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Regularity and normality on soft ideal topological spaces

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Abstract. Molodtsov [14] introduced the soft set theory as a new mathematical approach to remove some uncertainties in mathematics in some fields such as social science, medical science,engineering, economic etc in 1999. Shabir and Naz [21] introduced the topological structure of soft sets and studied many properties in 2011. In this paper, in order to contribute to the study of soft topological spaces, we introduced the soft I-regularity and the soft I-normality by using the concepts of I-regularity and I-normality. Then we investigate some properties of these new notions. Moreover we obtain relations between soft I-regularity and the soft I-normality with respect to soft Lindelöf spaces.

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1. Introduction

Several concepts, vague sets [6], interval mathematics [2], rough sets [17] and fuzzy sets [23], bring up to remove uncertainties in mathematics. But these concepts are not sufficient to solve some problems in fields such as social science, medical science, engineering, economics etc. To solve these problems, Molodtsov [14] introduced soft set theory in 1999. This theory has been applied to many fields such as optimization theory, basic mathematical analysis etc.

In 2011, Shabir and Naz [21] applied this theory to topological structure. They introduced and studied some concepts such as soft topological space, soft interior, soft closure, soft subspace and soft separation axioms etc. Kharal and Ahmad [11] defined the notion of soft mappings on soft classes. Then *Aygünoğlu* and *Aygün* [3] introduced soft continuity of soft mapping, soft product topology and studied soft compactness. Moreover, they generalized Tychonoff theorem to soft topological spaces. Cagman et al. in [4] and Hazra et al. in [7] defined soft topologies in different manners. Nazmul and Samanta [16] studied the neighbourhood properties in a soft topological space. Rong [19] introduced some new concepts in soft topological spaces such as soft first-countable spaces, soft second-countable spaces and soft Lindelöf spaces. The notion of soft ideal was first given by R. Sahin and A. Kucuk [20]. Then H. I. Mustafa and F. M. Sleim in [15] introduced a different version of soft ideal. By the light of this definition Kandil et all. [8, 9, 10] gave the concept of soft $\star$-topology which was finer than soft topology, soft semi I-compactness and introduced the definition of soft connected via soft ideals.

In this paper, we introduced the notion of soft I-regularity and soft I-normality by the definition of soft ideal and then examined some properties of these notions. Then we give the relations between soft I-regularity and soft I-normality with respect to soft Lindelöf spaces.

2. Preliminaries

Throughout this paper, X will be a nonempty initial universal set and A will be a set of parameters. Let P(X) denote the power set of X and S(X) denote the set of all soft sets over X.

Definition 2.1 ([14])**.** A pair (F,A), where F is a mapping from A to P(X), is called a soft set over X.

Definition 2.2 ([10])**.** Let (F_1, A) and (F_2, A) be two soft sets over a common universe X. Then (F_1, A) is said to be a soft subset of (F_2, A) if $F_1(\alpha) \subset F_2(\alpha)$, for all $\alpha \in A$. This relation is denoted by $(F_1, A)\widetilde{\subset}(F_2, A)$.

(F_1, A) is said to be soft equal to (F_2, A) if $F_1(\alpha) = F_2(\alpha)$, for all $\alpha \in A$. This relation is denoted by $(F_1, A) = (F_2, A)$.

Definition 2.3 ([13])**.** A soft set (F,A) over X is said to be a null soft set if $F(\alpha) = \varnothing$, for all $\alpha \in A$, and denoted by $\widetilde{\varnothing}$.

Definition 2.4 ([13])**.** A soft set (F,A) over X is said to be an absolute soft set if $F(\alpha) = X$, for all $\alpha \in A$, and denoted by $\widetilde{X}$.

Definition 2.5 ([1])**.** The complement of a soft set (F,A) is defined as $(F, A)^c = (F^c, A)$, where $F^c(\alpha) = (F(\alpha))^c = X - F(\alpha)$, for all $\alpha \in A$.

Clearly, we have $(\widetilde{X})^c = \widetilde{\varnothing}$ and $(\widetilde{\varnothing})^c = \widetilde{X}$.

Definition 2.6 ([21])**.** The difference of two soft sets (F,A) and (G,A) is defined by $(F, A) - (G, A) = (F - G, A)$, where $(F - G)(\alpha) = F(\alpha) - G(\alpha)$, for all $\alpha \in A$.

Definition 2.7 ([13])**.** The union of two soft sets (F,A) and (G,A) over the common universe X is the soft set (H,A), $H(\alpha) = F(\alpha) \cup G(\alpha)$ where for all $\alpha \in A$.

Definition 2.8 ([18])**.** The intersection of two soft sets (F,A) and (G,A) over X is the soft set (H,A) , $H(\alpha) = F(\alpha) \cap G(\alpha)$ where for all $\alpha \in A$.

