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Characterizations of Γ-semigroups by the properties of their interval valued T-fuzzy ideals

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Abstract. This research paper is based on the introduction and initiation of interval valued T-fuzzy ideals, interval valued T-fuzzy bi-ideals, interval valued T-fuzzy quasi ideals and interval valued T-fuzzy interior ideals in Γ-semigroups along with their characteristic traits. Furthermore several specifications of regular Γ-semigroups and intra-regular Γ-semigroups along with their specific qualities in terms of these ideals have been thoroughly explored at their different stages.

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Keywords: Interval valued T-fuzzy sub-Γ-semigroup, Interval valued T-fuzzy quasi ideal, Interval valued T-fuzzy bi-ideal and interval valued T-fuzzy generalized bi-ideal, Regular, Intra-regular and semisimple Γ-semigroup.

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1. Introduction

It were Anthony and Sherwood [1] who re-nagotiated and defined the fuzzy subgroups employing statistical triangular norm in 1979. Earlier on Schweizer and Sklar [19] while in an effort to generalize the ordinary triangular inequality in the metric space to the more common and generalized probable metric space came very close to delineate this concept. Furthermore, in the sequential progression Bedregal and Takahashi [2] suggested a generalization of t-norms for interval values co-extensive with Gehrke [8] interval t-norm from a t-norm in a manner that always secure and gaurentees that the output interval of the interval t-norm is the narrowest containing the real result of the t-norm.

The concept of Γ in algebra was first introduced by Nobusawa [12] in 1964. Similarly the notion of Γ-semigroup was pioneered by Sen in [20]. The fact of matter is that the Γ-semigroup is a generalized abstraction of semigroup along with ternary semigroup. Consequently, a lot many researchers have continued to work on Γ-semigroups and its structure. Diverse mathematicians have explored the Γ-semigroups. Some of the outstanding nagotiaters of this exercise are Dutta and Adhikar [7], Hila [9], Chinram [5], Sardar and Majumder [16, 17, 18] who have specified and delineated sub-Γ-semigroups, bi-ideals, interior ideals and semiprime ideals in terms of fuzzy subsets. They also explored their different characteristics directly and by way of operator semigroups of a Γ-semigroup.

The concept of generalized fuzzy sub-Γ-semigroups, generalized fuzzy ideals and generalized fuzzy bi-ideals in a Γ-semigroups was introduced by [14, 15] S. K. Sardar, B. Davvaz, S. K. Majumder and S. Kayal. Zadeh [22] first of all introduced the theory of fuzzy sets. Since its launching, this theory has developed in multilateral directions and discovered applications in multifarious fields. The concept of a fuzzy subset was extended by an interval valued fuzzy subset by L. A. Zadeh [23]. Subsequently, Biswas [3], defined the interval valued fuzzy subgroups of Rosenfields's nature and explored some basic properties. In [11], Narayanan and Manikantan initiated the concepts of an interval valued fuzzy semigroups and various interval valued fuzzy ideals in semigroups.

Interval valued subsets were proposed as a natural extension of fuzzy sets, thirty years back. Interval valued fuzzy set is termed as interval valued membership function, that is, fuzzy set in a state that the membership gradation of each element of the fuzzy set is resulteted by a closed subinteval of the interval [0,1]. These sets, manifest, a more appropriate description of uncertainty than traditional fuzzy sets. Interval valued Γ-fuzzy semigroups are applicable in diverse fields: control systems, computer engineerings, information sciences and technologies. We have inspired by Narayana and Manikantan [11] and prsued the study of Γ-semigroups in terms of interval valued fuzzy subsets. This research is a continuation of these Paradigms and indepth exploration of its different perimeters.

2. Preliminaries

Definition 2.1. Let $G = \{x, y, z, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets, then G is called a Γ-semigroup if there exists a mapping $G \times \Gamma \times G \longrightarrow G$ (images to be denoted by $a\alpha b$) satisfying

(1) $x\gamma y \in G$,
(2) $(x\alpha y)\beta z = x\alpha(y\beta z)$, for all $x, y, z \in G$ and $\alpha, \beta, \gamma \in \Gamma$.

A non-empty subset A of a Γ-semigroup G is called sub-Γ-semigroup of G if $A\Gamma A \subseteq A$. A left (right) ideal of a Γ-semigroup G is a non-empty subset A of G such that $G\Gamma A \subseteq A$ ($A\Gamma G \subseteq A$). If A is both a left and a right ideal of a Γ-semigroup G, then we say that A is a Γ-ideal of G. Let A be a non-empty subset of a Γ-semigroup G. By $L[A]$, we mean the left Γ-ideal of G generated by A (that is, the intersection of all left Γ-ideals of G containing A). Similarly, $R[A]$ and $J[A]$ denote the right and two-sided Γ-ideals generated by A, respectively. $L[a] = \{a\} \cup G\Gamma a$, $R[a] = \{a\} \cup a\Gamma G$ and $J[a] = \{a\} \cup a\Gamma G \cup G\Gamma a \cup G\Gamma a\Gamma G$ are principal left, right and two-sided ideals of a Γ-semigroup G generated by a, respectively. Let G be a Γ-semigroup. A sub-Γ-semigroup A of G is called a bi-Γ-ideal of G if $A\Gamma G\Gamma A \subseteq A$. Let G be a Γ-semigroup, a subset A of G is called a generalized bi-Γ-ideal of G if $A\Gamma G\Gamma A \subseteq A$. A non-empty subset A of a Γ-semigroup G is called an interior Γ-ideal of G if $G\Gamma A\Gamma G \subseteq A$.

Let G be a Γ-semigroup, a non-empty subset A of G is called quasi ideal of G if $(G\Gamma A) \cap (A\Gamma G) \subseteq A$. A Γ-semigroup G is called left (right) simple if G has no proper left (right) ideals. Equivalently for all $x, y \in G$ there exist $a, b \in G$ and $\gamma \in \Gamma$ such that $x = a\gamma y$ and $y = b\gamma x$ ($x = y\gamma a$ and $y = x\gamma b$). Let G be a Γ-semigroup, an element $x \in G$ is called regular if there exist $g \in G$ and α, $\beta \in \Gamma$ such that $x = x\alpha g\beta x$. A Γ-semigroup G is called regular if every element of G is regular. Let G be a Γ-semigroup, an element $x \in G$ is called intra-regular if there exist $a, b \in G$ and α, β, $\gamma \in \Gamma$ such that $x = a\alpha x\beta x\gamma b$. G is called intra-regular if each element of G is intra-regular (cf. [4, 6, 7, 10, 13, 16]).

Theorem 2.2 ([21]). *The following conditions in a Γ-semigroup G are equivalent.*
(1) *G is regular.*
(2) *$R \cap L = R\Gamma L$, for every right ideal R and every left ideal L of G.*
(3) *For every right ideal R and for every left ideal L of G, we have*
(*i*) *$R\Gamma R = R$*
(*ii*) *$L\Gamma L = L$*
(*iii*) *$R \cap L = R\Gamma L$, is the quasi-ideal of a Γ-semigroup G.*
(4) *Every quasi-ideal Q of Γ-semigroup G has the form $Q = Q\Gamma G\Gamma Q$.*

3. Basic concepts in fuzzy subsets and fuzzy Ideals in Γ-semigroups

A fuzzy subset μ of a universal set G is a function from G into $[0, 1]$. Let μ and λ be two fuzzy subsets of G. Define $\mu \cap \lambda$ and $\mu \cup \lambda$ as: $(\mu \cap \lambda)(x) = \mu(x) \wedge \lambda(x)$ and $(\mu \cup \lambda)(x) = \mu(x) \vee \lambda(x)$ for all $x \in G$. Let μ, λ be two fuzzy subsets of G. Then their product $\mu \circ \lambda$ is defined as

$$(\mu \circ \lambda)(x) = \begin{cases} \bigvee\limits_{x = y\gamma z} [\mu(y) \wedge \lambda(z)] & \text{if there exist } y, z \in G \text{ and } \gamma \in \Gamma \text{ such that } x = y\gamma z. \\ 0 & \text{otherwise} \end{cases}$$

A fuzzy subset μ of a Γ-semigroup G is called fuzzy sub-Γ-semigroup of G if $\mu(x\gamma y) \geq \mu(x) \wedge \mu(y)$ for all x, $y \in G$ and $\gamma \in \Gamma$. A fuzzy subset μ of a Γ-semigroup G is called fuzzy left (right) ideal of G if $\mu(x\gamma y) \geq \mu(y)$ ($\mu(x\gamma y) \geq \mu(x)$) for all x, $y \in G$ and $\gamma \in \Gamma$. A fuzzy subset μ of a Γ-semigroup G is called a fuzzy ideal of G if it is both a fuzzy left ideal and a fuzzy right ideal of G. A fuzzy subset μ of a Γ-semigroup G is called a fuzzy quasi ideal of G if $(\mu \circ \chi) \cap (\chi \circ \mu) \subseteq \mu$, where χ is the characteristic function of G. A fuzzy sub-Γ-semigroup μ of a Γ-semigroup G is called a fuzzy bi-ideal of G if $\mu(x\beta s\gamma y) \geq \mu(x) \wedge \mu(y)$ for all x, s, $y \in G$ and β, $\gamma \in \Gamma$. A fuzzy subset μ of a Γ-semigroup G is called a fuzzy generalized bi-ideal of G if $\mu(x\beta s\gamma y) \geq \mu(x) \wedge \mu(y)$ for all x, s, $y \in G$ and β, $\gamma \in \Gamma$. A fuzzy subset μ of a Γ-semigroup G is called a fuzzy interior ideal of G if $\mu(x\alpha a\beta y) \geq \mu(a)$ for all x, a, $y \in G$ and α, $\beta \in \Gamma$ (cf. [13, 16, 17, 22]).

4. Basic concepts in i-v fuzzy subsets

By an interval number $\widetilde{t}$ we mean (cf. [23]) an interval $[t^l, t^u]$, where $0 \leq t^l \leq t^u \leq 1$. The set of all interval numbers is denoted by $D[0, 1]$. The interval $[t, t]$ is defined with the number $t \in [0, 1]$. By interval numbers $\widetilde{t}_i = [t_i^l, t_i^u]$, $\widetilde{t} = [t^l, t^u]$

and $\widetilde{r} = [r^l, r^u] \in D[0,1]$, $i \in I$. We define $\inf \widetilde{t} = [\wedge_{i\in I} t_i^l, \wedge_{i\in I} t_i^u]$, $\sup \widetilde{t} = [\vee_{i\in I} t_i^l, \vee_{i\in I} t_i^u]$ and

(1) $\widetilde{t} \leq \widetilde{r}$ if and only if $t^l \leq r^l$ and $t^u \leq r^u$.

(2) $\widetilde{t} = \widetilde{r}$ if and only if $t^l = r^l$ and $t^u = r^u$.

(3) $\widetilde{t} < \widetilde{r}$ if and only if $\widetilde{t} \leq \widetilde{r}$ and $\widetilde{t} \neq \widetilde{r}$.

(4) $\widetilde{t} \subseteq \widetilde{r}$ if and only if $r^l \leq t^l \leq t^u \leq r^u$.

(5) $k\widetilde{t} = [kt^l, kt^u]$, whenever $0 \leq k \leq 1$.

