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An application of interval valued intuitionistic fuzzy soft matrix theory in medical diagnosis

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Abstract. Soft set theory is a new emerging mathematical tool to deal with uncertainties.In this paper, we introduce the concept of reduced intuitionistic fuzzy soft matrix and define different types of reduction along with it. Then based on these matrices, we develope an algorithm which is a new approach in the medical diagnosis field by employing interval valued intuitionistic fuzzy soft matrices.

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1. Introduction

In many complicated problems arising in the fields of engineering, social science,economics,medical science etc involving uncertainties, classical methods are found to be inadequate in recent times. In real life scenario, we face so many uncertainties, in all walks of life. Zadeh's classical concept of fuzzy set ([19]) is strong to deal with such type of problems. Since the initiation of fuzzy set theory, there are suggestions for higher order fuzzy sets for different applications in many fields. Among higher fuzzy sets intuitionistic fuzzy set introduced by Atanassov ([1],[2]) have been found to be very useful and applicable. Atanassov and Gargov ([3]) extended the intuitionistic fuzzy set ([1],[2]) to the interval valued intuitionistic fuzzy set which is characterized by a membership function and a non-membership function whose values are intervals rather than real numbers.

Soft set theory has received much attention since its introduction by Molodtsov ([13]). The concept and basic properties of soft set theory are presented in ([13],[9]). Later on Maji et al. ([10]) proposed the theory of fuzzy soft set. Majumdar et al. ([12]) further generalised the concept of fuzzy soft sets. Maji et al. ([11]) extended fuzzy soft sets to intuitionistic fuzzy soft sets which is based on the combination of the intuitionistic fuzzy set ([1]) and soft set. Yang et al. ([17]) presented the concept of the interval valued fuzzy soft sets by combining the interval valued fuzzy sets and soft set.

Due to the increasing complexity of the scientific environment and the lack of data about the problem domain, in the process of decision making under an intuitionistic fuzzy environment, a decision maker may provide their preferences over alternatives with interval valued intuitionistic fuzzy values. Jiang et al. ([8]) presented the concept of interval valued intuitionistic fuzzy soft set theory which is an extension of the intuitionistic fuzzy soft set theory ([11]).

Matrices play an important role in the broad area of science and engineering. However,the classical matrix theory cannot solve the problems involving various types of uncertainties. In ([18]), Yong et al. initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. Borah et al. ([6]) extended fuzzy soft matrix theory and its application. Intuitionistic fuzzy soft matrix theory and its applications are studied in ([4],[5],[7],[14]). In ([16]) Rajarajeswari et al. introduced interval valued intuitionistic fuzzy soft matrix and its types and also studied some operations on the basis of weight.

In this paper, we propose a new algorithm which can be applied to the domain of medical diagnosis by employing interval valued intuitionistic fuzzy soft matrices. We have implemented the idea to convert interval-valued intuitionistic fuzzy membership degree and interval-valued intuitionistic fuzzy non-membership degree into one intuitionistic fuzzy value. For this we define Reduced Intuitionistic Fuzzy Soft Matrix and its type.

2. Preliminaries

In this section,we recall some basic notion of fuzzy soft set theory and fuzzy soft matrices.

Definition 2.1 ([13]). Suppose that U is an initial Universe set and E is a set of parameters, let $P(U)$ denotes the power set of U. A pair(F, E) is called a soft set over U where F is a mapping given by $F : E \to P(U)$. Clearly, a soft set is a mapping from parameters to $P(U)$ and it is not a set, but a parameterized family of subsets of the Universe.

Definition 2.2 ([10]). Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by $F : A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3 ([18]). Let $U = \{c_1, c_2, c_3, \ldots, c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3 \ldots, e_n\}$. Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E). Then fuzzy soft set (F, A) in a matrix form as $A_{m\times n} = [a_{ij}]_{m\times n}$ or $A = [a_{ij}] \quad i = 1, 2, \ldots, m, j = 1, 2, 3, \ldots, n$

where $a_{ij} = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases}$

$\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$.

