i})\rangle,\langle x,\mathcal{U}(P^1_{n_i}),\mathcal{U}(P^2_{n_i})\rangle)$ of x that intersects only a finite number of members of $\mathcal{V}_{n_i}$. Then $M \cap (\cap_{i=1}^{k} P_{n_i})$ is an intuitionistic rough neighbourhood of x that intersects only a finite number of members of $\mathcal{A}$. Therefore $\mathcal{A}$ is intuitionistic rough locally finite, and so $\mathcal{A}$ is the desired intuitionistic rough locally finite refinement of $\mathcal{U}$.

(iii)⇒ (iv) Let $\mathcal{U}$ be an intuitionistic rough open cover of $(X,\mathcal{R})$. For each $x \in X$, let $U_x \in \mathcal{U}$ where $U_x = (\langle x,\mathcal{L}(U^1_x),\mathcal{L}(U^2_x)\rangle,\langle x,\mathcal{U}(U^1_x),\mathcal{U}(U^2_x)\rangle)$ such that $x \in U_x$. Since $(X,\mathcal{R})$ is intuitionistic rough regular, for each $x \in X$, there exists an intuitionistic rough neighbourhood $V_x = (\langle x,\mathcal{L}(V^1_x),\mathcal{L}(V^2_x)\rangle,\langle x,\mathcal{U}(V^1_x),\mathcal{U}(V^2_x)\rangle)$ of x such that $I\mathcal{R}cl(V_x) \subseteq U_x$. Then $\{V_x : x \in X\}$ is an intuitionistic rough open cover of X, and so, by (iii), it has an intuitionistic rough locally finite refinement $\{A_\alpha : \alpha \in \Lambda\}$ where $A_\alpha = (x,\mathcal{L}(A_\alpha),\mathcal{U}(A_\alpha))$. By Proposition 3.18, $\{I\mathcal{R}cl(A_\alpha) : \alpha \in \Lambda\}$

is intuitionistic rough locally finite. For each $\alpha \in \Lambda$, there exists $x \in X$ such that $A_\alpha \subseteq V_x$. Therefore since $I\mathcal{R}cl(V_x) \subseteq U_x$ for each $x \in X$, $I\mathcal{R}cl(A_\alpha) \subseteq U_x$. Thus $\{I\mathcal{R}cl(A_\alpha) : \alpha \in \Lambda\}$ is intuitionistic rough locally finite closed refinement of $\mathcal{U}$.

(iv)$\Rightarrow$ (i) Let $\mathcal{U}$ be an intuitionistic rough open cover of $(X, \mathcal{R})$. Then there exists an intuitionistic rough locally finite closed refinement $\mathcal{A}$ of $\mathcal{U}$. For each $x \in X$, let V_x be an intuitionistic rough neighbourhood of x that intersects only a finite number of members of $\mathcal{A}$. Then $\{V_x : x \in X\}$ is an intuitionistic rough open cover of X, so there exists an intuitionistic rough locally finite closed refinement $\mathcal{C}$ of $\{V_x : x \in X\}$. For each $A \in \mathcal{A}$ where $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$, let $A^* = \overline{\cup\{C \in \mathcal{C} : A \cap C = \varnothing_\sim\}}$ where $C = (\langle x, \mathcal{L}(C^1), \mathcal{L}(C^2)\rangle, \langle x, \mathcal{U}(C^1), \mathcal{U}(C^2)\rangle)$. Since $\mathcal{C}$ is intuitionistic rough locally finite, by Proposition 3.19, $I\mathcal{R}cl(\cup\{C \in \mathcal{C} : A \cap C = \varnothing_\sim\}) = \cup\{I\mathcal{R}cl(C) \in \mathcal{C} : A \cap C = \varnothing_\sim\}$. Therefore $\cup\{C \in \mathcal{C} : A \cap C = \varnothing_\sim\}$ is intuitionistic rough closed, and so A^* is intuitionistic rough open. For each $A \in \mathcal{A}$, $A \subseteq A^*$. Therefore $\{A^* : A \in \mathcal{A}\}$ is an intuitionistic rough cover of X. We claim that $\{A^* : A \in \mathcal{A}\}$ is intuitionistic rough locally finite.

