

A new approach for ranking of exponential fuzzy number with use optimism

SALIM REZVANI

Received 22 July 2014; Revised 1 August 2014; Accepted 9 September 2014

ABSTRACT. The main aim of this paper is to propose a new approach for the ranking of generalized exponential trapezoidal fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems. Also, with the help of several counter examples it is proved that ranking method proposed by Chen and Chen 2009 is incorrect.

2010 AMS Classification: 06D72, 08A72, 47S40

Keywords: Ranking function, Generalized exponential trapezoidal fuzzy numbers.

Corresponding Author: Salim Rezvani (salim_rezvani@yahoo.com)

1. INTRODUCTION

In most of cases in our life, the data obtained for decision making are only approximately known. In 1965, Zadeh [26] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [8]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. Ranking fuzzy numbers were first proposed by Jain [10] and Liou [13] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [2] reviewed some of these ranking methods for ranking fuzzy subsets and fuzzy numbers. Chen [3] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade [9] presented the mean value of a fuzzy number. Chu and Tsao [6] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [7] presented

a centroid-index method for ranking fuzzy numbers. Liang et al. [12] and Wang and Lee [25] also used the centroid concept in developing their ranking index. Chen and Chen [4] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α -levels of trapezoidal fuzzy numbers. Chen and Chen [5] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Also Some of the interesting Approach Ranking Of Trapezoidal Fuzzy Number can be found in Amit Kumar [11]. Moreover, S, Rezvani ([14]–[23]) proposed a method for ranking in fuzzy numbers.

The main aim of this paper is to propose a new approach for the ranking of generalized exponential trapezoidal fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems. Also, with the help of several counter examples it is proved that ranking method proposed by Chen 2009 is incorrect.

2. PRELIMINARIES

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R , whose membership function μ_A satisfies the following conditions,

- (i) μ_A is a continuous mapping from R to the closed interval $[0,1]$,
- (ii) $\mu_A(x) = 0, -\infty < x \leq c$,
- (iii) $\mu_A(x) = L(x)$ is strictly increasing on $[c, a]$,
- (iv) $\mu_A(x) = w, a \leq x \leq b$,
- (v) $\mu_A(x) = R(x)$ is strictly decreasing on $[b, d]$,
- (vi) $\mu_A(x) = 0, d \leq x < \infty$

Where $0 < w \leq 1$ and a, b, c , and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by $A = (c, a, b, d; w)_{LR}$.

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by $A = (c, a, b, d)_{LR}$.

However, these fuzzy numbers always have a fix range as $[c, d]$. Here, we define its general form as follows:

$$(2.1) \quad f_A(x) = \begin{cases} we^{-[(a-x)/\gamma]} & x \leq a, \\ w & a \leq x \leq b, \\ we^{-[(x-b)/\beta]} & b \leq x, \end{cases}$$

where $0 < w \leq 1$, a, b are real numbers, and γ, β are positive real numbers. we denote this type of generalized exponential fuzzy number as $A = (a, b, \gamma, \beta; w)_E$. Especially, when $w = 1$, we denote it as $A = (a, b, \gamma, \beta)_E$.

we define the representation of generalized exponential fuzzy number based on the integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number $A = (a, b, \gamma, \beta)_E$, where $0 < w \leq 1$, and γ, β are positive real numbers, a, b are real numbers as in formula (2.1). Now, let two monotonic functions be

$$(2.2) \quad L(x) = we^{-(a-x)/\gamma}, \quad R(x) = we^{-(x-b)/\beta}$$

then the inverse functions of function L and R are L^{-1} and R^{-1} respectively. the h-level graded mean value of generalized exponential fuzzy number $A = (a, b, \gamma, \beta; w)_E$ can be express as

$$(2.3) \quad h[L^{-1}(h) + R^{-1}(h)]/2$$

Definition 1. Let $A = (a, b, \gamma, \beta; w)_E$, be generalized exponential number, then the graded mean integration representation of A is define by

$$(2.4) \quad P(A) = \left(\int_0^w h \left(\frac{L^{-1}(h) + R^{-1}(h)}{2} \right) dh \right) / \int_0^w h \, dh.$$

