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Some new properties on soft separation axioms

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Abstract. Firstly we showed that if a soft topological space (X, τ, E) is a soft T_4 space, then (X, τ, E) may not be a soft T_2 space (also may not be a soft T_3 space from Theorem 2.30). In this case we described a new soft separation axiom which is called soft $n - T_4$ space. Then we indicated that if a soft topological space (X, τ, E) is a soft $n - T_4$ space, then (X, τ, E) is a soft T_3 space. Finally, we showed that a soft discrete topological space (X, τ, E) may not be a soft T_3 space. In this case we described a new soft topological space which is called soft single point space. Then we introduced that a soft single point space is soft subspace of soft discrete space. Hence, we indicated that a soft single point space is soft $n - T_4$ space.

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Keywords: Soft set, soft closed, soft topology, soft separation axioms, soft $\mathrm{T}_i, i = 0, 1, 2, 3, 4$, soft n-$\mathrm{T}_4$ space, soft discrete space, soft single point space.

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1. Introduction

None mathematical tools can successfully deal with the several kinds of uncertainties in complicated problems in engineerig, economics, environment, sociology, medical science, etc, so Molodtsov [8] introduced the concept of a soft set in order to solve these problems in 1999. However, there are some theories such as theory of probability, theory of fuzzy sets [13], theory of intuitionistic fuzzy sets [1], theory of vague sets [4], theory of interval mathematics [5] and the theory of rough sets [9], which can be taken into account as mathematical tools for dealing with uncertainties. But these theories have their own difficulties. Maji et al. [6] introduced a few operators for soft set theory and made a more detailed theoretical study of the soft set theory. Recently, study on the soft set theory and its applications in different fields has been making progress rapidly [2, 10, 12]. Shabir and Naz [11] introduced the concept of soft topological spaces which are defined over an inital universe with fixed set of parameter. They indicated that a soft topological space gives a parameterized family of topological spaces and introduced the concept of soft open sets, soft closed sets, soft interior point, soft closure, soft discrete topological spaces and soft separation axioms. They indicated that if a soft topological space (X, τ, E) is a soft T_i space then (X, τ, E) is a soft T_{i-1} space for $i = 1, 2$. In this case, Won Keun Min indicated that if a soft topological space (X, τ, E) is a soft T_3 space then (X, τ, E) is a soft T_2 space.

In the present paper, firstly we show that if a soft topological space (X, τ, E) is a soft T_4 space, then (X, τ, E) may not be a soft T_2 space (also may not be a soft T_3 space from Theorem2.30) . In this case, we describe a new soft separation axiom which is called soft $n - T_4$ space. Then we indicate that if a soft topological space (X, τ, E) is a soft $n - T_4$ space, then (X, τ, E) is a soft T_3 space. Finally, we show that a soft discrete topological space (X, τ, E) may not be a soft T_3 space. In this case, we describe a new soft topological space which is called soft single point space. Then we introduce that a soft single point space is soft subspace of soft discrete space. Hence, we indicate that a soft single point space is soft $n - T_4$ space.

2. Preliminaries

Definition 2.1 ([8])**.** Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A, F(e)$ may be considered as the set of $e-$approximate elements of the soft set (F, A). Clearly, a soft set is not a set

Definition 2.2 ([6])**.** For two soft sets (F, A) and (G, B) over a common universe U, (F, A) is a soft subset of (G, B), denoted by $(F, A)\tilde{\subseteq}(G, B)$, if $A \subset B$ and $e \in A$, $F(e) \subseteq G(e)$. (F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A), $(F, A)\tilde{\supseteq}(G, B)$.

Definition 2.3 ([6])**.** Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4 ([6])**.** A soft set (F, A) over U is said to be a NULL soft set denoted by $\tilde{\varnothing}$ if for all $e \in A$, $F(e) = \varnothing$ (null set).