It is clear that $(D[0,1], \leq, \vee, \wedge)$ is a complete lattice (a partially ordered set in which all subsets have both a supremum and infimum) with $0 = [0,0]$ as the least element and $1 = [1,1]$ as the greatest element. By an interval valued fuzzy subset (briefly, an i-v fuzzy subset) on G we mean the set $F = \{(x, [\mu^l(x), \mu^u(x)])|x \in G\}$, where μ^l and μ^u are two fuzzy sets on G such that $\mu^l(x) \leq \mu^u(x)$ for all $x \in G$. Putting $\hat{\mu}(x) = [\mu^l(x), \mu^u(x)]$, we see that $F = \{(x, \hat{\mu}(x))|x \in G\}$, where $\hat{\mu} : G \to D[0,1]$.

Definition 4.1 ([19]). A mapping $T : [0,1] \times [0,1] \to [0,1]$ is called a t-norm if it satisfies the following:

(1) $T(1,x) = x$. (1 acts as an identity element)

(2) $T(x,y) = T(y,x)$. (Commutativity)

(3) $T(x, T(y,z)) = T(T(x,y), z)$. (Associativity)

(4) If $w \leq x$ and $y \leq z$, then $T(w,y) \leq T(x,z)$ for all $x,y,z,w \in [0,1]$. (Monotonicity) The first, second and fourth conditions give

$T(0,x) \leq T(0,1) = 0.$

Definition 4.2 ([2]). A mapping $\triangle : D[0,1] \times D[0,1] \longrightarrow D[0,1]$ is called an interval triangular norm if $\triangle$ satisfies the following properties:

(1) for each $\widetilde{t}, \widetilde{r} \in D[0,1]$, $\widetilde{t} \triangle \widetilde{r} = \widetilde{r} \triangle \widetilde{t}$. (Symmetry)

(2) for each $\widetilde{t}, \widetilde{r}, \widetilde{s} \in D[0,1]$, $\widetilde{t} \triangle (\widetilde{r} \triangle \widetilde{s}) = (\widetilde{t} \triangle \widetilde{r}) \triangle \widetilde{s}$. (Associativity)

(3) for each $\widetilde{t}_1, \widetilde{r}_1, \widetilde{t}_2, \widetilde{r}_2 \in D[0,1]$, if $\widetilde{t}_1 \leq \widetilde{t}_2 (t_1^l \leq t_2^l$ and $t_1^u \leq t_2^u)$ and $\widetilde{r}_1 \leq \widetilde{r}_2 (r_1^l \leq r_2^l$ and $r_1^u \leq r_2^u)$, then $\widetilde{t}_1 \triangle \widetilde{r}_1 \leq \widetilde{t}_2 \triangle \widetilde{r}_2$ (KM-monotonicity).

(4) for each $\widetilde{t}_1, \widetilde{r}_1, \widetilde{t}_2, \widetilde{r}_2 \in D[0,1]$, if $\widetilde{t}_1 \subseteq \widetilde{t}_2 (t_2^l \leq t_1^l \leq t_1^u \leq t_2^u)$ and $\widetilde{r}_1 \subseteq \widetilde{r}_2 (r_2^l \leq r_1^l \leq r_1^u \leq r_2^u)$, then $\widetilde{t}_1 \triangle \widetilde{r}_1 \subseteq \widetilde{t}_2 \triangle \widetilde{r}_2$ (Inclusion monotonicity).

(5) for each $\widetilde{t} \in D[0,1]$, $\widetilde{t} \triangle [1,1] = \widetilde{t}$. (1 acts as an identity element).

5. Interval valued T-fuzzy ideals in Γ-semigroups

In this section, we define interval valued T- fuzzy left, right, interior, quasi, bi and generalized bi-ideals of a Γ-semigroup G. In what follows, $\triangle$ will denote an interval t-norm and $\hat{\mu} \subseteq \hat{\lambda}$ if and only if $\hat{\mu}(x) \leq \hat{\lambda}(x)$ for all $x \in G$.

Definition 5.1. An i-v fuzzy subset $\hat{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy sub-Γ-semigroup of G if $\hat{\mu}(x\gamma y) \geq \hat{\mu}(x) \triangle \hat{\mu}(y)$ for all $x, y \in G$ and $\gamma \in \Gamma$.

Definition 5.2. An i-v fuzzy subset $\hat{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy left (right) ideal of G if $\hat{\mu}(x\gamma y) \geq \hat{\mu}(y)$ $(\hat{\mu}(x\gamma y) \geq \hat{\mu}(x))$ for all x, $y \in G$ and $\gamma \in \Gamma$.

Definition 5.3. An i-v fuzzy subset $\hat{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy two sided ideal of G or an i-v T-fuzzy ideal of G if it is both an i-v T-fuzzy left ideal and an i-v T-fuzzy right ideal of G.

Definition 5.4. An i-v fuzzy sub-Γ-semigroup $\hat{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy bi-ideal of G if $\hat{\mu}(x\alpha y\beta z) \geq \hat{\mu}(x) \triangle \hat{\mu}(z)$ for all $x, y, z \in G$ and $\alpha, \beta \in \Gamma$.

Definition 5.5. An i-v fuzzy subset $\hat{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy generalized bi-ideal of G if $\hat{\mu}(x\alpha y\beta z) \geq \hat{\mu}(x) \triangle \hat{\mu}(z)$ for all $x, y, z \in G$ and $\alpha, \beta \in \Gamma$.

Remark 5.6. Every i-v T-fuzzy bi-ideal of a Γ-semigroup G is an i-v T-fuzzy generalized bi-ideal of G but converse is not true.

Definition 5.7. An i-v fuzzy subset $\hat{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy interior ideal of G if $\hat{\mu}(x\alpha a\beta y) \geq \hat{\mu}(a)$ for all $x, a, y \in G$ and $\alpha, \beta \in \Gamma$.

Definition 5.8. Let $\hat{\mu}$ and $\hat{\lambda}$ be two i-v T-fuzzy subsets of a Γ-semigroup G. Then $\hat{\mu} \triangle \hat{\lambda}$ is defined as $(\hat{\mu} \triangle \hat{\lambda})(x) = \hat{\mu}(x) \triangle \hat{\lambda}(x)$.

Definition 5.9. Let $\hat{\mu}$ be an i-v fuzzy subset of a Γ-semigroup G and let $\tilde{t} \in [0, 1]$. Then the set $\hat{\mu}_{\tilde{t}} = \{x \in G : \hat{\mu}(x) \geq \tilde{t}\}$ is called the level subset of $\hat{\mu}$.

Definition 5.10. Let $\hat{\mu}$ and $\hat{\lambda}$ be two i-v fuzzy subsets of a Γ-semigroup G. Then

$$(\hat{\mu}\circ\hat{\lambda})(x) = \begin{cases} \bigvee\limits_{x=y\gamma z} [\hat{\mu}(y) \triangle \hat{\lambda}(z)] & \text{if there exist } y, z \in G \text{ and } \gamma \in \Gamma \text{ such that } x = y\gamma z. \\ 0 & \text{otherwise} \end{cases}$$

Definition 5.11. An i-v fuzzy subset $\hat{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy quasi ideal of G if $(\hat{\mu} \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\mu}) \subseteq \hat{\mu}$, where $\hat{\chi}$ is the characteristic function of G.

6. Interval valued T-fuzzy Ideals

Lemma 6.1. *Let $\hat{\mu}$, $\hat{\lambda}$ and $\hat{\nu}$ be three i-v fuzzy subsets of a Γ-semigroup G. If $\hat{\mu} \subseteq \hat{\lambda}$ then $\hat{\mu} \circ \hat{\nu} \subseteq \hat{\lambda} \circ \hat{\nu}$ and $\hat{\nu} \circ \hat{\mu} \subseteq \hat{\nu} \circ \hat{\lambda}$.*

Proof. Let $x \in G$, then $(\hat{\mu} \circ \hat{\nu})(x) = [0, 0] = (\hat{\lambda} \circ \hat{\nu})(x)$, if x is not expressible as $x = y\gamma z$. Otherwise

$$\begin{aligned}(\hat{\mu} \circ \hat{\nu})(x) &= \bigvee_{x=y\gamma z} [\hat{\mu}(y) \triangle \hat{\nu}(z)] \\ &\leq \bigvee_{x=y\gamma z} [\hat{\lambda}(y) \triangle \hat{\nu}(z)] \\ &= (\hat{\lambda} \circ \hat{\nu})(x).\end{aligned}$$

Thus $\hat{\mu} \circ \hat{\nu} \subseteq \hat{\lambda} \circ \hat{\nu}$. Similarly we can show that $\hat{\nu} \circ \hat{\mu} \subseteq \hat{\nu} \circ \hat{\lambda}$. □

Lemma 6.2. *If $\hat{\mu}$ and $\hat{\lambda}$ be two i-v T-fuzzy sub-Γ-semigroups of a Γ-semigroup G. Then $\hat{\mu} \triangle \hat{\lambda}$ is also an i-v T-fuzzy sub-Γ-semigroup of G.*

Proof. If $\hat{\mu}$ and $\hat{\lambda}$ be two i-v fuzzy sub-Γ-semigroups of a Γ-semigroup G. Then $\hat{\mu}(x\gamma y) \geq \hat{\mu}(x) \triangle \hat{\mu}(y)$ and $\hat{\lambda}(x\gamma y) \geq \hat{\lambda}(x) \triangle \hat{\lambda}(y)$ for all $x, y \in G$ and $\gamma \in \Gamma$. Now

$$\begin{aligned}(\hat{\mu} \triangle \hat{\lambda})(x\gamma y) &= \hat{\mu}(x\gamma y) \triangle \hat{\lambda}(x\gamma y)\\ &\geq [\hat{\mu}(x) \triangle \hat{\mu}(y)] \triangle [\hat{\lambda}(x) \triangle \hat{\lambda}(y)]\\ &= [\hat{\mu}(x) \triangle \hat{\lambda}(x)] \triangle [\hat{\mu}(y) \triangle \hat{\lambda}(y)]\\ &= (\hat{\mu} \triangle \hat{\lambda})(x) \triangle (\hat{\mu} \triangle \hat{\lambda})(y).\end{aligned}$$

This implies that $\hat{\mu} \triangle \hat{\lambda}$ is an i-v T-fuzzy sub-Γ-semigroup of G. □

Theorem 6.3. *Let $\hat{\mu}$ be an i-v fuzzy subset of a Γ-semigroup G. Then $\hat{\mu}$ is an i-v T-fuzzy sub-Γ-semigroup of G if $\hat{\mu}_{\tilde{t}} (\neq \varnothing)$ is a sub-Γ-semigroup of G for all $\tilde{t} \in D[0,1]$.*

Proof. Assume that every non-empty level subset of $\hat{\mu}$ is a sub-Γ-semigroup of G. Let $a, b \in G$ and $\gamma \in \Gamma$ be such that $\hat{\mu}(a) \triangle \hat{\mu}(b) > \hat{\mu}(a\gamma b)$. Choose $\tilde{t} \in D[0,1]$ such that $\hat{\mu}(a) \triangle \hat{\mu}(b) \geq \tilde{t} > \hat{\mu}(a\gamma b)$. Implies $a \in \hat{\mu}_{\tilde{t}}$ and $b \in \hat{\mu}_{\tilde{t}}$ but $a\gamma b \notin \hat{\mu}_{\tilde{t}}$. Which is a contradiction. Hence $\hat{\mu}(a\gamma b) \geq \hat{\mu}(a) \triangle \hat{\mu}(b)$. Thus $\hat{\mu}$ is an i-v T-fuzzy sub-Γ-semigroup of G. The converse of above Theorem does not hold in general. □

Example 6.4. Let $G = \{0, a, b, c\}$ be the Γ-semigroup with $\Gamma = \{\gamma\}$ and the multiplication table defined as:

γ	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define an i-v fuzzy subset $\hat{\mu}$ in G by $\hat{\mu}(0) = [0.8,\ 0.9]$, $\hat{\mu}(a) = [0.6,\ 0.7]$, $\hat{\mu}(b) = [0.55,\ 0.59]$, $\hat{\mu}(c) = [0.5,\ 0.54]$. Using interval t-norm associated with Lukasiewiez t-norm, by routine calculations, it can be shown that $\hat{\mu}(x\gamma y) \geq \hat{\mu}(x) \triangle \hat{\mu}(y)$ for all $x,\ y \in G$ and $\gamma \in \Gamma$, which shows that $\hat{\mu}$ is an i-v T-fuzzy sub-Γ-semigroup of G. But its level subset $\hat{\mu}_{[0.55,\ 0.59]} = \{0,\ a,\ b\}$ is not a sub-Γ-semigroup of G, because $a\gamma b = c \notin \hat{\mu}_{[0.55,\ 0.59]}$.