Definition 2.4 ([11]). Let U be the Universal set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called intuitionistic fuzzy soft set over U where F is a mapping given by $F : A \to I^U$,where I^U denotes the collection of all intuitionistic fuzzy subsets of U.

Definition 2.5 ([14]). Let $U = \{c_1, c_2, c_3, \ldots, c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3 \ldots, e_n\}$. Let $A \subseteq E$ and (F, A) be an intuitionistic fuzzy soft set in the fuzzy soft class (U, E). Then intuitionistic fuzzy soft set (F, A) in a matrix form as $A_{m\times n} = [a_{ij}]_{m\times n}$
or $A = [a_{ij}]$ $i = 1, 2, \ldots m$, $j = 1, 2, 3, \ldots, n$ where $a_{ij} = \begin{cases} (\mu_j(c_i), \nu_j(c_i)) & \text{if } e_j \in A \\ (0, 1) & \text{if } e_j \notin A \end{cases}$
$\mu_j(c_i)$ and $\nu_j(c_i)$ represents the membership and non-membership of c_i in the intuitionistic fuzzy set $F(e_j)$.

Definition 2.6 ([17]). Let U be the Universal set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called interval valued intuitionistic fuzzy soft set over U where F is a mapping given by $F : A \to I^U$,where I^U denotes the collection of all interval valued intuitionistic fuzzy subsets of U.

Definition 2.7 ([16]). Let $U = \{c_1, c_2, c_3, \ldots, c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3 \ldots, e_n\}$. Let $A \subseteq E$ and (F, A) be an interval valued intuitionistic fuzzy soft set in the fuzzy soft class $(U, E)$where F is a mapping given by $F : A \to I^U$, where I^U denotes the collection of all interval valued intuitionistic fuzzy subsets of U. Then the interval valued intuitionistic fuzzy soft set (F, A) in a matrix form as $\hat{A}_{m\times n} = [a_{ij}]_{m\times n}$ or
$\hat{A} = [a_{ij}]$ $i = 1, 2, \ldots m$, $j = 1, 2, 3, \ldots, n$
where $a_{ij} = \begin{cases} ([\mu_{jL}(c_i), \mu_{jU}(c_i)][\nu_{jL}(c_i), \nu_{jU}(c_i)]) & \text{if } e_j \in A \\ ([0, 0][1, 1]) & \text{if } e_j \notin A \end{cases}$
$[\mu_{jL}(c_i), \mu_{jU}(c_i)]$ and $[\nu_{jL}(c_i), \nu_{jU}(c_i)]$ represents the membership and non-membership of c_i in the interval valued intuitionistic fuzzy set $F(e_j)$.
Note:If $\mu_{jL}(c_i) = \mu_{jU}(c_i)$and $\nu_{jL}(c_i) = \nu_{jU}(c_i)$ then the IVIFSM reduces to an IFSM([14])

Example 2.8. Suppose that there are four houses under consideration, namely the universe $U = \{h_1, h_2, h_3, h_4\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4\}$ where e_i stands for beautiful, large, cheap, and in green surroundings respectively. Consider the mapping F from parameter set $A = \{e_1, e_2\} \subseteq E$ to the set of all interval valued intuitionistic fuzzy subsets of power set U. Consider an interval valued intuitionistic fuzzy soft set (F,A) which describes the attractiveness of houses that is considering for purchase. Consider interval valued intuitionistic fuzzy soft set (F,A) as

$$\begin{aligned}(F, A) = \{F(e_1) &= \{(h_1, [0.6, 0.8][0.1, 0.2]), (h_2, [0.8, 0.9][0.05, 0.1]),\\ &\quad (h_3, [0.6, 0.7][0.2, 0.25]), (h_4, [0.5, 0.6][0.1, 0.2])\},\\ F(e_2) &= \{(h_1, [0.7, 0.8][0.15, 0.2]), (h_2, [0.6, 0.7][0.2, 0.25]),\\ &\quad (h_3, [0.5, 0.7][0.2, 0.25]), (h_4, [0.8, 0.9][0.1, 0.1])\}\}.\end{aligned}$$