Definition 2.9 ([24])**.** Let $\{(F_j, A) \mid j \in J\}$ be a nonempty family of soft sets over a common universe X. Then

(i) Intersection of this family, denoted by $\widetilde{\cap}_{j\in J}$, is defined by $\widetilde{\cap}_{j\in J}(F_j, A)$= $(\widetilde{\cap}_{j\in J}F_j, A)$,

where $(\cap_{j\in J}F_j)(\alpha)= \cap_{j\in J}(F_j(\alpha))$, for all $\alpha \in$ A.
(ii) Union of this family, denoted by $\widetilde{\cup}_{j\in J}$, is defined by $\widetilde{\cup}_{j\in J}(F_j, A) = (\widetilde{\cup}_{j\in J}F_j, A)$, where $(\cup_{j\in J}F_j)(\alpha) = \cup_{j\in J}(F_j(\alpha))$, for all $\alpha \in$ A.

Definition 2.10 ([5, 12]). A soft set (E, A) over X is said to be a soft point, which is called soft element in [16] , if there is $\alpha \in A$ such that $E(\alpha) = \{x\}$ for some $x \in X$ and $E(\beta) = \varnothing$ for all $\beta \in E\backslash\{\alpha\}$. This soft point is denoted by E^x_α

The soft point E^x_α is said to belongs to the soft set (F, A), denoted by $E^x_\alpha \widetilde{\in}(F, A)$, if $x \in F(\alpha)$. [5, 16]

From now on, the family of all soft points over X will be denoted by $SP(X, A)$.

Definition 2.11 ([5]). Two soft points E^x_α, E^y_β are said to be equal if $\alpha = \beta$ and $x = y$. Thus, $E^x_\alpha \neq E^y_\beta \Leftrightarrow x \neq y$ or $\alpha \neq \beta$.

Proposition 2.12 ([5, 16]). *Let $(F, A), (G, A) \in S(X)$ and let $E^x_\alpha \in SP(X, A)$. Then we have:*

(i) $E^x_\alpha \widetilde{\in}(F, A)$ iff $E^x_\alpha \widetilde{\notin} (F, A)^c$.
(ii) $E^x_\alpha \widetilde{\in} (F, A)\widetilde{\cup}(G, A)$ iff $E^x_\alpha \widetilde{\in} (F, A)$ or $E^x_\alpha \widetilde{\in} (G, A)$.
(iii) $E^x_\alpha \widetilde{\in} (F, A)\widetilde{\cap}(G, A)$ iff $E^x_\alpha \widetilde{\in} (F, A)$ and $E^x_\alpha \widetilde{\in} (G, A)$.
(iv) $(F, A)\widetilde{\subset}(G, A)$ iff $E^x_\alpha \widetilde{\in} (F, A)$ implies $E^x_\alpha \widetilde{\in} (G, A)$.

Definition 2.13 ([21]). Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if :
(i) $\widetilde{\varnothing}$, $\widetilde{X} \in \tau$;
(ii) the intersection of any two soft sets in τ belongs to τ;
(iii) the union of any number of soft sets in τ belongs to τ.

The triple (X, τ, A) is called a soft topological space over X. The members of τ are said to be $\tau-$ soft open sets or simply, soft open sets in X. A soft set over X is said to be soft closed in X if its complement belongs to τ.

Proposition 2.14 ([21]). *Let (X, τ, A) be a soft topological space. Then $\tau_\alpha = \{F(\alpha)|(F, A) \in \tau\}$ for each $\alpha \in A$, defines a topology on X.*

Definition 2.15 ([21]). Let Y be a nonempty subset of X, then $\widetilde{Y}$ denotes the soft set (Y,A) over X for which $Y(\alpha) = Y$, for all $\alpha \in A$.

Definition 2.16 ([21]). Let (F,A) be a soft set over X and Y be a non empty subset of X. Then the soft subset of (F,A) over Y denoted by $(^Y F, A)$ is defined as follows: $^Y F(\alpha) = Y \cap F(\alpha)$, for each $\alpha \in A$. In other words $(^Y F, A) = \widetilde{Y}\widetilde{\cap}(F, A)$.

Definition 2.17 ([21]). Let (X, τ, A) be a soft topological space over X and Y be a nonempty subset of X. Then $\tau_Y = \{(^Y F, A) \mid (F, A) \in \tau\}$,
is said to be the soft relative topology on Y and (Y, τ_Y , A) is called a soft subspace of (X, τ, A).

Definition 2.18 ([16]). Let (X, τ, A) be a soft topological space . A soft set (F,A) is said to be a soft neighbourhood (abbreviated as a soft nbd) of the soft set (H,A) if there exist a soft set $(G, A) \in \tau$ such that $(H, A)\widetilde{\subseteq}(G, A)\widetilde{\subseteq}(F, A)$. If $(H, A) = E^x_\alpha$, then (F,A) is said to be a soft nbd of the soft point E^x_α.

The soft neighbourhood system of a soft point E_α^x, denoted by $\mathcal{N}(E_\alpha^x)$, is the family of all its soft nbds.

The soft open neighbourhood system of a soft element E_α^x, denoted by $\mathcal{V}(E_\alpha^x)$, is the family of all its soft open nbds.

