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Some results on domination number in products of intuitionistic fuzzy graphs

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Abstract. In this paper, the operations like join, ringsum, cartesian product, lexicographic product, tensor product, strong product, $\alpha-$ product, $\beta-$ product, $\gamma-$ product on two Intuitionistic Fuzzy Graphs (IFGs) are defined. Also we investigate some domination parameters such as, independent domination, connected domination, total domination on join, cartesian product, lexicographic product, tensor product and strong product of two IFGs.

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1. Introduction

Every branch of mathematics employs some notion of product that enables the combination or decomposition of the structure of its elements. The operations on two fuzzy graphs were defined by J.N.Mordeson and C.S.Peng [5] in 1994. The domination in product fuzzy graphs was introduced by A.Somasundaram [10] in 2005. The concept of intuitionistic fuzzy graph was introduced by Krasmmir T.Atanassov [3] in 1994. Krasmmir T.Atanassov and A.Shannon [2] defined IFG using five types of Cartesian products. R.Parvathi and S.Thilagavathi [8] defined IFHG, using six types of Cartesian products of n vertices of IFHG. The aim of this paper is to introduce and analyze the theory of domination on join, cartesian product, lexicographic product, tensor product and strong product of two IFGs.

2. Preliminaries

In this section, some basic definitions relating to IFGs are given. Also, the definition of tensor product, strong product, $\alpha-$ product, $\beta-$ product and $\gamma-$ product in IFGs are introduced. Simple IFGs are taken into consideration throughout this paper.

Definition 2.1 ([1]). Let a set E be fixed. An Intuitionistic Fuzzy set (IFS) A in E is an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in E\}$, where the function $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ determine the degree of membership and the degree of non - membership of the element $x \in E$, respectively and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Notations

1. Hereafter, $\langle \mu(v_i), (\nu(v_i) \rangle$ or simply $\langle \mu_i, \nu_i \rangle$ denotes the degrees of membership and non-membership of the vertex $v_i \in V$ such that $0 \leq \mu_i + \nu_i \leq 1$.

2. $\langle \mu(v_{ij}), (\nu(v_{ij}) \rangle$ or simply $\langle \mu_{ij}, \nu_{ij} \rangle$ denotes the degrees of membership and non-membership of the edge $(v_i, v_j) \in V \times V$ such that $0 \leq \mu_{ij} + \nu_{ij} \leq 1$.

Definition 2.2 ([8]). Let X be a universal set and let V be an IFS over X in the form $V = \{\langle v_i, \mu_i, \nu_i \rangle | v_i \in V\}$ such that $0 \leq \mu_i + \nu_i \leq 1$. *Six types of cartesian products* of n elements of V over X are defined as

$$v_1 \times_1 v_2 \times_1 v_3 \times_1 \cdots \times_1 v_n = \left\{ \left\langle \langle v_1, v_2, \cdots, v_n \rangle, \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \right\rangle | \langle v_1, v_2, \cdots, v_n \rangle \in V \right\}$$

$$\begin{aligned} v_1 \times_2 v_2 \times_2 v_3 \times_2 \cdots \times_2 v_n = \Bigg\{ \Bigg\langle & \langle v_1, v_2, \ldots, v_n \rangle, \sum_{i=1}^{n} \mu_i - \sum_{i \neq j}^{n} \mu_i \mu_j \\ & + \sum_{i \neq j \neq k}^{n} \mu_i \mu_j \mu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \neq n}^{n} \mu_i \mu_j \mu_k \cdots \mu_n \\ & + (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \Bigg\rangle | \langle v_1, v_2, \cdots, v_n \rangle \in V \Bigg\} \end{aligned}$$

$$\begin{aligned} v_1 \times_3 v_2 \times_3 v_3 \times_3 \cdots \times_3 v_n = \Bigg\{ \Bigg\langle & \langle v_1, v_2, \cdots, v_n \rangle, \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \nu_i - \sum_{i \neq j}^{n} \nu_i \nu_j \\ & + \sum_{i \neq j \neq k}^{n} \nu_i \nu_j \nu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \neq n}^{n} \nu_i \nu_j \nu_k \cdots \nu_n \\ & + (-1)^{n-1} \prod_{i=1}^{n} \nu_i \Bigg\rangle | \langle v_1, v_2, \cdots, v_n \rangle \in V \Bigg\} \end{aligned}$$

$$v_1 \times_4 v_2 \times_4 v_3 \times_4 \cdots \times_4 v_n = \{\langle \langle v_1, v_2, \cdots, v_n \rangle, min(\mu_1, \mu_2, \cdots, \mu_n), max(\nu_1, \nu_2, \cdots, \nu_n) \rangle \mid \langle v_1, v_2, \cdots, v_n \rangle \in V\}$$

$$v_1 \times_5 v_2 \times_5 v_3 \times_5 \cdots \times_5 v_n = \{\langle \langle v_1, v_2, \cdots, v_n \rangle, max(\mu_1, \mu_2, \cdots, \mu_n), min(\nu_1, \nu_2, \cdots, \nu_n) \rangle \mid \langle v_1, v_2, \cdots, v_n \rangle \in V\}$$

$$v_1 \times_6 v_2 \times_6 v_3 \times_6 \cdots \times_6 v_n = \left\{ \left\langle \langle v_1, v_2, \ldots, v_n \rangle, \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \nu_i}{n} \right\rangle \mid \langle v_1, v_2, \cdots, v_n \rangle \in V \right\}$$

It must be noted that $v_i \times_t v_j$ is an IFS , where $t = 1, 2, 3, 4, 5, 6$.

