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Fuzzy soft Γ-semiring and fuzzy soft k-ideal over Γ-semiring

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Abstract. In this paper we introduce the notion of fuzzy soft $\Gamma-$semirings, fuzzy soft ideals and fuzzy soft k-ideals over $\Gamma-$semirings and study some of their algebraical properties.

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1. Introduction

The notion of a semiring was introduced by H.S.Vandiver[9] in 1934. Semiring is a well known Universal algebra. Semiring have been used for studying optimization theory, Graph theory, Matrices, Determinants, Theory of Automata, Formal language theory, Coding theory, Analysis of computer programmes, etc. Notion of $\Gamma-$semiring was introduced by M. Murali Krishna Rao[7] not only generalizes the notion of semiring and gammaring but also the notion of ternary semiring. The natural growth of gamma semiring is influenced by two things. One is the generalization of results of gamma rings and another is the generalization of results of semirings and ternary semirings. This notion provides an algebraic back ground to the non positive cones of the totally ordered rings. Molodtsov[6] introduced the concept of Soft set theory as a new mathematical tool for dealing with uncertainties. The theory of fuzzy sets most appropriate theory for dealing with uncertainty is first introduced by L.A. Zadeh[10]. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to Logic, Set theory, Group theory, Ring theory, Real analysis, Topology, Measure theory etc. The concept of fuzzy subgroup was introduced by A. Rosenfeld [8] in 1971. Then Maji et.al[5] extended soft set theory to fuzzy soft set theory. F. Feng et.al[3] initiated the study of soft semirings. Soft rings are defined by U. Acar et.al[1] and Jayanth Ghosh et.al[4] initiated the study of Fuzzy soft rings and Fuzzy soft ideals. In this paper we introduce the notion of fuzzy soft $\Gamma-$semirings, fuzzy soft ideals and fuzzy soft k-ideals over $\Gamma-$semirings and study some of their algebraic properties.

2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier.

Definition 2.1 ([7]). A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called semiring provided
(i) addition is a commutative operation.
(ii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for each $x \in S$.
(iii) multiplication distributes over addition both from the left and from the right.

Definition 2.2. [7] Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$ semiring, if there exists a mapping $M \times \Gamma \times M \to M$ is written (x, α, y) as $x\alpha y$ such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$
(i) $x\alpha(y + z) = x\alpha y + x\alpha z$
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Definition 2.3 ([7]). Let S be a $\Gamma-$semiring and A be a nonempty subset of S. A is called a $\Gamma-$subsemiring of S if A is a sub-semigroup of $(S, +)$ and $A\Gamma A \subseteq A$.

Definition 2.4 ([7]). Let S be a $\Gamma-$semiring. A subset A of S is called a left(right) ideals of S if A is closed under addition and $S\Gamma A \subseteq A(A\Gamma S \subseteq A)$. A is called an ideal of S if it is both a left ideal and right ideal.

Definition 2.5 ([10]). Let S be a non empty set, a mapping $f : S \to [0, 1]$ is called a fuzzy subset of S.

Definition 2.6 ([10]). Let f be a fuzzy subset of a nonempty subset S, for $t \in [0, 1]$ the set $f_t = \{x \in S \mid f(x) \geq t\}$ is called level subset of S w.r.t. f.

Definition 2.7 ([2]). Let S be a $\Gamma-$semiring . A fuzzy subset μ of S is said to be a fuzzy $\Gamma-$subsemiring of S if it satisfies the following conditions
(i) $\mu(x + y) \geq min\ \{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) \geq min\ \{\mu(x), \mu(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.8 ([2]). A fuzzy subset μ of a $\Gamma-$semiring S is called a fuzzy left(right) ideal of S if for all $x, y \in S, \alpha \in \Gamma$
(i) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) \geq \mu(y)\ (\mu(x))$.

Definition 2.9 ([2]). A fuzzy subset μ of a $\Gamma-$semiring S is called a fuzzy ideal of S if for all $x, y \in S, \alpha \in \Gamma$
(i) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) \geq max\ \{\mu(x), \mu(y)\}$.

Definition 2.10 ([2])**.** An ideal I of a $\Gamma-$semiring S is called $k-$ideal if for $x, y \in S,\ x + y \in I, y \in I \Rightarrow x \in I$.

Definition 2.11 ([5])**.** Let f and g be fuzzy subsets of S then $f \cup g, f \cap g$ are fuzzy subsets of S defined by
$f \cup g(x) = max\{f(x), g(x)\}, f \cap g(x) = min\{f(x), g(x)\}$ for all $x \in S$.

Definition 2.12 ([10])**.** A fuzzy subset $\mu : S \to [0, 1]$ is nonempty if μ is not the constant function.

Definition 2.13 ([10])**.** For any two fuzzy subsets λ and μ of S, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in S$.

Definition 2.14 ([6])**.** Let U be an initial Universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (f, E) is called soft set over U where f is a mapping given by $f : E \to P(U)$.

Definition 2.15 ([6])**.** For a soft set (f, A), the set $\{x \in A \mid f(x) \neq \varnothing\}$ is called Support of (f, A) denoted by $Supp(f, A)$. If $\mathrm{Supp}(f, A) \neq \varnothing$ then (f, A) is called non null soft set.