Definition 2.19 ([11])**.** Let $S(X)_A$ and $S(Y)_B$ be the families of all soft sets over X and Y with A and B be parameterized, respectively. The mapping φ_ψ is called a soft mapping from X to Y, denoted by $\varphi_\psi : S(X)_A \to S(Y)_B$, where φ: X $\to$ Y and ψ:A $\to$ B are two mappings.
(1) Let (F, A) $\in S(X)_A$, then the image of (F, A) under the soft mapping $\varphi_\psi((F,A))$ and defined by

$$\varphi_\psi((F,A))(k) = \begin{cases} \bigcup_{e\in\psi^{-1}(k)\cap A} \varphi((F,A)(e)), & if\ \psi^{-1}(k)\cap A \neq \varnothing \\ \varnothing, & otherwise. \end{cases} \tag{2.1}$$

(2) Let $(G,B) \in S(Y)_B$, then the pre-image of (G, B) under the soft mapping φ_ψ is the soft over X denoted by $\varphi_\psi^{-1}((G,B))$, where

$$\varphi_\psi^{-1}((G,B))(e) = \begin{cases} \varphi^{-1}((G,B)(\psi(e))), & if\ \psi(e) \in A \\ \varnothing, & otherwise. \end{cases} \tag{2.2}$$

The soft mapping is called injective (surjective), if φ and ψ are injective (surjective) [3, 24].

Theorem 2.20 ([11])**.** *Let X and Y crisp sets (F,A), $(F_i,A) =^\triangle (F_i,A_i) \in S(X)$ and (G,B), $(G_i,B) =^\triangle (G_i,B_i) \in S(Y)$, $\forall\ i \in \triangle$, where $\triangle$ is an index set. Then*
(1) If $(F_1,A) \subset (F_2,A)$ then $\varphi_\psi((F_1,A)) \subset \varphi_\psi((F_2,A))$.
(2) If $(G_1,A) \subset (G_2,A)$ then $\varphi_\psi^{-1}((G_1,A)) \subset \varphi_\psi^{-1}((G_2,A))$.
(3) $(F,A) \subset \varphi_\psi^{-1}(\varphi_\psi(F,A))$, the equality holds if φ_ψ is injective.
(4) $\varphi_\psi(\varphi_\psi^{-1}(G,A)) \subset (G,A)$, the equality holds if φ_ψ is surjective.
(5) $\varphi_\psi(\bigcup_{i\in\triangle})(F_i,A) = \bigcup_{i\in\triangle}\varphi_\psi(F_i,A)$.
(6) $\varphi_\psi(\bigcap_{i\in\triangle})(F_i,A) \subset \bigcap_{i\in\triangle}\varphi_\psi(F_i,A)$, the equality holds if φ_ψ is injective .
(7) $\varphi_\psi^{-1}(\bigcup_{i\in\triangle})(G_i,B) = \bigcup_{i\in\triangle}\varphi_\psi^{-1}(G_i,A)$.
(8) $\varphi_\psi^{-1}(\bigcap_{i\in\triangle})(G_i,B) = \bigcap_{i\in\triangle}\varphi_\psi^{-1}(G_i,A)$.
(9) $\varphi_\psi^{-1}(\widetilde{Y}) = \widetilde{X}$ and $\varphi_\psi^{-1}(\widetilde{\varnothing}) = \widetilde{\varnothing}$.
(10) $\varphi_\psi(\widetilde{X}) = \widetilde{Y}$ if φ_ψ is surjective.
(11) $\varphi_\psi(\widetilde{\varnothing}) = \widetilde{\varnothing}$.

Theorem 2.21 ([24])**.** *Let (X,τ,A) and (Y,τ_1,B) be two soft topological spaces and $\varphi_\psi : S(X)_A \to S(Y)_B$ be a function. Then the following statement are equivalent:*
(i) φ_ψ is soft continuous,
(ii) For each (G, B) $\in \tau_1$, $\varphi_\psi^{-1}(G,B) \in \tau$,
(iii) For each (F,B) is soft closed over Y, $\varphi_\psi^{-1}(F,B)$ is soft closed over X.

Definition 2.22. [3] Let (X,τ,A) and (Y,τ_1,B) be soft topological spaces, $\varphi_\psi : S(X)_A \to S(Y)_B$ be a function. Then the function φ_ψ is called soft open if $\varphi_\psi(G,A) \in \tau_1$ for each $(G,A) \in \tau$.

Definition 2.23 ([22]). Let (X, τ, A) and (Y, τ_1, B) be soft topological spaces and $\varphi_\psi : S(X)_A \to S(Y)_B$ be a function. If f is bijection, soft continuous and soft open mapping, then f is called soft homeomorphism from X to Y.

Definition 2.24 ([19]). Let (X, τ, A) be a soft topological space over X, let (G,A) be a soft closed set in X and soft point E^x_α such that $E^x_\alpha \widetilde{\notin} (G, A)$. If there exist soft open sets (F_1, A) and (F_2, A) such that $E^x_\alpha \widetilde{\in} (F_1, A)$, $(G, A) \widetilde{\subseteq} (F_2, A)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \widetilde{\varnothing}$, then (X, τ, A) is called a soft regular space.

Definition 2.25 ([19]). Let (X, τ, A) be a soft topological space over X, and let (G_1, A) and (G_2, A) be two disjoint soft closed sets. If there exist two soft open sets (F_1, A) and (F_2, A) such that $(G_1, A) \widetilde{\subseteq} (F_1, A)$, $(G_2, A) \widetilde{\subseteq} (F_2, A)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \widetilde{\varnothing}$, then (X, τ, A) is called a soft normal space.