Lemma 6.5. *If $\hat{\mu}$ and $\hat{\lambda}$ be two i-v T-fuzzy left (right) ideals of a Γ-semigroup G. Then*

(1) $\hat{\mu} \triangle \hat{\lambda}$ *is also an i-v* T*-fuzzy left (right) ideal of* G.

(2) $\hat{\mu} \cup \hat{\lambda}$ *is also an i-v* T*-fuzzy left (right) ideal of* G.

Proof. (1) Let $\hat{\mu}$ and $\hat{\lambda}$ be i-v T-fuzzy left ideals of a Γ-semigroup G. Then $\hat{\mu}(x\gamma y) \geq \hat{\mu}(y)$ and $\hat{\lambda}(x\gamma y) \geq \hat{\lambda}(y)$ for all $x, y \in G$ and $\gamma \in \Gamma$. Now $\hat{\mu}(x\gamma y) \triangle \hat{\lambda}(x\gamma y) \geq \hat{\mu}(y) \triangle \hat{\lambda}(y)$. Thus $(\hat{\mu} \triangle \hat{\lambda})(x\gamma y) \geq (\hat{\mu} \triangle \hat{\lambda})(y)$. Hence $\hat{\mu} \triangle \hat{\lambda}$ is an i-v T-fuzzy left ideal of G.

(2) Let $\hat{\mu}$ and $\hat{\lambda}$ be i-v T-fuzzy left ideals of a Γ-semigroup G. Then $\hat{\mu}(x\gamma y) \geq \hat{\mu}(y)$ and $\hat{\lambda}(x\gamma y) \geq \hat{\lambda}(y)$ for all $x, y \in G$ and $\gamma \in \Gamma$. Now $\hat{\mu}(x\gamma y) \vee \hat{\lambda}(x\gamma y) \geq \hat{\mu}(y) \vee \hat{\lambda}(y)$. Thus $(\hat{\mu} \cup \hat{\lambda})(x\gamma y) \geq (\hat{\mu} \cup \hat{\lambda})(y)$. Hence $\hat{\mu} \cup \hat{\lambda}$ is an i-v T-fuzzy left ideal of G. □

Lemma 6.6. *Let* $\hat{\mu}$ *be an i-v* T *fuzzy subset of a* Γ*-semigroup* G. *Then* $\hat{\mu}$ *is an i-v* T*- fuzzy sub-*Γ*-semigroup of* G *if and only if* $\hat{\mu} \circ \hat{\mu} \subseteq \hat{\mu}$.

Proof. Let $\hat{\mu}$ be an i-v T-fuzzy sub-Γ-semigroup of G and $x \in G$. If $(\hat{\mu} \circ \hat{\mu})(x) = [0, 0]$, then $(\hat{\mu} \circ \hat{\mu})(x) \leq \hat{\mu}(x)$. Otherwise,

$$(\hat{\mu} \circ \hat{\mu})(x) = \bigvee_{x=y\gamma z} [\hat{\mu}(y) \triangle \hat{\mu}(z)] \leq \bigvee_{x=y\gamma z} \hat{\mu}(y\gamma z) = \hat{\mu}(x)$$

Thus $\hat{\mu} \circ \hat{\mu} \subseteq \hat{\mu}$. Conversely, let $\hat{\mu} \circ \hat{\mu} \subseteq \hat{\mu}$ and $y, z \in G$, $\gamma \in \Gamma$. Then

$$\hat{\mu}(y\gamma z) \geq (\hat{\mu} \circ \hat{\mu})(y\gamma z) = \bigvee_{y\gamma z = a\gamma b} [\hat{\mu}(a) \triangle \hat{\mu}(b)] \geq \hat{\mu}(y) \triangle \hat{\mu}(z).$$

So, $\hat{\mu}$ is an i-v T-fuzzy sub-Γ-semigroup of G. □

Lemma 6.7. *Let* $\hat{\mu}$ *be an i-v* T *fuzzy subset of a* Γ*-semigroup* G. *Then* $\hat{\mu}$ *is an i-v* T*- fuzzy left (right) ideal of* G *if and only if* $\hat{\chi} \circ \hat{\mu} \subseteq \hat{\mu}$ $(\hat{\mu} \circ \hat{\chi} \subseteq \hat{\mu})$.

Proof. Let $\hat{\mu}$ is an i-v T- fuzzy left ideal of G and $x \in G$. If $(\hat{\chi} \circ \hat{\mu})(x) = [0, 0]$ then $(\hat{\chi} \circ \hat{\mu})(x) \leq \hat{\mu}(x)$. Otherwise,

$$\begin{aligned}(\hat{\chi} \circ \hat{\mu})(x) &= \bigvee_{x=y\gamma z} [\hat{\chi}(y) \triangle \hat{\mu}(z)] = \bigvee_{x=y\gamma z} [[1, 1] \triangle \hat{\mu}(z)] \\ &= \bigvee_{x=y\gamma z} \hat{\mu}(z) \leq \bigvee_{x=y\gamma z} \hat{\mu}(y\gamma z) = \hat{\mu}(x).\end{aligned}$$

Thus $\hat{\chi} \circ \hat{\mu} \subseteq \hat{\mu}$. Conversely, let $\hat{\chi} \circ \hat{\mu} \subseteq \hat{\mu}$ and $y,\ z \in G,\ \gamma \in \Gamma$. Then

$$\begin{aligned}\hat{\mu}(y\gamma z) \geq (\hat{\chi} \circ \hat{\mu})(y\gamma z) &= \bigvee_{y\gamma z = a\gamma b} [\hat{\chi}(a) \triangle \hat{\mu}(b)] \\ &\geq \hat{\chi}(y) \triangle \hat{\mu}(z) = [1, 1] \triangle \hat{\mu}(z) = \hat{\mu}(z).\end{aligned}$$

Thus $\hat{\mu}(y\gamma z) \geq \hat{\mu}(z)$. Thus $\hat{\mu}$ is an i-v T- fuzzy left ideal of G. □

Lemma 6.8. *An i-v T fuzzy subset $\overset{\wedge}{\mu}$ of a Γ-semigroup G is an i-v T-fuzzy generalized bi-ideal of G if and only if $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$.*

Proof. Let $\overset{\wedge}{\mu}$ be an i-v T-fuzzy generalized bi-ideal of G and $a \in G$ such that $a = v\alpha z$ where $\alpha \in \Gamma$. If $(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu})(a) = [0, 0]$ then $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$. Otherwise,

$$\begin{aligned}
(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu})(a) &= \bigvee_{a=v\alpha z} [(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi})(v) \bigtriangleup \overset{\wedge}{\mu}(z)] \\
&= \bigvee_{a=v\alpha z} [\bigvee_{v=x\beta y} [\overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\chi}(y)] \bigtriangleup \overset{\wedge}{\mu}(z)] \\
&= \bigvee_{a=v\alpha z} [\bigvee_{v=x\beta y} [\overset{\wedge}{\mu}(x) \bigtriangleup [1,1]] \bigtriangleup \overset{\wedge}{\mu}(z)] \\
&= \bigvee_{a=v\alpha z} [\bigvee_{v=x\beta y} [\overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\mu}(z)]] \\
&= \bigvee_{a=x\beta y\alpha z} [\overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\mu}(z)] \\
&\leq \bigvee_{a=x\beta y\alpha z} \overset{\wedge}{\mu}(x\beta y\alpha z) \\
&= \overset{\wedge}{\mu}(a).
\end{aligned}$$

Thus $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$. Conversely, let $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$ and $x,\ y \in G,\ \ \alpha,\ \beta \in \Gamma$. Then

$$\begin{aligned}
\overset{\wedge}{\mu}(x\alpha y\beta z) &\geq (\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu})(x\alpha y\beta z) \\
&= \bigvee_{x\alpha y\beta z=a\gamma b} [(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi})(a) \bigtriangleup \overset{\wedge}{\mu}(b)] \\
&\geq (\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi})(x\alpha y) \bigtriangleup \overset{\wedge}{\mu}(z) \\
&= \bigvee_{x\alpha y=a\delta c} [\overset{\wedge}{\mu}(a) \bigtriangleup \overset{\wedge}{\chi}(c)] \bigtriangleup \overset{\wedge}{\mu}(z) \\
&\geq \overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\chi}(y) \bigtriangleup \overset{\wedge}{\mu}(z) \\
&= \overset{\wedge}{\mu}(x) \bigtriangleup [1,1] \bigtriangleup \overset{\wedge}{\mu}(z) \\
&= \overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\mu}(z)
\end{aligned}$$

Thus $\overset{\wedge}{\mu}(x\alpha y\beta z) \geq \overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\mu}(z)$. So, $\overset{\wedge}{\mu}$ is an i-v T-fuzzy generalized bi-ideal of G. □

Lemma 6.9. *Let $\overset{\wedge}{\mu}$ is an i-v T fuzzy subset of a Γ-semigroup G. Then $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G if and only if $\overset{\wedge}{\mu} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$ and $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$.*

Proof. Let $\overset{\wedge}{\mu}$ is an i-v fuzzy subset of a Γ-semigroup G. Suppose $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G. So $\overset{\wedge}{\mu}$ is a fuzzy sub-Γ-semigroup of G. Hence by Lemma 6.6, $\overset{\wedge}{\mu} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$. Let $a \in G$. Suppose there exist $x,\ y,\ p,\ q \in G$ and $\beta,\ \gamma \in \Gamma$ such that $a = x\gamma y$ and $x = p\beta q$. Since $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G, we obtain $\overset{\wedge}{\mu}(p\beta q\gamma y) \geq \overset{\wedge}{\mu}(p) \bigtriangleup \overset{\wedge}{\mu}(y)$.