We would represent this interval valued intuitionistic fuzzy soft set in matrix form as

$$\begin{pmatrix} (([0.6,0.8][0.1,0.2]) & ([0.7,0.8][0.15,0.2]) & ([0.0,0.0][1.0,1.0]) & ([0.0,0.0][1.0,1.0]) \\ ([0.8,0.9][0.05,0.1]) & ([0.6,0.7][0.2,0.25]) & ([0.0,0.0][1.0,1.0]) & ([0.0,0.0][1.0,1.0]) \\ ([0.6,0.7][0.2,0.25]) & ([0.5,0.7][0.2,0.25]) & ([0.0,0.0][1.0,1.0]) & ([0.0,0.0][1.0,1.0]) \\ ([0.5,0.6][0.1,0.2]) & ([0.8,0.9][0.1,0.1]) & ([0.0,0.0][1.0,1.0]) & ([0.0,0.0][1.0,1.0]) \end{pmatrix}$$

3. Major section

Reduced intuitionistic fuzzy soft matrix (RIFSM). Based on weighted vector,we convert interval-valued intuitionistic fuzzy membership degree and interval-valued intuitionistic fuzzy non-membership degree into one intuitionistic fuzzy value. Value of weighted vector decide the possible reduction. As a result of, an IVIFSM will be transformed to an IFSM which will facilitate the making decision based on IVIFSM,we define the notion of reduced IFSM as follows:

Definition 3.1. Let $U = \{c_1, c_2, c_3, \ldots, c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3 \ldots, e_n\}$. Let $A \subseteq E$ and (F, A) be an interval valued intuitionistic fuzzy soft set in the fuzzy soft class $(U, E)$where F is a mapping given by $F : A \to I^U$,where I^U denotes the collection of all interval valued intuitionistic fuzzy subsets of U. Then the interval valued intuitionistic fuzzy soft set is
$F(e_j) = \{c_i, ([\mu_{jL}(c_i), \mu_{jU}(c_i)][\nu_{jL}(c_i), \nu_{jU}(c_i)]) \, , \forall e_j \in A \text{ and } \forall c_j \in U\}$
$[\mu_{jL}(c_i), \mu_{jU}(c_i)]$ and $[\nu_{jL}(c_i), \nu_{jU}(c_i)]$ represents the membership and non-membership of c_i in the interval valued intuitionistic fuzzy set $F(e_j)$.
Let w_1, w_2, w_3, w_4 in [0,1],$w_1 + w_2 = 1, w_3 + w_4 = 1$.
The Vector $W = (w_1, w_2, w_3, w_4)$ is called weighted vector. The intuitionistic fuzzy soft set (F_W, A) over U such that
$(F_w(e_j)) = \{c_i, w_1\mu_{jL}(c_i) + w_2\mu_{jU}(c_i), w_3\nu_{jL}(c_i) + w_4\nu_{jU}(c_i), \forall e_j \in A \text{ and } \forall c_i \in U\}$
is called the **Reduced Intuitionistic Fuzzy Soft Set(RIFSS)** of the interval valued intuitionistic fuzzy soft set with respect to the weighted vector. By adjusting the value of w_1, w_2, w_3, w_4 an interval valued intuitionistic fuzzy soft matrix can be converted into reduced intuitionistic fuzzy soft set.

