Fuzzy soft $\Gamma$-semiring and fuzzy soft $k$-ideal over $\Gamma$-semiring

M. Murali Krishna Rao

Received 16 June 2014; Revised 27 July 2014; Accepted 1 September 2014

Abstract. In this paper we introduce the notion of fuzzy soft $\Gamma$-semirings, fuzzy soft ideals and fuzzy soft $k$-ideals over $\Gamma$-semirings and study some of their algebraical properties.

2010 AMS Classification: 06Y60, 06B10

Keywords: Fuzzy soft $\Gamma$-semiring, Fuzzy soft ideal, Fuzzy soft $k$–ideal over $\Gamma$–semiring.

Corresponding Author: M. Murali Krishna Rao (mmkr@gitam.edu)

1. Introduction

The notion of a semiring was introduced by H.S. Vandiver [9] in 1934. Semiring is a well-known Universal algebra. Semiring have been used for studying optimization theory, Graph theory, Matrices, Determinants, Theory of Automata, Formal language theory, Coding theory, Analysis of computer programmes, etc. Notion of $\Gamma$-semiring was introduced by M. Murali Krishna Rao [7] not only generalizes the notion of semiring and gamma semiring but also the notion of ternary semiring. The natural growth of gamma semiring is influenced by two things. One is the generalization of results of gamma rings and another is the generalization of results of semirings and ternary semirings. This notion provides an algebraic back ground to the non positive cones of the totally ordered rings. Molodtsov [6] introduced the concept of Soft set theory as a new mathematical tool for dealing with uncertainties. The theory of fuzzy sets most appropriate theory for dealing with uncertainty is first introduced by L.A. Zadeh [10]. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to Logic, Set theory, Group theory, Ring theory, Real analysis, Topology, Measure theory etc. The concept of fuzzy subgroup was introduced by A. Rosenfeld [8] in 1971. Then Maji et.al [5] extended soft set theory to fuzzy soft set theory. F. Feng et.al [3] initiated the study of soft semirings. Soft rings are defined by U. Acar et.al [1] and Jayanth Ghosh et.al [4] initiated the study of Fuzzy soft rings and Fuzzy soft ideals. In this paper we introduce the notion of
fuzzy soft $\Gamma$–semirings, fuzzy soft ideals and fuzzy soft k-ideals over $\Gamma$–semirings and study some of their algebraic properties.

2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier.

**Definition 2.1 ([7]).** A set $S$ together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$, respectively) will be called semiring provided

(i) addition is a commutative operation.
(ii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0$ for each $x \in S$.
(iii) multiplication distributes over addition both from the left and from the right.

**Definition 2.2.** [7] Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call $M$ as a $\Gamma$–semi-ring, if there exists a mapping $M \times \Gamma \times M \rightarrow M$ is written $(x, \alpha, y)$ as $x \alpha y$ such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

(i) $x \alpha(y + z) = x \alpha y + x \alpha z$
(ii) $(x + y) \alpha z = x \alpha z + y \alpha z$
(iii) $x(\alpha + \beta)y = x \alpha y + x \beta y$
(iv) $x \alpha(y \beta z) = (x \alpha y) \beta z$.

**Definition 2.3 ([7]).** Let $S$ be a $\Gamma$–semi-ring and $A$ be a nonempty subset of $S$. $A$ is called a $\Gamma$–subsemi-ring of $S$ if $A$ is a sub-semigroup of $(S, +)$ and $A \Gamma A \subseteq A$.

**Definition 2.4 ([7]).** Let $S$ be a $\Gamma$–semi-ring. A subset $A$ of $S$ is called a left(right) ideals of $S$ if $A$ is closed under addition and $A \Gamma S \subseteq A(AS \subseteq A)$. $A$ is called an ideal of $S$ if it is both a left ideal and right ideal.

**Definition 2.5 ([10]).** Let $S$ be a non empty set, a mapping $f : S \rightarrow [0, 1]$ is called a fuzzy subset of $S$.

**Definition 2.6 ([10]).** Let $f$ be a fuzzy subset of a nonempty subset $S$, for $t \in [0, 1]$ the set $f_t = \{x \in S \mid f(x) \geq t\}$ is called level subset of $S$ w.r.t. $f$.

**Definition 2.7 ([2]).** Let $S$ be a $\Gamma$–semi-ring . A fuzzy subset $\mu$ of $S$ is said to be a fuzzy $\Gamma$–subsemi-ring of $S$ if it satisfies the following conditions

(i) $\mu(x + y) \geq \min \{\mu(x), \mu(y)\}$
(ii) $\mu(x \alpha y) \geq \min \{\mu(x), \mu(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

**Definition 2.8 ([2]).** A fuzzy subset $\mu$ of a $\Gamma$–semi-ring $S$ is called a fuzzy left(right) ideal of $S$ if for all $x, y \in S, \alpha \in \Gamma$

(i) $\mu(x + y) \geq \min \{\mu(x), \mu(y)\}$
(ii) $\mu(x \alpha y) \geq \mu(y) (\mu(x))$.

**Definition 2.9 ([2]).** A fuzzy subset $\mu$ of a $\Gamma$–semi-ring $S$ is called a fuzzy ideal of $S$ if for all $x, y \in S, \alpha \in \Gamma$

(i) $\mu(x + y) \geq \min \{\mu(x), \mu(y)\}$
(ii) $\mu(x \alpha y) \geq \max \{\mu(x), \mu(y)\}$. 

342
Definition 2.10 ([2]). An ideal $I$ of a $\Gamma$–semiring $S$ is called $k$–ideal if for $x, y \in S$, $x + y \in I, y \in I \Rightarrow x \in I$.

Definition 2.11 ([5]). Let $f$ and $g$ be fuzzy subsets of $S$ then $f \cup g, f \cap g$ are fuzzy subsets of $S$ defined by

$$f \cup g(x) = \max\{f(x), g(x)\}, f \cap g(x) = \min\{f(x), g(x)\} \quad \text{for all } x \in S.$$ 

Definition 2.12 ([10]). A fuzzy subset $\mu : S \to [0,1]$ is nonempty if $\mu$ is not the constant function.

Definition 2.13 ([10]). For any two fuzzy subsets $\lambda$ and $\mu$ of $S$, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in S$.

Definition 2.14 ([6]). Let $U$ be an initial Universe set and $E$ be the set of parameters. Let $P(U)$ denotes the power set of $U$. A pair $(f, E)$ is called soft set over $U$ where $f$ is a mapping defined by $f : E \to P(U)$.