Then

$$\begin{aligned}
(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu})(a) &= \bigvee_{a=x\gamma y} [(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi})(x) \bigtriangleup \overset{\wedge}{\mu}(y)] \\
&= \bigvee_{a=x\gamma y} [\bigvee_{x=p\beta q} [\overset{\wedge}{\mu}(p) \bigtriangleup \overset{\wedge}{\chi}(q)] \bigtriangleup \overset{\wedge}{\mu}(y)] \\
&= \bigvee_{a=x\gamma y} [\bigvee_{v=p\beta q} [\overset{\wedge}{\mu}(p) \bigtriangleup [1,1]] \bigtriangleup \overset{\wedge}{\mu}(y)] \\
&= \bigvee_{a=x\gamma y} [\bigvee_{v=p\beta q} [\overset{\wedge}{\mu}(p) \bigtriangleup \overset{\wedge}{\mu}(y)]] \\
&\leq \overset{\wedge}{\mu}(p\beta q\gamma y) \\
&= \overset{\wedge}{\mu}(x\gamma y) \\
&= \overset{\wedge}{\mu}(a).
\end{aligned}$$

Thus $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$. Otherwise, $(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu})(a) = [0,0] \leq \overset{\wedge}{\mu}(a)$. Thus $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$. Conversely, let us assume that $\overset{\wedge}{\mu} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$ and $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$. As $\overset{\wedge}{\mu} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$, so $\overset{\wedge}{\mu}$ is an i-v T fuzzy sub-Γ-semigroup of G. Let x, y, $z \in G$ and β, $\gamma \in \Gamma$ and $a = x\beta y\gamma z$. Also since $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$, we have

$$\begin{aligned}
\overset{\wedge}{\mu}(x\beta y\gamma z) &= \overset{\wedge}{\mu}(a) \\
&\geq (\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu})(a) \\
&= \bigvee_{a=x\beta y\gamma z} [(\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi})(x) \bigtriangleup \overset{\wedge}{\mu}(z)] \\
&\geq (\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi})(p) \bigtriangleup \overset{\wedge}{\mu}(z) (\text{ let } p = x\beta y) \\
&= \bigvee_{p=x\beta y} [\overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\chi}(y)] \bigtriangleup \overset{\wedge}{\mu}(z) \\
&= \bigvee_{p=x\beta y} [\overset{\wedge}{\mu}(x) \bigtriangleup [1,1]] \bigtriangleup \overset{\wedge}{\mu}(z) \\
&= \bigvee_{p=x\beta y} \overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\mu}(z) \\
&\geq \overset{\wedge}{\mu}(x) \bigtriangleup \overset{\wedge}{\mu}(z).
\end{aligned}$$

Hence $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G. □

Lemma 6.10. *Let $\overset{\wedge}{\mu}$ is an i-v fuzzy subset of a Γ-semigroup G. Then $\overset{\wedge}{\mu}$ is an i-v T-fuzzy interior ideal of G if and only if $\overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \subseteq \overset{\wedge}{\mu}$.*

Proof. Let $\hat{\mu}$ is an i-v T-fuzzy interior ideal of G and $a \in G$, $\alpha, \beta \in \Gamma$. If $(\hat{\chi} \circ \hat{\mu} \circ \hat{\chi})(a) = [0,0]$ then $(\hat{\chi} \circ \hat{\mu} \circ \hat{\chi})(a) \leq \hat{\mu}(a)$. Otherwise,

$$\begin{aligned}
(\hat{\chi} \circ \hat{\mu} \circ \hat{\chi})(a) &= \bigvee_{a=v\alpha z} [(\hat{\chi} \circ \hat{\mu})(v) \vartriangle \hat{\chi}(z)] \\
&= \bigvee_{a=v\alpha z} [\bigvee_{v=x\beta y} [\hat{\chi}(x) \vartriangle \hat{\mu}(y)] \vartriangle \hat{\chi}(z)] \\
&= \bigvee_{a=v\alpha z} [\bigvee_{v=x\beta y} [[1,1] \vartriangle \hat{\mu}(y)] \vartriangle [1,1]] \\
&= \bigvee_{a=v\alpha z} [\bigvee_{v=x\beta y} \hat{\mu}(y)] \\
&= \bigvee_{a=x\beta y\alpha z} \hat{\mu}(y) \\
&\leq \bigvee_{a=x\beta y\alpha z} \hat{\mu}(x\beta y\alpha z) \\
&= \hat{\mu}(a).
\end{aligned}$$

Thus $\hat{\chi} \circ \hat{\mu} \circ \hat{\chi} \subseteq \hat{\mu}$. Conversely, let $\hat{\chi} \circ \hat{\mu} \circ \hat{\chi} \subseteq \hat{\mu}$ and $x, y, z \in G$, $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
\hat{\mu}(x\alpha y\beta z) &= \hat{\mu}(a) \geq (\hat{\chi} \circ \hat{\mu} \circ \hat{\chi})(a) \\
&= \bigvee_{a=x\alpha y\beta z} [(\hat{\chi} \circ \hat{\mu})(x\alpha y) \vartriangle \hat{\chi}(z)] \\
&\geq (\hat{\chi} \circ \hat{\mu})(p) \vartriangle \hat{\chi}(z) \\
&= \bigvee_{p=x\alpha y} [[\hat{\chi}(x) \vartriangle \hat{\mu}(y)] \vartriangle \hat{\chi}(z)] \\
&= \bigvee_{p=x\alpha y} [[1,1] \vartriangle \hat{\mu}(y) \vartriangle [1,1]] \\
&= \bigvee_{p=x\alpha y} \hat{\mu}(y) \\
&\geq \hat{\mu}(y).
\end{aligned}$$

Thus $\hat{\mu}(x\alpha y\beta z) \geq \hat{\mu}(y)$. So $\hat{\mu}$ is an i-v T-fuzzy interior ideal of G. □

Remark 6.11. Every i-v T-fuzzy ideal is an i-v T-fuzzy interior ideal of a Γ-semigroup G but the converse is not true.

Lemma 6.12. *Let $\hat{\mu}$ is an i-v fuzzy subset of a regular Γ-semigroup G. Then the following conditions are equivalent.*

(1) $\hat{\mu}$ *is an i-v T-fuzzy ideal of G.*

(2) $\hat{\mu}$ *is an i-v T-fuzzy interior ideal of G.*

Proof. Suppose (1) holds. By Lemma 6.7, $\hat{\mu}$ is an i-v T-fuzzy ideal of G if and only if $\hat{\chi} \circ \hat{\mu} \subseteq \hat{\mu}$ and $\hat{\mu} \circ \hat{\chi} \subseteq \hat{\mu}$. By Lemma 6.10, $\hat{\mu}$ is an i-v T-fuzzy interior ideal of G if and only if $\hat{\chi} \circ \hat{\mu} \circ \hat{\chi} \subseteq \hat{\mu} \circ \hat{\chi} \subseteq \hat{\mu}$.

Conversely, suppose (2) holds. Let $a, b \in G$ and $\alpha \in \Gamma$. Since G is regular, so there exist elements $x, y \in G$ and $\beta, \gamma \in \Gamma$ such that $a = a\beta x\gamma a$ and $b = b\beta y\gamma b$, we have $\hat{\mu}(a\alpha b) = \hat{\mu}((a\beta x\gamma a)\alpha b) = \hat{\mu}((a\beta x)\gamma a\alpha b) \geq \hat{\mu}(a)$ and $\hat{\mu}(a\alpha b) = \hat{\mu}(a\alpha(b\beta y\gamma b)) = \hat{\mu}(a\alpha b\beta(y\gamma b)) \geq \hat{\mu}(b)$. Thus $\hat{\mu}$ is an i-v T-fuzzy ideal of G. □

Lemma 6.13. *Let A be a non-empty subset of a Γ-semigroup G. Then A is a bi-ideal (sub-Γ-semigroup, left ideal, right ideal, two sided ideal, interior ideal, generalized bi-ideal) of G if and only if the i-v characteristic function $\hat{\chi}_A$ of A is an i-v T-fuzzy bi-ideal (sub-Γ-semigroup, left ideal, right ideal, two sided ideal, interior ideal, generalized bi-ideal) of G, respectively.*

Proof. Let A be a bi-ideal of G and x, y, $z \in G$, α, β, $\gamma \in \Gamma$. If $x \notin A$ or $z \notin A$, then $\hat{\chi}_A(x) = [0,0]$ or $\hat{\chi}_A(z) = [0,0]$. This implies $\hat{\chi}_A(x\alpha y\beta z) \geq [0,0] = \hat{\chi}_A(x) \vartriangle \hat{\chi}_A(z)$ and $\hat{\chi}_A(x\gamma z) \geq [0,0] = \hat{\chi}_A(x) \vartriangle \hat{\chi}_A(z)$. If $x \in A$ and $z \in A$, then $\hat{\chi}_A(x) = \hat{\chi}_A(z) = [1,1]$. Since A is a bi-Γ-ideal. So $x\alpha y\beta z \in A\Gamma G\Gamma A \subseteq A$ and $x\gamma z \in A\Gamma A \subseteq A$. Thus $\hat{\chi}_A(x\alpha y\beta z) = [1,1] \geq \hat{\chi}_A(x) \vartriangle \hat{\chi}_A(z)$ and $\hat{\chi}_A(x\gamma z) = [1,1] \geq \hat{\chi}_A(x) \vartriangle \hat{\chi}_A(z)$. Hence $\hat{\chi}_A$ is an i-v T-fuzzy bi-ideal of G.

Conversely, assume that $\hat{\chi}_A$ is an i-v T-fuzzy bi-ideal of G and x, $z \in A$. Then $\hat{\chi}_A(x) = \hat{\chi}_A(z) = [1,1]$. This implies $\hat{\chi}_A(x) \vartriangle \hat{\chi}_A(z) = [1,1]$. Now, for any $y \in G$ and α, β, $\gamma \in \Gamma$, we have $\hat{\chi}_A(x\alpha y\beta z) \geq \hat{\chi}_A(x) \vartriangle \hat{\chi}_A(z) = [1,1]$ and $\hat{\chi}_A(x\gamma z) \geq \hat{\chi}_A(x) \vartriangle \hat{\chi}_A(z) = [1,1]$. Thus $x\alpha y\beta z \in A$ and $x\gamma z \in A$. Hence A is a bi-Γ-ideal of G. □

A Γ-semigroup G is called an i-v T-fuzzy left (right) simple if every i-v T-fuzzy left (right) ideal of G is a constant function.

Definition 6.14. A Γ-semigroup G is called an i-v T-fuzzy two-sided simple if every i-v T-fuzzy two-sided ideal of G is a constant function.

Lemma 6.15. *For a Γ-semigroup G, the following conditions are equivalent.*
(1) *G is left (right) simple.*
(2) *G is i-v -fuzzy left (right) simple.*

Proof. (1)⇒(2) First assume that G is left simple. Let $\hat{\mu}$ be an i-v T-fuzzy left ideal of G and a, $b \in G$, $\gamma \in \Gamma$ such that $b = x\gamma a$ and $a = y\gamma b$. Since $\hat{\mu}$ is an i-v T-fuzzy left ideal of G, so $\hat{\mu}(a) = \hat{\mu}(y\gamma b) \geq \hat{\mu}(b) = \hat{\mu}(x\gamma a) \geq \hat{\mu}(a)$. Thus $\hat{\mu}(a) = \hat{\mu}(b)$. Since a and b are any elements of G, this means that $\hat{\mu}$ is a constant function and so G is T-fuzzy left simple.