Definition 2.3 ([4, 9])**.** An *intuitionistic fuzzy graph* (IFG) is of the form $G = (V, E)$ where
(i) $V = \{v_1, v_2, ..., v_n\}$, such that $\mu_i : V \to [0, 1]$ and $\nu_i : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_i + \nu_i \leq 1$ for every $v_i \in V$, $i = 1, 2, \cdots, n$
(ii) $E \subset V \times V$ where $\mu_{ij} : V \times V \to [0, 1]$ and $\nu_{ij} : V \times V \to [0, 1]$ are such that

$$\mu_{ij} \leq \mu_i \oslash \mu_j, \quad \nu_{ij} \leq \nu_i \oslash \nu_j$$

and

$$0 \leq \mu_{ij} + \nu_{ij} \leq 1$$

where μ_{ij} and ν_{ij} are the membership and non- membership values of the edge (v_i, v_j); the values $\mu_i \oslash \mu_j$ and $\nu_i \oslash \nu_j$ can be determined by one of the six cartesian products $\times_t$, $t = 1, 2, 3, 4, 5, 6$ for all i and j given in Definition 2.2.

Note 1. When $\mu_{ij} = \nu_{ij} = 0$ for some i and j, there is no edge between v_i and v_j. Otherwise, there exists an edge between v_i and v_j.

Definition 2.4 ([7])**.** Let $G = (V, E)$ be an IFG, then the *vertex cardinality* of V is defined by $\sum_{v_i \in V} \left(\frac{1 + \mu_i - \nu_i}{2} \right)$.

Definition 2.5 ([7])**.** An edge (v_i, v_j) is said to be a *strong edge* of an IFG $G = (V, E)$, if $\mu_{ij} \geq \mu_{ij}^{\infty}$ and $\nu_{ij} \geq \nu_{ij}^{\infty}$.

Definition 2.6 ([7])**.** An IFG, $G = (V, E)$ is said to be *connected* IFG if there exist a path between every pair of vertices v_i, v_j in V. Connected IFG is also defined using strength of connectedness as follows:

(i) $\mu_{ij}^{\infty} > 0$, and $\nu_{ij}^{\infty} > 0$
(ii) $\mu_{ij}^{\infty} = 0$, and $\nu_{ij}^{\infty} > 0$
(iii) $\mu_{ij}^{\infty} > 0$, and $\nu_{ij}^{\infty} = 0$ for all $v_i, v_j \in V$.

Definition 2.7 ([6])**.** The *join* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 + G_2$, is an IFG $G = \Big(V_1 \cup V_2, E_1 \cup E_2 \cup E^{'}, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle\Big)$ where

(i) $E^{'}$ is the set of edges joining the vertices of V_1 and V_2

(ii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by

$$\langle \mu_r, \nu_r \rangle = \begin{cases} \langle \mu_i, \nu_i \rangle & \text{if } v_r \in V_1 \\ \langle \mu_p, \nu_p \rangle & \text{if } v_r \in V_2 \\ \langle max(\mu_i, \mu_p), min(\nu_i, \nu_p) \rangle & \text{if } v_r \in V_1 \cap V_2, V_1 \cap V_2 \neq \phi \end{cases}$$

(iii) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle \mu_{ij}, \nu_{ij} \rangle & \text{if } (v_r, v_s) \in E_1 \\ \langle \mu_{pq}, \nu_{pq} \rangle & \text{if } (v_r, v_s) \in E_2 \\ \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle & \text{if } \begin{cases} r \neq s, v_r \in V_1, v_s \in V_2, \\ (v_r, v_s) \notin E_1 \cup E_2 \end{cases} \\ \langle max(\mu_{ij}, \mu_{pq}), min(\nu_{ij}, \mu_{pq}) \rangle & \text{if } (v_r, v_s) \in E_1 \cap E_2 \\ \langle 0, 0 \rangle & \text{otherwise} \end{cases}$$

Definition 2.8. The *ringsum* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \oplus G_2$, is an IFG $G = (V_1 \cup V_2, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $E = ((E_1 \cup E_2) - (E_1 \cap E_2))$
(ii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by

$$\langle \mu_r, \nu_r \rangle = \begin{cases} \langle \mu_i, \nu_i \rangle & \text{if } v_r \in V_1 \\ \langle \mu_p, \nu_p \rangle & \text{if } v_r \in V_2 \\ \langle max(\mu_i, \mu_p), min(\nu_i, \nu_p) \rangle & \text{if } v_r \in V_1 \cap V_2 \end{cases}$$

(iii) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle \mu_{ij}, \nu_{ij} \rangle & \text{if } (v_r, v_s) \in E_1 \\ \langle \mu_{pq}, \nu_{pq} \rangle & \text{if } (v_r, v_s) \in E_2 \\ \langle 0, 0 \rangle & \text{if } (v_r, v_s) \in E_1 \cap E_2 \end{cases}$$

Note 2. *In the following definitions, the vertex sets under consideration are distinct.*

Definition 2.9. The *cartesian product* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \Box G_2$, is an IFG $G = (V, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2, V_1 \cap V_2 = \phi, i = 1, 2, \ldots m, p = 1, 2, \ldots n$
(ii) $E = (v_i u_p, v_j u_q)$, such that either one of the following is true :
- $(u_p, u_q) \in E_2$, when $i = j$
- $(v_i, v_j) \in E_1$, when $p = q$

(iii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by
$\langle \mu_r, \nu_r \rangle = \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle$ for all $v_r \in V$, $r = 1, 2, 3, \ldots m.n$
(iv) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle min(\mu_i, \mu_{pq}), max(\nu_i, \nu_{pq}) \rangle & \text{if } i = j, (u_p, u_q) \in E_2 \\ \langle min(\mu_p, \mu_{ij}), max(\nu_p, \nu_{ij}) \rangle & \text{if } p = q, (v_i, v_j) \in E_1 \\ \langle 0, 0 \rangle & \text{otherwise} \end{cases}$$

Definition 2.10. The *lexicographic product* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \circ G_2$, is an IFG $G = (V, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2, V_1 \cap V_2 = \phi, i = 1, 2, \ldots m, p = 1, 2, \ldots n$
(ii) $E = (v_i u_p, v_j u_q)$, such that either one of the following is true :
- $(v_i, v_j) \in E_1$, when $i \neq j$
- $(u_p, u_q) \in E_2$, when $i = j$