Theorem 1. Let $A = (a, b, \gamma, \beta; w)_E$, be generalized exponential number with $0 < w \leq 1$ and γ, β are positive real numbers, a, b are real numbers. then the graded mean integration representation of A is

$$(2.5) \quad P(A) = \frac{a+b}{2} + \frac{\beta-\gamma}{4}.$$

Proof.

$$(2.6) \quad L^{-1}(h) = a - \gamma \left(\ln \frac{w}{h} \right),$$

$$(2.7) \quad R^{-1}(h) = b + \beta \left(\ln \frac{w}{h} \right).$$

$$\begin{aligned} P(A) &= \frac{1}{2} \int_0^w h \left[a + b + \beta \left(\ln \frac{w}{h} \right) - \gamma \left(\ln \frac{w}{h} \right) \right] dh / \frac{1}{2} w^2 \\ &= \frac{a+b}{2} + \frac{\beta-\gamma}{2} \int_0^w h \left(\ln \frac{w}{h} \right) \\ &= \frac{a+b}{2} + \frac{\beta-\gamma}{2} \left[\int_0^w h \ln(w) \, dh - \int_0^w h \ln(h) \, dh \right] \\ &= \frac{a+b}{2} + \frac{\beta-\gamma}{2} \int_0^w h [\ln(w) - \ln(h)] \, dh \\ &= \frac{a+b}{2} + \frac{\beta-\gamma}{4}. \end{aligned}$$

□

Remark 1. When $\gamma = \beta$, $P(A) = \frac{a+b}{2}$.

3. ARITHMETIC OPERATIONS OF EXPONENTIAL FUZZY NUMBERS AND RANKING FUNCTION

Definition 2. suppose that $A_1 = (a_1, b_1, \gamma_1, \beta_1; w_1)_E$ and $A_2 = (a_2, b_2, \gamma_2, \beta_2; w_2)_E$ are two generalized exponential fuzzy numbers. Let $w = \min\{w_1, w_2\}$, according to the essential of the second function principle, some arithmetical operations results could be well define as follows.

(i) The addition of A_1 and A_2 is

$$(3.1) \quad A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, \gamma_1 + \gamma_2, \beta_1 + \beta_2; w)_E$$

where $\gamma_1, \gamma_2, a_1, a_2, b_1, b_2, \beta_1, \beta_2$ are all real numbers, and $\gamma_1, \gamma_2, \beta_1, \beta_2$ are positive.

(ii) The multiplication of A_1 and A_2 is

$$(3.2) \quad A_1 \otimes A_2 = (a, b, \gamma, \beta; w)_E.$$

Where $T = \{a_1 a_2, a_1 b_2, b_1 a_2, b_1 b_2\}$, $T_1 = \{\gamma_1 \gamma_2, \gamma_1 \beta_2, \beta_1 \gamma_2, \beta_1 \beta_2\}$

and $a = \min T = k^{th}$ element of T , and $b = \max T = l^{th}$ element of T , then $\gamma = k^{th}$ element of T_1 and $\beta = l^{th}$ element of T_1 , where $1 \leq k \leq 4, 1 \leq l \leq 4$.

(iii) $-A_2 = (-b_2, -a_2, \beta_2, \gamma_2; w)$, then

$$(3.3) \quad A_1 \ominus A_2 = A_1 \oplus (-A_2) = (a_1 - b_2, b_1 - a_2, \gamma_1 + \beta_2, \beta_1 + \gamma_2; w)_E.$$

(iv) Let $m \in R^+$, $A = (a, b, \gamma, \beta; w)_E$, then

$$(3.4) \quad m \otimes A = (ma, mb, m\gamma, m\beta; w)_E$$

if $m \in R^-$, $A = (a, b, \gamma, \beta; w)_E$, then

$$(3.5) \quad m \otimes A = (mb, ma, |m| \beta, |m| \gamma; w)_E$$

(v) $\frac{1}{A_2} = (\frac{1}{b_2}, \frac{1}{a_2}, \frac{1}{\beta_2}, \frac{1}{\gamma_2}; w)_E$, We have

$$(3.6) \quad \frac{A_1}{A_2} = A_1 \otimes (\frac{1}{A_2}) = (\frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{\gamma_1}{\beta_2}, \frac{\beta_1}{\gamma_2}; w)_E,$$

where if $a_1 b_1, a_2 b_2, \gamma_1, \gamma_2, \beta_1, \beta_2$ are all nonzero positive real numbers.