Definition 2.16 ([5])**.** Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (f, A) is called fuzzy soft set over U where f is a mapping given by $f : A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.17 ([4])**.** Let X be a group and (f, A) be a soft set over X. Then (f, A) is said to be soft group over X if and only if $f(a)$ is a subgroup of X for each $a \in A$.

Definition 2.18. [4] Let X be a group and (f, A) be fuzzy soft set over X. Then (f, A) is said to be fuzzy soft group over X if and only if for each $a \in A, x, y \in X$
(i) $f_a(x * y) \geq f_a(x) * f_a(y)$
(ii) $f_a(x^{-1}) \geq f_a(x)$
where f_a is the fuzzy subset of X corresponding to the parameter $a \in A$.

Definition 2.19 ([5])**.** Let $(f, A), (g, B)$ be fuzzy soft sets over U then (f, A) is said to be a fuzzy soft subset of (g, B) denoted by $(f, A) \subseteq (g, B)$ if $A \subseteq B$ and $f(a) \subseteq g(a)$ for all $a \in A$.

Definition 2.20 ([5])**.** Let $(f, A), (g, B)$ be fuzzy soft sets. The intersection of fuzzy soft sets (f, A) and (g, B) is denoted by $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cap g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.21 ([5])**.** Let $(f, A), (g, B)$ be fuzzy soft sets. The union of fuzzy soft sets (f, A) and (g, B) is denoted by $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cup g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.22 ([5]). Let $(f,A),(g,B)$ be fuzzy soft sets over U. "(f,A) and (g,B) is denoted by "$(f,A)\wedge(g,B)$" is defined by $(f,A)\wedge(g,B)=(h,C)$ where $C=A\times B$. $h_c(x)=min\ \{f_a(x),g_b(x)\}$ for all $c=(a,b)\in A\times B$ and $x\in U$.

Definition 2.23 ([5]). Let $(f,A),(g,B)$ be fuzzy soft sets over U. "(f,A) or (g,B)" is denoted by $(f,A)\vee(g,B)$ is defined by $(f,A)\vee(g,B)=(h,C)$ where $C=A\times B$ and $h_c(x)=Max\ \{f_a(x),g_b(x)\}$ for all $c=(a,b)\in A\times B, x\in U$.

Definition 2.24 ([2]). Let f and g be fuzzy subsets of $\Gamma-$semiring S. Then $f\circ g$ is defined by

$$f\circ g(z)=\begin{cases} \sup\limits_{z=x\alpha y}\{min\{f(x),g(y)\}\}, \\ 0, \quad \text{otherwise}\end{cases}$$

$x,y,z\in S,\alpha\in\Gamma$.

Definition 2.25 ([5]). Let S and T be two sets. Let $\phi:S\to T$ be any function. A fuzzy subset f of S is called ϕ invariant if $\phi(x)=\phi(y)\Rightarrow f(x)=f(y)$.

Definition 2.26 ([7]). A function $f:R\to S$ where R and S are $\Gamma-$semirings is said to be a $\Gamma-$semiring homomorphism if $f(a+b)=f(a)+f(b)$ and $f(a\alpha b)=f(a)\alpha f(b)$ for all $a,b\in R,\alpha\in\Gamma$.

3. Main results

In this section, the concepts of soft $\Gamma-$semiring, fuzzy soft $\Gamma-$semiring and fuzzy soft ideal over $\Gamma-$semiring are introduced and study the properties related to these notions.

Definition 3.1. Let S be a $\Gamma-$semiring and E be a parameter set and $A\subseteq E$. Let f be a mapping given by $f:A\to P(S)$ where $P(S)$ is the power set of S. Then (f,A) is called a soft $\Gamma-$semiring over S if and only if for each $a\in A$, $f(a)$ is $\Gamma-$subsemiring of S. i.e. (i) $x,y\in S\Rightarrow x+y\in f(a)$ (ii) $x,y\in S,\alpha\in\Gamma\Rightarrow x\alpha y\in f(a)$.

Definition 3.2. Let S be a $\Gamma-$semiring and E be a parameter set and $A\subseteq E$. Let f be a mapping given by $f:A\to[0,1]^S$ where $[0,1]^S$ denotes the collection of all fuzzy subsets of S. Then (f,A) is called a fuzzy soft $\Gamma-$semiring over S if and only if for each $a\in A, f(a)=f_a$ is the fuzzy $\Gamma-$subsemiring of S. i.e.,(i) $f_a(x+y)\geq min\{f_a(x),f_a(y)\}$ (ii) $f_a(x\alpha y)\geq min\{f_a(x),f_a(y)\}$ for all $x,y\in S,\alpha\in\Gamma$.

Definition 3.3. Let S be a $\Gamma-$semiring and E be a parameter set and $A\subseteq E$. Let f be a mapping given by $f:A\to P(S)$. Then (f,A) is called a soft left(right) ideal over S if and only if for each $a\in A,f(a)$ is a left(right) ideal of S. i.e., (i) $x,y\in f(a)\Rightarrow x+y\in f(a)$ (ii) $x,y\in f(a),\alpha\in\Gamma,r\in S\Rightarrow r\alpha x(x\alpha r)\in f(a)$.