Definition 2.15 ([6]). For a soft set $(f, A)$, the set $\{x \in A \mid f(x) \neq \emptyset\}$ is called Support of $(f, A)$ denoted by $\text{Supp}(f, A)$. If $\text{Supp}(f, A) \neq \emptyset$ then $(f, A)$ is called non null soft set.

Definition 2.16 ([5]). Let $U$ be an initial Universe set and $E$ be the set of parameters. Let $A \subseteq E$. A pair $(f, A)$ is called fuzzy soft set over $U$ where $f$ is a mapping given by $f : A \to I^U$ where $I^U$ denotes the collection of all fuzzy subsets of $U$.

Definition 2.17 ([4]). Let $X$ be a group and $(f, A)$ be a soft set over $X$. Then $(f, A)$ is said to be soft group over $X$ if and only if $f(a)$ is a subgroup of $X$ for each $a \in A$.

Definition 2.18. [4] Let $X$ be a group and $(f, A)$ be fuzzy soft set over $X$. Then $(f, A)$ is said to be fuzzy soft group over $X$ if and only if for each $a \in A, x, y \in X$

(i) $f_a(x + y) \geq f_a(x) + f_a(y)$

(ii) $f_a(x^{-1}) \geq f_a(x)$

where $f_a$ is the fuzzy subset of $X$ corresponding to the parameter $a \in A$.

Definition 2.19 ([5]). Let $(f, A), (g, B)$ be fuzzy soft sets over $U$ then $(f, A)$ is said to be a fuzzy soft subset of $(g, B)$ denoted by $(f, A) \subseteq (g, B)$ if $A \subseteq B$ and $f(a) \subseteq g(a)$ for all $a \in A$.

Definition 2.20 ([5]). Let $(f, A), (g, B)$ be fuzzy soft sets. The intersection of fuzzy soft sets $(f, A)$ and $(g, B)$ is denoted by $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} 
    f_c, & \text{if } c \in A \setminus B; \\
    g_c, & \text{if } c \in B \setminus A; \\
    f_c \cap g_c, & \text{if } c \in A \cap B.
\end{cases}$$

Definition 2.21 ([5]). Let $(f, A), (g, B)$ be fuzzy soft sets. The union of fuzzy soft sets $(f, A)$ and $(g, B)$ is denoted by $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} 
    f_c, & \text{if } c \in A \setminus B; \\
    g_c, & \text{if } c \in B \setminus A; \\
    f_c \cup g_c, & \text{if } c \in A \cap B.
\end{cases}$$
Definition 2.22 ([5]). Let \((f, A), (g, B)\) be fuzzy soft sets over \(U\). \(\langle f, A \rangle \) and \((g, B)\) is denoted by \(\langle f, A \rangle \wedge (g, B)\) is defined by \((f, A) \wedge (g, B) = (h, C)\) where \(C = A \times B\).

Definition 2.23 ([5]). Let \((f, A), (g, B)\) be fuzzy soft sets over \(U\). \(\langle f, A \rangle \vee (g, B)\) is defined by \((f, A) \vee (g, B) = (h, C)\) where \(C = A \times B\) and \(h_c(x) = \max \{f_a(x), g_b(x)\}\) for all \(c = (a, b) \in A \times B, x \in U\).

Definition 2.24 ([2]). Let \(f\) and \(g\) be fuzzy subsets of \(\Gamma\)-semiring \(S\). Then \(f \circ g\) is defined by

\[
f \circ g(z) = \begin{cases}
\sup_{z = x \circ y} \{\min\{f(x), g(y)\}\}, & \text{if } x, y, z \in S, \alpha \in \Gamma \\
0, & \text{otherwise}
\end{cases}
\]

Definition 2.25 ([5]). Let \(S\) and \(T\) be two sets. Let \(\phi : S \to T\) be any function. A fuzzy set \(f\) of \(S\) is called \(\phi\) invariant if \(\phi(x) = \phi(y) \Rightarrow f(x) = f(y)\).

Definition 2.26 ([7]). A function \(f : R \to S\) where \(R\) and \(S\) are \(\Gamma\)-semirings is said to be a \(\Gamma\)-semiring homomorphism if \(f(a + b) = f(a) + f(b)\) and \(f(ab) = f(a)f(b)\) for all \(a, b \in R, \alpha \in \Gamma\).

3. Main results

In this section, the concepts of soft \(\Gamma\)-semiring, fuzzy soft \(\Gamma\)-semiring and fuzzy soft ideal over \(\Gamma\)-semiring are introduced and study the properties related to these notions.

Definition 3.1. Let \(S\) be a \(\Gamma\)-semiring and \(E\) be a parameter set and \(A \subseteq E\). Let \(f\) be a mapping given by \(f : A \to P(S)\) where \(P(S)\) is the power set of \(S\). Then \((f, A)\) is called a \(\Gamma\)-semiring over \(S\) if and only if for each \(a \in A\), \(f(a)\) is \(\Gamma\)-subsemiring of \(S\). i.e. (i) \(x, y \in S \Rightarrow x + y \in f(a)\) (ii) \(x, y \in S, \alpha \in \Gamma \Rightarrow x \circ y \in f(a)\).

Definition 3.2. Let \(S\) be a \(\Gamma\)-semiring and \(E\) be a parameter set and \(A \subseteq E\). Let \(f\) be a mapping given by \(f : A \to [0, 1]^S\) where \([0, 1]^S\) denotes the collection of all fuzzy subsets of \(S\). Then \((f, A)\) is called a fuzzy soft \(\Gamma\)-semiring over \(S\) if and only if for each \(a \in A\), \(f(a) = f_a\) is the fuzzy \(\Gamma\)-subsemiring of \(S\). i.e. (i) \(f_a(x + y) \geq \min\{f_a(x), f_a(y)\}\) (ii) \(f_a(x \circ y) \geq \min\{f_a(x), f_a(y)\}\) for all \(x, y \in S, \alpha \in \Gamma\).

Definition 3.3. Let \(S\) be a \(\Gamma\)-semiring and \(E\) be a parameter set and \(A \subseteq E\). Let \(f\) be a mapping given by \(f : A \to P(S)\). Then \((f, A)\) is called a soft left(right) ideal over \(S\) if and only if for each \(a \in A\), \(f(a)\) is a left(right) ideal of \(S\). i.e., (i) \(x, y \in f(a) \Rightarrow x + y \in f(a)\) (ii) \(x, y \in f(a), \alpha \in \Gamma, r \in S \Rightarrow rax(xar) \in f(a)\).

