Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 9, No. 4, (April 2015), pp. 615–620
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Intuitionistic fuzzy pre-α-irresolute open and closed mappings

R. Renuka, V. Seenivasan

Received 27 April 2014; Revised 20 August 2014; Accepted 12 september 2014

Abstract. In this paper the concept of intuitionistic fuzzy pre-α-irresolute open and closed mappings are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these mappings are also given. We also study relationship between this function with other existing functions.

2010 AMS Classification: 54A40, 03E72

Keywords: Intuitionistic fuzzy α-open set, Intuitionistic fuzzy preopen set, Intuitionistic fuzzy pre-α-irresolute.

Corresponding Author: R. Renuka (renuka.autpc@gmail.com)

1. Introduction

Ever since the introduction of fuzzy sets by L.A.Zadeh[10], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C.L.Chang[2]. Atanassov[1] introduced the notion of intuitionistic fuzzy sets, Coker[3] introduced the intuitionistic fuzzy topological spaces. Intuitionistic fuzzy open, α-open, and preopen mappings were discussed in [3, 5]. In this paper, we introduce the concept of intuitionistic fuzzy pre-α-irresolute open and intuitionistic fuzzy pre-α-irresolute closed mappings as an extension of our work done in the paper [9]. Also we study some of their properties and establish their relationships with other existing mappings.

2. Preliminaries

Definition 2.1 ([1]). Let X be a nonempty fixed set and I the closed interval [0,1]. An intuitionistic fuzzy set(IFS) A is an object of the following form

$$A = \{<x, \mu_A(x), \nu_A(x)>|\ x \in X\}$$

where the mappings μ_A : X→I and ν_A : X→I denote the degree of membership(namely) μ_A(x) and the degree of nonmembership(namely) ν_A(x) for each element x∈X to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x∈X.

Definition 2.2 ([1]). Let A and B are IFSs of the form $A = \{<x, \mu_A(x), \nu_A(x)>| x\in X\}$ and $B = \{<x, \mu_B(x), \nu_B(x)>| x\in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
(ii) $\bar{A} = \{<x, \nu_A(x), \mu_A(x)>| x\in X\}$;
(iii) $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >| x\in X\}$;
(iv) $A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >| x\in X\}$.

We will use the notation $A = \{<x, \mu_A, \nu_A> | x\in X\}$ instead of $A= \{<x, \mu_A(x), \nu_A(x)> | x\in X\}$. The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{<x, \underline{0}, \underline{1} > | x\in X\}$ and $1_\sim = \{<x, \underline{1}, \underline{0} > | x\in X\}$.

Let X and Y are two non-empty sets and f: (X,τ)→(Y,σ) be a function.
If $B = \{<y, \mu_B(y), \nu_B(y)>| y\in Y\}$ is an IFS in Y,then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{<x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))>| x\in X\}$ Since μ_B, ν_B are fuzzy sets, we explain that $f^{-1}(\mu_B(x))=\mu_B(f(x))$.

Definition 2.3 ([3]). An intuitionistic fuzzy topology(IFT) in Coker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$;
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$;
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i\in J\} \subseteq \tau$.

In this paper by (X,τ) or simply by X we will denote the intuitionistic fuzzy topological space (IFTS). Each IFS which belongs to τ is called an intuitionistic fuzzy open set(IFOS) in X. The complement $\bar{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set(IFCS) in X.

Definition 2.4 ([3]). Let (X,τ) be an IFTS and $A = \{<x, \mu_A(x), \nu_A(x)> | x\in X\}$ be an IFS in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by

(i) $cl(A) = \bigcap \{C:C$ is an IFCS in X and $C \supseteq A\}$;
(ii) $int(A) = \bigcup \{D:D$ is an IFOS in X and $D \subseteq A\}$;

It can be also shown that cl(A) is an IFCS, int(A) is an IFOS in X and A is an IFCS in X if and only if cl(A) = A ; A is an IFOS in X if and only if int(A) = A.