Definition 3.4. Let S be a $\Gamma-$semiring and E be a parameter set, $A\subseteq E$ and $f:A\to P(R)$. Then (f,A) is called a soft ideal over S if and only if for each $a\in A,f(a)$ is an ideal of S. i.e., (i) $x,y\in f(a)\Rightarrow x+y\in f(a)$ (ii) $x\in f(a),\alpha\in\Gamma,r\in S\Rightarrow r\alpha x\in f(a)$ and $x\alpha r\in f(a)$.

Definition 3.5. Let S be a $\Gamma-$semiring and E be a parameter set and $A\subseteq E$. Let f be a mapping given by $f:A\to[0,1]^S$ where $[0,1]^S$ denotes the collection of all fuzzy subsets of S. Then (f,A) is called a fuzzy soft left(right) ideal over S if and only if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0,1]$ is a fuzzy left(right) ideal of S. i.e., (i) $f_a(x+y) \geq \min\{f_a(x), f_a(y)\}$ (ii) $f_a(x\alpha y) \geq f_a(y)(f_a(x))$ for all $x, y \in S, \alpha \in \Gamma$.

Definition 3.6. Let S be a $\Gamma-$semiring and E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to [0,1]^S$ where $[0,1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft ideal over S if and only if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0,1]$ is a fuzzy ideal of S. i.e., (i) $f_a(x+y) \geq \min\{f_a(x), f_a(y)\}$ (ii) $f_a(x\alpha y) \geq \max\{f_a(x), f_a(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

Definition 3.7. Let $(f, A), (g, B)$ be fuzzy soft ideals over a $\Gamma-$semiring S. The product (f, A) and (g, B) is defined as $((f \circ g), C)$ where $C = A \cup B$ and

$$(f \circ g)_c(x) = \begin{cases} f_c(x), & \text{if } c \in A \setminus B; \\ g_c(x), & \text{if } c \in B \setminus A; \\ \sup\limits_{x=a\alpha b} \{min\{f_c(a), g_c(b)\}\}, & \text{if } c \in A \cap B. \end{cases}$$

for all $c \in A \cup B$ and $x \in S, \alpha \in \Gamma$.

Theorem 3.1. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over a $\Gamma-$semiring S. Then $(f, A) \cup (g, B)$ is a fuzzy soft $\Gamma-$semiring over S.*

Proof. Let $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$.
$Case(i)$: if $A \cap B = \varnothing$.
Let $c \in C = A \cup B$. Then $c \in A$ or $c \in B$. If $c \in A$ then $h_c = f_c$ and if $c \in B$ then $h_c = g_c$, since (f, A) and (g, B) are fuzzy soft $\Gamma-$semirings over a $\Gamma-$semiring S. In both cases h_c is a fuzzy $\Gamma-$subsemiring of S.
$Case(ii)$: if $A \cap B \neq \varnothing$.
If $c \in A \setminus B$ then $h_c = f_c, h_c$ is a fuzzy $\Gamma-$subsemiring.
If $c \in B \setminus A$ then $h_c = gc, h_c$ is a fuzzy $\Gamma-$subsemiring.
If $c \in A \cap B$ then $h_c = f_c \cup g_c$.
Let $x, y \in S, \alpha \in \Gamma$. Then

$$\begin{aligned} h_c(x+y) &= f_c \cup g_c(x+y) \\ &= max\{f_c(x+y), g_c(x+y)\} \\ &\geq max\{min\{f_c(x), f_c(y)\}, min\{g_c(x), g_c(y)\}\} \\ &= min\{max\{f_c(x), g_c(x)\}, max\{f_c(y), g_c(y)\}\} \\ &= min\{f_c \cup g_c(x), f_c \cup g_c(y)\} \end{aligned}$$

$$\begin{aligned} h_c(x\alpha y) &= f_c \cup g_c(x\alpha y) \\ &= max\{f_c(x\alpha y), g_c(x\alpha y)\} \\ &\geq max\{min\{f_c(x), f_c(y)\}, min\{g_c(x), g_c(y)\}\} \\ &= min\{max\{f_c(x), g_c(x)\}, max\{f_c(y), g_c(y)\}\} \\ &= min\{f_c \cup g_c(x), f_c \cup g_c(y)\} \end{aligned}$$

Hence h_c is a fuzzy $\Gamma-$subsemiring of S. Therefore $(f, A) \cup (g, B)$ is a fuzzy soft $\Gamma-$semiring over S. □

Theorem 3.2. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over a $\Gamma-$semiring S. Then $(f, A) \cap (g, B)$ is a fuzzy soft $\Gamma-$semiring over S.*

Proof. Let $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$ and

$$h_c(x) = \begin{cases} f_c(x), & \text{if } c \in A \setminus B; \\ g_c(x), & \text{if } c \in B \setminus A; \\ f_c \cap g_c(x), & \text{if } c \in A \cap B. \end{cases}$$

for all $c \in C = A \cup B$, for all $x \in S$.
$Case(i)$: If $c \in A \setminus B$ then $h_c = f_c$, f_c is a fuzzy $\Gamma-$subsemiring of S since (f, A) is a fuzzy soft $\Gamma-$semiring over S.
$Case(ii)$: If $c \in B \setminus A$ then $h_c = g_b$, g_b is a fuzzy $\Gamma-$subsemiring of S, since (g, B) is a fuzzy soft $\Gamma-$semiring over S
$Case(iii)$: If $c \in A \cap B$ then $h_c(x) = f_c \cap g_c(x)$. Let $x, y \in S, \alpha \in \Gamma$. Then