Definition 2.5 ([6])**.** A soft set (F, A) over U is said to be an absolute soft set denoted by $\tilde{A}$ if for all $e \in A, F(e) = U$. Clearly $\tilde{A}^c = \widetilde{\varnothing}$ and $\widetilde{\varnothing}^c = \tilde{A}$

Definition 2.6 ([6])**.** The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if} \quad e \in A - B \\ G(e) & \text{if} \quad e \in B - A \\ F(e) \cup G(e) & \text{if} \quad e \in A \cap B. \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.7 ([3]). The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A)\widetilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.8 ([11]). Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$, $x \notin (F, E)$,if $x \notin F(\alpha)$ for some $\alpha \in E$

Definition 2.9 ([11]). Let (F, E) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F, E) over Y denoted by (F_Y, E), is defined as follows $F_Y(\alpha) = Y \cap F(\alpha)$, for all $\alpha \in E$. In other words $(F_Y, E) = \tilde{Y}\widetilde{\cap}(F, E)$.

Definition 2.10 ([11]). Let $x \in X$, then (x, E) denotes the soft set over X for which $x(e) = \{x\}$, for all $e \in E$.

Definition 2.11 ([11]). Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

(1) $\breve{\varnothing}, \breve{X}$ belong to τ
(2) the union of any number of soft sets in τ belongs to τ
(3) the intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.12 ([11]). Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition 2.13 ([11]). Let X be an initial universe set, E be the set of parameters and let τ be the collection of all soft sets which can be defined over X. Then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Proposition 2.14 ([11]). *Let (X, τ, E) be a soft space over X. Then the collection $\tau_\alpha = \{F(\alpha) | (F, E) \in \tau\}$ for each $\alpha \in E$, defines a topology on X.*

Definition 2.15 ([11]). Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ

Proposition 2.16 ([11]). *Let (X, τ, E) be a soft space over X. Then*

(1) $\tilde{\varnothing}$, $\tilde{X}$ are closed soft sets over X
(2) the intersection of any number of soft closed sets is a soft closed set over X
(3) the union of any two soft closed sets is a soft closed set over X.

Definition 2.17 ([11]). Let (X, τ, E) be a soft topological space over X and Y be a non-empty subset of X. Then $\tau_Y = \{(F_Y, E) | (F, E) \in T\}$ is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E). We can easily verify that τ_Y is, in fact, a soft topology on Y.

Proposition 2.18 ([11]). *Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and (F, E) be a soft open set in Y. If $\widetilde{Y} \in \tau$ then $(F, E) \in \tau$*

Theorem 2.19 ([11]). *Let(Y, τ_Y, E) be a soft subspace of soft topological space (X, τ, E) and (F, E) be a soft set over X, then*

(1) *(F,E) is soft open in Y if and only if $(F,E)=\widetilde{Y}\cap(G,E)$ for some $(G,E)\in\tau$*
(2) *(F,E) is soft closed in Y if and only if $(F,E)=\widetilde{Y}\cap(G,E)$ for some soft closed set (G,E) in X.*

Definition 2.20 ([11])**.** Let (X,τ,E) be a soft topological space over X and $x,y\in X$ such that $x\neq y$. If there exist soft open sets (F,E) and (G,E) such that $x\in(F,E)$, $y\notin(F,E)$ or $y\in(G,E)$, $x\notin(G,E)$, then (X,τ,E) is called a soft T_0 space.

Definition 2.21 ([11])**.** Let (X,τ,E) be a soft topological space over X and $x,y\in X$ such that $x\neq y$. If there exist soft open sets $(F,E)$and (G,E) such that $x\in(F,E)$, $y\notin(F,E)$ and $y\in(G,E)$, $x\notin(G,E)$, then (X,τ,E) is called a soft T_1 space.