(iii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by
$\langle \mu_r, \nu_r \rangle = \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle$ for all $v_r \in V$, $r = 1, 2, 3, \ldots m.n$
(iv) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle min(\mu_i, \mu_{pq}), max(\nu_i, \nu_{pq}) \rangle & \text{if } i = j, (u_p, u_q) \in E_2 \\ \langle min(\mu_p, \mu_{ij}), max(\nu_p, \nu_{ij}) \rangle & \text{if } p = q, (v_i, v_j) \in E_1 \\ \langle min(\mu_p, \mu_q, \mu_{ij}), max(\nu_p, \nu_q, \nu_{ij}) \rangle & \text{if } i \neq j,\ p \neq q,\ (v_i, v_j) \in E_1 \\ \langle 0, 0 \rangle & \text{otherwise} \end{cases}$$

Definition 2.11. The *tensor product* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \otimes G_2$, is an IFG $G = (V, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2, V_1 \cap V_2 = \phi, i = 1, 2, \ldots m, p = 1, 2, \ldots n$
(ii) $E = (v_i u_p, v_j u_q)$ if $i \neq j$, $p \neq q, (v_i, v_j) \in E_1$ and $(u_p, u_q) \in E_2$
(iii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by
$\langle \mu_r, \nu_r \rangle = \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle$ for all $v_r \in V$, $r = 1, 2, 3, \ldots m.n$

(iv) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle min(\mu_{ij}, \mu_{pq}), max(\nu_{ij}, \nu_{pq}) \rangle & \text{if } \begin{cases} i \neq j, p \neq q, (v_i, v_j) \in E_1, \\ (u_p, u_q) \in E_2 \end{cases} \\ \langle 0, 0 \rangle & \text{otherwise} \end{cases}$$

Definition 2.12. The *strong product* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \boxtimes G_2$, is an IFG $G = (V, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2, V_1 \cap V_2 = \phi$, $i = 1, 2, \ldots m$, $p = 1, 2, \ldots n$

(ii) $E = (v_i u_p, v_j u_q)$, such that either one of the following is true :
- $(u_p, u_q) \in E_2$, when $i = j$, $p \neq q$
- $(v_i, v_j) \in E_1$, when $p = q$, $i \neq j$
- $(v_i, v_j) \in E_1$ and $(u_p, u_q) \in E_2$, when $i \neq j, p \neq q$

(iii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by
$\langle \mu_r, \nu_r \rangle = \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle$ for all $v_r \in V$, $r = 1, 2, 3, \ldots m.n$

(iv) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle min(\mu_i, \mu_{pq}), max(\nu_i, \nu_{pq}) \rangle & \text{if } i = j, (u_p, u_q) \in E_2 \\ \langle min(\mu_p, \mu_{ij}), max(\nu_p, \nu_{ij}) \rangle & \text{if } p = q, (v_i, v_j) \in E_1 \\ \langle min(\mu_{ij}, \mu_{pq}), max(\nu_{ij}, \nu_{pq}) \rangle & \text{if } \begin{cases} i \neq j, p \neq q, (v_i, v_j) \in E_1, \\ (u_p, u_q) \in E_2 \end{cases} \end{cases}$$

Definition 2.13. The *α-product* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \odot G_2$, is an IFG $G = (V, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2, V_1 \cap V_2 = \phi, i = 1, 2, \ldots m, p = 1, 2, \ldots n$

(ii) $E = (v_i u_p, v_j u_q)$, such that either one of the following is true:
- $(v_i, v_j) \in E_1$ and $(u_p, u_q) \notin E_2$
- $(u_p, u_q) \in E_2$ and $(v_i, v_j) \notin E_1$

(iii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by
$\langle \mu_r, \nu_r \rangle = \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle$ for all $v_r \in V$, $r = 1, 2, 3, \ldots m.n$

(iv) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle min(\mu_i, \nu_j, \mu_{pq}), max(\nu_i, \nu_j, \nu_{pq}) \rangle & \text{if } (v_i, v_j) \notin E_1 \text{ and } (u_p, u_q) \in E_2 \\ \langle min(\mu_p, \mu_q, \mu_{ij}), max(\nu_p, \nu_q, \nu_{ij}) \rangle & \text{if } (v_i, v_j) \in E_1 \text{ and } (u_p, u_q) \notin E_2 \\ \langle 0, 0 \rangle & \text{if } (v_i, v_j) \in E_1 \text{ and } (u_p, u_q) \in E_2 \end{cases}$$

Definition 2.14. The *β-product* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 * G_2$, is an IFG $G = (V, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2, V_1 \cap V_2 = \phi, i = 1, 2, \ldots m, p = 1, 2, \ldots n$

(ii) $E = (v_i u_p, v_j u_q)$, such that either one of the following is true:
- $(v_i, v_j) \in E_1$, when $p \neq q$, $i \neq j$

- $(u_p, u_q) \in E_2$, when $i \neq j$, $p \neq q$

(iii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by
$\langle \mu_r, \nu_r \rangle = \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle$ for all $v_r \in V, r = 1, 2, 3, \ldots m.n$

(iv) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle min(\mu_i, \mu_j, \mu_{pq}), max(\nu_i, \nu_j, \nu_{pq}) \rangle & \text{if } \begin{cases} i \neq j, (v_i, v_j) \notin E_1, \\ (u_p, u_q) \in E_2 \end{cases} \\ \langle min(\mu_p, \mu_q, \mu_{ij}), max(\nu_p, \nu_q, \nu_{ij}) \rangle & \text{if } \begin{cases} p \neq q, (u_p, u_q) \notin E_2, \\ (v_i, v_j) \in E_1 \end{cases} \\ \langle min(\mu_{ij}, \mu_{pq}), max(\nu_{ij}, \nu_{pq}) \rangle & \text{if } \begin{cases} i \neq j, p \neq q, (v_i, v_j) \in E_1, \\ (u_p, u_q) \in E_2 \end{cases} \\ \langle 0, 0 \rangle & \text{otherwise} \end{cases}$$

Definition 2.15. The *γ-product* of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \boxdot G_2$, is an IFG $G = (V, E, \langle \mu_r, \nu_r \rangle, \langle \mu_{rs}, \nu_{rs} \rangle)$ where