$$\begin{aligned} h_c(x+y) &= (f \cap g)_c(x+y) \\ &= min\{f_c(x+y), g_c(x+y)\} \\ &\geq min\{min\{f_c(x), f_c(y)\}, min\{g_c(x), g_c(y)\}\} \\ &= min\{min\{f_c(x), g_c(x)\}, min\{f_c(y), g_c(y)\}\} \\ &= min\{(f \cap g)_c(x), (f \cap g)_c(y)\} \end{aligned}$$

$$\begin{aligned} h_c(x\alpha y) &= (f \cap g)_c(x\alpha y) \\ &= min\{f_c(x\alpha y), g_c(x\alpha y)\} \\ &\geq min\{min\{f_c(x), f_c(y)\}, min\{g_c(x), g_c(y)\}\} \\ &= min\{min\{f_c(x), g_c(x)\}, min\{f_c(y), g_c(y)\}\} \\ &= min\{(f \cap g)_c(x), (f \cap g)_c(y)\} \end{aligned}$$

Hence h_c is a fuzzy $\Gamma-$subsemiring of S. Therefore $(f, A) \cap (g, B)$ is a fuzzy soft $\Gamma-$semiring over S. □

Theorem 3.3. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over a $\Gamma-$semiring S. Then $(f, A) \wedge (g, B)$ is a fuzzy soft $\Gamma-$semiring over S.*

Proof. By definition 2.22, $(f, A) \wedge (g, B) = (h, C)$ where $C = A \times B$.
Let $c = (a, b) \in C = A \times B$ and $x, y \in S, \alpha \in \Gamma$. Then

$$\begin{aligned} h_c(x+y) &= min\{f_a(x+y), g_b(x+y)\} \\ &\geq min\{min\{f_a(x), f_a(y)\}, min\{g_b(x), g_b(y)\}\} \\ &= min\{min\{f_a(x), g_b(x)\}, min\{f_a(y), g_b(y)\}\} \\ &= min\{f_a \wedge g_b(x), f_a \wedge g_b(y)\} \\ &= min\{h_c(x), h_c(y)\} \end{aligned}$$

$$\begin{aligned} h_c(x\alpha y) &= min\{f_a(x\alpha y), g_b(x\alpha y)\} \\ &\geq min\{min\{f_a(x), f_a(y)\}, min\{g_b(x), g_b(y)\}\} \\ &= min\{min\{f_a(x), g_b(x)\}, min\{f_a(y), g_b(y)\}\} \\ &= min\{h_c(x), h_c(y)\} \end{aligned}$$

Hence h_c is a fuzzy $\Gamma-$subsemiring of S. Therefore (h, C) is a fuzzy soft $\Gamma-$semiring over S. □

The following theorem can be proved easily.

Theorem 3.4. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over a $\Gamma-$semiring S. Then $(f, A) \vee (g, B)$ is a fuzzy soft $\Gamma-$semiring over S.*

Definition 3.8. Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over S. Then (g, B) is a fuzzy soft $\Gamma-$subsemiring of (f, A) if it satisfies the following conditions
(i) $B \subset A$ (ii) $g_b(x) \leq f_a(x)$ for all $x \in Supp(g, B)$.

Theorem 3.5. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over S. Then the following statements are true*
(i) If $g_b \subset f_b$ for all $b \in B \subset A$ then (g,B) is a fuzzy soft $\Gamma-$subsemiring of (f, A).
(ii) (f, A) and (g, B) are fuzzy soft $\Gamma-$subsemirings of $(f, A) \cup (g, B)$.

Proof. (i) Since $g_b \subset f_b$ for all $b \in B \subset A$ and by definition 3.12, (g, B) is a fuzzy soft $\Gamma-$subsemiring of (f, A).
(ii) Let $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$. By theorem 3.8, (h, C) is a fuzzy soft $\Gamma-$semiring over S. Since $A \subset C$ and $B \subset C$, by definition 3.12, (f, A) and (g, B) are fuzzy soft $\Gamma-$subsemirings of $(f, A) \cup (g, B)$. □

Theorem 3.6. *Let S be a $\Gamma-$semiring and E be a parameter set and $A \subseteq E$. Then (f, A) is a fuzzy soft left ideal over S if and only if for each $a \in A$, $(f_a)_t(t \in Im(f_a))$ is a left ideal of S where f_a is the fuzzy subset of S.*

Proof. Suppose (f, A) is a fuzzy soft left ideal over S. Let $a \in A$ and $t \in Im(f_a)$ and $x, y \in (f_a)_t, \alpha \in \Gamma, r \in S$. Then $f_a(x + y) \geq min\{f_a(x), f_a(y)\} \geq min\{t, t\} = t$ and $f_a(r\alpha x) \geq f_a(x) \geq t$ which implies that $x + y, r\alpha x \in (f_a)_t$. Therefore for each $t \in Im(f_a), (f_a)_t$ is a left ideal of S. Conversely, suppose that $(f_a)_t$ is a left ideal of S for each $t \in Im(f_a)$ and corresponding to each $a \in A$ and $x, y \in S$. Suppose $f_a(x+y) < min\{f_a(x), f_a(y)\} = t_1$(say). Then $x, y \in (f_a)_{t_1}$, but $x+y \notin (f_a)_{t_1}$ which is a contradiction. So $f_a(x + y) \geq min\{f_a(x), f_a(y)\}$. Suppose $f_a(x\alpha y) < f_a(y) = t_2$(say). Then $y \in (f_a)_{t_2}$, but $x\alpha y \notin (f_a)_{t_2}$ which is a contradiction. Therefore $f_a(x\alpha y) \geq f_a(y)$. Hence f_a is a fuzzy left ideal of S. Therefore (f, A) is fuzzy soft left ideal. □