(2)⇒(1) Conversely, assume that (2) holds. Let A be any left Γ-ideal of G. Then, by Lemma 6.14, $\hat{\chi}_A$ is an i-v T-fuzzy left ideal of G. Thus $\hat{\chi}_A$ is a constant function. Let $x \in G$. Then, $\hat{\chi}_A(x) = [1,1]$ and so, $x \in A$. This implies that $G \subseteq A$, that is, $G = A$. Hence G is left simple. □

7. Interval valued T-fuzzy Quasi-ideal

Lemma 7.1. *Let A be a non-empty subset of a Γ-semigroup G. Then A is a quasi-ideal of G if and only if the i-v characteristic function $\hat{\chi}_A$ of A is an i-v T-fuzzy quasi-ideal of G.*

Proof. Assume that A is a quasi-ideal of G and $a \in A$. Then $((\hat{\chi}_A \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\chi}_A))(a) \leq [1,1] = \hat{\chi}_A(a)$. If $a \notin A$, then $\hat{\chi}_A(a) = [0,0]$. On the other hand, if $((\hat{\chi}_A \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\chi}_A))(a) = [1,1]$, then for $\gamma \in \Gamma$, $\bigvee_{a=p\gamma q} [\hat{\chi}_A(p) \triangle \hat{\chi}(q)] = (\hat{\chi}_A \circ \hat{\chi})(a) = [1,1]$ and for $\alpha \in \Gamma$, $\bigvee_{a=r\alpha t} [\hat{\chi}(r) \triangle \hat{\chi}_A(t)] = (\hat{\chi} \circ \hat{\chi}_A)(a) = [1,1]$. This implies that b, c, d, $e \in G$ and β, $\delta \in \Gamma$ with $a = b\beta c = d\delta e$ such that $\hat{\chi}_A(b) = [1,1]$ and $\hat{\chi}_A(e) = [1,1]$, so $b \in A$ and $e \in A$. Hence $a = b\beta c = d\delta e \in A\Gamma G \cap G\Gamma A \subseteq A$, which contradicts the fact that $a \notin A$. Thus we have $(\hat{\chi}_A \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\chi}_A) \subseteq \hat{\chi}_A$ and so $\hat{\chi}_A$ is an i-v T-fuzzy quasi ideal of G. Conversely, assume that $\hat{\chi}_A$ is an i-v T-fuzzy quasi ideal of G and $a \in A\Gamma G \cap G\Gamma A$. Then there exist s, $t \in G$, β, $\gamma \in \Gamma$ and b, $c \in A$ such that $a = b\beta s = t\gamma c$. Thus we have

$$\begin{aligned}(\hat{\chi}_A \circ \hat{\chi})(a) &= \bigvee_{a=p\alpha q} [\hat{\chi}_A(p) \triangle \hat{\chi}(q)] \\ &\geq \hat{\chi}_A(b) \triangle \hat{\chi}(s) \\ &= [1,1] \triangle [1,1] = [1,1],\end{aligned}$$

and so $(\hat{\chi}_A \circ \hat{\chi})(a) = [1,1]$. Similarly, we have $(\hat{\chi} \circ \hat{\chi}_A)(a) = [1,1]$. Since $\hat{\chi}_A(a) \geq ((\hat{\chi}_A \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\chi}_A))(a) = [1,1]$. We have $a \in A$, β, $\gamma \in \Gamma$ and so $A\Gamma G \cap G\Gamma A \subseteq A$. Hence A is a quasi ideal of G. □

Proposition 7.2. *Every i-v T-fuzzy one-sided ideal of a Γ-semigroup G is an i-v T-fuzzy quasi ideal of G.*

Proof. Let $\hat{\mu}$ is an i-v T- fuzzy one-sided ideal of G then $\hat{\chi} \circ \hat{\mu} \subseteq \hat{\mu}$. If $((\hat{\mu} \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\mu}))(x) = [0,0]$, then $((\hat{\mu} \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\mu}))(x) = [0,0] \leq \hat{\mu}(x)$. Otherwise,

$$\begin{aligned}((\hat{\mu} \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\mu}))(x) &= (\hat{\mu} \circ \hat{\chi})(x) \triangle (\hat{\chi} \circ \hat{\mu})(x) \\ &= \bigvee_{x=p\gamma q} [\hat{\mu}(p) \triangle \hat{\chi}(q)] \triangle \bigvee_{x=p\gamma q} [\hat{\chi}(p) \triangle \hat{\mu}(q)] \\ &\leq \bigvee_{x=p\gamma q} [\hat{\mu}(p) \triangle [1,1]] \triangle \bigvee_{x=p\gamma q} [[1,1] \triangle \hat{\mu}(q)] \\ &= \bigvee_{x=p\gamma q} [\hat{\mu}(p) \triangle [1,1] \triangle [1,1] \triangle \hat{\mu}(q)] \\ &= \bigvee_{x=p\gamma q} [\hat{\mu}(p) \triangle \hat{\mu}(q)] \leq \hat{\mu}(x).\end{aligned}$$

Thus $(\hat{\mu} \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\mu} \subseteq \hat{\mu}(x)$, which implies that $\hat{\mu}$ is an i-v T-fuzzy quasi ideal of G. □

Proposition 7.3. *Every i-v T-fuzzy quasi ideal of a Γ-semigroup G is an i-v T-fuzzy bi-ideal of G.*

Proof. Let $\hat{\mu}$ be an i-v T- fuzzy quasi ideal of G and $x \in G$. If $(\hat{\mu} \circ \hat{\mu})(x) = [0,0]$ then $(\hat{\mu} \circ \hat{\mu})(x) = [0,0] \leq \hat{\mu}(x)$. Otherwise,

$$\begin{aligned}
(\hat{\mu} \circ \hat{\mu})(x) &= \bigvee_{x=m\beta n} [\hat{\mu}(m) \triangle \hat{\mu}(n)] = \bigvee_{x=m\beta n} [[\hat{\mu}(m) \triangle [1,1]] \triangle [[1,1] \triangle \hat{\mu}(n)]] \\
&= \bigvee_{x=m\beta n} [[\hat{\mu}(m) \triangle \hat{\chi}(n)] \triangle [\hat{\chi}(m) \triangle \hat{\mu}(n)]] \\
&\leq \bigvee_{x=m\beta n} [\hat{\mu}(m) \triangle \hat{\chi}(n)] \triangle \bigvee_{x=m\beta n} [\hat{\chi}(m) \triangle \hat{\mu}(n)] \\
&= (\hat{\mu} \circ \hat{\chi})(x) \triangle (\hat{\chi} \circ \hat{\mu})(x) \\
&= ((\hat{\mu} \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\mu}))(x) \\
&\leq \hat{\mu}(x)
\end{aligned}$$

because $\hat{\mu}$ is an i-v T- fuzzy quasi ideal of G. Hence $\hat{\mu} \circ \hat{\mu} \subseteq \hat{\mu}$, so $\hat{\mu}$ is an i-v T- fuzzy sub-Γ-semigroup of G. If $(\hat{\mu} \circ \hat{\chi} \circ \hat{\mu})(x) = [0,0]$ then $(\hat{\mu} \circ \hat{\chi} \circ \hat{\mu})(x) = [0,0] \leq \hat{\mu}(x)$. Otherwise,

$$\begin{aligned}
(\hat{\mu} \circ \hat{\chi} \circ \hat{\mu})(x) &= \bigvee_{x=p\gamma q} [(\hat{\mu} \circ \hat{\chi})(p) \triangle \hat{\mu}(q)] \\
&= \bigvee_{x=p\gamma q} [\bigvee_{p=u\beta v} [\hat{\mu}(u) \triangle \hat{\chi}(v)] \triangle \hat{\mu}(q)] \\
&= \bigvee_{x=u\beta v\gamma q} [[\hat{\mu}(u) \triangle [1,1]] \triangle \hat{\mu}(q)] \\
&= \bigvee_{x=u\beta v\gamma q} [[\hat{\mu}(u) \triangle [1,1] \triangle [1,1]] \triangle \hat{\mu}(q)] \\
&= \bigvee_{x=u\beta v\gamma q} [[\hat{\mu}(u) \triangle [1,1]] \triangle [[1,1] \triangle \hat{\mu}(q)]] \\
&\leq \bigvee_{x=u\beta v\gamma q} [[\hat{\mu}(u) \triangle [1,1]] \triangle \bigvee_{x=u\beta v\gamma q} [[1,1] \triangle \hat{\mu}(q)]] \\
&= \bigvee_{x=u\beta v\gamma q} [[\hat{\mu}(u) \triangle \hat{\chi}(v\gamma q)] \triangle \bigvee_{x=u\beta v\gamma q} [\hat{\chi}(u\beta v) \triangle \hat{\mu}(q)]] \\
&= (\hat{\mu} \circ \hat{\chi})(x) \triangle (\hat{\chi} \circ \hat{\mu})(x) \\
&= ((\hat{\mu} \circ \hat{\chi}) \triangle (\hat{\chi} \circ \hat{\mu}))(x) \\
&\leq \hat{\mu}(x)
\end{aligned}$$

because $\overset{\wedge}{\mu}$ is an i-v T- fuzzy quasi ideal of G. Thus $\overset{\wedge}{\mu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$, which implies that $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G. □

Remark 7.4. The converse of the Proposition 7.2 and Proposition 7.3 do not hold in general.

Example 7.5. Let $G = \{0,\ a,\ b,\ c\}$ be a Γ-semigroup with $\Gamma = \{\gamma\}$ and the multiplication table defined as:

γ	0	a	b	c
0	0	0	0	0
a	0	a	b	0
b	0	0	0	0
c	0	c	0	0

Then $Q = \{0, a\}$ is a quasi Γ-ideal of G and is not a left (right) ideal of G. Define an i-v fuzzy subset $\overset{\wedge}{\mu}$ of G by $\overset{\wedge}{\mu}(0) = \overset{\wedge}{\mu}(a) = [0.7, 0.8]$ and $\overset{\wedge}{\mu}(b) = \overset{\wedge}{\mu}(c) = [0, 0]$. Using interval t-norm associated with Lukasiewiez t-norm, it can be verified that $\overset{\wedge}{\mu}$ is an i-v T-fuzzy quasi ideal of G and not an i-v T-fuzzy left (right)ideal of G, because $\overset{\wedge}{\mu}(c) = \overset{\wedge}{\mu}(c\gamma a) \ngeq \overset{\wedge}{\mu}(a)$.

Example 7.6. Let $G = \{0, a, b, c\}$ be a Γ-semigroup with $\Gamma = \{\gamma\}$ and the multiplication table defined as:

γ	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	a
c	0	0	a	b

Then $B = \{0, b\}$ is a bi-Γ-ideal of G and is not a quasi Γ-ideal of G. Define an i-v fuzzy subset $\overset{\wedge}{\mu}$ of G by $\overset{\wedge}{\mu}(0) = \overset{\wedge}{\mu}(b) = [0.7, 0.8]$ and $\overset{\wedge}{\mu}(a) = \overset{\wedge}{\mu}(c) = [0, 0]$. Using interval t-norm associated with Lukasiewiez t-norm, it can be verified that $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G and not an i-v T-fuzzy quasi ideal of G.