Let $x \in X$. There exists an intuitionistic rough neighbourhood $W = (\langle x, \mathcal{L}(W^1), \mathcal{L}(W^2)\rangle, \langle x, \mathcal{U}(W^1), \mathcal{U}(W^2)\rangle)$ of x that intersects only a finite number of members $C_1, C_2, ..., C_n$ of $\mathcal{C}$. Since $\mathcal{C}$ intuitionistic rough covers X, $W \subseteq \cup_{i=1}^n C_i$. Therefore if $W \cap A^* \neq \varnothing_\sim$, then there exists $k(1 \leq k \leq n)$ such that $C_k \cap A^* \neq \varnothing_\sim$. But $C_k \cap A^* \neq \varnothing_\sim$ implies $C_k \cap A \neq \varnothing_\sim$. Since each C_i intersects only a finite number of members of $\mathcal{A}$, $W \cap A^* = \varnothing_\sim$ for all but a finite number of members of $\{A^* : A \in \mathcal{A}\}$. Therefore $\{A^* : A \in \mathcal{A}\}$ is intuitionistic rough locally finite. Now for each $A \in \mathcal{A}$, choose $U_A \in \mathcal{U}$ where $U_A = (\langle x, \mathcal{L}(U_A^1), \mathcal{L}(U_A^2)\rangle, \langle x, \mathcal{U}(U_A^1), \mathcal{U}(U_A^2)\rangle)$ such that $A \subseteq U_A$. Then $\{A^* \cap U_A : A \in \mathcal{A}\}$ is an intuitionistic rough locally finite open refinement of $\mathcal{U}$. □

4. Characterizations of intuitionistic rough nearly paracompactness

Definition 4.1. Let $(X, \mathcal{R})$ be any intuitionistic rough topological space and let $A = (\langle x, \mathcal{L}(A^1), \mathcal{L}(A^2)\rangle, \langle x, \mathcal{U}(A^1), \mathcal{U}(A^2)\rangle)$ be an intuitionistic rough set in $(X, \mathcal{R})$. Then A is called:

(i) an *intuitionistic rough regular open set* if $A = I\mathcal{R}int(I\mathcal{R}cl(A))$;
(ii) an *intuitionistic rough regular closed set* if $A = I\mathcal{R}cl(I\mathcal{R}int(A))$.

Definition 4.2. Let $(X, \mathcal{R})$ be an intuitionistic rough topological space. If a family $\{G_i = (\langle x, \mathcal{L}(G_i^1), \mathcal{L}(G_i^2)\rangle, \langle x, \mathcal{U}(G_i^1), \mathcal{U}(G_i^2)\rangle) : i \in J\}$ of an intuitionistic rough regular open sets in $(X, \mathcal{R})$ satisfies the condition $\cup\{G_i : i \in J\} = X_\sim$, then it is called an *intuitionistic rough regular open cover* of $(X, \mathcal{R})$.

Remark 4.3. Every intuitionistic rough regular open set is an intuitionistic rough open set.

Definition 4.4. Let $(X, \mathcal{R})$ be an intuitionistic rough topological space. A space $(X, \mathcal{R})$ is said to be an *intuitionistic rough nearly paracompact* if every intuitionistic rough regular open cover of X has an intuitionistic rough locally finite open refinement.

Definition 4.5. Let $(X, \mathcal{R})$ be an intuitionistic rough topological space. A space $(X, \mathcal{R})$ is said to be an *intuitionistic rough almost regular* if for any intuitionistic rough regular closed set $F = (\langle x, \mathcal{L}(F^1), \mathcal{L}(F^2)\rangle, \langle x, \mathcal{U}(F^1), \mathcal{U}(F^2)\rangle)$ and any point $x \notin F$, there exist disjoint intuitionistic rough open sets containing F and x respectively.