Corollary 3.7. *Let μ be a fuzzy set in a $\Gamma-$semiring S and $A = [0, 1]$. Then (f, A) is a fuzzy soft left ideal over S if and only if μ is a fuzzy left ideal of S.*

Proof. Follows from theorem 3.14. □

Theorem 3.8. *Let S be a $\Gamma-$semiring and E be a parameterset and $A \subseteq E$. Then (f, A) is a fuzzy soft left(right) ideal over S if and only if for each $a \in A$ the corresponding fuzzy set f_a of S satisfy the following conditions*
(1) $f_a(x + y) \geq min\{f_a(x), f_a(y)\}$
(2) $\chi_S \circ f_a \subseteq f_a(f_a \circ \chi_S \subseteq f_a)$
where χ_S stands for characteristic function of S.

Proof. Suppose (f, A) is a fuzzy soft left ideal over S. Then, for each $a \in A$, f_a is fuzzy left ideal of S. Let $z \in S$. Then

$$\chi_S \circ f_a(z) = \sup_{z=x\alpha y} \{min\{\chi_S(x), f_a(y)\}\} = \sup_{z=x\alpha y} \{f_a(y)\} \leq f_a(x\alpha y) = f_a(z)$$

If z cannot be expressed as $z = x\alpha y$ where $x, y \in S, \alpha \in \Gamma$ then $\chi_S \circ f_a(z) = 0 \leq f_a(z)$. Conversely, suppose that for each $a \in A$, f_a satisfy the given two conditions.

Let $x, y \in S, \alpha \in \Gamma$. Then we have

$$f_a(x\alpha y) \geq \chi_S \circ f_a(x\alpha y) = \sup_{x\alpha y = p\beta q} min\{\chi_S(p), f_a(q)\} \geq min\{\chi_S(x), f_a(y)\} = f_a(y).$$

This shows that for each $a \in A, f_a$ is a fuzzy left ideal of S. So (f, A) is a fuzzy soft left ideal over S. Similarly we can prove the result for a fuzzy soft right ideal over S. □

Theorem 3.9. *Let S be a $\Gamma-$semiring and E be a parameter set and $A \subseteq E$. We define a fuzzy subset α_a of S corresponding to $a \in A$ by*

$$\begin{aligned} \alpha_a(x) &= s \quad \text{if } x \in I(a) \\ &= t \quad \text{otherwise} \end{aligned}$$

for all $x \in S$ and $s, t \in [0, 1]$ with $s < t$. Then (α, A) is a fuzzy soft left ideal over S. if and only if (I, A) is a soft left ideal over S

Proof. Let (α, A) be a fuzzy soft left ideal over S and $x, y \in I(a)$ and $r \in S, \gamma \in \Gamma$. Then $\alpha_a(x) = s = \alpha_a(y)$ and hence

$$\alpha_a(x + y) \geq min\{\alpha_a(x), \alpha_a(y)\} = min\{s, s\} = s.$$

So $x + y \in I(a)$. Also, $\alpha_a(r\gamma x) \geq \alpha_a(x) = s$. Therefore $r\gamma x \in I(a)$. Hence $I(a)$ is a left ideal of S. Thus (I, A) is a soft left ideal over S. Conversely, (I, A) be soft left ideal over S for each $a \in A, I(a)$ is a left ideal of S. Let $x, y \in S$ then the following four cases arise for consideration.
Case $(i):\ x, y \in I(a)$.
Then $x + y \in I(a), x\gamma y \in I(a)$ and hence $\alpha_a(x + y) = s, \alpha_a(x) = s = \alpha_a(y) = \alpha_a(x\gamma y)$. Therefore $\alpha_a(x + y) = min\{\alpha_a(x), \alpha_a(y)\} = s$ and $\alpha_a(x\gamma y) = s = \alpha_a(y)$.
Case $(ii):\ x \in I(a), y \notin I(a)$.
Then $x + y \notin I(a)$ and hence $\alpha_a(x) = s, \alpha_a(y) = t$.Therefore $\alpha_a(x + y) = t$. Now

$$t = \alpha_a(x + y) \geq min\{\alpha_a(x), \alpha_a(y)\} = min\{s, t\} = s.$$

Hence $y \cdot x \in I(a)$ implies that $\alpha_a(y\gamma x) = s = \alpha_a(x)$.
Case$(iii):\ x, y \notin I(a)$.
Then $x + y \notin I(a)$ and $x\gamma y \notin I(a)$ and hence $\alpha_a(x) = t = \alpha_a(y)$.
$t = \alpha_a(x{+}y) \geq min\{\alpha_a(x), \alpha_a(y)\} = min\{t, t\} = t$. Therefore $\alpha_a(x\gamma y) = t = \alpha_a(y)$. Hence α_a is a fuzzy left ideal of S for each $a \in A$. Therefore (α, A) is a fuzzy soft left ideal over S. □

Definition 3.9. A fuzzy set μ of $\Gamma-$semiring M is said to be normal fuzzy if μ is a $\Gamma-$sub semiring of M and $\mu(0) = 1$.