(i) $V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2, V_1 \cap V_2 = \phi$, $i = 1, 2, \ldots m$, $p = 1, 2, \ldots n$

(ii) $E = (v_i u_p, v_j u_q)$, such that either $(v_i, v_j) \in E_1$ or $(u_p, u_q) \in E_2$

(iii) $\langle \mu_r, \nu_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by
$\langle \mu_r, \nu_r \rangle = \langle min(\mu_i, \mu_p), max(\nu_i, \nu_p) \rangle$ for all $v_r \in V$, $r = 1, 2, 3, \ldots m.n$

(iv) $\langle \mu_{rs}, \nu_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \nu_{rs} \rangle = \begin{cases} \langle min(\mu_i, \mu_j, \mu_{pq}), min(\nu_i, \nu_j, \nu_{pq}) \rangle & \text{if} (v_i, v_j) \notin E_1 \text{ and } (u_p, u_q) \in E_2 \\ \langle min(\mu_p, \mu_q, \mu_{ij}), min(\nu_p, \nu_q, \nu_{ij}) \rangle & \text{if } (u_p, u_q) \notin E_2 \text{ and } (v_i, v_j) \in E_1 \\ \langle min(\mu_{ij}, \mu_{pq}), max(\nu_{ij}, \nu_{pq}) \rangle & \text{if } (v_i, v_j) \in E_1 \text{ and } (u_p, u_q) \in E_2 \\ \langle 0, 0 \rangle & \text{otherwise} \end{cases}$$

Example 2.16. Consider the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, where $V_1 = \{v_1, v_2\}, E_1 = \{(v_1, v_2)\}$ and $V_2 = \{u_1, u_2, u_3\}, E_2 = \{(u_1, u_2), (u_2, u_3)\}$ in Figure 1.

The graph of $G_1 + G_2$ is shown in Figure 2.

Example 2.17. Consider the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, in Figure 1. The graph of $G_1 \square G_2$ is displayed in Figure 3. Figure 4 depicts the graph of $G_1 \circ G_2$.

Example 2.18. Consider the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, in Figure 1. The graph of $G_1 \otimes G_2$ is displayed in Figure 5.

The graph of $G_1 \boxtimes G_2$ is displayed in Figure 6

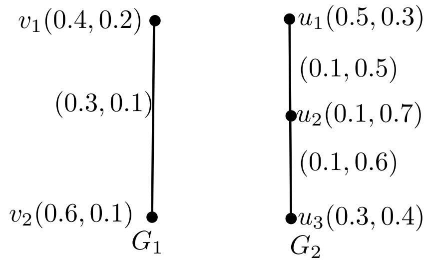

FIGURE 1.

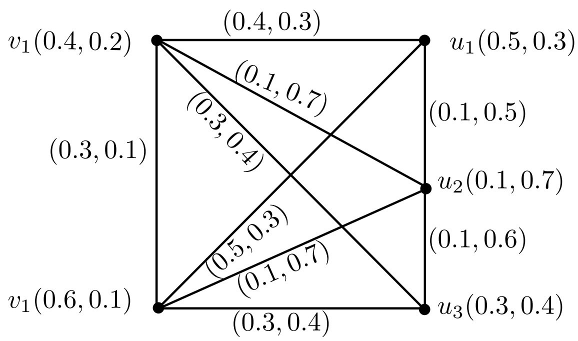

FIGURE 2. $G_1 + G_2$

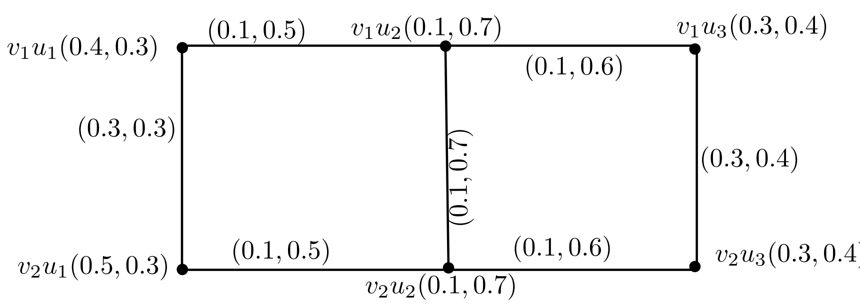

FIGURE 3. $G_1 \Box G_2$

Example 2.19. Consider the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, in Figure 1. The graph of $G_1 \odot G_2$ is displayed in Figure 7. The graph of $G_1 * G_2$ is displayed in Figure 8.

Example 2.20. Consider the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, in Figure 1. The graph of $G_1 \boxdot G_2$ is displayed in Figure 9.