Theorem 2.22 ([11])**.** *Let (X,τ,E) be a soft topological space over X and Y be a non-empty subset of X. If $(X,\tau,E)$is a soft T_1-space then (Y,τ_Y,E) is a soft T_1 space.*

Definition 2.23 ([11])**.** Let (X,τ,E) be a soft toplogical space over X and $x,y\in X$ such that $x\neq y$. If there exist soft open sets (F,E) and (G,E) such that $x\in(F,E)$, $y\in(G,E)$ and $(F,E)\widetilde{\cap}(G,E)=\widetilde{\varnothing}$, then (X,τ,E) is called a soft T_2 space.

Definition 2.24 ([11])**.** Let (X,τ,E) be a soft toplogical space over X, (G,E) be a soft closed set in X and $x\in X$ such that $x\notin(G,E)$. If there exist soft open sets (F_1,E) and (F_2,E) such that $x\in(F_1,E)$, $(G,E)\widetilde{\subset}(F_2,E)$ and $(F_1,E)\widetilde{\cap}(F_2,E)=\widetilde{\varnothing}$, then (X,τ,E) is called a soft regular space.

Definition 2.25 ([11])**.** Let (X,τ,E) be a soft topological space over X. Then (X,τ,E) is said to be a soft T_3-space if it is soft regular and soft T_1-space.

Definition 2.26 ([11])**.** Let (X,τ,E) be a soft topological space over X, (F,E) and (G,E) soft closed sets such that $(F,E)\widetilde{\cap}(G,E)=\tilde{\varnothing}$. If there exist soft open sets (F_1,E) and (F_2,E) such that $(F,E)\tilde{\subset}(F_1,E)$, $(G,E)\tilde{\subset}(F_2,E)$ and $(F_1,E)\widetilde{\cap}(F_2,E)=\tilde{\varnothing}$, then (X,τ,E) is called a soft normal space.

Definition 2.27 ([11])**.** Let (X,τ,E) be a soft topological space over X. Then (X,τ,E) is said to be a soft T_4 space if it is soft normal and soft T_1 space.

Theorem 2.28 ([11])**.** *if (X,τ,E) is a soft T_1-space then (X,τ,E) is a soft T_0 space.*

Theorem 2.29 ([11])**.** *If (X,τ,E) is a soft T_2-space then (X,τ,E) is a soft T_1 space.*

Theorem 2.30 ([7])**.** *A soft T_3 space is soft T_2.*

3. Main results

Definition 3.1. Let X be an initial universe set, E be the set of parameters, $x\in X$ and A is subset of X. Let (A,E) is defined as $A(e)=A$, for all $e\in E$. Then $\tau=\{(A,E)|\,\forall A\subset X\}$ is a soft topology over X. Here because all soft single points are open set, τ is called the soft single topology on X and (X,τ,E) is said to be a soft single point space over X.

Proposition 3.2. *Let X be an initial universe set, E be the set of parameters. If (X, τ, E) is a soft single point space, then each soft element of (X, τ, E) is both soft open and soft closed set.*

Proposition 3.3. *Let X be an initial universe set, E be the set of parameters. If (X, τ, E) is a soft single point space, then (X, τ_e) is a discrete space for all $e \in E$.*

Theorem 3.4. *Let (X, τ, E) be a soft single point space over X. Then (X, τ, E) is a soft T_1 space.*

Proof. Let (X, τ, E) be a soft single point space over X and let $x, y \in X$ such that $x \neq y$. Then there exist soft open sets $(x, E), (y, E)$ such that $x \in (x, E) \in \tau$, $y \notin (x, E)$ and $y \in (y, E) \in \tau$, $x \notin (y, E)$. □

Theorem 3.5. *Let (X, τ, E) be a soft single point space over X. Then, (X, τ, E) is a soft T_2 space.*

Proof. Let (X, τ, E) be a soft single point space over X and let $x, y \in X$ such that $x \neq y$. Then there exist soft open sets (x, E) and (y, E) such that $x \in (x, E) \in \tau$, $y \in (y, E) \in \tau$ and $(x, E) \widetilde{\cap} (y, E) = \widetilde{\varnothing}$. □