Definition 3.4. Let \(S\) be a \(\Gamma\)-semiring and \(E\) be a parameter set, \(A \subseteq E\) and \(f : A \to P(R)\). Then \((f, A)\) is called a soft ideal over \(S\) if and only if for each \(a \in A\), \(f(a)\) is an ideal of \(S\). i.e., (i) \(x, y \in f(a) \Rightarrow x + y \in f(a)\) (ii) \(x \in f(a), \alpha \in \Gamma, r \in S \Rightarrow rax(xar) \in f(a)\).

Definition 3.5. Let \(S\) be a \(\Gamma\)-semiring and \(E\) be a parameter set and \(A \subseteq E\). Let \(f\) be a mapping given by \(f : A \to [0, 1]^S\) where \([0, 1]^S\) denotes the collection of all fuzzy subsets of \(S\). Then \((f, A)\) is called a fuzzy soft left(right) ideal over \(S\) if and only if
for each $a \in A$, the corresponding fuzzy subset $f_a : S \rightarrow [0, 1]$ is a fuzzy left(right) ideal of $S$. i.e., (i) $f_a(x + y) \geq \min \{f_a(x), f_a(y)\}$ (ii) $f_a(xay) \geq f_a(y)(f_a(x)$ for all $x, y \in S, \alpha \in \Gamma$.

**Definition 3.6.** Let $S$ be a $\Gamma$–semiring and $E$ be a parameter set and $A \subseteq E$. Let $f$ be a mapping given by $f : A \rightarrow [0, 1]^S$ where $[0, 1]^S$ denotes the collection of all fuzzy subsets of $S$. Then $(f, A)$ is called a fuzzy soft ideal over $S$ if and only if for each $a \in A$, the corresponding fuzzy subset $f_a : S \rightarrow [0, 1]$ is a fuzzy ideal of $S$. i.e., (i) $f_a(x + y) \geq \min \{f_a(x), f_a(y)\}$ (ii) $f_a(xay) \geq \max \{f_a(x), f_a(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

**Definition 3.7.** Let $(f, A), (g, B)$ be fuzzy soft ideals over a $\Gamma$–semiring $S$. The product $(f, A)$ and $(g, B)$ is defined as $((f \circ g), C)$ where $C = A \cup B$ and

$$
(f \circ g)_c(x) = \begin{cases} 
  f_c(x), & \text{if } c \in A \setminus B; \\
  g_c(x), & \text{if } c \in B \setminus A; \\
  \sup \{\min \{f_c(a), g_c(b)\}\}, & \text{if } c \in A \cap B.
\end{cases}
$$

for all $c \in A \cup B$ and $x \in S, \alpha \in \Gamma$.

**Theorem 3.1.** Let $(f, A)$ and $(g, B)$ be fuzzy soft $\Gamma$–semirings over a $\Gamma$–semiring $S$. Then $(f, A) \cup (g, B)$ is a fuzzy soft $\Gamma$–semiring over $S$.

**Proof.** Let $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$.

Case (i) : if $A \cap B = \emptyset$.
Let $c \in C = A \cup B$. Then $c \in A$ or $c \in B$. If $c \in A$ then $h_c = f_c$ and if $c \in B$ then $h_c = g_c$, since $(f, A)$ and $(g, B)$ are fuzzy soft $\Gamma$–semirings over a $\Gamma$–semiring $S$. In both cases $h_c$ is a fuzzy $\Gamma$–subsemiring of $S$.

Case (ii) : if $A \cap B \neq \emptyset$.
If $c \in A \setminus B$ then $h_c = f_c$, $h_c$ is a fuzzy $\Gamma$–subsemiring.
If $c \in B \setminus A$ then $h_c = g_c$, $h_c$ is a fuzzy $\Gamma$–subsemiring.
If $c \in A \cap B$ then $h_c = f_c \cup g_c$.

Let $x, y \in S, \alpha \in \Gamma$. Then

$$
\begin{align*}
  h_c(x + y) &= f_c \cup g_c(x + y) \\
            &= \max \{f_c(x + y), g_c(x + y)\} \\
            &\geq \max \{\min \{f_c(x), f_c(y)\}, \min \{g_c(x), g_c(y)\}\} \\
            &= \min \{\max \{f_c(x), g_c(x)\}, \max \{f_c(y), g_c(y)\}\} \\
            &= \min \{f_c \cup g_c(x), f_c \cup g_c(y)\}
\end{align*}
$$

$$
\begin{align*}
  h_c(xay) &= f_c \cup g_c(xay) \\
          &= \max \{f_c(xay), g_c(xay)\} \\
          &\geq \max \{\min \{f_c(x), f_c(y)\}, \min \{g_c(x), g_c(y)\}\} \\
          &= \min \{\max \{f_c(x), g_c(x)\}, \max \{f_c(y), g_c(y)\}\} \\
          &= \min \{f_c \cup g_c(x), f_c \cup g_c(y)\}
\end{align*}
$$

Hence $h_c$ is a fuzzy $\Gamma$–subsemiring of $S$. Therefore $(f, A) \cup (g, B)$ is a fuzzy soft $\Gamma$–semiring over $S$. 

345
Theorem 3.2. Let \((f, A)\) and \((g, B)\) be fuzzy soft \(\Gamma\)-semirings over a \(\Gamma\)-semiring \(S\). Then \((f, A)\cap (g, B)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\).

Proof. Let \((f, A)\cap (g, B) = (h, C)\) where \(C = A \cup B\) and

\[
h_c(x) = \begin{cases} 
  f_c(x), & \text{if } c \in A \setminus B; \\
  g_c(x), & \text{if } c \in B \setminus A; \\
  f_c \cap g_c(x), & \text{if } c \in A \cap B.
\end{cases}
\]

for all \(c \in C = A \cup B\), for all \(x \in S\).

Case(i) : If \(c \in A \setminus B\) then \(h_c = f_c\), \(f_c\) is a fuzzy \(\Gamma\)-subsemiring of \(S\) since \((f, A)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\).

Case(ii) : If \(c \in B \setminus A\) then \(h_c = g_b\), \(g_b\) is a fuzzy \(\Gamma\)-subsemiring of \(S\), since \((g, B)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\).