Definition 2.26 ([3]). Let (X, τ, A) be a soft topological space. A family $\mathcal{C} = \{(G_i, A) \mid i \in \Lambda\}$ of soft open sets in (X, τ, A) is called a soft open cover of $\widetilde{X}$, if it is satisfies $\widetilde{\bigcup}_{i \in \Lambda} (G_i, A) = \widetilde{X}$. A subfamily of a soft open cover $\mathcal{C}$ of $\widetilde{X}$ is called a subcover of $\mathcal{C}$, if it is also a soft open cover of $\widetilde{X}$.

Definition 2.27 ([19]). Let (X, τ, A) be a soft topological space. (X, τ, A) is a soft Lindelöf space if each soft open covering $\mathcal{C}$ of $\widetilde{X}$ has a countable subcover.

Definition 2.28 ([20]). A soft ideal I is a non-empty collection of soft sets over X which satisfies the following conditions :
(i) $\widetilde{X} \notin I$
(ii) $(F, A) \in I$ and $(G, A) \widetilde{\subset} (F, A)$ implies $(G, A) \in I$
(ii) $(F, A) \in I$ and $(G, A) \in I$ implies $(F, A) \widetilde{\cup} (G, A) \in I$.
Then, Mustafa at all [15] gave the definition of soft ideal by removing the condition (i). Throughout this paper we will use the definition of soft ideal as it is stated in Mustafa at all.

Definition 2.29 ([8]). Let (X, τ, A) be a soft topological space and I be a soft ideal over X with the same set of parameters A. Then

$$(F, A)^*(I, \tau) = \widetilde{\bigcup} \{E^x_\alpha \in \widetilde{X} : (U, A) \widetilde{\cap} (F, A) \notin \widetilde{I}, \forall (U, A) \in \tau \text{ containing } E^x_\alpha\}$$

is called the soft local function of (F, A) with respect to I and τ.

Definition 2.30 ([8]). Let (X, τ, A) be a soft topological space, I be a soft ideal over X with same set of parameters A and $cl^\star : S(X) \to S(X)$ be the soft closure operator such that $cl^\star(F, A) = (F, A) \widetilde{\cup} (F, A)^*(I, \tau)$. Then, there exists a unique soft topology over X with the same set of parameters A, finer than τ, called the $\star$-soft topology, denoted by $\tau^\star$.

Lemma 2.31 ([10]). *If I is a soft ideal on X and Y is a subset of X, then $I_Y = \{\widetilde{Y} \widetilde{\cap} (I, A) \mid (I, A) \in I\}$ is a soft ideal on Y.*

Theorem 2.32 ([9]). *Let (X, τ_1, A) be a soft ideal topological space, (Y, τ_2, B) be a soft topological space and $\varphi_\psi : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $\varphi_\psi(I) = \{\varphi_\psi((F, A)) : (F, A) \in I\}$ is a soft ideal on Y.*

Theorem 2.33 ([8]). *Let (X, τ, A) be a soft topological space and I be a soft ideal over X with the same set of parameters A. Then,*
$\beta(I, \tau) = \{(F, A) - (G, A) : (F, A) \in \tau, (G, A) \in I\}$ *is a soft basis for the soft topology* $\tau^\star$.

3. Soft I-regular space

In this section we introduce the notion of soft I-regularity and investigate some properties of soft I-regular spaces.

Definition 3.1. Let (X, τ, A) be a soft topological space on X with I be soft ideal and let (G,A) be a soft closed set over X such that $E^x_\alpha \widetilde{\notin} (G, A)$ for $x \in X$. If there exist disjoint soft open sets (U, A) and (V, A) such that $E^x_\alpha \widetilde{\in} (U, A), (G, A) - (V, A) \in I$ and then (X, τ, A) is called a soft I- regular space.

Remark 3.2. (a) If $I = \{\widetilde{\varnothing}\}$, then soft regularity and soft I- regularity coincides.
(b) If (X, τ, A) is a soft regular space then (X, τ, A) is a soft-I-regular space.

Example 3.3. (*i*) Let (X, τ, A) be a soft topological space with soft ideal I. where $X = \{a, b\}, A = \{\alpha, \beta\}, \tau = \{\widetilde{\varnothing}, \widetilde{X}, \{(\alpha, \{a\}), (\beta, \{b\})\}, \{(\alpha, \{b\}), (\beta, \{a\})\}, \{(\alpha, \varnothing), (\beta, \{a\})\}, \{(\alpha, \{a\}), (\beta, X)\}\}$ and $I = \{\widetilde{\varnothing}, \{(\alpha, \{b\}), (\beta, \varnothing)\}\}$. Then, (X, τ, A) is a soft I-regular space.
(*ii*) Let (X, τ, A) be a soft topological space with a soft ideal I where $X = \{a, b, c\}$, $A = \{\alpha, \beta\}, \tau = \{\widetilde{\varnothing}, \widetilde{X}, \{(\alpha, \{a\}), (\beta, \{b\})\}, \{(\alpha, \{a, c\}), (\beta, \{b, c\})\}, \{(\alpha, \{a, b\}), (\beta, \{a, b\})\}\}$ and $I = \{\widetilde{\varnothing}, \{(\alpha, \{b\}), (\beta, \varnothing)\}, \{(\alpha, \varnothing), (\beta, \{a\})\}, \{(\alpha, \{b\}), (\beta, \{a\})\}\}$. Then, (X, τ, A) is not a soft I- regular space. Because there are no two disjoint soft open sets (U,A) and (V,A) such that $E^a_\alpha \widetilde{\in} (U, A)$ and (F,A) $= \{(\alpha, \{b, c\}), (\beta, \{a, c\})\} - (V, A) \in I$.