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function, $\Re: F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists,

(1) $A > B$ iff $\Re(A) > \Re(B)$.

(2) $A < B$ iff $\Re(A) < \Re(B)$.

(3) $A = B$ iff $\Re(A) = \Re(B)$.

Remark 2. ([14]) For all fuzzy numbers A, B, C and D , we have

- (1) $A > B \Rightarrow A \oplus C > B \oplus C$.
- (2) $A > B \Rightarrow A \ominus C > B \ominus C$.
- (3) $A \sim B \Rightarrow A \oplus C \sim B \oplus C$.
- (4) $A > B, C > D \Rightarrow A \oplus C > B \oplus D$.

4. SHORTCOMINGS OF CHEN AND CHEN APPROACH [5]

In this section, on the basis of reasonable properties of fuzzy quantities [22] and on the basis of height of fuzzy numbers, are pointed out. Let A and B be any two fuzzy numbers. Then

$A > B \Rightarrow A \ominus B > B \ominus B$ (Using Remark 2.),

That is

$$\Re(A) > \Re(B) \Rightarrow \Re(A \ominus B) > \Re(B \ominus B).$$

In this subsection, several examples are chosen to prove that the ranking function proposed by Chen and Chen does not satisfy the reasonable property,

$$(4.1) \quad A > B \rightarrow A \ominus B > B \ominus B$$

for the ordering of fuzzy quantities i.e., according to Chen and Chen approach

$A > B$, not result $A \ominus B > B \ominus B$

Example 1. Let $A = (0.1, 0.3, 0.3, 0.5; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$, but

$$B \ominus A < A \ominus A \text{ that is, } B > A \text{ not result } B \ominus A > A \ominus A.$$

Example 2. Let $A = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$, but

$$B \ominus A < A \ominus A \text{ that is, } B > A \text{ not result } B \ominus A > A \ominus A.$$

Example 3. Let

$$A = (-0.8, -0.6, -0.4, -0.2; 0.35) \text{ and } B = (-0.4, -0.3, -0.2, -0.1; 0.7)$$

be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $A > B$, but

$$A \ominus B < B \ominus B \text{ that is, } A > B \text{ not result } A \ominus B > B \ominus B.$$

Example 4. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two

generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$, but

$B \ominus A < A \ominus A$ that is, $B > A$ not result $B \ominus A > A \ominus A$.

In some cases, Chen and Chen approach [8] states that the ranking of fuzzy numbers depends upon height of fuzzy numbers while in several cases the ranking does not depend upon the height of fuzzy numbers.

Let $A_1 = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $A_2 = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen [5] there may be two cases.

Case(1) If $(a_1 + b_1 + \gamma_1 + \beta_1) \neq 0$, then

$$(4.2) \quad \begin{cases} A < B & \text{if } w_1 < w_2 \\ A > B & \text{if } w_1 > w_2 \\ A \sim B & \text{if } w_1 \sim w_2. \end{cases}$$

Case(2) If $(a_1 + b_1 + \gamma_1 + \beta_1) = 0$, then $A \sim B$ for all values of w_1 and w_2 .

According to Chen and Chen [5] in first case ranking of fuzzy numbers depends upon height and in second case ranking does not depend upon the height which is contradiction.

Example 5. Let $A = (1, 1, 1, 1; w_1)$ and $B = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $A < B$ if $w_1 < w_2$, $A > B$ if $w_1 > w_2$ and $A = B$ if $w_1 = w_2$.

Example 6. Let $A = (-0.4, -0.2, -0.1, 0.7; w_1)$ and $B = (-0.4, -0.2, -0.1, 0.7; w_2)$ be two generalized triangular fuzzy numbers. Then $A = B$ for all values of w_1 and w_2 .

5. PROPOSED APPROACH

In this section, on the basis of property of ranking function, discussed in Section 3, a new approach is proposed for the ranking of generalized exponential trapezoidal fuzzy numbers.