Theorem 3.6. *Let (X, τ, E) be a soft single point space over X. Then, (X, τ, E) is a soft T_3 space.*

Proof. Let (X, τ, E) be a soft single point space over X, (G, E) be a soft closed set in X and $x \in X$ such that $x \notin (G, E)$. From Proposition 3.2, there exist soft open sets (x, E) and (G, E) such that $x \in (x, E)$, $(G, E) \widetilde{\subset} (G, E)$ and $(x, E) \widetilde{\cap} (G, E) = \widetilde{\varnothing}$. Also, from Theorem 3.4, (X, τ, E) is a soft T_1 space, so (X, τ, E) is a soft T_3 space. □

Theorem 3.7. *Let (X, τ, E) be a soft single point space over X. Then, (X, τ, E) is a soft T_4 space.*

Proof. Let (X, τ, E) be a soft single point space over X and let (F, E) and (G, E) be soft closed sets in X such that $(F, E) \widetilde{\cap} (G, E) = \tilde{\varnothing}$. From Proposition3.2, there exist soft open sets (F, E) and (G, E) such that $(F, E) \tilde{\subset} (F, E), (G, E) \tilde{\subset} (G, E)$.
Since $(F, E) \widetilde{\cap} (G, E) = \tilde{\varnothing}$, (X, τ, E) is called a soft normal space. Also from Theorem 3.4 (X, τ, E) is a soft T_1 space, so (X, τ, E) is a soft T_4 space. □

Corollary 3.8. *Let (X, τ, E) be a soft single point space over X. Then (X, τ, E) is a soft T_i space for $i = 0, 1, 2, 3, 4$.*

Proposition 3.9. *Let X be an initial universe set, E be the set of parameters. If (X, τ, E) is a soft discrete space, then each soft element of (X, τ, E) is both soft open and soft closed set.*

Theorem 3.10. *Let (X, τ, E) be a soft discrete space over X. Then, (X, τ, E) is a soft T_4 space.*

Proof. The proof is similar to the Proof of Theorem 3.7. □

Proposition 3.11. *Let X be an initial universe set, E be the set of parameters. Then Soft single point space (X, τ_1, E) is soft subspace of soft discrete space (X, τ_2, E).*

Example 3.12. Let (X, τ, E) be a soft discrete space over X. Then (X, τ, E) is not a soft T_3 space. Because, let $X = \{x, y, z\}, E = \{e_1, e_2\}$ and let τ be soft discrete topology over X. There exists a closed set $\{\varnothing, \{x\}\}$. We know $x \notin \{\varnothing, \{x\}\}$, $\{\varnothing, \{x\}\}\widetilde{\subset}\{\varnothing, \{x\}\} \in \tau$ and $x \in (x, E) = \{\{x\}, \{x\}\} \in \tau$ and then, $\{\varnothing, \{x\}\}\widetilde{\cap}\{\{x\}, \{x\}\} = \{\varnothing, \{x\}\} \neq \widetilde{\varnothing}$.

Theorem 3.13. *Let (X, τ, E) be a soft discrete space over X. Then (X, τ, E) is a soft T_2 space.*

Proof. The proof is similar to the Proof of Theorem 3.5. □

Example 3.14. Let $X = \{x, y, z\}$, $E = \{e_1, e_2\}$ and

$$\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$$

where
$F_1(e_1) = \{x, y\}$, $F_1(e_2) = \{x, z\}$,
$F_2(e_1) = \{y, z\}$, $F_2(e_2) = \{y, x\}$,
$F_3(e_1) = \{x, z\}$, $F_3(e_2) = \{y, z\}$.
$F_4(e_1) = \{y\}$, $F_4(e_2) = \{x\}$
$F_5(e_1) = \{z\}$, $F_5(e_2) = \{y\}$
$F_6(e_1) = \{x\}$, $F_6(e_2) = \{z\}$
Then (X, τ, E) is a soft topological space over X. We note that (X, τ, E) is a soft T_1 space because there exist soft open sets (F_1, E), (F_2, E), (F_3, E) such that $x \in (F_1, E)$, $y \notin (F_1, E)$ and $y \in (F_2, E)$, $x \notin (F_2, E)$; $x \in (F_1, E)$, $z \notin (F_1, E)$ and $z \in (F_3, E)$, $x \notin (F_3, E))$; $y \in (F_2, E)$, $z \notin (F_2, E)$ and $z \in (F_3, E)$, $y \notin (F_3, E)$.