Definition 3.2. Let $U = \{c_1, c_2, c_3, \ldots, c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3 \ldots, e_n\}$. Let $A \subseteq E$ and (F, A) be an interval valued intuitionistic fuzzy soft set in the fuzzy soft class (U, E),where F is a mapping given by $F : A \to I^U$,where I^U denotes the collection of all interval valued intuitionistic fuzzy subsets of U.
Let w_1, w_2, w_3, w_4 in [0,1],$w_1 + w_2 = 1, w_3 + w_4 = 1$.
Then the reduced intuitionistic fuzzy soft set (F_W, A) over U in a matrix form as
$\hat{A}_{m\times n} = [a_{ij}]_{m\times n}$ or $\hat{A} = [a_{ij}]$ $\quad i = 1, 2, \ldots m, j = 1, 2, 3, \ldots, n$
where $a_{ij} = \begin{cases} (w_1\mu_{jL}(c_i) + w_2\mu_{jU}(c_i), w_3\nu_{jL}(c_i) + w_4\nu_{jU}(c_i)) & \text{if } e_j \in A \\ (0, 1) & \text{if } e_j \notin A \end{cases}$
so we can identify any Reduced intuitionistic fuzzy soft set(F_W, A) in the Intuitionistic fuzzy soft class (U,E) by its Reduced intuitionistic fuzzy soft matrix $\hat{A}_{m\times n}$. The set of all $m \times n$ Reduced Intuitionistic Fuzzy Soft Matrices will be denoted by $\mathbf{RIFSM}_{m\times n}$

Definition 3.3. Let $\hat{A}_{m\times n} = [a_{ij}] \in \mathbf{RIFSM}_{m\times n}$
Then $\hat{A}_P$ is called **Pessimistic Reduced Intuitionistic Fuzzy Soft Matrix** if $w_1 = 1, w_2 = 0, w_3 = 0, w_4 = 1$,
The Pessimistic Reduced Intuitionistic Fuzzy Soft set
$F_P(e_j) = \{(c_i, (\mu_{jL}(c_i), \nu_{jU}(c_i)), \forall e_j \in A \text{ and } \forall c_j \in U\}$ over U in a matrix form as $\hat{A}_P = [a_{ij}]_{m\times n}$ or $\hat{A}_P = [a_{ij}]$ $\quad i = 1, 2, \ldots m,\ j = 1, 2, 3, \ldots, n$
where $a_{ij} = \begin{cases} (\mu_{jL}(c_i), \nu_{jU}(c_i)) & \text{if } e_j \in A \\ (0, 1) & \text{if } e_j \notin A \end{cases}$
so we can identify any Pessimistic Reduced Intuitionistic Fuzzy Soft Set(F_P, A) in the Intuitionistic fuzzy soft class (U,E) by its Pessimistic Reduced Intuitionistic Fuzzy Soft Matrix $\hat{A}_P$. The set of all $m \times n$ Pessimistic Reduced Intuitionistic Fuzzy Soft Matrices will be denoted by $\mathbf{PRIFSM}_{m\times n}$

Definition 3.4. Let $\hat{A}_{m\times n} = [a_{ij}] \in \mathbf{RIFSM}_{m\times n}$
Then $\hat{A}_O$ is called **Optimistic Reduced Intuitionistic Fuzzy Soft Matrix** if $w_1 = 0, w_2 = 1, w_3 = 1, w_4 = 0$,
The Optimistic Reduced Intuitionistic Fuzzy Soft Set
$F_O(e_j) = \{(c_i, (\mu_{jU}(c_i), \nu_{jL}(c_i)), \forall e_j \in A \text{ and } \forall c_j \in U\}$ over U in a matrix form as $\hat{A}_O = [a_{ij}]_{m\times n}$ or $\hat{A}_O = [a_{ij}]$ $\quad i = 1, 2, \ldots m,\ j = 1, 2, 3, \ldots, n$
where $a_{ij} = \begin{cases} (\mu_{jU}(c_i), \nu_{jL}(c_i)) & \text{if } e_j \in A \\ (0, 1) & \text{if } e_j \notin A \end{cases}$
so we can identify any Optimistic Reduced Intuitionistic Fuzzy Soft Set(F_P, A) in the Intuitionistic fuzzy soft class (U,E) by its Optimistic Reduced Intuitionistic Fuzzy Soft Matrix $\hat{A}_O$. The set of all $m \times n$ Optimistic Reduced Intuitionistic Fuzzy Soft Matrices will be denoted by $\mathbf{ORIFSM}_{m\times n}$