Let $A_1 = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $A_2 = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two generalized trapezoidal fuzzy numbers. Then

(1) $A > B$ if $RM(A \ominus B) > RM(B \ominus B)$,

(2) $A < B$ if $RM(A \ominus B) < RM(B \ominus B)$,

(3) $A = B$ if $RM(A \ominus B) = RM(B \ominus B)$.

5.1. Method to find value of $\Re(A)$ and $\Re(B)$. Let $A = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $B = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two any generalized exponential trapezoidal fuzzy number. Then with use [11], find the values of

* step 1. Find $w = \min\{w_1, w_2\}$

* step 2. Let $\mu_A(x)$ and $\mu_B(x)$ are the membership function of A and B respectively, Then

$$(5.1) \quad \mu_A(x) = \begin{cases} w_1 L_1 e^{-[(a_1-x)/\gamma_1]} & x \leq a_1, \\ w_1 & a_1 \leq x \leq b_1, \\ w_1 R_1 e^{-[(x-b_1)/\beta_1]} & b_1 \leq x, \end{cases}$$

and

$$(5.2) \quad \mu_B(x) = \begin{cases} w_2 L_2 e^{-[(a_2-x)/\gamma_2]} & x \leq a_2, \\ w_2 & a_2 \leq x \leq b_2, \\ w_2 R_2 e^{-[(x-b_2)/\beta_2]} & b_2 \leq x, \end{cases}$$

Put $w = \min\{w_1, w_2\}$, Then

$$(5.3) \quad \mu_A(x) = \begin{cases} w L_1 e^{-[(a_1-x)/\gamma_1]} & x \leq a_1, \\ w & a_1 \leq x \leq b_1, \\ w R_1 e^{-[(x-b_1)/\beta_1]} & b_1 \leq x, \end{cases}$$

and

$$(5.4) \quad \mu_B(x) = \begin{cases} w L_2 e^{-[(a_2-x)/\gamma_2]} & x \leq a_2, \\ w & a_2 \leq x \leq b_2, \\ w R_2 e^{-[(x-b_2)/\beta_2]} & b_2 \leq x, \end{cases}$$

With use equation (6), (7), we have

$$L_1(h) = w L_1 e^{-[(a_1-x)/\gamma_1]} \Rightarrow L_1^{-1}(h) = a_1 - \gamma_1 \left(\ln \frac{w}{h} \right),$$

$$R_1(h) = w R_1 e^{-[(x-b_1)/\beta_1]} \Rightarrow R_1^{-1}(h) = b_1 + \beta_1 \left(\ln \frac{w}{h} \right).$$

and

$$L_2(h) = w L_2 e^{-[(a_2-x)/\gamma_2]} \Rightarrow L_2^{-1}(h) = a_2 - \gamma_2 \left(\ln \frac{w}{h} \right),$$

$$R_2(h) = w R_2 e^{-[(x-b_2)/\beta_2]} \Rightarrow R_2^{-1}(h) = b_2 + \beta_2 \left(\ln \frac{w}{h} \right).$$

So

$$\begin{aligned} \Re(A) &= \alpha \int_0^w [a_1 - \gamma_1 \left(\ln \frac{w}{h} \right)] dh + (1 - \alpha) \int_0^w [b_1 + \beta_1 \left(\ln \frac{w}{h} \right)] dh \\ &= \alpha [a_1 w - \int_0^w \gamma_1 (\ln w - \ln h) dh] + (1 - \alpha) [b_1 w - \int_0^w \beta_1 (\ln w - \ln h) dh] \end{aligned}$$

$$(5.5) \quad = \alpha [a_1 w - \gamma_1 w] + (1 - \alpha) [b_1 w + \beta_1 w] = \alpha w [a_1 - \gamma_1] + (1 - \alpha) w [b_1 + \beta_1].$$

and

$$\begin{aligned} \Re(B) &= \alpha \int_0^w [a_2 - \gamma_2 \left(\ln \frac{w}{h} \right)] dh + (1 - \alpha) \int_0^w [b_2 + \beta_2 \left(\ln \frac{w}{h} \right)] dh \\ &= \alpha [a_2 w - \int_0^w \gamma_2 (\ln w - \ln h) dh] + (1 - \alpha) [b_2 w - \int_0^w \beta_2 (\ln w - \ln h) dh] \end{aligned}$$

$$(5.6) \quad = \alpha[a_2w - \gamma_2w] + (1 - \alpha)[b_2w + \beta_2w] = \alpha w[a_2 - \gamma_2] + (1 - \alpha)w[b_2 + \beta_2].$$