Case(iii) : If \(c \in A \cap B\) then \(h_c(x) = f_c \cap g_c(x)\). Let \(x, y \in S, \alpha \in \Gamma\). Then

\[
h_c(x+y) = (f \cap g)_c(x+y)
\]

\[
= \min\{f_c(x+y), g_c(x+y)\}
\]

\[
\geq \min\{\min\{f_c(x), f_c(y)\}, \min\{g_c(x), g_c(y)\}\}
\]

\[
= \min\{\min\{f_c(x), g_c(x)\}, \min\{f_c(y), g_c(y)\}\}
\]

\[
= \min\{(f \cap g)_c(x), (f \cap g)_c(y)\}
\]

Hence \(h_c\) is a fuzzy \(\Gamma\)-subsemiring of \(S\). Therefore \((f, A)\cap (g, B)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\). \(\square\)

Theorem 3.3. Let \((f, A)\) and \((g, B)\) be fuzzy soft \(\Gamma\)-semirings over a \(\Gamma\)-semiring \(S\). Then \((f, A)\cap (g, B)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\).

Proof. By definition 2.22, \((f, A)\cap (g, B) = (h, C)\) where \(C = A \times B\).

Let \(c = (a, b) \in C = A \times B\) and \(x, y \in S, \alpha \in \Gamma\). Then

\[
h_c(x+y) = \min\{f_a(x+y), g_b(x+y)\}
\]

\[
\geq \min\{\min\{f_a(x), f_a(y)\}, \min\{g_b(x), g_b(y)\}\}
\]

\[
= \min\{\min\{f_a(x), g_b(x)\}, \min\{f_a(y), g_b(y)\}\}
\]

\[
= \min\{\min\{f_a \cap g_b(x), f_a \cap g_b(y)\}\}
\]

\[
= \min\{h_c(x), h_c(y)\}
\]

Hence \(h_c\) is a fuzzy \(\Gamma\)-subsemiring of \(S\). Therefore \((h, C)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\). \(\square\)

The following theorem can be proved easily.

Theorem 3.4. Let \((f, A)\) and \((g, B)\) be fuzzy soft \(\Gamma\)-semirings over a \(\Gamma\)-semiring \(S\). Then \((f, A)\vee (g, B)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\).
Definition 3.8. Let \((f, A)\) and \((g, B)\) be fuzzy soft \(\Gamma\)-semirings over \(S\). Then \((g, B)\) is a fuzzy soft \(\Gamma\)-subsemiring of \((f, A)\) if it satisfies the following conditions

(i) \(B \subseteq A\) (ii) \(g_b(x) \leq f_a(x)\) for all \(x \in \text{Supp}(g, B)\).

Theorem 3.5. Let \((f, A)\) and \((g, B)\) be fuzzy soft \(\Gamma\)-semirings over \(S\). Then the following statements are true

(i) If \(g_b \subset f_b\) for all \(b \in B \subseteq A\) then \((g, B)\) is a fuzzy soft \(\Gamma\)-subsemiring of \((f, A)\).

(ii) \((f, A)\) and \((g, B)\) are fuzzy soft \(\Gamma\)-subsemirings of \((f, A) \cup (g, B)\).

Proof. (i) Since \(g_b \subset f_b\) for all \(b \in B \subseteq A\) and by definition 3.12, \((g, B)\) is a fuzzy soft \(\Gamma\)-subsemiring of \((f, A)\).

(ii) Let \((f, A) \cup (g, B) = (h, C)\) where \(C = A \cup B\). By theorem 3.8, \((h, C)\) is a fuzzy soft \(\Gamma\)-semiring over \(S\). Since \(A \subseteq C\) and \(B \subseteq C\), by definition 3.12, \((f, A)\) and \((g, B)\) are fuzzy soft \(\Gamma\)-subsemirings of \((f, A) \cup (g, B)\).

Theorem 3.6. Let \(S\) be a \(\Gamma\)-semiring and \(E\) be a parameter set and \(A \subseteq E\). Then \((f, A)\) is a fuzzy soft left ideal over \(S\) if and only if for each \(a \in A\), \((f_a)_t (t \in \text{Im}(f_a))\) is a left ideal of \(S\) where \(f_a\) is the fuzzy subset of \(S\).

Proof. Suppose \((f, A)\) is a fuzzy soft left ideal over \(S\). Let \(a \in A\) and \(t \in \text{Im}(f_a)\) and \(x, y \in (f_a)_t, \alpha \in \Gamma, r \in S\). Then \(f_a(x + y) \geq \min\{f_a(x), f_a(y)\} \geq \min\{t, t\} = t\) and \(f_a(r \alpha x) \geq f_a(x) \geq t\) which implies that \(x + y, r \alpha x \in (f_a)_t\). Therefore for each \(t \in \text{Im}(f_a), (f_a)_t\) is a left ideal of \(S\). Conversely, suppose that \((f_a)_t\) is a left ideal of \(S\) for each \(t \in \text{Im}(f_a)\) and corresponding to each \(a \in A\) and \(x, y \in S\). Suppose \(f_a(x + y) < \min\{f_a(x), f_a(y)\} = t_1\) (say). Then \(x, y \in (f_a)_{t_1}\), but \(x + y \notin (f_a)_{t_1}\) which is a contradiction. So \(f_a(x + y) \geq \min\{f_a(x), f_a(y)\}\). Suppose \(f_a(x \alpha y) < f_a(y) = t_2\) (say). Then \(y \in (f_a)_{t_2}\), but \(x \alpha y \notin (f_a)_{t_2}\) which is a contradiction. Therefore \(f_a(x \alpha y) \geq f_a(y)\). Hence \(f_a\) is a fuzzy left ideal of \(S\). Therefore \((f, A)\) is fuzzy soft left ideal.

Corollary 3.7. Let \(\mu\) be a fuzzy set in a \(\Gamma\)-semiring \(S\) and \(A = [0, 1]\). Then \((f, A)\) is a fuzzy soft left ideal over \(S\) if and only if \(\mu\) is a fuzzy left ideal of \(S\).


Theorem 3.8. Let \(S\) be a \(\Gamma\)-semiring and \(E\) be a parameters set and \(A \subseteq E\). Then \((f, A)\) is a fuzzy soft left(right) ideal over \(S\) if and only if for each \(a \in A\) the corresponding fuzzy set \(f_a\) of \(S\) satisfy the following conditions

1. \(f_a(x + y) \geq \min\{f_a(x), f_a(y)\}\)
2. \(\chi_S \circ f_a \subseteq f_a \circ \chi_S \subseteq f_a\)

where \(\chi_S\) stands for characteristic function of \(S\).