The following example show that there is no relation between e-parameter regular space and soft I-regular space.

Example 3.4. Let X $=\{a, b, c\}$, $\tau = \{\widetilde{\varnothing}, \widetilde{X}, \{(\alpha, \{b, c\}), (\beta, \{a, c\})\}\}$ and $I = \{\widetilde{\varnothing}, \{(\alpha, \{\varnothing\}), (\beta, \{b\})\}, \{(\alpha, \{a\}), (\beta, \{\varnothing\})\}, \{(\alpha, \{a\}), (\beta, \{b\})\}\}$. Then, (X, τ, A) is soft I-regular space. But if we take $\tau(\alpha) = \{\varnothing, X, \{b, c\}\}$ then $(X, \tau(\alpha))$ is not a α-parameter regular space.

Theorem 3.5. (X, τ, A) *is a soft I- regular space if and only if* $(X, \tau^\star, A)$ *is soft I-regular space.*

Proof. Let (X, τ, A) be a soft I- regular space and (F,A) be a soft $\star$-closed set over X such that $E^x_\alpha \widetilde{\notin} (F, A)$. Since $(F, A)^c$ is a soft $\star$-open set, $(F, A)^c = (H, A)$ - (I, A) where $(H, A) \in \tau$ and $(I, A) \in I$. Then $(H, A)^c$ is soft closed such that $E^x_\alpha \widetilde{\notin} (H, A)^c$. Since (X, τ, A) is a soft I-regular space, there exist disjoint soft open sets (U, A) and (V, A) such that $E^x_\alpha \widetilde{\in} (U, A)$, $(H, A)^c$-(V,A) $\in I$. Then $((F, A)$ -$(I, A))$-$(V, A) \in I$. By definition of soft ideal, (F, A)-$(V, A) \in I$.

Conversely, let $(X, \tau^\star, A)$ be a soft I- regular space and (F,A) be a soft-closed set over X such that $E^x_\alpha \widetilde{\notin} (F, A)$. Since $\tau \subseteq \tau^\star$, (F,A) is a soft $\star$-closed set over X. By hypothesis, there exist soft $\star$-open sets (U,A) and (V,A) such that $E^x_\alpha \widetilde{\in} (U, A)$ and (F, A)-$(V, A) \in I$. Since (U, A) is a soft $\star$-open set, $(U, A) = (H_1, A)$ - (I_1, A)

where $(H_1, A) \in \tau$ and $(I_1, A) \in I$. Then $E^x_\alpha \widetilde{\in} (H_1, A)$. Similarly, $(V, A) = (H_2, A)$ - (I_2, A) where $(H_2, A) \in \tau$ and $(I_2, A) \in I$. By hereditary of I, (F, A)- $(H_2, A) \in I$. So $(X, \tau, A,)$ is a soft I- regular space. □

Theorem 3.6. *If (X, τ, A) is a soft I- regular space and $Y \subset X$, then (Y, τ_Y, A) is a soft I_Y- regular space.*

Proof. Let $Y \subset X$ and (K,A) be a soft closed set on Y. Let $E^x_\alpha \widetilde{\in} \widetilde{Y}$ and $E^x_\alpha \widetilde{\notin} (K, A)$. Since (K,A) is soft closed on Y, we have $(K, A) = \widetilde{Y} \widetilde{\cap} (F, A)$ such that $E^x_\alpha \widetilde{\notin} (F, A)$ and (F,A) soft closed on X. By hypothesis, there exist disjoint soft open sets (U,A) and (V,A) such that $E^x_\alpha \widetilde{\in} (U, A)$ and $(F, A) - (V, A) \in I$. Clearly, $((K, A) - (V, A)) \widetilde{\cap} \widetilde{Y} \in I_Y$. Since $\widetilde{Y} \widetilde{\cap} (U, A)$ and $\widetilde{Y} \widetilde{\cap} (V, A)$ are disjoint soft open sets, (Y, τ_Y, A, I_Y) is soft I_Y- regular space. □

Theorem 3.7. *Let (X, τ, A) be a soft topological space with a soft ideal I. Then the followings are equivalent:*
(i) (X, τ, A) is a soft I- regular space.
(ii) For each $x \in X$ and soft open set (U,A) containing E^x_α, there is a soft open set (V,A) containing E^x_α such that $cl(V, A) - (U, A) \in I$.
(iii) For each $x \in X$ and soft closed set (F,A) not containing E^x_α, there is a soft open set (V,A) containing E^x_α such that $cl(V, A) \widetilde{\cap} (F, A) \in I$.