Proposition 2.5. *Let (X,τ) be an IFTS and A,B be IFSs in X. Then the following properties hold:*

(i) $cl\overline{A} = \overline{(int(A))}$, $int\overline{(A)} = \overline{(cl(A))}$;
(ii) $int(A) \subseteq A \subseteq cl(A)$.[3]

Definition 2.6 ([4]). An IFS A in an IFTS X is called

(i) an intuitionistic fuzzy pre open set(IFPOS) if A ⊆ int(clA).
(ii) an intuitionistic fuzzy α-open set (IFαOS) if and only if A ⊆ int(cl(intA)).
(iii) an intuitionistic fuzzy semi open set(IFSOS) if and only if A ⊆ cl(int(A)).

The complement of an IFPOS(IFαOS and IFSOS) A in X is called IFPCS(IFαCS and IFSCS) in X.

Definition 2.7 ([4])**.** Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called intuitionistic fuzzy continuous(IFα-continuous, IF pre continuous) if and only if $f^{-1}(B)$ is an IFOS (IFαOS, IFPOS) in X, for each IFOS B in Y.

Definition 2.8 ([8, 6])**.** Let (X,τ) be an IFTS and A = {<x, μ_A(x), ν_A(x)> | x∈X} be an IFS in X.
The intuitionistic fuzzy α-closure and intuitionistic fuzzy α-interior of A are defined by

(i) αcl(A) = $\bigcap$ {C:C is an IFαCS in X and C $\supseteq$ A};
(ii) αint(A) = $\bigcup$ {D:D is an IFαOS in X and D $\subseteq$ A}.

The intuitionistic fuzzy preclosure and intuitionistic fuzzy preinterior of A are defined by

(i) pcl(A) = $\bigcap$ {C:C is an IFPCS in X and C $\supseteq$ A};
(ii) pint(A) = $\bigcup$ {D:D is an IFPOS in X and D $\subseteq$ A}.

Definition 2.9 ([9])**.** A function f:(X,τ)→(Y,σ) from a intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy pre-α-irresolute if $f^{-1}(B)$ is an IFPOS in (X,τ) for each IFαOS B in (Y,σ).

Definition 2.10 ([3, 5])**.** Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,σ). Then, f is called an intuitionistic fuzzy open mapping(IFα-open mapping, IF preopen mapping) if $f(A)$ is an IFOS (IFαOS, IFPOS) in Y for every IFOS A in X.

Definition 2.11 ([7])**.** A mapping $f:(X,\tau)\to(Y,\sigma)$ from an intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy α-irresolute open(IFα-irresolute open) mapping if $f(A)$ is an IFαOS in Y for every IFαOS A in X.

3. Intuitionistic fuzzy pre-α-irresolute open and closed mappings

Definition 3.1. A mapping $f:(X,\tau)\to(Y,\sigma)$ from an intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy pre-α-irresolute open mapping(intuitionstic fuzzy pre-α-irresolute closed) if $f(A)$ is an IFαOS(IFαCS) in Y for every IFPOS(IFPCS) A in X.

Theorem 3.2. *Let (X,τ) and (Y,σ) be two IFTSs and let $f:(X,\tau)\to(Y,\sigma)$ be a mapping. Then the following conditions are equivqlent:*

(i) *f is an IFpre-α-irresolute open mapping.*
(ii) *$f(pintA)\subseteq \alpha int f(A)$ for each IFS A in X.*
(iii) *$pint(f^{-1}(B))\subseteq f^{-1}(\alpha int(B))$ for each IFS B in Y.*
(iv) *For any IFS A in X, IFS B in Y and let A be IFPCS such that $f^{-1}(B)\subseteq A$. Then there exists an IFαCS C in Y and $B\subseteq C$ such that $f^{-1}(C)\subseteq A$.*