Definition 3.10. Let (f, A) be fuzzy soft $\Gamma-$semiring over S. Then (f, A) is said to be normal fuzzy soft $\Gamma-$semiring if f_a is normal fuzzy of $\Gamma-$semiring over S, for all $a \in A$.

Theorem 3.10. *If (f, A) is a fuzzy soft $\Gamma-$semiring over S and for each $a \in A$, f_a^+ is defined by $f_a^+(x) = f_a(x) + 1 - f_a(0)$ for all $x \in S$ then (f^+, A) is a normal fuzzy soft $\Gamma-$semiring over S and (f, A) is subset of (f^+, A).*

Proof. Let $x, y \in S, \alpha \in \Gamma$ and $a \in A$. Then

$$\begin{aligned} f_a^+(x+y) &= f_a(x+y) + 1 - f_a(0) \\ &\geq min\{f_a(x), f_a(y)\} + 1 - f_a(0) \\ &= min\{f_a(x) + 1 - f_a(0), f_a(y) + 1 - f_a(0)\} \\ &= min\{f_a^+(x), f_a^+(y)\} \end{aligned}$$

$$\begin{aligned} f_a^+(x\alpha y) &= f_a(x\alpha y) + 1 - f_a(0) \\ &\geq min\{f_a(x), f_a(y)\} + 1 - f_a(0) \\ &= min\{f_a(x) + 1 - f_a(0), f_a(y) + 1 - f_a(0)\} \\ &= min\{f_a^+(x), f_a^+(y)\} \end{aligned}$$

If $x = 0$, then $f_a^+(0) = 1$ and $f_a \subset f_a^+$. Hence (f^+, A) is a normal fuzzy soft $\Gamma-$semiring over S and (f, A) is a subset of (f^+, A). □

Theorem 3.11. *Let (f, A) and (g, B) be fuzzy soft ideals over $\Gamma-$semiring S. Then $(f, A) \cap (g, B)$ is a fuzzy soft ideal over S.*

Proof. By definition 2.20, we have $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$.
Case (i) : $h_c = f_c$ if $c \in A \setminus B$. Then h_c is a fuzzy ideal of S since (f, A) is a fuzzy soft ideal over S.
Case (ii) : If $c \in B \setminus A$ then $h_c = g_c$. Therefore h_c is a fuzzy ideal of S since $(g, B)$is a fuzzy soft ideal over S.
Case (iii) : If $c \in A \cap B$, and $x, y \in S, \alpha \in \Gamma$ then $h_c = f_c \cap g_c$ and

$$\begin{aligned} h_c(x+y) &= min\{f_c(x+y), g_c(x+y)\} \\ &\geq min\{min\{f_c(x), f_c(y)\}, min\{g_c(x), g_c(y)\}\} \\ &= min\{min\{f_c(x), g_c(x)\}, min\{f_c(y), g_c(y)\}\} \\ &= min\{f_c \cap g_c(x), f_c \cap g_c(y)\} \\ &= min\{h_c(x), h_c(y)\} \end{aligned}$$

and

$$\begin{aligned} h_c(x\alpha y) &= min\{f_c(x\alpha y), g_c(x\alpha y)\} \\ &\geq min\{max\{f_c(x), f_c(y)\}, max\{g_c(x), g_c(y)\}\} \\ &= max\{min\{f_c(x), g_c(x)\}, min\{f_c(y), g_c(y)\}\} \\ &= max\{f_c \cap g_c(x), f_c \cap g_c(y)\} \\ &= max\{h_c(x), h_c(y)\} \end{aligned}$$

Hence h_c is a fuzzy ideal of S. Thus $(f, A) \cap (g, B)$ is a fuzzy soft ideal over S. □

Theorem 3.12. *Let (f, A) and (g, B) be two fuzzy soft ideals over $\Gamma-$semiring$S$. Then $(f, A) \cup (g, B)$ is a fuzzy soft ideal over S.*

Proof. By definition 2.21, we have $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cup g_c, & \text{if } c \in A \cap B. \end{cases}$$

Case(i): If $c \in A \setminus B$ then $h_c = f_c$, h_c is a fuzzy ideal of S since (f, A) is a fuzzy soft ideal over S.
Case(ii): If $c \in B \setminus A$ then $h_c = g_c$, h_c is a fuzzy ideal of S since (g, B) is a fuzzy soft ideal over S.
Case(iii): If $c \in A \cap B$ then for all $x, y \in S, \alpha \in \Gamma$,
$h_c(x) = f_c \cup g_c(x) = max\{f_c(x), g_c(x)\}$

$$\begin{aligned}
h_c(x+y) &= max\{f_c(x+y), g_c(x+y)\} \\
&\geq max\{min\{f_c(x), f_c(y)\}, min\{g_c(x), g_c(y)\}\} \\
&= min\{max\{f_c(x), g_c(x)\}, max\{f_c(y), g_c(y)\}\} \\
&= max\{(f \cup g)_c(x), (f \cup g)_c(y)\} \\
h_c(x\alpha y) &= (f \cup g)_c(x\alpha y) \\
&= max\{f_c(x\alpha y), g_c(x\alpha y)\} \\
&\geq max\{max\{f_c(x), f_c(y)\}, max\{g_c(x), g_c(y)\}\} \\
&= max\{max\{f_c(x), g_c(x)\}, max\{f_c(y), g_c(y)\}\}
\end{aligned}$$