Proof. $(i) \Rightarrow (ii)$ Let $x \in X$ and (U,A) be a soft open set containing E^x_α. Then, there exist disjoint soft open sets (V,A) and (W,A) such that $E^x_\alpha \widetilde{\in} (V, A)$ and $(\widetilde{X} - (U, A)) - (W, A) \in I$. Then $(\widetilde{X} - (U, A)) \widetilde{\subset} (W, A) \widetilde{\cup} (I, A)$. Now $(V, A) \widetilde{\cap} (W, A) = \widetilde{\varnothing}$ implies that $(V, A) \widetilde{\subset} \widetilde{X} - (W, A)$ and so $cl(V, A) \widetilde{\subset} \widetilde{X} - (W, A)$. Hence $cl(V, A) - (U, A) \widetilde{\subset} (\widetilde{X} - (W, A)) \widetilde{\cap} ((W, A) \widetilde{\cup} (I, A)) = (\widetilde{X} - (W, A)) \in I$.
$(ii) \Rightarrow (iii)$ Let (F,A) be a soft closed set on X such that $E^x_\alpha \widetilde{\notin} (F, A)$. Then, there exists a soft open set (V,A) containing E^x_α such that $cl(V, A) - (\widetilde{X} - (F, A)) \in I$ which implies that $cl(V, A) \widetilde{\cap} (F, A) \in I$.
$(iii) \Rightarrow (i)$ Let (F,A) be a soft closed set on X such that $E^x_\alpha \widetilde{\notin} (F, A)$. Then, there exists a soft open set (V,A) containing E^x_α such that $cl(V, A) \widetilde{\cap} (F, A) \in I$. If $cl(V, A) \widetilde{\cap} (F, A) = (I, A) \in I$, then $(F, A) - (\widetilde{X} - cl(V, A)) = (I, A) \in I$. (V,A) and $(\widetilde{X} - cl(V, A))$ are the required disjoint soft open sets such that $E^x_\alpha \widetilde{\in} (V, A)$ and $(F, A) - (\widetilde{X} - cl(V, A)) \in I$. Hence (X, τ, A) is soft I- regular space. □

Lemma 3.8. *Let X and Y crisp sets (F,A), (G,A) $\in S(X)$ and $\varphi_\psi : S(X)_A \to S(Y)_B$ is a injective function. Then $\varphi_\psi((F, A) - (G, A)) = \varphi_\psi((F, A)) - \varphi_\psi((G, A))$.*

Proof. $E^y_\beta \,\widetilde{\in}\, \varphi_\psi((F, A) - (G, A))$. Then,

$$\varphi_\psi(((F, A) - (G, A)))(\beta)$$
$$= \begin{cases} \bigcup\limits_{\alpha \in \psi^{-1}(\beta) \cap A} \varphi(((F, A) - (G, A))(\alpha)), & \text{if } \psi^{-1}(\beta) \cap A \neq \varnothing; \\ \varnothing, & \text{otherwise.} \end{cases}$$

y$\in \bigcup_{\alpha \in \psi^{-1}(\beta) \cap A} \varphi(((F(\alpha) - (G(\alpha)))$. Since φ is injective, y$\in \varphi(F(\alpha)) - \varphi(G(\alpha))$. Then y$\in \varphi(F(\alpha))$ and y$\notin \varphi(G(\alpha))$. So y $\in \varphi_\psi((F, A)(\beta)$ and y $\notin \varphi_\psi((G, A)(\beta)$.

And y $\in \varphi_\psi((F,A)(\beta) - \varphi_\psi((G,A)(\beta) = \varphi_\psi((F,A) - \varphi_\psi((G,A)))(\beta)$. Therefore E^y_β $\widetilde{\in}\varphi_\psi((F,A) - \varphi_\psi((G,A)))$. □

Theorem 3.9. *If (X,τ,A) is a soft I- regular space and $\varphi_\psi : S(X)_A \to S(Y)_B$ is a homeomorphism, then (Y,τ_1,B) is a soft $\varphi_\psi(I)$- regular space.*

Proof. Let y $\in$ Y and (F_1,B) be a soft closed set not containing E^y_β. Since φ_ψ is a soft homeomorphism, there exists a soft closed set (F,A) such that $\varphi_\psi((F,A)) = (F_1,B)$ and $\varphi_\psi(E^x_\alpha)= E^y_\beta$. By hypothesis, there exists a soft open set (U, A) containing E^x_α such that $Cl_X((U,A))$-$(F,A) \in I$. By Theorem 2.32 and Lemma 3.8, $\varphi_\psi(Cl((U,A))$-$\varphi_\psi((F,A)) = Cl(V,B)$-$(G,B) \in \varphi_\psi(I)$. Since φ_ψ is a soft homeomorphism, (V,B) is a soft open set containing E^y_β. So (Y,τ_1,B) is a soft $\varphi_\psi(I)$- regular space. □

4. Soft *I*-normal spaces

In this section we give the definition of soft *I*-normality and investigate some properties of soft *I*-normal spaces.

Definition 4.1. A soft ideal topological space (X,τ,A,I) is called a soft *I*-normal space if for every pair of disjoint soft closed sets (F,A) and (G,A) of $\widetilde{X}$, there exist disjoint soft open sets (U,A) and (V,A) such that $(F,A) - (U,A) \in I$ and $(G,A) - (V,A) \in I$.

Remark 4.2. If $I = \{\widetilde{\varnothing}\}$, then soft normality and soft $I-$ normality coincides.