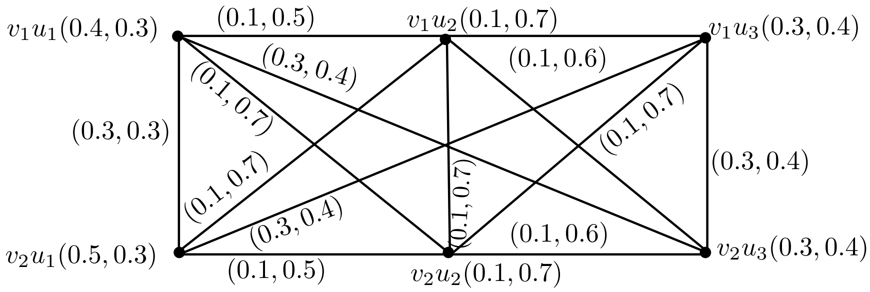

FIGURE 4. $G_1 \circ G_2$

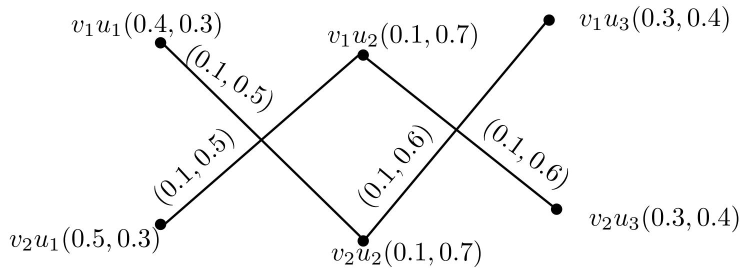

FIGURE 5. $G_1 \otimes G_2$

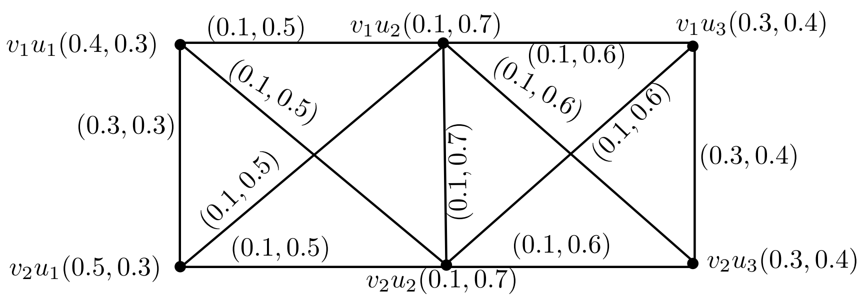

FIGURE 6. $G_1 \boxtimes G_2$

3. Domination in products of intuitionistic fuzzy graphs

Definition 3.1 ([7]). Let $G = (V, E)$ be an IFG on V. Let $u, v \in V$, u *is said to dominate* v in G if there exists a strong edge between them.

Definition 3.2 ([7]). A subset S of V is called a *dominating set* in G if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

Definition 3.3 ([7]). A dominating set S of an IFG is said to be a *minimal dominating set* if no proper subset of S is a dominating set.

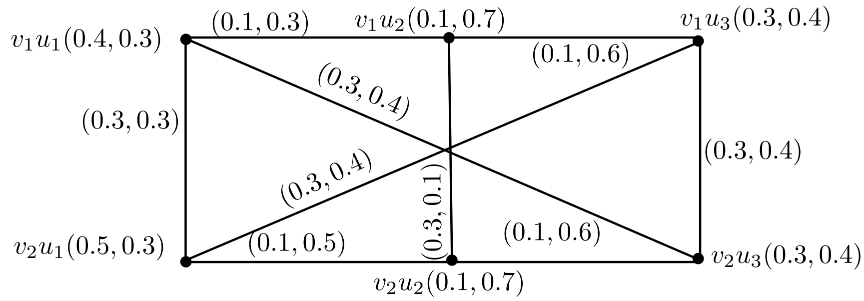

FIGURE 7. $G_1 \odot G_2$

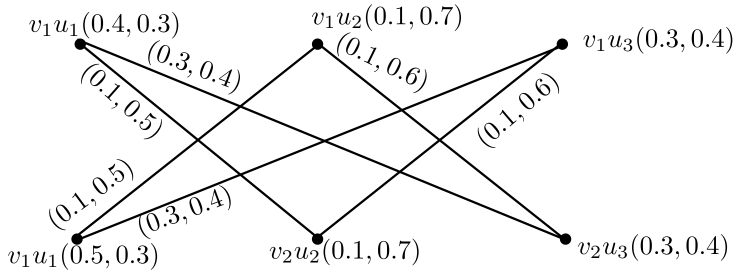

FIGURE 8. $G_1 * G_2$

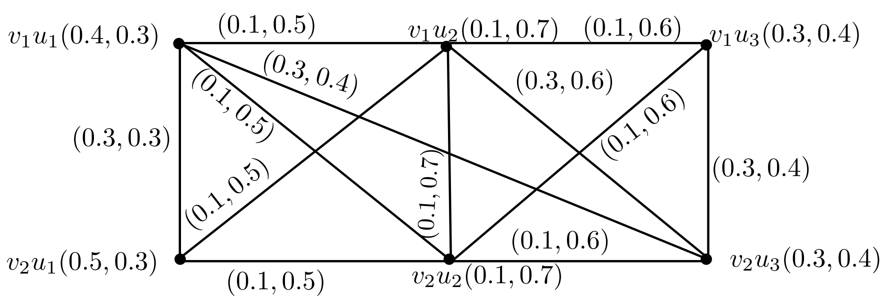

FIGURE 9. $G_1 \boxdot G_2$

Definition 3.4 ([7])**.** Minimum cardinality among all minimal dominating set is called *lower domination number* of G, and is denoted by $d(G)$.
Maximum cardinality among all minimal dominating set is called *upper domination number* of G, and is denoted by $D(G)$.

Definition 3.5 ([7])**.** Two vertices in an IFG, $G = (V, E)$ are said to be *independent* if there is no strong edge between them.

Definition 3.6 ([7])**.** A subset S of V is said to be *independent set* of G if $\mu_{ij} < \mu_{ij}^{\infty}$ and $\nu_{ij} < \nu_{ij}^{\infty}$ for all $v_i, v_j \in S$. An independent set S of G in an IFG is said to be *maximal independent*, if for every vertex $v_j \in V - S$, the set $S \cup \{v_j\}$ is not independent.

Definition 3.7. The minimum cardinality among all maximal independent set is called *lower independence number* of G, and it is denoted by $i(G)$.
The maximum cardinality among all maximal independent set is called *upper independence number* of G, and it is denoted by $I(G)$.

Definition 3.8 ([7])**.** Let $G = (V, E)$ be an IFG without isolated vertices. A subset D of V is a *total dominating set* if for every vertex $v_i \in V$, there exists a vertex $v_j \in D$, $v_i \neq v_j$, such that v_j dominates v_i.

Definition 3.9 ([7])**.** The minimum cardinality of a total dominating set is called *total domination number* of G, and it is denoted by $d_t(G)$.

Definition 3.10 ([11])**.** Let G be a connected IFG. A subset V' of V is called a *connected dominating set* of G, if

(i) For every $v_j \in V$- V', there exists $v_i \in V'$ such that $\mu_{ij} \geq \mu_{ij}^{\infty}$ and $\nu_{ij} \geq \nu_{ij}^{\infty}$
(ii) The sub graph $H = (V', E')$ of G=(V, E) induced by V' is connected.

Definition 3.11 ([7])**.** The minimum cardinality of a connected dominating set is called the *connected domination number* of G, and is denoted by $d_c(G)$.