Definition 3.5. Let $\hat{A}_{m\times n} = [a_{ij}] \in \mathbf{RIFSM}_{m\times n}$
Then $\hat{A}_N$ is called **Neutral Reduced Intuitionistic Fuzzy Soft Matrix** if $w_1 = 0.5, w_2 = 0.5, w_3 = 0.5, w_4 = 0.5$,
The Neutral Reduced Intuitionistic Fuzzy Soft Set
$F_N(e_j) = \{(c_i, (\frac{\mu_{jL}(c_i)+\mu_{jU}(c_i)}{2}, \frac{\nu_{jL}(c_i)+\nu_{jU}(c_i)}{2}), \forall e_j \in A \text{ and } \forall c_j \in U\}$ over U in a matrix form as $\hat{A}_N = [a_{ij}]_{m\times n}$ or $\hat{A}_N = [a_{ij}]$ $\quad i = 1, 2, \ldots m,\ j = 1, 2, 3, \ldots, n$
where $a_{ij} = \begin{cases} (\frac{\mu_{jL}(c_i)+\mu_{jU}(c_i)}{2}, \frac{\nu_{jL}(c_i)+\nu_{jU}(c_i)}{2}) & \text{if } e_j \in A \\ (0, 1) & \text{if } e_j \notin A \end{cases}$
so we can identify any Neutral Reduced intuitionistic fuzzy soft set(F_P, A) in the Intuitionistic fuzzy soft class (U,E) by its Neutral Reduced Intuitionistic Fuzzy Soft Matrix $\hat{A}_N$. The set of all $m \times n$ Neutral Reduced Intuitionistic Fuzzy Soft Matrices will be denoted by $\mathbf{NRIFSM}_{m\times n}$

Example 3.6. Consider the Example 2.8, by adjusting the value of weight w_1, w_2, w_3, w_4 we have reduced matrices as

$$\hat{A}_P = \begin{pmatrix} (0.6, 0.2) & (0.7, 0.2) & (0.0, 1.0) & (0.0, 1.0) \\ (0.8, 0.1) & (0.6, 0.25) & (0.0, 1.0) & (0.0, 1.0) \\ (0.6, 0.25) & (0.5, 0.25) & (0.0, 1.0) & (0.0, 1.0) \\ (0.5, 0.2) & (0.8, 0.1) & (0.0, 1.0) & (0.0, 1.0) \end{pmatrix},$$

$$\hat{A}_O = \begin{pmatrix} (0.8, 0.1) & (0.8, 0.15) & (0.0, 1.0) & (0.0, 1.0) \\ (0.9, 0.05) & (0.7, 0.2) & (0.0, 1.0) & (0.0, 1.0) \\ (0.7, 0.2) & (0.7, 0.2) & (0.0, 1.0) & (0.0, 1.0) \\ (0.6, 0.1) & (0.9, 0.1) & (0.0, 1.0) & (0.0, 1.0) \end{pmatrix} \text{ and}$$

$$\hat{A}_N = \begin{pmatrix} (0.7, 0.15) & (0.75, 0.175) & (0.0, 1.0) & (0.0, 1.0) \\ (0.85, 0.075) & (0.65, 0.22) & (0.0, 1.0) & (0.0, 1.0) \\ (0.65, 0.22) & (0.6, 0.22) & (0.0, 1.0) & (0.0, 1.0) \\ (0.55, 0.15) & (0.85, 0.1) & (0.0, 1.0) & (0.0, 1.0) \end{pmatrix}$$