Therefore

$$\Re(A) = \alpha w[a_1 - \gamma_1] + (1 - \alpha)w[b_1 + \beta_1],$$

$$\Re(B) = \alpha w[a_2 - \gamma_2] + (1 - \alpha)w[b_2 + \beta_2].$$

Indeed, The index of optimism (α) is representing the degree of optimism of a decision maker [15]. A larger value of α indicates a higher degree of optimism. For $\alpha = 0$ and $\alpha = 1$ value of $\Re(A)$ and $\Re(B)$ represents the view points of a pessimistic and optimistic decision maker respectively while for $\alpha = 0.5$ values of $\Re(A)$ and $\Re(B)$ represents the view points of a moderate decision maker.

Remark 3. We get for generalized exponential trapezoidal fuzzy numbers

$$\Re(A) = \alpha w[a_1 - \gamma_1] + (1 - \alpha)w[b_1 + \beta_1],$$

$$\Re(B) = \alpha w[a_2 - \gamma_2] + (1 - \alpha)w[b_2 + \beta_2].$$

and normalize fuzzy numbers

$$\Re(A) = \alpha[a_1 - \gamma_1] + (1 - \alpha)[b_1 + \beta_1],$$

$$\Re(B) = \alpha[a_2 - \gamma_2] + (1 - \alpha)[b_2 + \beta_2].$$

Remark 4. The arithmetic operations between two fuzzy numbers is obtained using the α -cut method [5] and the maximum value of h , that will be common for both fuzzy numbers, will be obtained by finding the minimum value of the height of the fuzzy numbers, due to which in $w = \min\{w_1, w_2\}$ is considered.

Theorem 2. Prove that ranking of generalized exponential fuzzy numbers does not depend upon the height of fuzzy numbers i.e., if A, B are two generalized exponential fuzzy numbers and C, D are normalize fuzzy numbers, obtained from A, B respectively, then

$$(i) \quad A > B \text{ iff } C > D,$$

$$(ii) \quad A < B \text{ iff } C < D,$$

$$(iii) \quad A \sim B \text{ iff } C \sim D,$$

Proof. Let $A = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $B = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two any generalized exponential trapezoidal fuzzy number and let $C = (a_1, b_1, \gamma_1, \beta_1; 1)$ and $D = (a_2, b_2, \gamma_2, \beta_2; 1)$ are normal exponential trapezoidal fuzzy number of A, B respectively and $w = \min\{w_1, w_2\}$. Then

(i) Let $A > B \Leftrightarrow \Re(A) > \Re(B) \Leftrightarrow \alpha w[a_1 - \gamma_1] + (1 - \alpha)w[b_1 + \beta_1] > \alpha w[a_2 - \gamma_2] + (1 - \alpha)w[b_2 + \beta_2] \Leftrightarrow \alpha[a_1 - \gamma_1] + (1 - \alpha)[b_1 + \beta_1] > \alpha[a_2 - \gamma_2] + (1 - \alpha)[b_2 + \beta_2] \Leftrightarrow \Re(C) > \Re(D) \Leftrightarrow C > D.$