Proof. Suppose \((f, A)\) is a fuzzy soft left ideal over \(S\). Then, for each \(a \in A\), \(f_a\) is fuzzy left ideal of \(S\). Let \(z \in S\). Then

\[
\chi_S \circ f_a(z) = \sup_{z = x \alpha y} \{\min\{\chi_S(x), f_a(y)\}\} = \sup_{z = x \alpha y} \{f_a(y)\} \leq f_a(x \alpha y) = f_a(z).
\]

If \(z\) cannot be expressed as \(z = x \alpha y\) where \(x, y \in S, \alpha \in \Gamma\) then \(\chi_S \circ f_a(z) = 0 \leq f_a(z)\). Conversely, suppose that for each \(a \in A\), \(f_a\) satisfy the given two conditions.
Let $x, y \in S, \alpha \in \Gamma$. Then we have
\[ f_a(x \alpha y) \geq \chi_S \circ f_a(x \alpha y) = \sup_{x \alpha y = p \beta q} \min \{ \chi_S(p), f_a(q) \} \geq \min \{ \chi_S(x), f_a(y) \} = f_a(y). \]
This shows that for each $a \in A, f_a$ is a fuzzy left ideal of $S$. So $(f, A)$ is a fuzzy soft left ideal over $S$. Similarly we can prove the result for a fuzzy soft right ideal over $S$.

**Theorem 3.9.** Let $S$ be a $\Gamma$–semiring and $E$ be a parameter set and $A \subseteq E$. We define a fuzzy subset $\alpha_a$ of $S$ corresponding to $a \in A$ by
\[ \alpha_a(x) = s \text{ if } x \in I(a) \]
\[ = t \text{ otherwise} \]
for all $x \in S$ and $s, t \in [0, 1]$ with $s < t$. Then $(\alpha, A)$ is a fuzzy soft left ideal over $S$ if and only if $(I, A)$ is a soft left ideal over $S$.

**Proof.** Let $(\alpha, A)$ be a fuzzy soft left ideal over $S$ and $x, y \in I(a)$ and $r \in S, \gamma \in \Gamma$. Then $\alpha_a(x) = s = \alpha_a(y)$ and hence
\[ \alpha_a(x + y) \geq \min \{ \alpha_a(x), \alpha_a(y) \} = \min \{ s, s \} = s. \]
So $x + y \in I(a)$. Also, $\alpha_a(r \gamma x) \geq \alpha_a(x) = s$. Therefore $r \gamma x \in I(a)$. Hence $I(a)$ is a left ideal of $S$. Thus $(I, A)$ is a soft left ideal over $S$. Conversely, $(I, A)$ be soft left ideal over $S$ for each $a \in A, I(a)$ is a left ideal of $S$. Let $x, y \in S$ then the following four cases arise for consideration.

*Case (i):* $x, y \in I(a)$.

Then $x + y \in I(a), x \gamma y \in I(a)$ and hence $\alpha_a(x + y) = s, \alpha_a(x) = s = \alpha_a(y) = \alpha_a(x \gamma y)$. Therefore $\alpha_a(x + y) = \min \{ \alpha_a(x), \alpha_a(y) \} = s$ and $\alpha_a(x \gamma y) = s = \alpha_a(y)$.

*Case (ii):* $x \in I(a), y \notin I(a)$.

Then $x + y \notin I(a)$ and hence $\alpha_a(x) = s, \alpha_a(y) = t$. Therefore $\alpha_a(x + y) = t$. Now
\[ t = \alpha_a(x + y) \geq \min \{ \alpha_a(x), \alpha_a(y) \} = \min \{ s, t \} = s. \]
Hence $y \cdot x \in I(a)$ implies that $\alpha_a(y \gamma x) = s = \alpha_a(x)$.

*Case (iii):* $x, y \notin I(a)$.

Then $x + y \notin I(a)$ and $x \gamma y \notin I(a)$ and hence $\alpha_a(x) = t = \alpha_a(y)$.
\[ t = \alpha_a(x + y) \geq \min \{ \alpha_a(x), \alpha_a(y) \} = \min \{ t, t \} = t. \]
Therefore $\alpha_a(x \gamma y) = t = \alpha_a(y)$. Hence $\alpha_a$ is a fuzzy left ideal of $S$ for each $a \in A$. Therefore $(\alpha, A)$ is a fuzzy soft left ideal over $S$.

**Definition 3.9.** A fuzzy set $\mu$ of $\Gamma$–semiring $M$ is said to be normal fuzzy if $\mu$ is a $\Gamma$–sub semiring of $M$ and $\mu(0) = 1$.

**Definition 3.10.** Let $(f, A)$ be fuzzy soft $\Gamma$–semiring over $S$. Then $(f, A)$ is said to be normal fuzzy soft $\Gamma$–semiring if $f_a$ is normal fuzzy of $\Gamma$–semiring over $S$, for all $a \in A$.

**Theorem 3.10.** If $(f, A)$ is a fuzzy soft $\Gamma$–semiring over $S$ and for each $a \in A$, $f_a^+$ is defined by $f_a^+(x) = f_a(x) + 1 - f_a(0)$ for all $x \in S$ then $(f^+, A)$ is a normal fuzzy soft $\Gamma$–semiring over $S$ and $(f, A)$ is subset of $(f^+, A)$.
Proof. Let \( x, y \in S, \alpha \in \Gamma \) and \( a \in A \). Then
\[
 f_+^a(x + y) = f_a(x + y) + 1 - f_a(0) \\
\geq \min\{f_a(x), f_a(y)\} + 1 - f_a(0) \\
= \min\{f_a(x) + 1 - f_a(0), f_a(y) + 1 - f_a(0)\} \\
= \min\{f_+^a(x), f_+^a(y)\}
\]
\[
f_+^a(xy) = f_a(xy) + 1 - f_a(0) \\
\geq \min\{f_a(x), f_a(y)\} + 1 - f_a(0) \\
= \min\{f_a(x) + 1 - f_a(0), f_a(y) + 1 - f_a(0)\} \\
= \min\{f_+^a(x), f_+^a(y)\}
\]
If \( x = 0 \), then \( f_+^a(0) = 1 \) and \( f_a \subset f_+^a \). Hence \( (f^+, A) \) is a normal fuzzy soft \( \Gamma \)-semiring over \( S \) and \( (f, A) \) is a subset of \( (f^+, A) \). \( \square \)

**Theorem 3.11.** Let \((f, A)\) and \((g, B)\) be fuzzy soft ideals over \( \Gamma \)-semiring \( S \). Then \((f, A) \cap (g, B)\) is a fuzzy soft ideal over \( S \).

Proof. By definition 2.20, we have \((f, A) \cap (g, B) = (h, C)\) where \( C = A \cup B \).

Case (i): \( h_c = f_c \) if \( c \in A \setminus B \). Then \( h_c \) is a fuzzy ideal of \( S \) since \((f, A)\) is a fuzzy soft ideal over \( S \).