Application of interval valued intuitionistic fuzzy soft matrix theory in medical diagnosis. Let us assume that there is a set of m patients $P = \{p_1, p_2, p_3, \ldots, p_m\}$ with a set of n symptoms $S = \{s_1, s_2, s_3, s_4, \ldots, s_n\}$ related to a set of k diseases $D = \{d_1, d_2, d_3, \ldots, d_k\}$. We apply interval valued intuitionistic fuzzy soft set technology to diagnose which patient is suffering from what disease. We construct a interval valued intuitionistic fuzzy soft set (F, P) over S where F is a mapping $F : P \to IVIF^S, IVIF^S$ is the collection of all interval valued intuitionistic fuzzy subsets of S. This interval valued intuitionistic fuzzy soft set gives a relation matrix $\hat{A}$ called patient symptom matrix, then construct another interval valued intuitionistic fuzzy soft set (G, S) over D where G is a mapping $G : S \to IVIF^D$, $IVIF^D$ is the collection of all interval valued intuitionistic fuzzy subsets of D. This interval valued intuitionistic fuzzy soft set gives a relation matrix $\hat{B}$ called symptom-disease matrix, where each element denotes the weight of the symptoms for a certain disease.

By adjusting the value of w_1, w_2, w_3, w_4 an interval-valued intuitionistic fuzzy matrix can be converted into one of the Reduced intuitionistic fuzzy soft matrix. We compute the complements $(F, P)^c$ and $(G, S)^c$ and their matrices $\hat{A}^c$ and $\hat{B}^c$. Compute $\hat{A} * \hat{B}$ which is the maximum membership of occurrence of Symptoms of the diseases. Compute $\hat{A}^c * \hat{B}^c$ which is the maximum membership of non occurrence of Symptoms of the diseases. Using Definition 3.1 and 3.2 in ([15]), compute $V(\hat{A} * \hat{B}), V(\hat{A}^c * \hat{B}^c)$ and the Score matrix. Finally find max (S_i),then conclude that the patient p_i is suffering from disease d_j. Incase $max(S_i)$ occurs for more than one value, then reassess the symptoms to break the tie.

Algorithm. Step1:Input the interval valued intuitionistic fuzzy soft set (F, E) , (G, E) and obtain the interval valued intuitionistic fuzzy soft matrices $\hat{A}$,$\hat{B}$ corresponding to (F, E) and (G, E) respectively .

Step2: Input the weighted vector W=(w_1, w_2, w_3, w_4),convert the interval-valued intuitionistic fuzzy matrix into one of the Reduced intuitionistic fuzzy soft matrix. Obtain the intuitionistic fuzzy soft complement matrices $\hat{A}^c, \hat{B}^c$ corresponding to $(F, E)^c$ and $(G, E)^c$ respectively.

Step3:Compute $(\hat{A} * \hat{B}), (\hat{A}^c * \hat{B}^c), V(\hat{A} * \hat{B}), V(\hat{A}^c * \hat{B}^c)$.

Step4:Compute the Score matrix.

Step5:Find p for which $max(S_i)$.
Then we conclude that the patient p_i is suffering from disease d_j. Incase $\max(S_i)$ occurs for more than one value, then reassess the symptoms to break the tie.

Case Study. Suppose there are four patients p_1, p_2, p_3, p_4 in a hospital with symptoms temperature, stomach problem and body pain. Let the possible diseases related to the above symptoms be viral fever, typhoid and malaria. Now take $P = \{p_1, p_2, p_3, p_4\}$ as the universal set where p_1, p_2, p_3 and p_4 represent patients. Let $S = \{s_1, s_2, s_3\}$ as the set of symptoms where s_1, s_2, s_3 represents symptoms temperature,stomach problem and body pain respectively. Suppose that $IVIFS$set(G, S) over P, where G is a mapping $G : S \to I^P$,gives a collection of an approximate description of patient symptoms in the hospital. Let