(ii) Let $A < B \Leftrightarrow \Re(A) < \Re(B) \Leftrightarrow \alpha w[a_1 - \gamma_1] + (1 - \alpha)w[b_1 + \beta_1] < \alpha w[a_2 - \gamma_2] +$

$$(1 - \alpha)w[b_2 + \beta_2] \Leftrightarrow \alpha[a_1 - \gamma_1] + (1 - \alpha)[b_1 + \beta_1] < \alpha[a_2 - \gamma_2] + (1 - \alpha)[b_2 + \beta_2] \Leftrightarrow \mathfrak{R}(C) < \mathfrak{R}(D) \Leftrightarrow C < D.$$

$$\text{(iii) Let } A \sim B \Leftrightarrow \mathfrak{R}(A) \sim \mathfrak{R}(B) \Leftrightarrow \alpha w[a_1 - \gamma_1] + (1 - \alpha)w[b_1 + \beta_1] \sim \alpha w[a_2 - \gamma_2] + (1 - \alpha)w[b_2 + \beta_2] \Leftrightarrow \alpha[a_1 - \gamma_1] + (1 - \alpha)[b_1 + \beta_1] \sim \alpha[a_2 - \gamma_2] + (1 - \alpha)[b_2 + \beta_2] \Leftrightarrow \mathfrak{R}(C) \sim \mathfrak{R}(D) \Leftrightarrow C \sim D. \quad \square$$

6. EXAMPLES AND RESULTS

In this section, the correct ordering of fuzzy numbers, are obtained. Also, in the Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [23].

Example 7. Let $A = (0.1, 0.3, 0.3, 0.5; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers. Since

$$\text{* step 1. } w = \min\{w_1, w_2\} = \min\{1, 1\} = 1$$

* step 2.

$$\mathfrak{R}(A) = \alpha[0.1 - 0.3] + (1 - \alpha)[0.3 + 0.5] = 0.8 - \alpha ,$$

and

$$\mathfrak{R}(B) = \alpha[0.2 - 0.3] + (1 - \alpha)[0.3 + 0.4] = 0.7 - 0.8\alpha .$$

For a pessimistic decision maker, with $\alpha = 0$,

$$\mathfrak{R}(A) = 0.8 ,$$

and

$$\mathfrak{R}(B) = 0.7 .$$

Then $\mathfrak{R}(A) > \mathfrak{R}(B) \Rightarrow A > B$.

For a pessimistic decision maker, with $\alpha = 1$,

$$\mathfrak{R}(A) = -0.2 ,$$

and

$$\mathfrak{R}(B) = -0.1 .$$

Then $\mathfrak{R}(A) < \mathfrak{R}(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 0.5$,

$$\mathfrak{R}(A) = 0.3 ,$$

and

$$\mathfrak{R}(B) = 0.3 .$$

Then $\mathfrak{R}(A) = \mathfrak{R}(B) \Rightarrow A \sim B$.

Example 8. Let $A = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two

generalized trapezoidal fuzzy numbers. Since

$$* \text{ step 1. } w = \min\{w_1, w_2\} = \min\{0.8, 1\} = 0.8$$

* step 2.

$$\Re(A) = 0.8\alpha[0.1 - 0.3] + 0.8(1 - \alpha)[0.3 + 0.5] = 0.64 - 0.8\alpha ,$$

and

$$\Re(B) = 0.8\alpha[0.1 - 0.3] + 0.8(1 - \alpha)[0.3 + 0.5] = 0.64 - 0.8\alpha .$$

Then $\Re(A) = \Re(B) \Rightarrow \forall \alpha \ A \sim B$.

Example 9. Let

$A = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $B = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized trapezoidal fuzzy numbers. Since

$$* \text{ step 1. } w = \min\{w_1, w_2\} = \min\{0.35, 0.7\} = 0.35$$

* step 2.

$$\Re(A) = 0.35\alpha[-0.8 + 0.4] + 0.35(1 - \alpha)[-0.6 - 0.2] = 0.14\alpha - 0.28 ,$$

and

$$\Re(B) = 0.35\alpha[-0.4 + 0.2] + 0.35(1 - \alpha)[-0.3 - 0.1] = 0.07\alpha - 0.14 .$$

For a pessimistic decision maker, with $\alpha = 0$,

$$\Re(A) = -0.28 ,$$

and

$$\Re(B) = -0.14 .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 1$,

$$\Re(A) = -0.14 ,$$

and

$$\Re(B) = -0.07 .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 0.5$,

$$\Re(A) = -0.21 ,$$

and

$$\Re(B) = -0.105 .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

Example 10. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be

two generalized trapezoidal fuzzy numbers. Since

$$* \text{ step 1. } w = \min\{w_1, w_2\} = \min\{0.35, 0.7\} = 0.35$$

* step 2.