Proof. (i)⇒(ii) $pintA \subseteq A \Rightarrow f(pintA) \subseteq f(A)$. But $pintA$ is an IFPOS in X, $f(pintA)$ is an IFαOS in Y. Hence $f(pintA) = \alpha int f(pintA) \subseteq \alpha int f(A)$.
(ii))⇒(iii) Let $A = f^{-1}(B)$. By(ii) $f(pint(f^{-1}(B)) \subseteq \alpha int f(f^{-1}(B)) \subseteq \alpha int(B)$. $\Rightarrow pint(f^{-1}(B)) \subseteq f^{-1}(f(pint(f^{-1}(B))) \subseteq f^{-1}(\alpha int(B))$. Thus $pint(f^{-1}(B) \subseteq f^{-1}(\alpha int(B))$.
(iii)⇒(iv) Let A be IFPCS in X and B be an IFS in Y such that $f^{-1}(B) \subseteq A$. Hence $\overline{f^{-1}(B)} \supseteq \overline{A}$. $\Rightarrow \overline{A} \subseteq \overline{f^{-1}(B)} = f^{-1}(\overline{B})$. But $\overline{A}$ is an IFPOS. Thus, $\overline{A} = pint(\overline{A}) \subseteq pint(f^{-1}(\overline{B})) \subseteq f^{-1}(\alpha int(\overline{B}))$ Hence $A \supseteq \overline{f^{-1}(\alpha int(\overline{B})} = f^{-1}(\alpha cl(B))$. Let $\alpha cl(B) = C$ then $f^{-1}(C) \subseteq A$.
(iv)⇒(i) Let D be an IFPOS in X, $B = \overline{f(D)}$ and $A = \overline{D}$. Then A is an IFPCS. Hence $f^{-1}(B) = f^{-1}(\overline{f(D)}) = \overline{f^{-1}(f(D))} \subseteq \overline{D} = A$. Then there exists an IFαCS C and $B \subseteq C$ such that $f^{-1}(C) \subseteq A = \overline{D}$. Thus, $D \subseteq \overline{f^{-1}(C)}$. $\Rightarrow f(D) \subseteq f(f^{-1}(\overline{C})) \subseteq \overline{C}$. On the otherhand by $B \subseteq C$, $f(D) = \overline{B} \supseteq \overline{C}$. Hence $f(D) = \overline{C}$. Since $\overline{C}$ is an IFαOS, we have $f(D)$ is an IFαOS in Y. □

Theorem 3.3. *Every IFpre-α-irresolute open mapping is IFα-irresolute open mapping.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ from an intuitionistic fuzzy topological space (X, τ) to another intuitionistic fuzzy topological space (Y, σ) is IFpre-α-irresolute open mapping. Let A be IFαOS in X. Since every IFαOS is an IFPOS, A is an IFPOS in X. As f is an IFpre-α-irresolute open $f(A)$ is an IFαOS in Y. Hence f is IFα-irresolute open mapping. □

Remark 3.4. However the converse need not be true as shown by the following example.

Example 3.5. Let X = {a,b,c} = Y, τ ={$0_\sim$,A,$1_\sim$}, σ={$0_\sim$,B,$1_\sim$} be IFTs on X and Y respectively where $A = \{< x, (\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.6}) >; x \in X\}$, $B = \{< y, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.2}), (\frac{a}{0.6}, \frac{b}{0.3}, \frac{c}{0.6}) >; y \in Y\}$. $C = \{< x, (\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.7})(\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.3}) >; x \in X\}$ be an IFS in X. Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. A is an IFOS in X and hence IFαOS in X. $f(A) = \{< y, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{0.6}, \frac{b}{0.3}, \frac{c}{0.5}) >; y \in Y\}$ And int(cl(int$f(A)$)) = $1_\sim$. Thus $f(A) \subseteq$ int(cl(int$f(A)$)). Hence $f(A)$ is an IFαOS in Y, which implies f is IFα-open mapping. And $C \subseteq int(clC) = 1_\sim$. Hence C is IFPOS in X. $f(C) = \{< y, (\frac{a}{0.7}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.3}) >; y \in Y\}$ And int(cl(int$f(C)$))= $0_\sim$. Thus $f(C) \nsubseteq$ int(cl(int$f(C)$)). Hence $f(C)$ is not IFαOS in Y. So, f is not IFpre-α-irresolute open mapping.