Definition 4.6. Let $(X, \mathcal{R})$ be an intuitionistic rough topological space. A space $(X, \mathcal{R})$ is said to be an *intuitionistic rough almost paracompact* iff for every intuitionistic rough open cover of the space there exists intuitionistic rough locally finite family of intuitionistic rough open sets which refines it and the intuitionistic rough closures of whose members cover the space.

Proposition 4.7. *For an intuitionistic rough almost regular space $(X, \mathcal{R})$, the following are equivalent:*

(i) $(X, \mathcal{R})$ *is intuitionistic rough nearly paracompact.*
(ii) *Every intuitionistic rough regular open cover of $(X, \mathcal{R})$ has intuitionistic rough regular open, intuitionistic rough locally finite refinement.*
(iii) $(X, \mathcal{R})$ *is intuitionistic rough almost paracompact.*
(iv) *For every intuitionistic rough regular open cover of $(X, \mathcal{R})$, there exists intuitionistic rough locally finite family of intuitionistic rough open sets which refines it and the intuitionistic rough closures of whose members cover the space.*
(v) *Every intuitionistic rough regular open cover of $(X, \mathcal{R})$ has intuitionistic rough locally finite refinement.*
(vi) *Every intuitionistic rough regular open cover of $(X, \mathcal{R})$ has an intuitionistic rough locally finite, intuitionistic rough closed refinement.*

Proof. (i) $\Rightarrow$ (ii) Let $\mathcal{U}$ be any intuitionistic rough regular open cover of $(X, \mathcal{R})$. Then there exists intuitionistic rough locally finite open refinement $\mathcal{V}$ of $\mathcal{U}$. Consider the family $\mathcal{W} = \{I\mathcal{R}int(I\mathcal{R}cl(V)) : V \in \mathcal{V}\}$. Then $\mathcal{W}$ is an intuitionistic rough locally finite regular open refinement of $\mathcal{U}$.

(ii) $\Rightarrow$ (iii) Let $\mathcal{G} = \{G_\lambda = (\langle x, \mathcal{L}(G^1_\lambda), \mathcal{L}(G^2_\lambda)\rangle, \langle x, \mathcal{U}(G^1_\lambda), \mathcal{U}(G^2_\lambda)\rangle) : \lambda \in \Lambda\}$ be any intuitionistic rough open covering of X. Then, $\{I\mathcal{R}int(I\mathcal{R}cl(G_\lambda)) : \lambda \in \Lambda\}$ is an intuitionistic rough regular open covering of X. By hypothesis there exists an intuitionistic rough locally finite open refinement

$$\{H_\lambda = (\langle x, \mathcal{L}(H^1_\lambda), \mathcal{L}(H^2_\lambda)\rangle, \langle x, \mathcal{U}(H^1_\lambda), \mathcal{U}(H^2_\lambda)\rangle) : \lambda \in \Lambda\}$$