Example 3.15. Let us consider the soft topology (X, τ, E) on Example 3.14**.** Now we show that (X, τ, E) is not a soft T_2 space. For $x \neq y$, $x \in (F_1, E)$, $y \in (F_2, E)$ and $(F_1, E)\widetilde{\cap}(F_2, E) = \{\{y\}, \{x\}\} \neq \widetilde{\varnothing}$. Then (X, τ, E) is not a soft T_2 space.

Example 3.16. Let us consider the soft topology (X, τ, E) on Example 3.14**.** We know that (X, τ, E) is a soft T_1 space from example 3.14. Now we show that (X, τ, E) is not a soft T_3 space. For this,

$$\tau' = \left\{\widetilde{X}, \widetilde{\varnothing}, (F_1, E)', (F_2, E)', (F_3, E)', (F_4, E)', (F_5, E)', (F_6, E)'\right\}$$

where,
$F_1'(e_1) = \{z\}$, $F_1'(e_2) = \{y\}$,
$F_2'(e_1) = \{x\}$, $F_2'(e_2) = \{z\}$,
$F_3'(e_1) = \{y\}$, $F_3'(e_2) = \{x\}$.
$F_4'(e_1) = \{x, z\}$, $F_4'(e_2) = \{y, z\}$
$F_5'(e_1) = \{x, y\}$, $F_5'(e_2) = \{x, z\}$
$F_6'(e_1) = \{y, z\}$, $F_6'(e_2) = \{x, y\}$
Then, $x \in (F_1, E) \in \tau$, $y \notin (G, E) = (F_1, E)' = \{\{z\}, \{y\}\}$ and $y \in (F_2, E) \in \tau$, $(G, E)\widetilde{\subset}(F_5, E)$. And then,

$$(F_2, E)\widetilde{\cap}(F_5, E) = \{\{y, z\}, \{y, x\}\}\widetilde{\cap}\{\{z\}, \{y\}\} = \{\{z\}, \{y\}\} \neq \widetilde{\varnothing}.$$

Thus, (X, τ, E) is not a soft regular space, so (X, τ, E) is not a soft T_3 space.

Example 3.17. Let us consider the soft topology (X, τ, E) on Example 3.14**.** We know that (X, τ, E) is a soft T_1 space from Example 3.14. Now we show that (X, τ, E) is a soft T_4 space. Here, (F_4, E), (F_5, E), (F_6, E) are soft closed sets such that $(F_5, E)\tilde{\cap}(F_6, E) = \tilde{\varnothing}$, $(F_5, E)\tilde{\cap}(F_4, E) = \tilde{\varnothing}$, $(F_4, E)\tilde{\cap}(F_6, E) = \tilde{\varnothing}$. Then there exist soft open sets (F_4, E), (F_5, E), (F_6, E) such that $(F_5, E)\tilde{\subset}(F_5, E)$, $(F_6, E)\tilde{\subset}(F_6, E)$, $(F_4, E)\tilde{\subset}(F_4, E)$ and $(F_5, E)\tilde{\cap}(F_6, E) = \tilde{\varnothing}$, $(F_5, E)\tilde{\cap}(F_4, E) = \tilde{\varnothing}$, $(F_4, E)\tilde{\cap}(F_6, E) = \tilde{\varnothing}$. And then, (X, τ, E) is a soft normal space. Therefore, (X, τ, E) is a soft T_4 space.