Proposition 7.7. *Let $\overset{\wedge}{\lambda}$ and $\overset{\wedge}{\mu}$ be i-v T-fuzzy right and left ideals of a Γ-semigroup G, respectively. Then $\overset{\wedge}{\lambda} \vartriangle \overset{\wedge}{\mu}$ is an i-v T-fuzzy quasi ideal of G.*

Proof. Since $((\overset{\wedge}{\lambda} \vartriangle \overset{\wedge}{\mu}) \circ \overset{\wedge}{\chi}) \vartriangle (\overset{\wedge}{\chi} \circ (\overset{\wedge}{\lambda} \vartriangle \overset{\wedge}{\mu})) \subseteq (\overset{\wedge}{\lambda} \circ \overset{\wedge}{\chi}) \vartriangle (\overset{\wedge}{\chi} \circ \overset{\wedge}{\mu}) \subseteq \overset{\wedge}{\lambda} \vartriangle \overset{\wedge}{\mu}$, which implies that $\overset{\wedge}{\lambda} \vartriangle \overset{\wedge}{\mu}$ is an i-v T-fuzzy quasi ideal of G. □

8. Interval valued T-fuzzy ideals in regular Γ-semigroups

Lemma 8.1. *Every i-v T-fuzzy generalized bi-ideal of a regular Γ-semigroup G is an i-v T-fuzzy bi-ideal of G.*

Proof. Let $\overset{\wedge}{\mu}$ be any i-v T-fuzzy generalized bi-ideal of a regular Γ-semigroup G. Then for any $a, b \in G$ and $\gamma \in \Gamma$ there exists $x \in G$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$. Now $\overset{\wedge}{\mu}(a\gamma b) = \overset{\wedge}{\mu}((a\alpha x\beta a)\gamma b) = \overset{\wedge}{\mu}(a\alpha(x\beta a)\gamma b) \geq \overset{\wedge}{\mu}(a) \vartriangle \overset{\wedge}{\mu}(b)$. Implies $\overset{\wedge}{\mu}(a\gamma b) \geq \overset{\wedge}{\mu}(a) \vartriangle \overset{\wedge}{\mu}(b)$, which implies that $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G. □

Theorem 8.2. *For a Γ-semigroup G, the following conditions are equivalent.*
(1) *G is regular.*
(2) $\hat{\mu} \triangle \hat{\mu} \subseteq \hat{\mu} \circ \hat{\chi} \circ \hat{\mu}$ *for every i-v T-fuzzy quasi-ideal $\hat{\mu}$ of G.*
(3) $\hat{\mu} \triangle \hat{\mu} \subseteq \hat{\mu} \circ \hat{\chi} \circ \hat{\mu}$ *for every i-v T-fuzzy bi-ideal $\hat{\mu}$ of G.*
(4) $\hat{\mu} \triangle \hat{\mu} \subseteq \hat{\mu} \circ \hat{\chi} \circ \hat{\mu}$ *for every i-v T-fuzzy generalized bi-ideal $\hat{\mu}$ of G.*

Proof. (1) $\Rightarrow$ (4) Let G be a regular Γ-semigroup, $\hat{\mu}$ be any i-v T-fuzzy generalized bi-ideal of G. Let $a \in G$ then there exist $x \in G$ and α, $\beta \in \Gamma$ such that $a = a\alpha x\beta a$. Hence we have

$$\begin{aligned}
(\hat{\mu} \circ \hat{\chi} \circ \hat{\mu})(a) &= \bigvee_{a=y\gamma z} [(\hat{\mu} \circ \hat{\chi})(y) \triangle \hat{\mu}(z)] \\
&\geq (\hat{\mu} \circ \hat{\chi})(a\alpha x) \triangle \hat{\mu}(a) \\
&= \bigvee_{a\alpha x=p\alpha q} [\hat{\mu}(p) \triangle \hat{\chi}(q)] \triangle \hat{\mu}(a) \\
&\geq [\hat{\mu}(a) \triangle \hat{\chi}(x)] \triangle \hat{\mu}(a) \\
&= [\hat{\mu}(a) \triangle [1,1]] \triangle \hat{\mu}(a) \\
&= \hat{\mu}(a) \triangle \hat{\mu}(a) = (\hat{\mu} \triangle \hat{\mu})(a).
\end{aligned}$$

Thus $\hat{\mu} \circ \hat{\chi} \circ \hat{\mu} \supseteq \hat{\mu} \triangle \hat{\mu}$. (4) $\Rightarrow$ (3) $\Rightarrow$ (2) Straightforward. (2) $\Rightarrow$ (1) Let A be any quasi-ideal of G. Then we have $A\Gamma G\Gamma A \subseteq A\Gamma(G\Gamma G)\cap(G\Gamma G)\Gamma A \subseteq A\Gamma G\cap G\Gamma A \subseteq A$. Let $a \in A$. Since $\hat{C}_A$ is an i-v T-fuzzy quasi-ideal of G, so we have

$$\begin{aligned}
((\hat{\chi}_A \circ \hat{\chi}) \circ \hat{\chi}_A)(a) &= \bigvee_{a=y\gamma z} [(\hat{\chi}_A \circ \hat{\chi})(y) \triangle \hat{\chi}_A(z)] \\
&\geq \hat{\chi}_A(a) \triangle \hat{\chi}_A(a) = [1,1] \triangle [1,1] = [1,1].
\end{aligned}$$

This implies that there exist elements b, $c \in G$ and $\beta \in \Gamma$ such that $a = b\beta c$ and $(\hat{\chi}_A \circ \hat{\chi})(b) = [1,1]$ and $\hat{\chi}_A(c) = [1,1]$. Thus we have $\bigvee_{b=p\alpha q} [\hat{\chi}_A(p) \triangle \hat{\chi}(q)] = (\hat{\chi}_A \circ \hat{\chi})(b) = [1,1]$.This implies that there exist elements d, $e \in G$ and $\delta \in \Gamma$ such that $b = d\delta e$ and $\hat{\chi}_A(d) = [1,1]$. Thus d, $c \in A$ and $e \in G$ such that $a = b\beta c = (d\delta e)\beta c \in A\Gamma G\Gamma A$. Therefore, $A \subseteq A\Gamma G\Gamma A$, and so $A = A\Gamma G\Gamma A$. Hence it follows from Theorem 2.2, that G is regular. □

Theorem 8.3. *Every i-v T-fuzzy ideal $\hat{\mu}$ of a regular Γ-semigroup G satisfies $\hat{\mu} \triangle \hat{\mu} \subseteq \hat{\mu} \circ \hat{\mu} \subseteq \hat{\mu}$.*

Proof. Let $\hat{\mu}$ be an i-v T-fuzzy ideal of a regular Γ-semigroup G. Then $\hat{\mu}$ is an i-v T-fuzzy bi-ideal of G by Proposition 7.2 and Proposition 7.3. Since G is regular, by Theorem 8.2, we have $\hat{\mu} \triangle \hat{\mu} \subseteq \hat{\mu} \circ \hat{\chi} \circ \hat{\mu} = \hat{\mu} \circ (\hat{\chi} \circ \hat{\mu}) \subseteq \hat{\mu} \circ \hat{\mu} \subseteq \hat{\mu} \circ \hat{\chi} \subseteq \hat{\mu}$ by Lemma 6.7 and so $\hat{\mu} \triangle \hat{\mu} \subseteq \hat{\mu} \circ \hat{\mu} \subseteq \hat{\mu}$. □

Lemma 8.4. *Let A and B be any non-empty subsets of a Γ-semigroup G. Then the following properties hold.*

(1)$\overset{\wedge}{\chi}_A \bigtriangleup \overset{\wedge}{\chi}_B = \overset{\wedge}{\chi}_{A\cap B}$.

(2)$\overset{\wedge}{\chi}_A \circ \overset{\wedge}{\chi}_B = \overset{\wedge}{\chi}_{A\Gamma B}$.

Proof. (1) Let $a \in G$. Suppose that $a \in A \cap B$. Then $a \in A$ and $b \in B$, which implies $\overset{\wedge}{\chi}_A(a) = \overset{\wedge}{\chi}_B(b) = [1,1]$. Then $(\overset{\wedge}{\chi}_A \bigtriangleup \overset{\wedge}{\chi}_B)(a) = \overset{\wedge}{\chi}_A(a) \bigtriangleup \overset{\wedge}{\chi}_B(a) = [1,1] \bigtriangleup [1,1] = [1,1] = \overset{\wedge}{\chi}_{A\cap B}(a)$. Again, if $a \notin A \cap B$. Then $a \notin A$ or $a \notin B$, which implies $\overset{\wedge}{\chi}_A(a) = [0,0]$ or $\overset{\wedge}{\chi}_B(a) = [0,0]$. Then $(\overset{\wedge}{\chi}_A \bigtriangleup \overset{\wedge}{\chi}_B)(a) = \overset{\wedge}{\chi}_A(a) \bigtriangleup \overset{\wedge}{\chi}_B(a) = [0,0] \bigtriangleup [0,0] = [0,0] = \overset{\wedge}{\chi}_{A\cap B}(a)$.

(2) Let $a \in G$. Suppose that $a \in A\Gamma B$. Then $a = x\gamma y$ for some $x \in A$, $y \in B$ and $\gamma \in \Gamma$.Then

$$\begin{aligned}(\overset{\wedge}{\chi}_A \circ \overset{\wedge}{\chi}_B)(a) &= \bigvee_{a=u\delta v} [\overset{\wedge}{\chi}_A(u) \bigtriangleup \overset{\wedge}{\chi}_B(v)] \\ &\geq \overset{\wedge}{\chi}_A(x) \bigtriangleup \overset{\wedge}{\chi}_B(y) = [1,1] \bigtriangleup [1,1] = [1,1].\end{aligned}$$

So $(\overset{\wedge}{\chi}_A \circ \overset{\wedge}{\chi}_B)(a) = [1,1]$. Since $a \in A\Gamma B$, $\overset{\wedge}{\chi}_{A\Gamma B}(a) = [1,1]$. In this case, when $a \notin A\Gamma B$ then we have $(\overset{\wedge}{\chi}_A \circ \overset{\wedge}{\chi}_B)(a) = [0,0] = \overset{\wedge}{\chi}_{A\Gamma B}(a)$. Thus we obtain, $\overset{\wedge}{\chi}_A \circ \overset{\wedge}{\chi}_B = \overset{\wedge}{\chi}_{A\Gamma B}$. □

Theorem 8.5. *For a Γ-semigroup G, the following are equivalent.*

(1) *G is regular.*

(2) $\overset{\wedge}{\lambda} \bigtriangleup \overset{\wedge}{\mu} = \overset{\wedge}{\lambda} \circ \overset{\wedge}{\mu}$ *for every i-v T-fuzzy left ideal $\overset{\wedge}{\mu}$ and i-v T-fuzzy right ideal $\overset{\wedge}{\lambda}$ of G.*