Hence h_c is a fuzzy ideal of S. Therefore (h, C) is a fuzzy soft ideal over S. □

Theorem 3.13. *Let (f, A) and (g, B) be fuzzy soft ideals over a $\Gamma-$semiring S. Then $(f, A) \wedge (g, B)$ is a fuzzy soft ideal over S.*

Proof. By definition 2.22, $(f, A) \wedge (g, B) = (h, C)$ where $C = A \times B$.
Let $c = (a, b) \in C = A \times B$ and $x, y \in S, \alpha \in \Gamma$. Then

$$\begin{aligned}
h_c(x+y) &= f_a(x+y) \wedge g_b(x+y) \\
&= min\{f_a(x+y), g_b(x+y)\} \\
&\geq min\{min\{f_a(x), f_a(y)\}, min\{g_b(x), g_b(y)\}\} \\
&= min\{min\{f_a(x), g_b(x)\}, min\{f_a(y), g_b(y)\}\} \\
&= min\{f_a \wedge g_b(x), f_a \wedge g_b(y)\} \\
&= min\{h_c(x), h_c(y)\} \\
h_c(x\alpha y) &= f_a(x\alpha y) \wedge g_b(x\alpha y) \\
&= min\{f_a(x\alpha y), g_b(x\alpha y)\} \\
&\geq min\{max\{f_a(x), f_a(y)\}, max\{g_b(x), g_b(y)\}\} \\
&= max\{min\{f_a(x), g_b(x)\}, min\{f_a(y), g_b(y)\}\} \\
&= max\{h_c(x), h_c(y)\}
\end{aligned}$$

Hence h_c is a fuzzy soft ideal over S. Therefore $(h, A \times B)$ is a fuzzy soft ideal over S. □

Theorem 3.14. *Let (f, A) and (g, B) be fuzzy soft ideals over a $\Gamma-$semiring S. Then $(f, A) \vee (g, B)$ is a fuzzy soft ideal over S.*

Proof. The proof is similar to that of theorem 3.23 and using definition 2.23. □

4. Fuzzy soft k-ideal

In this section the concept of fuzzy soft k-ideal in $\Gamma-$semiring is introduced and study the properties related to this notion.

Definition 4.1. A fuzzy ideal μ of a $\Gamma-$semiring S is said to be fuzzy $k-$ideal of S if $\mu(x) \geq min\{\mu(x+y), \mu(y)\}$ for all $x, y \in S$.

Definition 4.2. Let (f, A) be fuzzy soft ideal over $\Gamma-$semiring S. Then (f, A) is said to be fuzzy soft k-ideal if f_a is fuzzy $k-$ideal of $\Gamma-$semiring S, for all $a \in A$.

Theorem 4.1. *Let (f, A) and (g, B) be two fuzzy soft $k-$ideals over a $\Gamma-$semiring S. Then $(f, A) \cap (g, B)$ is a fuzzy soft $k-$ideal if it is non null.*

Proof. By theorem 3.21, $(f, A) \cap (g, B)$ is a fuzzy soft ideal over S. Let $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$. If $c \in A \setminus B$ then $h_c = f_c$, h_c is a fuzzy $k-$ideal of S since (f, A) is a fuzzy soft $k-$ideal over S. If $c \in B \setminus A$ then $h_c = g_c$, h_c is a fuzzy k-ideal of S since (g, B) is a fuzzy soft $k-$ideal. If $c \in A \cap B$ then $h_c = (f \cap g)_c$ and

$$\begin{array}{rcc} h_c(x) & = & (f \cap g)_c(x) \\ & = & min\{f_c(x), g_c(x)\} \\ & \geq & min\{min\{f_c(x+y), f_c(y)\}, min\{g_c(x+y), g_c(y)\}\} \\ & = & min\{min\{f_c(x+y), g_c(x+y)\}, min\{f_c(y), g_c(y)\}\} \\ & = & min\{f_c \cap g_c(x+y), f_c \cap g_c(y)\} \end{array}$$

for all $x, y \in S$. Hence $f_c \cap g_c$ is a fuzzy $k-$ideal of S.
Therefore $(f, A) \cap (g, B)$ is a fuzz soft $k-$ideal over S. □

Theorem 4.2. *Let (f, A) and (g, B) be two fuzzy soft $k-$ideals over a $\Gamma-$semiring S. Then $(f, A) \cup (g, B)$ is a fuzzy soft $k-$ideal over S.*