Case (ii): If \( c \in B \setminus A \) then \( h_c = g_c \). Therefore \( h_c \) is a fuzzy ideal of \( S \) since \((g, B)\) is a fuzzy soft ideal over \( S \).

Case (iii): If \( c \in A \cap B \), and \( x, y \in S, \alpha \in \Gamma \) then \( h_c = f_c \cap g_c \) and
\[
h_c(x + y) = \min\{f_c(x + y), g_c(x + y)\} \\
\geq \min\{\min\{f_c(x), f_c(y)\}, \min\{g_c(x), g_c(y)\}\} \\
= \min\{\min\{f_c(x), g_c(x)\}, \min\{f_c(y), g_c(y)\}\} \\
= \min\{f_c \cap g_c(x), f_c \cap g_c(y)\} \\
= \min\{h_c(x), h_c(y)\}
\]

and
\[
h_c(xy) = \min\{f_c(xy), g_c(xy)\} \\
\geq \min\{\max\{f_c(x), f_c(y)\}, \max\{g_c(x), g_c(y)\}\} \\
= \max\{\min\{f_c(x), g_c(x)\}, \min\{f_c(y), g_c(y)\}\} \\
= \max\{f_c \cap g_c(x), f_c \cap g_c(y)\} \\
= \max\{h_c(x), h_c(y)\}
\]

Hence \( h_c \) is a fuzzy ideal of \( S \). Thus \((f, A) \cap (g, B)\) is a fuzzy soft ideal over \( S \). \( \square \)

**Theorem 3.12.** Let \((f, A)\) and \((g, B)\) be two fuzzy soft ideals over \( \Gamma \)-semiring \( S \). Then \((f, A) \cup (g, B)\) is a fuzzy soft ideal over \( S \).

Proof. By definition 2.21, we have \((f, A) \cup (g, B) = (h, C)\) where \( C = A \cup B \)
\[
h_c = \begin{cases} 
  f_c, & \text{if } c \in A \setminus B; \\
  g_c, & \text{if } c \in B \setminus A; \\
  f_c \cup g_c, & \text{if } c \in A \cap B.
\end{cases}
\]

Case(i): If \( c \in A \setminus B \) then \( h_c = f_c, \) \( h_c \) is a fuzzy ideal of \( S \) since \((f, A)\) is a fuzzy soft ideal over \( S \).

Case(ii): If \( c \in B \setminus A \) then \( h_c = g_c, \) \( h_c \) is a fuzzy ideal of \( S \) since \((g, B)\) is a fuzzy soft ideal over \( S \).

Case(iii): If \( c \in A \cap B \) then for all \( x, y \in S, \alpha \in \Gamma \),
\[
h_c(x) = f_c \cup g_c(x) = \max\{f_c(x), g_c(x)\}
\]
\[ h_c(x + y) = \max \{ f_c(x + y), g_c(x + y) \} \geq \max \{ \min \{ f_c(x), f_c(y) \}, \min \{ g_c(x), g_c(y) \} \} = \min \{ \max \{ f_c(x), f_c(y) \}, \max \{ g_c(x), g_c(y) \} \} = \max \{ (f \cup g)_c(x), (f \cup g)_c(y) \} \]

\[ h_c(x \alpha y) = (f \cup g)_c(x \alpha y) = \max \{ f_c(x \alpha y), g_c(x \alpha y) \} \geq \max \{ \max \{ f_c(x), f_c(y) \}, \max \{ g_c(x), g_c(y) \} \} = \max \{ \max \{ f_c(x), f_c(y) \}, \max \{ f_c(y), g_c(y) \} \} \]

Hence, \( h_c \) is a fuzzy soft ideal of \( S \). Therefore \( (h, C) \) is a fuzzy soft ideal over \( S \). \( \Box \)

**Theorem 3.13.** Let \( (f, A) \) and \( (g, B) \) be fuzzy soft ideals over a \( \Gamma \)-semiring \( S \). Then \( (f, A) \land \ (g, B) \) is a fuzzy soft ideal over \( S \).

**Proof.** By definition 2.22, \( (f, A) \land \ (g, B) = (h, C) \) where \( C = A \times B \).

Let \( c = (a, b) \in C = A \times B \) and \( x, y \in S, \alpha \in \Gamma \). Then

\[ h_c(x + y) = f_a(x + y) \land g_b(x + y) \geq \min \{ f_a(x), f_a(y) \}, \min \{ g_b(x), g_b(y) \} \]  
\[ = \min \{ f_a(x), g_b(x) \}, \min \{ f_a(y), g_b(y) \} \]  
\[ = \min \{ h_c(x), h_c(y) \} \]  

\[ h_c(x \alpha y) = f_a(x \alpha y) \land g_b(x \alpha y) \geq \min \{ f_a(x), f_a(y) \}, \max \{ g_b(x), g_b(y) \} \]  
\[ = \max \{ \min \{ f_a(x), f_a(y) \}, \min \{ f_a(y), g_b(y) \} \} \]  
\[ = \max \{ h_c(x), h_c(y) \} \]  

Hence, \( h_c \) is a fuzzy soft ideal over \( S \). Therefore \( (h, A \times B) \) is a fuzzy soft ideal over \( S \). \( \Box \)

**Theorem 3.14.** Let \( (f, A) \) and \( (g, B) \) be fuzzy soft ideals over a \( \Gamma \)-semiring \( S \). Then \( (f, A) \lor \ (g, B) \) is a fuzzy soft ideal over \( S \).

**Proof.** The proof is similar to that of theorem 3.23 and using definition 2.23. \( \Box \)

4. FUZZY SOFT K-IDEAL

In this section the concept of fuzzy soft k-ideal in \( \Gamma \)-semiring is introduced and study the properties related to this notion.

**Definition 4.1.** A fuzzy ideal \( \mu \) of a \( \Gamma \)-semiring \( S \) is said to be fuzzy \( k \)-ideal of \( S \) if \( \mu(x) \geq \min \{ \mu(x + y), \mu(y) \} \) for all \( x, y \in S \).

**Definition 4.2.** Let \( (f, A) \) be fuzzy soft ideal over \( \Gamma \)-semiring \( S \). Then \( (f, A) \) is said to be fuzzy soft \( k \)-ideal if \( f_a \) is fuzzy \( k \)-ideal of \( \Gamma \)-semiring \( S \), for all \( a \in A \).

**Theorem 4.1.** Let \( (f, A) \) and \( (g, B) \) be two fuzzy soft \( k \)-ideals over a \( \Gamma \)-semiring \( S \). Then \( (f, A) \land \ (g, B) \) is a fuzzy soft \( k \)-ideal if it is non null.