$$\Re(A) = 0.35\alpha[0.2 - 0.6] + 0.35(1 - \alpha)[0.4 + 0.8] = 0.42 - 0.56\alpha ,$$

and

$$\Re(B) = 0.35\alpha[0.1 - 0.3] + 0.35(1 - \alpha)[0.2 + 0.4] = 0.21 - 0.28\alpha .$$

For a pessimistic decision maker, with $\alpha = 0$,

$$\Re(A) = 0.42 ,$$

and

$$\Re(B) = 0.21 .$$

Then $\Re(A) > \Re(B) \Rightarrow A > B$.

For a pessimistic decision maker, with $\alpha = 1$,

$$\Re(A) = -0.14 ,$$

and

$$\Re(B) = -0.07 .$$

Then $\Re(A) < \Re(B) \Rightarrow A < B$.

For a pessimistic decision maker, with $\alpha = 0.5$,

$$\Re(A) = 0.14 ,$$

and

$$\Re(B) = 0.07 .$$

Then $\Re(A) > \Re(B) \Rightarrow A > B$.

Example 11. Let $A = (1, 1, 1, 1; w_1)$ and $B = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers. Since

$$* \text{ step 1. } w = \min\{w_1, w_2\} = w$$

* step 2.

$$\Re(A) = \alpha w[1 - 1] + (1 - \alpha)w[1 + 1] = 2w(1 - \alpha) ,$$

and

$$\Re(B) = \alpha w[1 - 1] + (1 - \alpha)w[1 + 1] = 2w(1 - \alpha) .$$

Then $\Re(A) = \Re(B) \Rightarrow \forall \alpha A \sim B$.

Example 12. Let $A = (-0.4, -0.2, -0.1, 0.7; w_1)$ and $B = (-0.4, -0.2, -0.1, 0.7; w_2)$ be two generalized trapezoidal fuzzy numbers. Since

* step 1. $w = \min\{w_1, w_2\} = w$

* step 2.

$$\mathfrak{R}(A) = \alpha w[-0.4 + 0.1] + (1 - \alpha)w[-0.2 + 0.7] = 0.5 - 0.8\alpha w ,$$

and

$$\mathfrak{R}(B) = \alpha w[-0.4 + 0.1] + (1 - \alpha)w[-0.2 + 0.7] = 0.5 - 0.8\alpha w .$$

Then $\mathfrak{R}(A) = \mathfrak{R}(B) \Rightarrow \forall \alpha \ A \sim B$.

6.1. Testimony of proposed ranking function. Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [24]

Table (1): Fulfilment of the axioms for the ordering in the first and second class [24]

Index	A_1	A_2	A_3	A_4	A'_4	A_5	A_6	A'_6	A_7
Y_1	Y	Y	Y	Y	Y	Y	N	N	N
Y_2	Y	Y	Y	Y	Y	Y	N	N	N
Y_3	Y	Y	Y	N	N	Y	N	N	N
Y_4	Y	Y	Y	Y	Y	Y	N	N	N
C	Y	Y	Y	N	N	Y	N	N	N
FR	Y	Y	Y	Y	Y	Y	Y	Y	N
CL	Y	Y	Y	Y	Y	Y	Y	Y	N
LW^λ	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_1^λ	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_2^λ	Y	Y	Y	Y	Y	Y	Y	Y	N
K	Y	Y	Y	N	N	N	N	N	N
W	Y	Y	Y	Y	N	N	N	N	N
J^k	Y	Y	Y	Y	Y	N	N	N	N
CH^k	Y	Y	Y	Y	Y	N	N	N	N
KP^k	Y	Y	Y	Y	Y	N	N	N	N
α	Y	Y	Y	Y	Y	Y	Y	Y	N
Proposed Approach	Y	Y	Y	Y	Y	Y	Y	Y	Y

REFERENCES

- [1] S. Abbasbandy and T. Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, Comput. Math. Appl. 57 (2009) 413–419.
- [2] G. Bortolan and R. Degani, A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems 15(1) (1985) 1–19.
- [3] S.-H. Chen, Ranking fuzzy numbers withmaximizing set and minimizing set, Fuzzy Sets and Systems 17(2) (1985) 113–129.
- [4] S. J. Chen and S. M. Chen, Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers, Applied Intelligence 26 (2007) 1–11.
- [5] S. M. Chen and J. H. Chen, Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads, Expert Systems with Applications 36(3) (2009) 6833–6842.
- [6] T. C. Chu and C. T. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, Comput. Math. Appl. 43 (2002) 111–117.