Theorem 3.12. *Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two IFGs with $V_1 \cap V_2 = \phi$. Then*

(i) $d(G_1 + G_2) = min\left(d(G_1), d(G_2), \frac{1+\mu_i-\nu_i}{2} + \frac{1+\mu_p-\nu_p}{2}\right)$ *where* $v_i \in V_1, u_p \in V_2$.
(ii) $d_i(G_1 + G_2) = min\left(d_i(G_1), d_i(G_2)\right)$
(iii) $d_t(G_1+G_2) = min\left(d_t(G_1), d_t(G_2), \frac{1+\mu_i-\nu_i}{2} + \frac{1+\mu_p-\nu_p}{2}\right)$ *where* $v_i \in V_1, u_p \in V_2$.
(iv) *If both G_1 and G_2 have isolated vertices, then* $d_t(G_1 + G_2) = \min_{v_i \in V_1}\left(\frac{1+\mu_i-\nu_i}{2}\right) + \min_{u_p \in V_2}\left(\frac{1+\mu_p-\nu_p}{2}\right)$.
(v) *If G_1 and G_2 be a connected IFG, then* $d_c(G_1 + G_2) = min\left(d_c(G_1), d_c(G_2), \frac{1+\mu_i-\nu_i}{2} + \frac{1+\mu_p-\nu_p}{2}\right)$ *where* $v_i \in V_1, u_p \in V_2$.
(vi) *If both G_1 and G_2 be a disconnected IFG, then* $d_c(G_1 + G_2) = \min_{v_i \in V_1}\left(\frac{1+\mu_i-\nu_i}{2}\right) + \min_{u_p \in V_2}\left(\frac{1+\mu_p-\nu_p}{2}\right)$.

Proof. (i) From the definition of $G_1 + G_2$, it is obvious that any edge of the form (v_i, u_p), where $v_i \in V_1$, $u_p \in V_2$ is a strong edge. Hence, any vertex of V_1 dominates all the vertices of V_2. Let D be any minimal dominating set of $G_1 + G_2$. Then D takes either one of the following forms:

(1) $D = D_1$, if D_1 is the minimal dominating set of G_1,

(2) $D = D_2$, if D_2 is the minimal dominating set of G_2,

(3) $D = v_i, u_p$, where $v_i \in V_1, u_p \in V_2, \{v_i\}$ is not a dominating set of G_1 and $\{u_p\}$ is not a dominating set of G_2.
Hence
$d(G_1 + G_2) = min\left(d(G_1), d(G_2), \frac{1+\mu_i-\nu_i}{2} + \frac{1+\mu_p-\nu_p}{2}\right)$ where $v_i \in V_1$, $u_p \in V_2$.

(ii) By the definition of $G_1 + G_2$, every vertex of V_1 dominates every vertex of V_2, any independent set in $G_1 + G_2$ is either a subset of V_1 or a subset of V_2. Hence any minimal independent dominating set D of $G_1 + G_2$ is of the forms:

(1) $D = D_1$, if D_1 is the minimal independent dominating set of G_1,

(2) $D = D_2$, if D_2 is the minimal independent dominating set of G_2.
Thus $d_i(G_1 + G_2) = min\,(d_i(G_1), d_i(G_2))$.

(iii) Both G_1 and G_2 have no isolate vertices, $d_t(G_1)$ and $d_t(G_2)$ exists. Any minimal total dominating set D of $G_1 + G_2$, is of the following forms:

(1) $D = D_1$, if D_1 is the minimal total dominating set of G_1,

(2) $D = D_2$, if D_2 is the minimal total dominating set of G_2.

(3) $D = v_i, u_p$ where $v_i \in V_1, u_p \in V_2, \{v_i\}$ is not a dominating set of G_1 and $\{u_p\}$ is not a total dominating set of G_2.
Hence
$d_t(G_1 + G_2) = min\left(d_t(G_1), d_t(G_2), \frac{1+\mu_i-\nu_i}{2} + \frac{1+\mu_p-\nu_p}{2}\right)$ where $v_i \in V_1, u_p \in V_2$.

In, $(1), (2)$ & (3) of all the above cases, $'='$ refers to crisp set equality.

(iv) If G_1 and G_2 have isolated vertices. Then $d_t(G_1)$ and $d_t(G_2)$ do not exist. Hence any total dominating set of $G_1 + G_2$, has nonempty intersection with both V_1 and V_2. Thus $d_t(G_1 + G_2) = \min_{v_i \in V_1}\left(\frac{1+\mu_i-\nu_i}{2}\right) + \min_{u_p \in V_2}\left(\frac{1+\mu_p-\nu_p}{2}\right)$.

(v) For any two IFGs G_1 and G_2, the IFG $G_1 + G_2$ is connected and hence $d_c(G_1 + G_2)$ exists. The proof of (v) and (vi) is similar to that of (iii) and (iv).

□

Theorem 3.13. *Let D_1 and D_2 be dominating sets of two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ respectively. Then $D_1 \times D_2$ is a dominating set of $G_1 \circ G_2$, where $'\times'$ refers to the cartesian product, in crisp sense.*

Proof. Let $D_1 \subseteq V_1$ be a dominating set of G_1 and $D_2 \subseteq V_2$ be a dominating set of G_2. Let $v_i u_p \notin D_1 \times D_2$, then $v_i \notin D_1$ or $u_p \notin D_2$.
Case(i) $v_i \notin D_1$ and $u_p \in D_2$.