Remark 3.18 ([11])**.** A soft T_4 space need not be a soft T_3 space

Example 3.19. We can see it from Example 3.17 and Example 3.16. Also we can see it from Theorem3.10 and Example3.12.

Remark 3.20. A soft T_4 space need not be a soft T_2 space.

Example 3.21. We can see it from Example 3.17 and Example 3.15

Soft T_4 space may not be a soft T_3 space and soft T_2 space so that, we should describe a new soft separation axiom which is both soft T_3 space and soft T_2 space.

Definition 3.22. Let (X, τ, E) be a soft topological space over X and $x, y \in X$. Let (F, E) and (G, E) soft closed sets such that $x \in (F, E)$ and $(F, E)\tilde{\cap}(G, E) = \tilde{\varnothing}$. If there exist soft open sets (F_1, E) and (F_2, E) such that $y \in (F_2, E)$, $(F, E)\tilde{\subset}(F_1, E)$, $(G, E)\tilde{\subset}(F_2, E)$ and $(F_1, E)\tilde{\cap}(F_2, E) = \tilde{\varnothing}$, then (X, τ, E) is a soft n-normal space.

Definition 3.23. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft n-normal space and soft T_1 space, then (X, τ, E) is a soft $n - T_4$ space.

Example 3.24. Let (X, τ, E) be a soft discrete space over X and let $X = \{x, y, z\}$, $E = \{e_1, e_2\}$, then (X, τ, E) is not a soft $n - T_4$ space. Take soft closed sets $\{\{x\}, \{x, y\}\}$ and $\{\{y\}, \varnothing\}$ such that $x \in \{\{x\}, \{x, y\}\}$ and $\{\{x\}, \{x, y\}\} \tilde{\cap} \{\{y\}, \varnothing\} = \tilde{\varnothing}$. In this case,

$$y \in \{\{y\}, \{y\}\} \in \tau, \{\{y\}, \varnothing\}\tilde{\subset}\{\{y\}, \{y\}\} \in \tau, \{\{x\}, \{x, y\}\}\tilde{\subseteq}\{\{x\}, \{x, y\}\} \in \tau.$$

then,

$$\{\{x\}, \{x, y\}\}\tilde{\cap}\{\{y\}, \{y\}\} = \{\varnothing, \{y\}\} \neq \tilde{\varnothing}.$$

Hence, (X, τ, E) is not a soft n-normal space, so it is not a soft n-T_4 space.

Theorem 3.25. *Let (X, τ, E) be a soft single point space over X and $x, y \in X$. Then (X, τ, E) is a soft n-T_4 space.*

Proof. Let (X, τ, E) be a soft single point space over X. And let (F, E) and (G, E) be soft closed sets such that $x \in (F, E)$ and $(F, E)\tilde{\cap}(G, E) = \tilde{\varnothing}$. Then there exist soft open sets (F, E) and (G, E) such that $(F, E)\tilde{\subset}(F, E), (G, E)\tilde{\subset}(G, E)$ and $(F, E)\tilde{\cap}(G, E) = \tilde{\varnothing}$. And then (X, τ, E) is a soft n-normal space. Also from theorem 3.4 (X, τ, E) is a soft T_1 space, so (X, τ, E) is a soft n-T_4 space. □

Now we will give the following lemma to establish our one of the main theorems in this section.

Lemma 3.26. *Let (X, τ, E) be a soft topological space over X. And let (Y, E) be a soft closed set in X and let $(G, E)\tilde{\subset}(Y, E)$. Then, (G, E) is a soft closed set in subspace Y if and only if (G, E) is a soft closed set in X.*

Proof. $\Longrightarrow$ Since (G,E) is a soft closed set in subspace Y, there exist a soft closed set (F,E) in X such that $(G,E) = \widetilde{Y}\widetilde{\cap}(F,E)$ from Theorem2.19. Because (Y,E) and (F,E) are soft closed set in X, (G,E) is a soft closed set in X.