**Proof.** By theorem 3.21, \( (f, A) \land \ (g, B) \) is a fuzzy soft ideal over \( S \). Let \( (f, A) \land \ (g, B) = (h, C) \) where \( C = A \cup B \). If \( c \in A \setminus B \) then \( h_c = f_c \), \( h_c \) is a fuzzy \( k \)-ideal of \( S \) since \( (f, A) \) is a fuzzy soft \( k \)-ideal over \( S \). If \( c \in B \setminus A \) then \( h_c = g_c \), \( h_c \) is a fuzzy 350
k-ideal of $S$ since $(g, B)$ is a fuzzy soft $k$–ideal. If $c \in A \cap B$ then $h_c = (f \cap g)_c$ and

$$h_c(x) = (f \cap g)_c(x) = \min\{f_c(x), g_c(x)\} \geq \min\{\min\{f_c(x + y), f_c(y)\}, \min\{g_c(x + y), g_c(y)\}\} = \min\{\min\{f_c(x + y), g_c(x + y)\}, \min\{f_c(x), g_c(x)\}\} = \min\{f_c \cap g_c(x + y), f_c \cap g_c(y)\}$$

for all $x, y \in S$. Hence $f_c \cap g_c$ is a fuzzy $k$–ideal of $S$.

Thus $(f, A) \cap (g, B)$ is a fuzzy soft $k$–ideal over $S$. \quad \square

**Theorem 4.2.** Let $(f, A)$ and $(g, B)$ be two fuzzy soft $k$–ideals over a $\Gamma$–semiring $S$. Then $(f, A) \cup (g, B)$ is a fuzzy soft $k$–ideal over $S$.

**Proof.** By theorem 3.22, $(f, A) \cup (g, B)$ is a fuzzy soft ideal over $S$. By definition 2.21, $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$.

If $c \in A \setminus B$ then $h_c = f_c$, $h_c$ is a fuzzy $k$–ideal since $(f, A)$ is a fuzzy soft $k$–ideal.

If $c \in B \setminus A$ then $h_c = g_c$, $h_c$ is a fuzzy $k$–ideal since $(g, B)$ is a fuzzy soft $k$–ideal.

If $c \in A \cap B$ then $h_c = f_c \cup g_c$. Clearly $h_c$ is a fuzzy ideal. Let $x, y \in S$. Then

$$h_c(x) = (f \cup g)_c(x) = \max\{f_c(x), g_c(x)\} \geq \max\{\min\{f_c(x + y), f_c(y)\}, \min\{g_c(x + y), g_c(x)\}\} = \max\{\min\{f_c(x + y), g_c(x + y)\}, \max\{f_c(x), g_c(x)\}\} = \min\{\max\{f_c(x + y), g_c(x + y)\}, \max\{f_c(x), g_c(x)\}\}$$

Hence $h_c$ is a fuzzy $k$–ideal of $S$. Therefore $(h, C)$ is a fuzzy soft $k$–ideal over $S$. \quad \square

**Theorem 4.3.** Let $(f, A)$ and $(g, B)$ be fuzzy soft $k$–ideals over a $\Gamma$–semiring $S$. Then “$(f, A)$ or $(g, B)$” denoted by $(f, A) \lor (g, B) = (h, A \times B)$ where $h_c = f_a \cup g_b$ for all $c = (a, b) \in A \times B$ is a fuzzy soft $k$–ideal over $S$.

**Proof.** Let $c = (a, b) \in A \times B, x \in S$. Then, by theorem 3.24, $(f, A) \lor (g, B)$ is a fuzzy soft left ideal over $S$ and

$$h_c(x) = f_a(x) \cup g_b(x) = \max\{f_a(x), g_b(x)\} \geq \max\{\min\{f_a(x + y), f_a(y)\}, \min\{g_b(x + y), g_b(y)\}\} = \max\{\min\{f_a(x + y), g_b(x + y)\}, \min\{f_a(y), g_b(y)\}\} = \min\{\max\{f_a(x + y), g_b(x + y)\}, \max\{f_a(y), g_b(y)\}\} = \min\{f_a \cup g_b(x + y), f_a \cup g_b(y)\}$$

Hence $h_c$ is a fuzzy $k$–ideal of $S$. Therefore $(h, A \times B)$ is a fuzzy soft $k$–ideal over $S$. \quad \square

**Theorem 4.4.** Let $(f, A)$ and $(g, B)$ be fuzzy soft $k$–ideals over a $\Gamma$–semiring $S$. Then $(f, A) \land (g, B)$ is a fuzzy soft $k$–ideal over $S$.

**Proof.** By definition 2.22, $(f, A) \land (g, B) = (h, C)$ where $C = A \times B$.

Let $c = (a, b) \in C = A \times B$ and $x, y \in S$. Then, by theorem 3.23, $(f, A) \land (g, B)$ is a
fuzzy soft ideal over $S$ and

$$h_c(x) = f_a(x) \land g_b(x)$$

$$= \min\{f_a(x), g_b(x)\}$$

$$\geq \min\{\min\{f_a(x+y), f_a(y)\}, \min\{g_b(x+y), g_b(y)\}\}$$

$$= \min\{\min\{f_a(x+y), g_b(x+y)\}, \min\{f_a(y), g_b(y)\}\}$$

$$= \min\{f_a \land g_b(x+y), f_a \land g_b(y)\}$$

Hence $h_c$ is a fuzzy $k$–ideal over $S$. Therefore $(f, A) \land (g, B)$ is a fuzzy soft $k$–ideal over $S$.

**Theorem 4.5.** Let $S$ be a $\Gamma$–semiring and $E$ be a parameter set and $A \subseteq E$. Then $(f, A)$ is a fuzzy soft $k$–ideal over $S$ if and only if for each $a \in A$, $(f_a)_t(t \in Im(f_a))$ is a $k$–ideal of $S$ where $f_a$ is the fuzzy subset of $S$.

**Proof.** Suppose $(f, A)$ is a fuzzy soft $k$–ideal over $S$. Let $a \in A$ and $t \in Im(f_a)$ and $x, y \in (f_a)_t, t \in \Gamma, r \in S$. Then $f_a(x + y) \geq \min\{f_a(x), f_a(y)\}$ and $f_a(x \land y) \geq \max\{f_a(x), f_a(y)\}$.