Let $v_j \in D_1$ be such that v_j dominates v_i. Then

$$\mu_{ij} \geq \mu_{ij}^{\infty} \geq max\left(min\left(\mu_{ij}\right)\right), \nu_{ij} \geq \nu_{ij}^{\infty} \geq min\left(max\left(\nu_{ij}\right)\right)$$

Now $v_j u_p \in D_1 \times D_2$ and

$$\begin{aligned}
\mu_{ip,jp} &= \mu_{ij} \wedge \mu_p \\
&\geq max(min(\mu_{ij})) \wedge \mu_p \\
&= max\left(min\left(\mu_i \wedge \mu_j \wedge \mu_p\right)\right) \\
&= max\left(min\left((\mu_i \wedge \mu_p), (\mu_j \wedge \mu_p)\right)\right) \\
&= max\left(min\left(\mu_{ip}, \mu_{jp}\right)\right) \\
\text{That is,} \quad \mu_{ip,jp} &\geq max\left(min\left(\mu_{ip,jq}\right)\right)
\end{aligned}$$

$$\begin{aligned}
\nu_{ip,jp} &= \nu_{ij} \vee \nu_p \\
&\geq min\left(max(\nu_{ij})\right) \vee \nu_p \\
&= min\left(max\left(\nu_i \vee \mu_j \vee \nu_p\right)\right) \\
&= min\left(max\left((\nu_i \vee \nu_p), (\nu_j \vee \nu_p)\right)\right) \\
&= min\left(max\left(\nu_{ip}, \nu_{jp}\right)\right) \\
\text{That is,} \quad \nu_{ip,jp} &\geq min(max(\nu_{ip,jp}))
\end{aligned}$$

Hence $v_j u_p$ dominates $v_i u_p$ in $G_1 \circ G_2$.
Case(ii) $v_i \in D_1$ and $u_p \notin D_2$.
Let $u_q \in D_2$ be such that u_q dominates u_p. Then

$$\mu_{pq} \geq \mu_{pq}^{\infty} \geq max(min(\mu_{pq})), \nu_{pq} \geq \nu_{pq}^{\infty} \geq min(max(\nu_{pq}))$$

Now $v_i u_q \in D_1 \times D_2$ and

$$\begin{aligned}
\mu_{ip,iq} &= \mu_i \wedge \mu_{pq} \\
&\geq \mu_i \wedge max(min(\mu_{pq})) \\
&= \mu_i \wedge max(min(\mu_p \wedge \mu_q)) \\
&= max(min(\mu_i \wedge \mu_p \wedge \mu_q)) \\
&= max(min((\mu_i \wedge \mu_p), (\mu_i \wedge \mu_q))) \\
&\geq= (min(\mu_{ip}, \mu_{iq})) \\
\text{That is,} \quad \mu_{ip,iq} &\geq max(min(\mu_{ip,iq}))
\end{aligned}$$

$$\begin{aligned}
\nu_{ip,iq} &= \nu_i \vee \nu_{pq} \\
&\geq \mu_i \vee max(min(\mu_{pq})) \\
&= \nu_i \vee max(min(\nu_p \vee \nu_q)) \\
&= min\,(max\,(\nu_i \vee \nu_q \vee \nu_q)) \\
&= min\,(max\,((\nu_i \vee \nu_q), (\nu_i \vee \ \nu_q))) \\
&= min\,(max\,(\nu_{ip}, \nu_{iq})) \\
\text{That is,} \quad \nu_{ip,iq} &\geq min\,(max\,(\nu_{ip,iq}))
\end{aligned}$$

Hence v_iu_q dominates v_iu_p in $G_1 \circ G_2$.
Case(iii) $v_i \notin D_1$ and $u_p \notin D_2$.
Let $v_j \in D_1$ and $u_p \in D_2$ be such that v_j dominates v_i in G_1 and u_q dominates u_p. Then $\mu_{ij} \geq \mu_{ij}^{\infty} \geq max(min\mu_{ij})$, $\nu_{ij} \geq \nu_{ij}^{\infty} \geq min(max\nu_{ij})$ and $\mu_{pq} \geq \mu_{pq}^{\infty} \geq max(min(\mu_{pq})$, $\nu_{pq} \geq \nu_{pq}^{\infty} \geq min(max(\nu_{pq})$
. Now $v_ju_q \in D_1 \times D_2$ and

$$\begin{aligned}
\mu_{ip,jq} &= \mu_{ij} \wedge \mu_p \wedge \mu_q \\
&\geq max(min(\mu_{ij} \wedge \mu_p \wedge \mu_q)) \\
&= max\,(min\,(\mu_i \wedge \mu_j \wedge \mu_p \wedge \mu_q)) \\
&= max\,(min\,((\mu_i \wedge \mu_p), (\mu_j \wedge \mu_q))) \\
&= max\,(min\,(\mu_{ip}, \mu_{jq})) \\
\text{That is,} \quad \mu_{ip,jq} &\geq max\,(min\,(\mu_{ip,jq}))
\end{aligned}$$

$$\begin{aligned}
\nu_{ip,jp} &= \nu_{ij} \vee \nu_p \vee \nu_q \\
&\geq min\,(max\,(\nu_{ij} \vee \nu_p \vee \nu_q)) \\
&= min\,(max\,(\nu_i \vee \mu_j \vee \nu_p \vee \nu_q)) \\
&= min\,(max\,((\nu_i \vee \nu_p), (\nu_i \vee \nu_q))) \\
&= min\,(max\,(\nu_{ip}, \nu_{jq})) \\
\text{That is,} \quad \nu_{ip,jp} &\geq min\,(max\,(\nu_{ip,jq}))
\end{aligned}$$

Hence (v_ju_q) dominates (v_iu_p) in $G_1 \circ G_2$.
Thus $D_1 \times D_2$ is a dominating set of $G_1 \circ G_2$.