$\Longleftarrow$ Since (G,E) is a soft closed set in X and $(G,E) = \widetilde{Y}\widetilde{\cap}(G,E)$, (G,E) is a soft closed set in subspace Y from Theorem2.19. □

Theorem 3.27. *Let (X,τ,E) be a soft topological space over X and Y be a non empty soft set of X. If (X,τ,E) is a soft n-T_4 space and $\widetilde{Y}$ be a soft closed set, then (Y,τ_Y,E) is a soft n-T_4 space.*

Proof. Let (X,τ,E) is a soft n-T_4-space and $\widetilde{Y}$ be a soft closed set in X. Because (X,τ,E) is a soft T_1 space, (Y,τ_Y,E) is a soft T_1 space from Theorem2.22. Let (F,E) and (G,E) soft closed sets in Y such that $x \in (F,E)$. Then (F,E) and (G,E) are soft closed sets in X from Lemma3.26. Because (X,τ,E) is a soft n-T_4 space, $(F,E)\widetilde{\cap}(G,E) = \tilde{\varnothing}$. Since (X,τ,E) is a soft n-normal space, there exist soft open sets (F_1,E) and (F_2,E) such that $y \in (F_2,E)$. $(F,E)\tilde{\subset}(F_1,E)$, $(G,E)\tilde{\subset}(F_2,E)$ and $(F_1,E)\widetilde{\cap}(F_2,E) = \tilde{\varnothing}$. $\widetilde{Y}\widetilde{\cap}(F_1,E)$ and $\widetilde{Y}\widetilde{\cap}(F_2,E)$ are soft open sets in Y from Theorem2.19. In this case, $y \in \widetilde{Y}\widetilde{\cap}(F_2,E)$, $(F,E)\tilde{\subset}\widetilde{Y}\widetilde{\cap}(F_1,E)$, $(G,E)\tilde{\subset}\widetilde{Y}\widetilde{\cap}(F_2,E)$ and $(\widetilde{Y}\widetilde{\cap}(F_1,E))\widetilde{\cap}(\widetilde{Y}\widetilde{\cap}(F_2,E)) = \tilde{\varnothing}$. Hence, (Y,τ_Y,E) is a soft n-normal space, so (Y,τ_Y,E) is a soft n-T_4 space. □

Theorem 3.28. *Soft $n-T_4$ space is soft T_3 space.*

Proof. Let (X,τ,E) be a soft $n-T_4$ space over X and let $x \in X$. And let (F,E) and (G,E) be soft closed sets such that $x \in (F,E)$ and $(F,E)\widetilde{\cap}(G,E) = \tilde{\varnothing}$. Then, there exist soft open sets (F_1,E) and (F_2,E) such that $y \in (F_2,E)$, $(F,E)\tilde{\subset}(F_1,E)$, $(G,E)\tilde{\subset}(F_2,E)$ and $(F_1,E)\widetilde{\cap}(F_2,E) = \tilde{\varnothing}$. Because of $x \in (F,E)$ and $(F,E)\widetilde{\cap}(G,E) = \tilde{\varnothing}$, $x \notin (G,E)$ (for all $\alpha \in E$, $x \notin G(\alpha)$). Then, $(G,E)\tilde{\subset}(F_2,E) \in \tau$, $x \in (F,E)\tilde{\subset}(F_1,E) \in \tau$ and $(F_1,E)\widetilde{\cap}(F_2,E) = \tilde{\varnothing}$. And then, (X,τ,E) is a soft regular space. Also (X,τ,E) is a soft T_1 space, so (X,τ,E) is a soft T_3 space. □

Corollary 3.29. *Soft $n-T_4$ space $\Longrightarrow$ soft T_3 space. $\Longrightarrow$ Soft T_2 space $\Longrightarrow$ soft T_1 space $\Longrightarrow$ soft T_0 space.*

Proof. See Theorem 3.28, Theorem 2.30, Theorem 2.29 and Theorem 2.28 respectively. □

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