$$\begin{aligned}(G, S) = \{G(s_1) = \{&(p_1, [0.8, 0.9][0.05, 0.1]), (p_2, [0.0, 0.05][0.75, 0.8]),\\ &(p_3, [0.8, 0.9][0.05, 0.1]), (p_4, [0.6, 0.8][0.05, 0.1])\}\\ G(s_2) = \{&(p_1, [0.6, 0.7][0.05, 0.1]), (p_2, [0.4, 0.45][0.3, 0.4]),\\ &(p_3, [0.8, 0.9][0.1, 0.1]), (p_4, [0.5, 0.55][0.3, 0.4])\}\\ G(s_3) = \{&(p_1, [0.2, 0.2][0.8, 0.8]), (p_2, [0.6, 0.7][0.05, 0.1]),\\ &(p_3, [0.0, 0.2][0.5, 0.6]), (p_4, [0.3, 0.4][0.3, 0.4])\}\}\end{aligned}$$

By adjusting the value of w_1, w_2, w_3, w_4 as $w_1 = 1, w_2 = 0, w_3 = 0, w_4 = 1$ an interval valued intuitionistic fuzzy matrix can be converted into Pessimistic Reduced intuitionistic fuzzy soft matrix.

$$\hat{A}_P = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{pmatrix} (0.8, 0.1) & (0.6, 0.1) & (0.2, 0.8) \\ (0.0, 0.8) & (0.4, 0.4) & (0.6, 0.1) \\ (0.8, 0.1) & (0.8, 0.1) & (0.0, 0.6) \\ (0.6, 0.1) & (0.5, 0.4) & (0.3, 0.4) \end{pmatrix} \end{array}$$

Next consider the set $S = \{s_1, s_2, s_3\}$ as universal set where s_1, s_2, s_3 represents symptoms temperature,stomach problem and body pain respectively and the set $D = \{d_1, d_2, d_3\}$ where d_1, d_2 and d_3 represent the diseases viral fever, typhoid and malaria respectively. Let Suppose that $IVIFS$set(F, D) over S, where F is a mapping $F : D \to I^S$,gives an approximate description of interval valued intuitionistic fuzzy soft medical knowledge of the three diseases and their symptoms. Let

$$\begin{aligned}&(F, D)\\ &= \{F(d_1) = \{(s_1, [0.6, 0.7][0.1, 0.2]), (s_2, [0.3, 0.4][0.4, 0.5]), (s_3, [0.1, 0.1][0.8, 0.8])\}\\ &\quad F(d_2) = \{(s_1, [0.6, 0.7][0.2, 0.2]), (s_2, [0.2, 0.3][0.4, 0.6]), (s_3, [0.2, 0.3][0.5, 0.7])\}\\ &\quad F(d_3) = \{(s_1, [0.3, 0.5][0.3, 0.4]), (s_2, [0.7, 0.8][0.1, 0.2]), (s_3, [0.7, 0.8][0.2, 0.2])\}\}\end{aligned}$$

This interval valued intuitionistic fuzzy soft set can be converted into Pessimistic Reduced Intuitionistic Fuzzy Soft Matrix as

$$\hat{B}_P = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (0.6, 0.2) & (0.6, 0.2) & (0.3, 0.4) \\ (0.3, 0.5) & (0.2, 0.6) & (0.7, 0.2) \\ (0.1, 0.8) & (0.2, 0.7) & (0.7, 0.2) \end{pmatrix} \end{array}$$

Then the intuitionistic fuzzy soft complement matrices are

$$\hat{A}^c_P = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{pmatrix} (0.1, 0.8) & (0.1, 0.6) & (0.8, 0.2) \\ (0.8, 0.0) & (0.4, 0.4) & (0.1, 0.6) \\ (0.1, 0.8) & (0.1, 0, 8) & (0.6, 0.0) \\ (0.1, 0.6) & (0.4, 0.5) & (0.4, 0.3) \end{pmatrix} \end{array}$$

$$\hat{B}^c_P = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (0.2, 0.6) & (0.2, 0.6) & (0.4, 0.3) \\ (0.5, 0.3) & (0.6, 0.2) & (0.2, 0.7) \\ (0.8, 0.1) & (0.7, 0.2) & (0.2, 0.7) \end{pmatrix} \end{array}$$