Proof. Assume that (1) holds. Let $\overset{\wedge}{\mu}$ be any i-v T-fuzzy left ideal of G and $\overset{\wedge}{\lambda}$ be any i-v T-fuzzy right ideal of G. Then for any $x \in G$, we have

$$\begin{aligned}(\overset{\wedge}{\lambda} \circ \overset{\wedge}{\mu})(x) &= \bigvee_{x=p\gamma q} [\overset{\wedge}{\lambda}(p) \bigtriangleup \overset{\wedge}{\mu}(q)] \\ &\leq \bigvee_{x=p\gamma q} [\overset{\wedge}{\lambda}(p\gamma q) \bigtriangleup \overset{\wedge}{\mu}(p\gamma q)] \\ &= \overset{\wedge}{\lambda}(x) \bigtriangleup \overset{\wedge}{\mu}(x) = (\overset{\wedge}{\lambda} \bigtriangleup \overset{\wedge}{\mu})(x).\end{aligned}$$

This implies that $\overset{\wedge}{\lambda} \circ \overset{\wedge}{\mu} \subseteq \overset{\wedge}{\lambda} \bigtriangleup \overset{\wedge}{\mu}$. Let $a \in G$. Then there exist $x \in G$ and α, $\beta \in \Gamma$ such that $a = a\alpha x\beta a$, so

$$(\overset{\wedge}{\lambda} \circ \overset{\wedge}{\mu})(a) = \bigvee_{a=b\beta c} [\overset{\wedge}{\lambda}(b) \bigtriangleup \overset{\wedge}{\mu}(c)] \geq \overset{\wedge}{\lambda}(a\alpha x) \bigtriangleup \overset{\wedge}{\mu}(a) \geq \overset{\wedge}{\lambda}(a) \bigtriangleup \overset{\wedge}{\mu}(a) = (\overset{\wedge}{\lambda} \bigtriangleup \overset{\wedge}{\mu})(a),$$

and so $\overset{\wedge}{\lambda} \circ \overset{\wedge}{\mu} \supseteq \overset{\wedge}{\lambda} \bigtriangleup \overset{\wedge}{\mu}$. Hence $\overset{\wedge}{\lambda} \bigtriangleup \overset{\wedge}{\mu} = \overset{\wedge}{\lambda} \circ \overset{\wedge}{\mu}$. Conversely, assume that (2) holds. Let R and L be any right and left ideals of G, respectively. In order to show that $R \cap L \subseteq R\Gamma L$, let $a \in R \cap L$. By Lemma 6.14, the characteristic function $\overset{\wedge}{\chi}_R$ and $\overset{\wedge}{\chi}_L$ of R and L are i-v T-fuzzy right and left ideals of G, respectively. Thus we have

$\overset{\wedge}{\chi}_{R\Gamma L}(a) = (\overset{\wedge}{\chi}_R \circ \overset{\wedge}{\chi}_L)(a) = (\overset{\wedge}{\chi}_R \triangle \overset{\wedge}{\chi}_L)(a) = \overset{\wedge}{\chi}_{R\cap L}(a) = [1,1]$. Implies $a \in R\Gamma L$. Thus $R\cap L \subseteq R\Gamma L$. But $R\Gamma L \subseteq R\cap L$ always holds. Hence $R\Gamma L = R\cap L$. So by Theorem 2.2, G is regular. □

Theorem 8.6. *Every i-v T-fuzzy bi-ideal of a regular Γ-semigroup G is an i-v T-fuzzy quasi-ideal of G.*

Proof. Let $\overset{\wedge}{\mu}$ be any i-v T-fuzzy bi-ideal of a regular Γ-semigroup G. Then $\overset{\wedge}{\chi}\circ\overset{\wedge}{\mu}$ (resp. $\overset{\wedge}{\mu}\circ\overset{\wedge}{\chi}$) is an i-v T-fuzzy left (resp. right) ideal of G. By Theorem 8.5, we have $(\overset{\wedge}{\mu}\circ\overset{\wedge}{\chi}) \triangle (\overset{\wedge}{\chi}\circ\overset{\wedge}{\mu}) = (\overset{\wedge}{\mu}\circ\overset{\wedge}{\chi}) \circ (\overset{\wedge}{\chi}\circ\overset{\wedge}{\mu}) = \overset{\wedge}{\mu}\circ(\overset{\wedge}{\chi}\circ\overset{\wedge}{\chi})\circ\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}\circ\overset{\wedge}{\chi}\circ\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}$ by lemma 6.8 (since $\overset{\wedge}{\mu}$ is an i-v T-fuzzy bi-ideal of G). Thus $\overset{\wedge}{\mu}$ is an i-v T-fuzzy quasi-ideal of G. □

Theorem 8.7. *For a Γ-semigroup G, the following conditions are equivalent.*

(1) *G is regular.*

(2) $\overset{\wedge}{\mu}\triangle\overset{\wedge}{\lambda}\triangle\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}\circ\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu}$ *for every i-v T-fuzzy quasi-ideal $\overset{\wedge}{\mu}$ and every i-v T-fuzzy two-sided ideal $\overset{\wedge}{\lambda}$ of G.*

(3) $\overset{\wedge}{\mu}\triangle\overset{\wedge}{\lambda}\triangle\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}\circ\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu}$ *for every i-v T-fuzzy quasi-ideal $\overset{\wedge}{\mu}$ and every i-v T-fuzzy interior ideal $\overset{\wedge}{\lambda}$ of G.*

(4) $\overset{\wedge}{\mu}\triangle\overset{\wedge}{\lambda}\triangle\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}\circ\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu}$ *for every i-v T-fuzzy bi-ideal $\overset{\wedge}{\mu}$ and every i-v T-fuzzy two-sided ideal $\overset{\wedge}{\lambda}$ of G.*

(5) $\overset{\wedge}{\mu}\triangle\overset{\wedge}{\lambda}\triangle\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}\circ\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu}$ *for every i-v T-fuzzy bi-ideal $\overset{\wedge}{\mu}$ and every i-v T-fuzzy interior ideal $\overset{\wedge}{\lambda}$ of G.*

(6) $\overset{\wedge}{\mu}\triangle\overset{\wedge}{\lambda}\triangle\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}\circ\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu}$ *for every i-v T-fuzzy generalized bi-ideal $\overset{\wedge}{\mu}$ and every i-v T-fuzzy two-sided ideal $\overset{\wedge}{\lambda}$ of G.*

(7) $\overset{\wedge}{\mu}\triangle\overset{\wedge}{\lambda}\triangle\overset{\wedge}{\mu} \subseteq \overset{\wedge}{\mu}\circ\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu}$ *for every i-v T-fuzzy generalized bi-ideal $\overset{\wedge}{\mu}$ and every i-v T-fuzzy interior ideal $\overset{\wedge}{\lambda}$ of G.*

Proof. (1)⇒(7) Let G be a regular Γ-semigroup and $\overset{\wedge}{\mu}$ and $\overset{\wedge}{\lambda}$ be any T-fuzzy generalized bi-ideal and i-v T-fuzzy interior ideal of G, respectively. Then for any $a \in G$, there exist $x \in G$ and α, β, $\gamma \in \Gamma$ such that $a = a\alpha x\beta a$ $(= a\alpha x\beta a\gamma x\delta a)$.Thus

$$\begin{aligned}(\overset{\wedge}{\mu}\circ\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu})(a) &= \bigvee_{a=y\beta z}[\overset{\wedge}{\mu}(y)\triangle(\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu})(z)] \geq \overset{\wedge}{\mu}(a)\triangle(\overset{\wedge}{\lambda}\circ\overset{\wedge}{\mu})(x\beta a\gamma x\delta a)\\ &= \overset{\wedge}{\mu}(a)\triangle(\bigvee_{x\beta a\gamma x\delta a=p\gamma q}[\overset{\wedge}{\lambda}(p)\triangle\overset{\wedge}{\mu}(q)])\\ &\geq \overset{\wedge}{\mu}(a)\triangle(\overset{\wedge}{\lambda}(x\beta a\gamma x)\triangle\overset{\wedge}{\mu}(a))\\ &\geq \overset{\wedge}{\mu}(a)\triangle\overset{\wedge}{\lambda}(a)\triangle\overset{\wedge}{\mu}(a)\\ &= (\overset{\wedge}{\mu}\triangle\overset{\wedge}{\lambda}\triangle\overset{\wedge}{\mu})(a),\end{aligned}$$

and so $\hat{\mu} \circ \hat{\lambda} \circ \hat{\mu} \supseteq \hat{\mu} \bigtriangleup \hat{\lambda} \bigtriangleup \hat{\mu}$. (7)⇒(5)⇒(3)⇒(2) and (7)⇒(6)⇒(4)⇒(2) are clear. (2)⇒(1) Let $\hat{\mu}$ be an i-v T-fuzzy quasi-ideal of G. Then, as $\hat{\chi}$ is an i-v T-fuzzy two-sided ideal of G, we have $\hat{\mu} \bigtriangleup \hat{\mu} = \hat{\mu} \bigtriangleup \hat{\chi} \bigtriangleup \hat{\mu} \subseteq \hat{\mu} \circ \hat{\chi} \circ \hat{\mu}$. Thus it follows, from Theorem 8.2, that G is regular. □

Theorem 8.8. *For a Γ-semigroup G, the following conditions are equivalent.*

(1) *G is regular.*

(2) *$\hat{\mu} \bigtriangleup \hat{\lambda} \subseteq \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy quasi-ideal $\hat{\mu}$ and every i-v T-fuzzy left (right) ideal $\hat{\lambda}$ of G.*

(3) *$\hat{\mu} \bigtriangleup \hat{\lambda} \subseteq \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy bi-ideal $\hat{\mu}$ and every i-v T-fuzzy left (right) ideal $\hat{\lambda}$ of G.*

(4) *$\hat{\mu} \bigtriangleup \hat{\lambda} \subseteq \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy generalized bi-ideal $\hat{\mu}$ and every i-v T-fuzzy left (right) ideal $\hat{\lambda}$ of G.*

Proof. (1)⇒(4) Let $\hat{\mu}$ and $\hat{\lambda}$ be any i-v T-fuzzy generalized bi-ideal and i-v T-fuzzy left ideal of G, respectively. For any $a \in G$, there exist $x \in G$ and α, $\beta \in \Gamma$ such that $a = a\alpha x\beta a$. Thus we have

$$\begin{aligned}(\hat{\mu} \circ \hat{\lambda})(a) &= \bigvee_{a=y\gamma z} [\hat{\mu}(y) \bigtriangleup \hat{\lambda}(z)] \\ &\geq \hat{\mu}(a) \bigtriangleup \hat{\lambda}(x\beta a) \\ &\geq \hat{\mu}(a) \bigtriangleup \hat{\lambda}(a) = (\hat{\mu} \bigtriangleup \hat{\lambda})(a).\end{aligned}$$

and so $\hat{\mu} \circ \hat{\lambda} \supseteq \hat{\mu} \bigtriangleup \hat{\lambda}$. (4)⇒(3)⇒(2) are clear. (2)⇒(1) Since every i-v T-fuzzy right ideal of G is an i-v T-fuzzy quasi-ideal of G, so (2) implies $\hat{\mu} \bigtriangleup \hat{\lambda} \subseteq \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy right ideal $\hat{\mu}$ and every i-v T-fuzzy left ideal $\hat{\lambda}$ of G. But $\hat{\mu} \circ \hat{\lambda} \subseteq \hat{\mu} \bigtriangleup \hat{\lambda}$ always holds, so we have $\hat{\mu} \bigtriangleup \hat{\lambda} = \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy right ideal $\hat{\mu}$ and every i-v T-fuzzy left ideal $\hat{\lambda}$ of G. Thus it follows from Theorem 8.5, that G is regular. □