Theorem 3.6. *Every IFpre-α-irresolute open mapping is IFα-open mapping.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ from an intuitionistic fuzzy topological space (X, τ) to another intuitionistic fuzzy topological space (Y, σ) is IFpre-α-irresolute open mapping. Let A be IFOS in X. Since every IFOS is an IFPOS, A is an IFPOS in X. As f is an IFpre-α-irresolute open $f(A)$ is an IFαOS in Y. Hence f is IFα-open mapping. □

Remark 3.7. However the converse need not be true as shown by the following example.

Example 3.8. Let X = {a,b,c} = Y, τ ={$0_\sim$,A,$1_\sim$}, σ={$0_\sim$,B, C, B∪C, B∩C,$1_\sim$} be IFTs on X and Y respectively where $A = \{< x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}) >; x \in X\}$, $B = \{< y, (\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.6}), (\frac{a}{0.5}, \frac{b}{0.7}, \frac{c}{0.4}) >; y \in Y\}$, $C = \{< y, (\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.3})(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.7}) >; y \in Y\}$. $D = \{< x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}) >; x \in X\}$ be an IFS in X.

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c$. A is an IFOS in X. $f(A) = \{< y, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}) >; y \in Y\}$ And int(cl(int$f(A)$)) $= B \cup C$. Thus $f(A) \subseteq$ int(cl(int$f(A)$)). Hence $f(A)$ is an IFαOS in Y, which implies f is IFα-open mapping. D is an IFS in X. And $D \subseteq int(clD) = 1_\sim$. Hence D is IFPOS in X. $f(D) = \{< y, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}) >; y \in Y\}$ And int(cl(int$f(D)$))$= B \cup C$. Thus $f(D) \nsubseteq$ int(cl(int$f(D)$)). Hence $f(D)$ is not IFαOS in Y. So, f is not IFpre-α-irresolute open mapping.

Theorem 3.9. *Let $(X, \tau), (Y, \sigma)$ be IFTSs. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two maps. If $g \circ f$:$(X, \tau) \to (Z, \delta)$ is an IFpre-α-irresolute open and f is surjective, IFpre-α-irresolute function then g is IF-α-irresolute open mapping.*

Proof. Let B be any IFαOS in Y. Since f is an IFpre-α-irresolute function, $f^{-1}(B)$ is IFPOS in X. Since $g \circ f$ is IFpre-α-irresolute open, $(g \circ f)(f^{-1}(B)) = g(B)$ is an IFαOS in (Z, δ). Hence g is an IFα irresolute open mapping. □

Theorem 3.10. *Let $(X, \tau), (Y, \sigma), (Z, \delta)$ be IFTSs. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two mappings. Then the following statements hold:*
(i) *If f is an IFpre-α-irresolute open and g is an IFα-irresolute open mappings, then $g \circ f$:$(X, \tau) \to (Z, \delta)$ is an IFpre-α-irresolute open mapping.*
(ii) *If f is an IF preopen mapping and g is an IFpre-α-irresolute open mapping then $g \circ f$ is an IFα-open mapping.*

Proof. (i) Let A be an IFPOS in X. Since f is IFpre-α-irresolute open, $f(A)$ is an IFαOS in Y. Now $(g \circ f)(A) = g(f(A))$. Since g is IFα-irresolute open, $g(f(A))$ is IFαOS in Z. Hence $g \circ f$ is IFpre-α-irresolute open mapping.
(ii) Let A be an IFOS in X. Since f is IFpreopen, $f(A)$ is an IFPOS in Y. Now $(g \circ f)(A) = g(f(A))$. Since g is IFpre-α-irresolute open, $g(f(A))$ is IFαOS in Z. Hence $g \circ f$ is IFα-open mapping. □

Theorem 3.11. *Let $(X, \tau), (Y, \sigma)$ be IFTSs. A function $f : (X, \tau) \to (Y, \sigma)$ is IFpre-α-irresolute closed mapping if and only if $\alpha clf(A) \subseteq f(pclA)$ for each IFS A in IFTS X.*