Example 4.3. Let $\mathbb{R}$ be real line and A $= \{\alpha,\beta\}$. Let ψ=$\{(F,A) : F(\alpha) = (a,b], F(\beta) = \{a\}, a < b\}$ and τ be the soft topology generated by ψ as a base. $I = \{(G,A) \mid G(\alpha) \textit{ and } G(\beta) \textit{ are finite}\}$. Then $(\mathbb{R},\tau,A)$ is a soft *I*-normal space.

The following example shows that there is no relation between e-parameter normal space and soft *I*-normal space.

Example 4.4. Let X $=\{a,b,c\}$,

$\tau = \{\widetilde{\varnothing},\ \widetilde{X},\ \{(\alpha,\{a\}),\ (\beta,\{b\})\},\ \{(\alpha,\{a,c\}),\ (\beta,\{a,b\})\},\ \{(\alpha,\{a,b\}),(\beta,\{b,c\})\}\}$ and $I = \{\widetilde{\varnothing},\ \{(\alpha,\{\varnothing\}),(\beta,\{a\})\},\ \{(\alpha,\{c\}),(\beta,\{\varnothing\})\},\ \{(\alpha,\{c\}),(\beta,\{a\})\}\}$. Then, (X,τ,A) is soft *I*-normal space. But if we take $\tau(\beta) = \{\varnothing, X, \{b\}, \{a,b\}, \{b,c\}\}$ then $(X,\tau(\beta))$ is not a β-parameter normal space.

Theorem 4.5. *If (X,τ,A) is a soft I-normal space and $\widetilde{Y} \in S(X)$ is soft closed, then (Y,τ_Y,A) is a soft I_Y - normal space.*

Proof. Let (F,A) and (G,A) be disjoint soft closed sets on Y. Since $\widetilde{Y}$ is soft closed, (F,A) and (G,A) be disjoint soft closed sets on X. By hypothesis there exist disjoint soft open sets (U,A) and (V,A) such that (F,A) - $(U,A) \in I$ and $(G,A) - (V,A)$ $\in I$. Then $(F,A) - (U,A) = (I,A) \in I$ and $(F,A) \widetilde{\subset} (U,A) \widetilde{\cup} (I,A)$. Therefore $(F,A) - (\widetilde{Y}\widetilde{\cap}(U,A)) \widetilde{\subset} (\widetilde{Y}\widetilde{\cap}(I,A)) \in I_Y$. Similarly $(G,A) - (\widetilde{Y}\widetilde{\cap}(V,A)) \in I_Y$. Hence Y is a soft I_Y - normal space. □

Theorem 4.6. *Let (X,τ,A) be a soft ideal topological space. Then the followings are equivalent:*
(i) *X is a soft I-normal space.*

(ii) *For every soft closed set (F, A) and soft open set (U, A) containing (F, A), there exists a soft open set (V, A) such that $(F, A) - (V, A) \in I$ and $cl(V, A) - (U, A) \in I$.*
(iii) *For each pair of disjoint soft closed sets (F, A) and (G, A), there exists a soft open set (U, A) such that $(F, A) - (U, A) \in I$ and $cl(U, A)\widetilde{\cap}(G, A) \in I$.*

Proof. $(i) \Rightarrow (ii)$ Let (F, A) be a soft closed set and (U, A) be a soft open subsets of X. Since $\widetilde{X} - (U, A)$ is soft closed and $(F, A) \widetilde{\cap} (\widetilde{X} - (U, A))=\widetilde{\varnothing}$. By hypothesis, there exist disjoint soft open sets (V_1, A) and (V_2, A) such that $(F, A) - (V_1, A)\in I$ and $(\widetilde{X} - (U, A)) - (V_2, A) \in I$. Then $cl(V_1, A) \widetilde{\subset}\widetilde{X} - (V_2, A)$ and $(\widetilde{X} - (U, A) \widetilde{\cap}$ $cl(V_1, A) \widetilde{\subset} (\widetilde{X} - (U, A)) \widetilde{\cap} (\widetilde{X} - (V_1, A))$. $cl(V_1, A) - (U, A) \in I$, by definition of soft ideal I.
$(ii) \Rightarrow (iii)$ Obvious by the hypothesis.
$(iii) \Rightarrow (i)$ Let (F, A) and (G, A) be disjoint soft closed sets. By the hypothesis there exists a soft open set (U, A) such that $(F, A) - (U, A)\in I$ and $cl(U, A)\widetilde{\cap}(G, A) \in I$. Let $(V, A)= \widetilde{X} - cl(U, A)$. Since (V, A) is soft open and $(U, A)\widetilde{\cap}(V, A) = \widetilde{\varnothing}$, then X is a soft I - normal space. □

Theorem 4.7. *If (X, τ, A) is a soft I- normal space and $\varphi_\psi : S(X)_A \to S(Y)_B$ is a homeomorphism, then (Y, τ_1, B) is a soft $\varphi_\psi(I)$- normal space.*