- [7] Y. Deng and Q. Liu, A TOPSIS-based centroid-index ranking method of fuzzy numbers and its applications in decision making, *Cybernetics and Systems* 36 (2005) 581–595.
- [8] D. Dubois and H. Prade, Operations on fuzzy numbers, *Internat. J. Systems Sci.* 9(6) (1978) 613–626.
- [9] D. Dubois and H. Prade, Ranking fuzzy numbers in the setting of possibility theory, *Inform. Sci.* 30(3) (1983) 183–224.
- [10] R. Jain, Decision making in the presence of fuzzy variables, *IEEE Transactions on Systems, Man and Cybernetics* 6(10) (1976) 698–703.
- [11] Amit Kumar, Pushpinder Singh and Parmpreet Kaur, A new approach for ranking of L-R type generalized fuzzy numbers, *Tamsui Oxford Journal of Information and Mathematical Sciences* 27(2) (2011) 197–211.
- [12] C. Liang, J. Wu and J. Zhang, Ranking indices and rules for fuzzy numbers based on gravity center point, Paper presented at the 6th world Congress on Intelligent Control and Automation, Dalian, China, (2006) 21–23.
- [13] T. S. Liou and M. J. Wang, Ranking fuzzy numbers with integral value, *Fuzzy Sets and Systems* 50(3) (1992) 247–255.
- [14] S. Rezvani, Graded mean representation method with triangular fuzzy number, *World Applied Sciences Journal* 11(7) (2010) 871–876.
- [15] S. Rezvani, Three-tier FMCDM Problems with trapezoidal fuzzy number, *World Applied Sciences Journal* 10(9) (2010) 1106–1113.
- [16] S. Rezvani, Multiplication operation on trapezoidal fuzzy numbers, *Journal of Physical Sciences*, Vol. 15 (2011), 17–26.
- [17] S. Rezvani, A new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers, *International Journal of Applied Operational Research* 2(3) (2012) 83–90.
- [18] S. Rezvani, A new approach ranking of exponential trapezoidal fuzzy numbers, *Journal of Physical Sciences* 16 (2012) 45–57.
- [19] S. Rezvani, A new method for ranking in areas of two generalized trapezoidal fuzzy numbers, *International Journal of Fuzzy Logic Systems* 3(1) (2013) 17–24.
- [20] S. Rezvani, Ranking generalized trapezoidal fuzzy numbers with Euclidean distance by the incentre of centroids, *Mathematica Aeterna*, (2013), Vol. 3, no. 2, 103 - 114.
- [21] S. Rezvani, Ranking method of trapezoidal intuitionistic fuzzy numbers, *Ann. Fuzzy Math. Inform.* 5(3) (2013) 515–523.
- [22] S. Rezvani, Representation of trapezoidal fuzzy numbers with shape function, *Ann. Fuzzy Math. Inform.* 8(1) (2014) 89–112.
- [23] S. Rezvani, A new method for ranking fuzzy numbers with using TRD distance based on mean and standard deviation, *International Journal of Mechatronics, Electrical and Computer Technology* 4(12) (2014) 840–856.
- [24] X. Wang and E. E. Kerre, Reasonable properties for the ordering of fuzzy quantities (I), *Fuzzy Sets and Systems* 118 (2001) 375–385.
- [25] Y. J. Wang and H. S. Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points, *Comput. Math. Appl.* 55 (2008) 2033–2042.
- [26] L. A. Zadeh, Fuzzy set, *Information and Control* 8(3) (1965) 338–353.

SALIM REZVANI (salim_rezvani@yahoo.com)

Department of Mathematics Marlik Higher Education Institute of Nowshahr, Nowshahr, Iran