Proof. $f(A) \subseteq f(pclA)$. And $f(pclA)$ is IFαCS in Y, since f is IFpre-α-irresolute closed mapping. Therefore $f(pclA) = \alpha clf(pclA)$. Thus $\alpha clf(A) \subseteq \alpha clf(pclA) = f(pclA)$. Hence $\alpha clf(A) \subseteq f(pclA)$.
Conversely, let A be IFPCS in X. Then $\alpha clf(A) \subseteq f(pclA) = f(A)$. Thus $\alpha clf(A) \subseteq f(A)$. But $f(A) \subseteq \alpha clf(A)$. Hence $\alpha clf(A) = f(A)$. $\Rightarrow f(A)$ is IFαCS in Y. Hence f is IFpre-α-irresolute closed mapping. □

Theorem 3.12. *Let $(X, \tau), (Y, \sigma)$ be IFTSs. If A function $f : (X, \tau) \to (Y, \sigma)$ is IFpre-α-irresolute closed mapping if and only if for each IFS B in Y and each IFPOS A in X with $A \supseteq f^{-1}(B)$, there exists an IFαOS C in Y with $C \supseteq B$ such that $f^{-1}(C) \subseteq A$.*

Proof. Let A be any arbitrary IFPOS A in X with $A \supseteq f^{-1}(B)$ where B is an IFS in Y. Then $\overline{A}$ is IFPCSin X. Since f is IFpre-α-irresolute closed mapping $f(\overline{A})$ is IFαCS in Y. Then $\overline{f(\overline{A})} = C(\text{say})$ is IFαOS in Y. Since $f^{-1}(B) \subseteq A$, $B \subseteq C$. Moreover we have $f^{-1}(C) = f^{-1}(\overline{f(\overline{A})} = \overline{f^{-1}(f(\overline{A})} \subseteq A$. Thus $f^{-1}(C) \subseteq A$.
Conversely Let A be IFPCS in X. Then $\overline{f(A)} = B$ (say) is an IFS in Y and $\overline{A}$ is IFPOS in X such that $f^{-}1(B) \subseteq \overline{A}$. By hypothesis, there is an IFαOS C of Y such that $B \subseteq C$ and $f^{-}1C \subseteq \overline{A}$. Therefore $A \subseteq \overline{f^{-}1(C)}$. Hene $\overline{C} \subseteq f(A) \subseteq f(\overline{f^{-}1(C)})$. $\Rightarrow f(A) = \overline{C}$. Since $\overline{C}$ is IFαCS of Y, $f(A)$ is an IFαCS in Y. Hence f is an IFpre-α-irresolute closed mapping. ☐

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.
[3] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88 (1997) 81–89.
[4] H. Gurcay, D. Coker and A. H. Es, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 5 (1997) 365–378.
[5] J. K. Joen, Y. B. Jun and J. H. Park, Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy precontinuity, Int. J. Math. Math. Sci. 19 (2005) 3091–3101.
[6] B. Krsteska and S. E. Abbas, Intuitionistic fuzzy strongly preopen (preclosed) mappings, Math. Morav. 10 (2006) 47–53.
[7] R. Renuka and V. Seenivasan, Intuitionistic fuzzy α-irresolute open and closed mappings, (submitted).
[8] V. Seenivasan and R. Renuka, On Intuitionistic fuzzy semi-α-irresolute functions, Ann. Fuzzy Math. Inform. 6(2) (2013) 315–323.
[9] V. Seenivasan and R. Renuka, Intuitionistic fuzzy pre-α-irresolute functions, Far East J. Math. Sci. 72(2) (2013) 251–267.
[10] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338–353.

R. Renuka (renuka.autpc@gmail.com)
Department of Mathematics, University College of Engineering Panruti (A Constituent College of Anna University Chennai) Panruti-607 106, Tamilnadu, India

V. Seenivasan (seenujsc@yahoo.co.in)
Department of Mathematics, University College of Engineering Panruti (A Constituent College of Anna University Chennai) Panruti-607 106, Tamilnadu, India