Let $x \in (f_a)_t, r \in S, a \in \Gamma$. Then $f_a(xr) \geq \max\{f_a(x), f_a(r)\} = f_a(x) \geq t$ and $f_a(xr) \geq \max\{f_a(r), f_a(x)\} = f_a(x) \geq t$. Hence $x, x, x \in (f_a)_t$. Therefore, for each $t \in Im(f_a), (f_a)_t$ is an ideal of $S$. Let $x + y \in (f_a)_t, y \in (f_a)_t$. Then $f_a(x + y) \geq t, f_a(y) \geq t$ and hence $f_a(x) \geq t$ since $f_a$ is a fuzzy $k$–ideal. Therefore $x \in (f_a)_t$. Hence $(f_a)_t$ is $k$–ideal of $S$. Conversely, suppose that $(f_a)_t$ is a $k$–ideal of $S$ for each $t \in Im(f_a)$ and corresponding to each $a \in A$ and $x, y \in SoA \subseteq \Gamma$. Suppose $f_a(x + y) < \min\{f_a(x), f_a(y)\} = t_1$(say). Then $x, x \in (f_a)_t, x + y \notin (f_a)_t$, which is a contradiction. So $f_a(x + y) \geq \min\{f_a(x), f_a(y)\}$.

Suppose $f_a(x \land y) < \max\{f_a(x), f_a(y)\} = t_2$(say). Then $x, x \in (f_a)_t, x \land y \notin (f_a)_t$, which is a contradiction. Therefore $f_a(x \land y) \geq \max\{f_a(x), f_a(y)\}$. Let $x, y \in S$ and $a \in A$, $\min\{f_a(x + y), f_a(y)\} = t_1$(say). Then $y, x \in (f_a)_t$ implies that $x \in (f_a)_t$, so that $f_a(x) \geq t_1 = \min\{f_a(x + y), f_a(y)\}$. Hence $f_a$ is a fuzzy $k$–ideal of $S$. Therefore $(f, A)$ is fuzzy soft $k$–ideal over $S$.

**Definition 4.3.** A fuzzy ideal $f$ of a $\Gamma$–semiring $S$ is said to be a $k$–fuzzy ideal of $S$ if $(f(x + y)) = f(0)$ and $f(y) = f(0)$ then $f(x) = f(0)$ for all $x, y \in S$.

**Definition 4.4.** Let $(f, A)$ be fuzzy soft ideal over $\Gamma$–semiring $S$. Then $(f, A)$ is said to be $k$–fuzzy soft ideal if $f_a$ is a $k$–fuzzy ideal of $\Gamma$–semiring $S$, for all $a \in A$.

**Theorem 4.6.** Let $(f, A)$ be a fuzzy soft $k$–ideal over $\Gamma$–semiring $S$ then $(f, A)$ is a $k$–fuzzy soft ideal.

**Proof.** Let $a \in A$. Then $f_a$ is a fuzzy $k$–ideal since $(f, A)$ is fuzzy soft $k$–ideal. Let $x, y \in S$ such that $f_a(x + y) = f_a(0)$ and $f_a(y) = f_a(0)$. Then

$$f_a(x) \geq \min\{f_a(x + y), f_a(y)\}$$

$$\geq \min\{f_a(0), f_a(0)\}$$

$$= f_a(0)$$

Therefore $f_a(x) \geq f_a(0)$. Also, we have $f_a(0) \geq f_a(x)$. Hence $f_a(x) = f_a(0)$. Therefore $f_a$ is a $k$–fuzzy ideal of $S$.

Hence $(f, A)$ is a $k$–fuzzy soft ideal over $\Gamma$–semiring $S$. 

352
5. Fuzzy soft ideal of a fuzzy soft $\Gamma-$semiring

In this section the concept of fuzzy soft ideal of a fuzzy soft $\Gamma-$semiring is introduced and studied the properties related to this notion.

**Definition 5.1.** Let $(f, A)$ be a fuzzy soft $\Gamma-$semiring over $\Gamma-$semiring $S$. A non null fuzzy soft set $(g, B)$ over $S$ is called a fuzzy soft ideal of $(f, A)$ if it satisfies the following conditions

(i) $(g, B)$ is a fuzzy soft subset of $(f, A)$

(ii) $(g, B)$ is a fuzzy soft ideal over $S$.

**Theorem 5.1.** Let $(f, A)$ and $(g, B)$ be two fuzzy soft ideals of a fuzzy soft $\Gamma-$semiring $(h, C)$ over $\Gamma-$semiring $S$. Then $(f, A) \cap (g, B)$ is a fuzzy soft ideal of $(h, C)$ if it is non null.

**Proof.** By theorem 3.21, $(f, A) \cap (g, B)$ is a fuzzy soft ideal over $S$. By definition 5.1, $(f, A)$ and $(g, B)$ are fuzzy soft subsets of $(h, C)$. Hence $(f, A) \cap (g, B)$ is a fuzzy soft ideal of $(h, C)$. □

The following theorem can be proved easily.

**Theorem 5.2.** Let $(f, A)$ and $(g, B)$ be two fuzzy soft ideals of a fuzzy soft $\Gamma-$semiring $(h, C)$ over $\Gamma-$semiring $S$. Then $(f, A) \cup (g, B)$ is a fuzzy soft ideal of $(h, C)$ if it is non null.

**Theorem 5.3.** Let $(f, A)$ and $(g, B)$ be fuzzy soft $\Gamma-$semirings over $\Gamma-$semiring $S$. Let $(f_1, C)$ and $(g_1, D)$ be fuzzy soft ideals of $(f, A)$ and $(g, B)$ respectively. Then $(f_1, C) \cap (g_1, D)$ is a fuzzy soft ideal of $(f, A) \cap (g, B)$ if it is non null.

**Proof.** Since $(f_1, C)$ and $(g_1, D)$ are fuzzy soft ideals of $(f, A)$ and $(g, B)$ respectively, we have $(f_1, C)$ and $(g_1, D)$ are fuzzy soft ideals over $S$. By theorem 3.21, $(f_1, C) \cap (g_1, D)$ is a fuzzy soft ideal over $S$ and by theorem 3.9, $(f, A) \cap (g, B)$ is a fuzzy soft $\Gamma-$semiring over $S$. Hence $(f_1, C) \cap (g_1, D)$ is a fuzzy soft ideal of $(f, A) \cap (g, B)$. □

**Acknowledgements.** This paper is dedicated to my teacher Prof. K.L.N. Swamy, Former Rector, Andhra University, Visakhapatnam, Andhra Pradesh, India.

**References**


M. Murali Krishna Rao (mmkr@gitam.edu)
Department of mathematics, GIT, GITAM University, Visakhapatnam-530 045, Andhra Pradesh, India