Theorem 8.9. *For a Γ-semigroup G, the following conditions are equivalent.*

(1) *G is regular.*

(2) *$\hat{\nu} \bigtriangleup \hat{\mu} \bigtriangleup \hat{\lambda} \subseteq \hat{\nu} \circ \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy right ideal $\hat{\nu}$, every i-v T-fuzzy quasi-ideal $\hat{\mu}$ and every i-v T-fuzzy left ideal $\hat{\lambda}$ of G.*

(3) *$\hat{\nu} \bigtriangleup \hat{\mu} \bigtriangleup \hat{\lambda} \subseteq \hat{\nu} \circ \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy right ideal $\hat{\nu}$, every i-v T-fuzzy bi-ideal $\hat{\mu}$ and every i-v T-fuzzy left ideal $\hat{\lambda}$ of G.*

(4) *$\hat{\nu} \bigtriangleup \hat{\mu} \bigtriangleup \hat{\lambda} \subseteq \hat{\nu} \circ \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy right ideal $\hat{\nu}$, every i-v T-fuzzy generalized bi-ideal $\hat{\mu}$ and every i-v T-fuzzy left ideal $\hat{\lambda}$ of G.*

Proof. (1)⇒(4) Let $\overset{\wedge}{\nu}$, $\overset{\wedge}{\mu}$ and $\overset{\wedge}{\lambda}$ be i-v T-fuzzy right ideal, i-v T-fuzzy generalized bi-ideal and i-v T-fuzzy left ideal of G, respectively.

Then for any $a \in G$, there exist $x \in G$ and α, $\beta \in \Gamma$ such that $a = a\alpha x\beta a$. Thus we have

$$\begin{aligned}
(\overset{\wedge}{\nu} \circ \overset{\wedge}{\mu} \circ \overset{\wedge}{\lambda})(a) &= \bigvee_{a=y\gamma z} [\overset{\wedge}{\nu}(y) \triangle (\overset{\wedge}{\mu} \circ \overset{\wedge}{\lambda})(z)] \\
&\geq \overset{\wedge}{\nu}(a\alpha x) \triangle (\overset{\wedge}{\mu} \circ \overset{\wedge}{\lambda})(a) \\
&\geq \overset{\wedge}{\nu}(a) \triangle (\bigvee_{a=p\delta q} [\overset{\wedge}{\mu}(p) \triangle \overset{\wedge}{\lambda}(q)]) \\
&\geq \overset{\wedge}{\nu}(a) \triangle (\overset{\wedge}{\mu}(a) \triangle \overset{\wedge}{\lambda}(x\beta a)) \\
&\geq \overset{\wedge}{\nu}(a) \triangle (\overset{\wedge}{\mu}(a) \triangle \overset{\wedge}{\lambda}(a)) \\
&= (\overset{\wedge}{\nu} \triangle \overset{\wedge}{\mu} \triangle \overset{\wedge}{\lambda})(a),
\end{aligned}$$

and so $\overset{\wedge}{\nu} \circ \overset{\wedge}{\mu} \circ \overset{\wedge}{\lambda} \supseteq \overset{\wedge}{\nu} \triangle \overset{\wedge}{\mu} \triangle \overset{\wedge}{\lambda}$. (4)⇒(3)⇒(2) are clear. (2)⇒(1) Let $\overset{\wedge}{\nu}$ and $\overset{\wedge}{\lambda}$ be any i-v T-fuzzy right ideal and i-v T-fuzzy left ideal $\overset{\wedge}{\lambda}$ of G, respectively. Then, as $\overset{\wedge}{\chi}$ is an i-v T-fuzzy quasi-ideal of G, we have $\overset{\wedge}{\nu} \triangle \overset{\wedge}{\lambda} = \overset{\wedge}{\nu} \triangle \overset{\wedge}{\chi} \triangle \overset{\wedge}{\lambda} \subseteq \overset{\wedge}{\nu} \circ \overset{\wedge}{\chi} \circ \overset{\wedge}{\lambda} \subseteq \overset{\wedge}{\nu} \circ \overset{\wedge}{\lambda}$. Since $\overset{\wedge}{\nu} \circ \overset{\wedge}{\lambda} \subseteq \overset{\wedge}{\nu} \triangle \overset{\wedge}{\lambda}$ always holds, so $\overset{\wedge}{\nu} \triangle \overset{\wedge}{\lambda} = \overset{\wedge}{\nu} \circ \overset{\wedge}{\lambda}$ for every i-v T-fuzzy right ideal $\overset{\wedge}{\nu}$ and every i-v T-fuzzy left ideal $\overset{\wedge}{\lambda}$ of G. Hence it follows from Theorem 8.5, that G is regular. □

9. Interval valued T-fuzzy ideals in Intra-regular Γ-semigroup

Definition 9.1. An i-v fuzzy subset $\overset{\wedge}{\mu}$ of a Γ-semigroup G is called an i-v T-fuzzy semiprime if $\overset{\wedge}{\mu}(a) \geq \overset{\wedge}{\mu}(a\gamma a)$ for all $a \in G$ and for all $\gamma \in \Gamma$.

Theorem 9.2. *For a Γ-semigroup G, the following conditions are equivalent.*

(1) *G is intra-regular.*

(2) *Every i-v T-fuzzy two-sided ideal $\overset{\wedge}{\mu}$ of G is an i-v T-fuzzy semiprime.*

(3) *Every i-v T-fuzzy interior ideal $\overset{\wedge}{\mu}$ of G is an i-v T-fuzzy semiprime.*

(4) *$\overset{\wedge}{\mu}(a) = \overset{\wedge}{\mu}(a\gamma a)$ for every i-v T-fuzzy two-sided ideal $\overset{\wedge}{\mu}$ of G, for all $a \in G$ and for all $\gamma \in \Gamma$.*

(5) *$\overset{\wedge}{\mu}(a) = \overset{\wedge}{\mu}(a\gamma a)$ for every i-v T-fuzzy interior ideal $\overset{\wedge}{\mu}$ of G, for all $a \in G$ and for all $\gamma \in \Gamma$.*

Proof. (1)⇒(5) Let G be an intra-regular Γ-semigroup and $\overset{\wedge}{\mu}$ be an i-v T-fuzzy interior ideal of G. For any $a \in G$, there exist x, $y \in G$ and α, β, $\gamma \in \Gamma$ such that $a = x\alpha a\beta a\gamma y$. Hence

$$\begin{aligned}
\overset{\wedge}{\mu}(a) &= \overset{\wedge}{\mu}(x\alpha a\beta a\gamma y) \geq \overset{\wedge}{\mu}(a\beta a) = \overset{\wedge}{\mu}(a\beta(x\alpha a\beta a\gamma y)) \\
&= \overset{\wedge}{\mu}((a\beta x)\alpha a\beta(a\gamma y)) \geq \overset{\wedge}{\mu}(a).
\end{aligned}$$

Thus $\hat{\mu}(a) = \hat{\mu}(a\beta a)$.

(5)⇒(4)⇒(2) and (5)⇒(3)⇒(2) are clear.

(2)⇒(1) Let $a \in G$ and $J[a\gamma a]$ be the two-sided ideal of G generated by $a\gamma a$. Then the characteristic function $\hat{\chi}_{J[a\gamma a]}$of $J[a\gamma a]$ is an i-v T-fuzzy two-sided ideal of G. Thus by hypothesis $\hat{\chi}_{J[a\gamma a]}(a) \geq \hat{\chi}_{J[a\gamma a]}(a\gamma a) = [1, 1]$. This implies that $a \in J[a\gamma a]$. Hence G is an intra-regular Γ-semigroup. □

Theorem 9.3. *Let G be an intra-regular Γ-semigroup. Then for any i-v T-fuzzy interior ideal $\hat{\mu}$ of G, for any $a, b \in G$ and $\gamma \in \Gamma$, $\hat{\mu}(a\gamma b) = \hat{\mu}(b\gamma a)$.*

Proof. Let G be an intra-regular Γ-semigroup and $\hat{\mu}$ be an i-v T-fuzzy interior ideal of G. Let a, $b \in G$ and $\gamma \in \Gamma$. Then by Theorem 9.2, we have

$$\begin{aligned}\hat{\mu}(a\gamma b) &= \hat{\mu}((a\gamma b)\gamma(a\gamma b)) = \hat{\mu}(a\gamma(b\gamma a)\gamma b)\\ &\geq \hat{\mu}(b\gamma a) = \hat{\mu}((b\gamma a)\gamma(b\gamma a))\\ &= \hat{\mu}(b\gamma(a\gamma b)\gamma a) \geq \hat{\mu}(a\gamma b).\end{aligned}$$

Thus $\hat{\mu}(a\gamma b) = \hat{\mu}(b\gamma a)$. □

Theorem 9.4. *For a Γ-semigroup G, the following conditions are euivalent.*

(1) *G is intra-regular.*

(2) *$L \cap R \subseteq L\Gamma R$ for every left ideal L and every right ideal R of G.*

(3) *$\hat{\mu} \triangle \hat{\lambda} \subseteq \hat{\mu} \circ \hat{\lambda}$ for every i-v T-fuzzy left ideal $\hat{\mu}$ and every i-v T-fuzzy right ideal $\hat{\lambda}$ of G.*

Proof. (1)⇔(2) It is well-known. (1)⇒(3) Let G be an intra-regular Γ-semigroup. Let $\hat{\mu}$ and $\hat{\lambda}$ are any i-v T-fuzzy left and i-v T-fuzzy right ideals of G, respectively. For any $a \in G$, there exist x, $y \in G$ and α, β, $\gamma \in \Gamma$ such that $a = x\alpha a\beta a\gamma y$. Hence we have

$$\begin{aligned}(\hat{\mu} \circ \hat{\lambda})(a) &= \bigvee_{a=p\delta q} [\hat{\mu}(p) \triangle \hat{\lambda}(q)]\\ &\geq \hat{\mu}(x\alpha a) \triangle \hat{\lambda}(a\gamma y)\\ &\geq \hat{\mu}(a) \triangle \hat{\lambda}(a)\\ &= (\hat{\mu} \triangle \hat{\lambda})(a).\end{aligned}$$

Thus $\hat{\mu} \triangle \hat{\lambda} \subseteq \hat{\mu} \circ \hat{\lambda}$. (3)⇒(2) Let L and R be any left ideal and right ideal of G, respectively. Then $\tilde{\chi}_L$and $\tilde{\chi}_R$are i-v T-fuzzy left and i-v T-fuzzy right ideals of G, respectively. Let a be an element of $L \cap R$. Then $a \in L$ and $a \in R$. By hypothesis we have $\tilde{\chi}_{L\Gamma R}(a) = (\tilde{\chi}_L \circ \tilde{\chi}_R)(a) \geq (\tilde{\chi}_L \triangle \tilde{\chi}_R)(a) = \tilde{\chi}_{L\cap R}(a) = [1, 1]$. Thus $a \in L\Gamma R$. This implies that $L \cap R \subseteq L\Gamma R$. □

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