Then the product matrices are

$$\hat{A}_P * \hat{B}_P = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (0.6, 0.2) & (0.6, 0.2) & (0.6, 0.2) \\ (0.3, 0.5) & (0.2, 0.6) & (0.6, 0.2) \\ (0.6, 0.2) & (0.6, 0.2) & (0.7, 0.2) \\ (0.6, 0.2) & (0.6, 0.2) & (0.5, 0.4) \end{pmatrix} \end{array}$$

$$\hat{A}^c_P * \hat{B}^c_P = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (0.8, 0.2) & (0.7, 0.2) & (0.2, 0.7) \\ (0.7, 0.1) & (0.4, 0.4) & (0.4, 0.3) \\ (0.6, 0.1) & (0.6, 0.2) & (0.2, 0.7) \\ (0.4, 0.3) & (0.4, 0.3) & (0.2, 0.6) \end{pmatrix} \end{array}$$

$$V(\hat{A}_P * \hat{B}_P) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} 0.4 & 0.4 & 0.4 \\ -0.2 & -0.4 & 0.4 \\ 0.4 & 0.4 & 0.5 \\ 0.4 & 0.4 & 0.1 \end{pmatrix} \end{array}$$

$$V(\hat{A}^c_P * \hat{B}^c_P) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} 0.6 & 0.5 & -0.5 \\ 0.0 & 0.0 & 0.1 \\ -0.5 & 0.4 & -0.5 \\ 0.1 & 0.1 & -0.4 \end{pmatrix} \end{array}$$

Calculate the score matrix

$$\hat{S} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} -0.02 & -0.1 & 0.9 \\ -0.2 & -0.4 & 0.3 \\ 0.9 & 0.0 & 1.0 \\ 0.3 & 0.3 & 0.5 \end{pmatrix} \end{array}$$

It is clear from the above matrix that all the patients p_1, p_2, p_3, p_4 have maximun score 0. 9,0. 3,1. 0,0. 5 for the disease malaria(d_3). So we conclude that all the four patients are suffered by malaria fever.

4. A comparative study

In this paper, we have discussed three types of reduced intuitionistic fuzzy soft matrix. In the above case study, an interval valued intuitionistic fuzzy soft matrix is converted into pessimistic reduced intuitionistic fuzzy soft matrix by taking the weighted vector as $w_1 = 1, w_2 = 0, w_3 = 0, w_4 = 1$. If we choose the weighted vector as $w_1 = 0, w_2 = 1, w_3 = 1, w_4 = 0$ and $w_1 = 0.5, w_2 = 0.5, w_3 = 0.5, w_4 = 0.5$ an interval valued intuitionistic fuzzy soft matrix is converted into optimistic reduced intuitionistic fuzzy soft matrix and neutral reduced intuitionistic fuzzy soft matrix respectively.
The maximum score by all the reductions are

$$\hat{S} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} \textit{pessimistic} & \textit{optimistic} & \textit{neutral} \end{array} \\ \begin{pmatrix} 0.9 & 1.2 & 1.1 \\ 0.3 & 0.7 & 0.5 \\ 1 & 1.3 & 1.15 \\ 0.5 & 0.85 & 0.65 \end{pmatrix} \end{array}$$

We see that all the four patients have maximum score for malaria disease by all the reductions. So we conclude that all the four patients are suffered by malaria fever only.

5. Conclusions

From this paper we can conclude that the interval valued intuitionistic fuzzy soft matrix can be reduced to intuitionistic fuzzy soft matrix, which holds a value instead of an interval value. This output is very supportive and encouraging in making a fair conclusion as it contains only one value. This feature highlights to be applied to many domains like pattern recognition, face identification, speech recognition, predictive analytics and many more decision making process. Thus, this result motivates us to work more towards this goal. Further our work is supported by a decision making problem in medical diagnosis. In the future,we will use **RIFSM** which are proposed in this paper in decision making with special operators. We will compare the results with other types of disease diagnosis.

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