of $\{I\mathcal{R}int(I\mathcal{R}cl(G_\lambda)) : \lambda \in \Lambda\}$ such that $H_\lambda \subseteq I\mathcal{R}int(I\mathcal{R}cl(G_\lambda))$ for each $\lambda \in \Lambda$. Since $H_\lambda \subseteq I\mathcal{R}int(I\mathcal{R}cl(G_\lambda)) = G_\lambda$, therefore $M_\lambda = H_\lambda \cap G_\lambda$. Thus $\{M_\lambda = (\langle x, \mathcal{L}(M^1_\lambda), \mathcal{L}(M^2_\lambda)\rangle, \langle x, \mathcal{U}(M^1_\lambda), \mathcal{U}(M^2_\lambda)\rangle) : \lambda \in \Lambda\}$ is an intuitionistic rough locally finite family of intuitionistic rough open sets which refines $\mathcal{G}$. We shall prove that $\cup\{I\mathcal{R}cl(M_\lambda) : \lambda \in \Lambda\} = X_\sim$. Let $x \in X$. Then $x \in H_\lambda$ for some $\lambda \in \Lambda$. Now, $I\mathcal{R}cl(M_\lambda) = I\mathcal{R}cl(H_\lambda \cap G_\lambda) = I\mathcal{R}cl(H_\lambda)$. Thus $x \in I\mathcal{R}cl(H_\lambda) = I\mathcal{R}cl(M_\lambda)$. Then $\{M_\lambda : \lambda \in \Lambda\}$ is an intuitionistic rough locally finite family of intuitionistic rough open sets of X whose intuitionistic rough closures cover X and which is an intuitionistic rough refinement of $\mathcal{G}$. Hence $(X, \mathcal{R})$ is an intuitionistic rough almost paracompact space.

(iii) $\Rightarrow$ (iv) The proof is obvious.

(iv) $\Rightarrow$ (v) Let $\mathcal{G}$ be any intuitionistic rough regular open covering of X. Let $x \in X$. Then $x \in G$ for some $G \in \mathcal{G}$ where $G = (\langle x, \mathcal{L}(G^1), \mathcal{L}(G^2)\rangle, \langle x, \mathcal{U}(G^1), \mathcal{U}(G^2)\rangle)$.

Since $(X, \mathcal{R})$ is intuitionistic rough almost regular, there exists an intuitionistic rough regular open set $F = (x, \mathcal{L}(F), \mathcal{U}(F))$ such that $x \in F \subseteq I\mathcal{R}cl(F) \subseteq G$. Consider the intuitionistic rough regular open covering $\mathcal{F} = \{F\}$ of $(X, \mathcal{R})$. By hypothesis, there exists an intuitionistic rough locally finite family $\mathcal{D}$ of intuitionistic rough open sets of $(X, \mathcal{R})$ which refines $\mathcal{F}$ and the intuitionistic rough closures of whose members cover the space. The family $\{I\mathcal{R}cl(\mathcal{D})\}$ is then an intuitionistic rough locally finite family which refines $\mathcal{G}$ and covers X.

(v) $\Rightarrow$ (vi) Let $\mathcal{G}$ be any intuitionistic rough regular open covering of X. Then each $x \in X$ is contained in some $G \in \mathcal{G}$ where $G = (\langle x, \mathcal{L}(G^1), \mathcal{L}(G^2)\rangle, \langle x, \mathcal{U}(G^1), \mathcal{U}(G^2)\rangle)$. By intuitionistic rough almost regularity, there exists intuitionistic rough regular open set F such that $x \in F \subseteq I\mathcal{R}cl(F) \subseteq G$. Consider the intuitionistic rough regular open covering $\mathcal{F} = \{F\}$ of $(X, \mathcal{R})$. There exists intuitionistic rough locally finite refinement $\mathcal{D} = \{D\}$ of $\mathcal{D}$. $\{I\mathcal{R}cl(\mathcal{D})\}$ is then an intuitionistic rough locally finite closed refinement of $\mathcal{G}$.

(vi) $\Rightarrow$ (i) The proof is similar to that of Proposition 3.26 (iv) $\Rightarrow$ (i). □

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P. Saranya (pssaranyalakshmi@gmail.com)
Research Scholar, Department of Mathematics, Sri Sarada College for Women, salem-16, Tamil Nadu, India

M. K. Uma (mkuma70@yahoo.com)
Associate Professor, Department of Mathematics, Sri Sarada College for Women, salem-16, Tamil Nadu, India

E. Roja (AMV-1982@rediffmail.com)
Associate Professor, Department of Mathematics, Sri Sarada College for Women, salem-16, Tamil Nadu, India