□

Theorem 3.14. *Let D_1 and D_2 be minimum dominating sets of the IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. Then $d(G_1 \Box G_2) \leq min\,\{|D_1 \times V_2|, |V_1 \times D_2|\}$,where $'\times'$ refers to the cartesian product, in crisp sense.*

Proof. Let $D_1 \subseteq V_1$ be a dominating set of G_1 and $D_2 \subseteq V_2$ be a dominating set of G_2.We first prove that $D_1 \times V_2$ is a dominating set of $d(G_1 \times G_2)$. Let $v_iu_p \notin D_1 \times V_2$. Hence $v_i \notin D_1$. Since D_1 is a dominating set of G_1 there exist $v_j \in D_1$ suchthat

$\mu_{ij} \geq \mu_{ij}^{\infty} \geq max(min(\mu_{ij}))$ and $\nu_{ij} \geq \nu_{ij}^{\infty} \geq min(max(\nu_{ij}))$.
Now, $v_j u_p \in D_1 \times V_2$ and

$$\begin{aligned} \mu_{ip,jp} &= \mu_p \wedge \mu_{ij} \\ &\geq \mu_p \wedge max(min(\mu_{ij})) \\ &= \mu_p \wedge max(min(\mu_i \wedge \mu_j)) \\ &= max\,(min\,(\mu_i \wedge \mu_j \wedge \mu_p)) \\ &= max\,(min\,((\mu_i \wedge \mu_p), (\mu_j \wedge \mu_p))) \\ &= max\,(min\,(\mu_{ip}, \mu_{jp})) \\ \text{That is,} \quad \mu_{ip,jp} &\geq max\,(min\,(\mu_{ip,jp})) \end{aligned}$$

$$\begin{aligned} \nu_{ip,jq} &= \nu_p \vee \nu_{ij} \\ &\geq \nu_p \vee min(max(\nu_{ij})) \\ &= \nu_p \vee min(max(\nu_i \vee \nu_j)) \\ &= min\,(max\,(\nu_i \vee \nu_j \vee \nu_p)) \\ &= min\,(max\,((\nu_i \vee \nu_p), (\nu_j \vee \nu_p))) \\ &= min\,(max\,(\nu_{ip}, \nu_{jp})) \\ \text{That is,} \quad \nu_{ip,jq} &\geq min\,(max\,(\nu_{ip,jp})) \end{aligned}$$

Thus, $v_i u_p$ is dominated in $G_1 \times G_2$, so that $D_1 \times V_2$ is a dominating set of $G_1 \times G_2$.
Similarly $V_1 \times D_2$ is also a dominating set of $G_1 \times G_2$.
Hence $d(G_1 \times G_2) \leq min\,\{|D_1 \times V_2|\,, |V_1 \times D_2|\}$

□

Theorem 3.15. *Let D_1 and D_2 be dominating sets of connected IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ respectively .Then*

(i) *$G_1 \boxtimes G_2$ is connected*

(ii) *If D_1 is connected, then $D_1 \times V_2$ is a connected dominating set of $G_1 \boxtimes G_2$.*

(iii) *If D_2 is connected, then $V_1 \times D_2$ is a connected dominating set of $G_1 \boxtimes G_2$.*

Proof. To prove $G_1 \boxtimes G_2$ is connected, it is enough to prove that for any two arbitary distinct vertices $v_i u_p, v_j u_q$ in $G_1 \boxtimes G_2$ such that $\mu_{ip,jq} > 0$ and $\nu_{ip,jq} > 0$
Case(i) $v_i = v_j$. G_2 is a conneced IFG. Then there exist a path $p = u_1, u_2, \cdots, u_p$ such that $(\mu_{pq}, \nu_{pq}) > 0$ for each two vertices u_p, u_q of path p. This implies that, $\mu_{ip,iq} = \mu_i \wedge \mu_{pq} > 0$ and $\nu_{ip,iq} = \nu_i \vee \nu_{pq} > 0$ and hence $p^{'} = v_i u_p, v_i u_1, v_i u_2 \cdots v_i u_q$ is the path between $v_i u_p$ and $v_i u_q$ in $G_1 \boxtimes G_2$
Case(ii)
$u_p = u_q$. G_1 is a conneced IFG. Then there exist a path $q = v_1, v_2, \cdots, v_i$ such that $(\mu_{ij}, \nu_{ij}) > 0$ for each two vertices v_i, v_j of path q. This implies that, $\mu_{ip,jq} = \mu_p \wedge \mu_{ij} > 0$ and $\nu_{ip,jq} = \nu_p \vee \nu_{ij} > 0$ and hence $q^{'} = v_1 u_p, v_2 u_p, v_3 u_p \cdots v_i u_p$

is the path between v_iu_p and v_ju_p in $G_1 \boxtimes G_2$

Case(iii)

$v_i \neq v_j, u_p \neq u_q$. By case (i), there exists a path between v_iu_p and $v_iu_q$in $G_1 \boxtimes G_2$ and by case (ii), there exist a path between v_iu_p and $v_ju_p$in $G_1 \boxtimes G_2$. The union of these two disjoint paths is a path between v_iu_q, v_ju_p in $G_1 \boxtimes G_2$.

By theorem 3.14, $D_1 \times V_2$ and $V_1 \times D_2$ are dominating sets and the proof of connectivity of $D_1 \times V_2$ and $V_1 \times D_2$ is similar. □

Theorem 3.16. *Let $G_1 = (V_1, E_1$ and $G_2 = (V_2, E_2)$ be an IFGs without isolated vertices. D_1 and D_2 be minimum total dominating set of G_1 and G_2. Then $d_t(G_1 \otimes G_2) \geq \frac{1+\mu_i-\nu_i}{2} + \frac{1+\mu_p-\nu_p}{2}$, where $v_i \in D_1, u_p \in D_2$*

Proof. Let D_1 and D_2 be minimum total dominating sets of G_1 and G_2. Let v_iu_p be an arbitrary vertex of $G_1 \otimes G_2$. Then there are vertices $v_j \in D_1$ and $u_q \in D_2$ such that $(\mu_{ij}, \nu_{ij}) > 0$ in E_1 and $(\mu_{pq}, \nu_{pq}) > 0$ in E_2. Therefore $\mu_{ip,jq} > 0$ and $\nu_{ip,jq} > 0$ in $G_1 \otimes G_2$ Thus, $d_t(G_1 \otimes G_2) \geq \frac{1+\mu_i-\nu_i}{2} + \frac{1+\mu_p-\nu_p}{2}$, where $v_i \in D_1, u_p \in D_2$ □

4. Conclusion

In this paper, the concepts of domination, total domination and connected domination on join, lexicographic product, cartesian product, tensor product and strong product of two IFGs have been defined. Domination in IFGs have found many applications in network analysis, pattern clustering, routings. Further, the authors proposed to investigate other domination parameters on product of two IFGs and bipolar IFGs.

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