Proof. Let (F_1, B) and (G_1, B) be disjoint soft closed sets over Y. Since φ_ψ is a soft homeomorphism, there exist disjoint soft closed sets (F, A) and (G,A) such that $\varphi_\psi((F, A)) = (F_1, B)$ and $\varphi_\psi((G, A)) = (G_1, B)$. By definition of soft normality, there exist disjoint soft open sets (U,A) and (V, A) such that (F, A)-$(U, A) \in I$ and (G, A)-$(V, A) \in I$. By Theorem 2.32 and Lemma 3.8, $\varphi_\psi((F, A))$-$\varphi_\psi((U, A))$ $\in \varphi_\psi(I)$ and $\varphi_\psi((G, A))$-$\varphi_\psi((V, A)) \in \varphi_\psi(I)$. Since φ_ψ is a soft homeomorphism, $\varphi_\psi((U, A))$=(U, B) and $\varphi_\psi((V, A))$=(V, B) are disjoint soft open sets. So (Y, τ_1, B) is a soft $\varphi_\psi(I)$- normal space. □

Theorem 4.8. *If (X, τ, A) is a soft Lindelöf and soft I- regular space, then (X, τ, A) is a soft I- normal space.*

Proof. Let (F_1, A) and (F_2, A) be disjoint soft closed sets. Since X is a soft *I*- regular space, for each $x \in X$ and soft closed set (F_1, A) containing E^x_α, there is a soft open set (V,A) containing E^x_α such that $cl(V_x, A)\widetilde{\cap}(F_2, A) \in I$. Let $\{(V^x_\alpha, A)|E^x_\alpha\widetilde{\in}(V^x_\alpha, A)\}$ $\cup$ $(F_1, A)^c$ is soft open cover of (X, τ, A). Since (X, τ, A) is soft Lindelöf, there exists a countable subcover $\{(V^{x_j}_{\alpha_j}, A)|E^{x_j}_{\alpha_j}\widetilde{\in}(V^{x_j}_{\alpha_j}, A), j \in J\} \cup (F_1, A)^c$. Then $(F_1, A) \subset$ $\{(V^{x_j}_{\alpha_j}, A)|E^{xj}_{\alpha_j}\widetilde{\in}(V^{x_j}_{\alpha_j}, A), j \in J\}$ and $cl(V^{x_j}_{\alpha_j}, A)\widetilde{\cap}(F_2, A) = I_j \in I$. Similarly, we can find a countable collection of $\{(U^{y_j}_{\alpha_j}, A)|E^{y_j}_{\alpha_j}\widetilde{\in}(U^{y_j}_{\alpha_j}, A), j \in J\}$ of soft open sets $(F_2, A) \subset \{(U^{y_j}_{\alpha_j}, A)|E^{y_j}_{\alpha_j}\widetilde{\in}(U^{y_j}_{\alpha_j}, A), j \in J\}$ and $cl(U^{y_j}_{\alpha_j}, A)\widetilde{\cap}(F_1, A) = I_j \in I$. Let $(G_j, A) = (U^{y_j}_{\alpha_j}, A)$ - $\{cl((V^{x_j}_{\alpha_j}, A))|E^{xj}_{\alpha_j}\widetilde{\in}cl((V^{x_j}_{\alpha_j}, A)), j \in J\}$ and $(H_j, A) = (V^{y_j}_{\alpha_j}, A)$ $\setminus$ $\{cl((U^{x_j}_{\alpha_j}, A))|E^{xj}_{\alpha_j}\widetilde{\in}cl((U^{x_j}_{\alpha_j}, A)), j \in J\}$. Let (G,A)= $\widetilde{\bigcup}\{(G_j, A)|j \in J\}$ and (H, A)= $\widetilde{\bigcup}\{(H_j, A)|j \in J\}$. Since (G_j, A) and (H_j, A) are soft open sets j $\in$ J, (G, A) and (H,A) are disjoint soft open sets such that (F_1, A) - (G,A) $\in$ *I* and (F_2, A) - (H,A) $\in$ *I*. Since $E^y_\alpha \in (F_1, A)$, there exists a $(V^{x_n}_{\alpha_n}, A)$ for some n $\in$ J. Also, $cl((V^{x_j}_{\alpha_j}, A)) \cap (F_1, A) = (I_j, A)$ for every j $\in$ J. $(F_1, A) \subset (X - cl((V^{x_j}_{\alpha_j}, A)) \cup (I_j, A)$ for every j $\in$ J. Then, $E^y_\alpha \notin cl(V^{x_j}_{\alpha_j}, A)$ or $E^y_\alpha \in (I_j, A)$ for every j $\in$ J. Hence E^y_α

$\in (G_n, A)$ or $E^y_\alpha \in \widetilde{\cap}\{(I_j, A)|j \in J\}$ =(I, A). Then $(F_1, A) \subset (G, A) \cup (I, A)$ which implies that (F_1, A) - (G,A) $\in I$. Similarly we can prove that (F_2, A) - (H,A) $\in I$. □

5. Conclusions

In this present paper, we introduce the soft I-regularity and the soft I-normality in soft topological spaces. Then we investigate some properties of soft I-regular spaces and soft I-normal spaces. We studied the relations between these spaces and we show that if (X, τ, A) is a soft Lindelöf space and soft I- regular space, then (X, τ, A) is a soft I- normal space. We believe that these results will contribute to study of topological spaces and will help researchers for futher studies on soft topological spaces.

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