Proof. By theorem 3.22, $(f, A) \cup (g, B)$ is a fuzzy soft ideal over S. By definition 2.21, $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$.
If $c \in A \setminus B$ then $h_c = f_c, h_c$ is a fuzzy $k-$ideal since (f, A) is a fuzzy soft $k-$ideal.
If $c \in B \setminus A$ then $h_c = g_c, h_c$ is a fuzzy $k-$ideal since (g, B) is a fuzzy soft $k-$ideal.
If $c \in A \cap B$ then $h_c = f_c \cup g_c$. Clearly h_c is a fuzzy ideal. Let $x, y \in S$. Then

$$\begin{array}{rcc} h_c(x) & = & (f_c \cup g_c)(x) \\ & = & max\{f_c(x), g_c(x)\} \\ & \geq & max\{min\{f_c(x+y), f_c(x)\}, min\{g_c(x+y), g_c(x)\}\} \\ & = & max\{min\{f_c(x+y), g_c(x+y)\}, min\{f_c(x), g_c(x)\}\} \\ & = & min\{max\{f_c(x+y), g_c(x+y)\}, max\{f_c(x), g_c(x)\}\} \\ & = & min\{f_c \cup g_c(x+y), f_c \cup g_c(x)\} \end{array}$$

Hence h_c is a fuzzy $k-$ideal of S. Therefore (h, C) is a fuzzy soft $k-$ideal over S. □

Theorem 4.3. *Let (f, A) and (g, B) be fuzzy soft $k-$ideals over a $\Gamma-$semiring S. Then "(f, A) or (g, B)" denoted by $(f, A) \vee (g, B) = (h, A \times B)$ where $h_c = f_a \cup g_b$ for all $c = (a, b) \in A \times B$ is a fuzzy soft $k-$ideal over S.*

Proof. Let $c = (a, b) \in A \times B, x \in S$. Then, by theorem 3.24, $(f, A) \vee (g, B)$ is a fuzzy soft left ideal over S and

$$\begin{array}{rcc} h_c(x) & = & f_a(x) \cup g_b(x) \\ & = & max\{f_a(x), g_b(x)\} \\ & \geq & max\{min\{f_a(x+y), f_a(y)\}, min\{g_b(x+y), g_b(y)\}\} \\ & = & max\{min\{f_a(x+y), g_b(x+y)\}, min\{f_a(y), g_b(y)\}\} \\ & = & min\{max\{f_a(x+y), g_b(x+y)\}, max\{f_a(y), g_b(y)\}\} \\ & = & min\{f_a \cup g_b(x+y), f_a \cup g_b(x)\} \end{array}$$

Hence h_c is a fuzzy $k-$ideal of S. Therefore $(h, A \times B)$ is a fuzzy soft $k-$ideal over S. □

Theorem 4.4. *Let (f, A) and (g, B) be fuzzy soft $k-$ideals over a $\Gamma-$semiring S. Then $(f, A) \wedge (g, B)$ is a fuzzy soft $k-$ideal over S.*

Proof. By definition 2.22, $(f, A) \wedge (g, B) = (h, C)$ where $C = A \times B$.
Let $c = (a, b) \in C = A \times B$ and $x, y \in S$. Then, by theorem 3.23, $(f, A) \wedge (g, B)$ is a

fuzzy soft ideal over S and

$$\begin{aligned} h_c(x) &= f_a(x) \wedge g_b(x) \\ &= min\{f_a(x), g_b(x)\} \\ &\geq min\{min\{f_a(x+y), f_a(y)\}, min\{g_b(x+y), g_b(y)\}\} \\ &= min\{min\{f_a(x+y), g_b(x+y)\}, min\{f_a(y), g_b(y)\}\} \\ &= min\{f_a \wedge g_b(x+y), f_a \wedge g_b(y)\} \end{aligned}$$

Hence h_c is a fuzzy $k-$ideal over S. Therefore $(f, A) \wedge (g, B)$ is a fuzzy soft $k-$ideal over S. □

Theorem 4.5. *Let S be a $\Gamma-$semiring and E be a parameter set and $A \subseteq E$. Then (f, A) is a fuzzy soft $k-$ideal over S if and only if for each $a \in A$, $(f_a)_t (t \in Im(f_a))$ is a $k-$ideal of S where f_a is the fuzzy subset of S.*

Proof. Suppose (f, A) is a fuzzy soft $k-$ideal over S. Let $a \in A$ and $t \in Im(f_a)$ and $x, y \in (f_a)_t, \alpha \in \Gamma, r \in S$. Then
$f_a(x+y) \geq min\{f_a(x), f(y)\} \geq min\{t, t\} = t$ and $f_a(x\alpha y) \geq max\{f_a(x), f_a(y)\} = max\{t, t\} = t$ and hence $x+y, x\alpha y \in (f_a)_t$. Let $x \in (f_a)_t, r \in S, \alpha \in \Gamma$. Then $f_a(x\alpha r) \geq max\{f_a(x), f_a(r)\} = f_a(x) \geq t$ and $f_a(r\alpha x) \geq max\{f_a(r), f_a(x)\} = f_a(x) \geq t$. Hence $r\alpha x, x\alpha r \in (f_a)_t$. Therefore, for each $t \in Im(f_a), (f_a)_t$ is an ideal of S. Let $x+y \in (f_a)_t, y \in (f_a)_t$. Then $f_a(x+y) \geq t, f_a(y) \geq t$ and hence $f_a(x) \geq t$ since f_a is a fuzzy $k-$ideal. Therefore $x \in (f_a)_t$. Hence $(f_a)_t$ is $k-$ideal of S. Conversely, suppose that $(f_a)_t$ is a $k-$ideal of S for each $t \in Im(f_a)$ and corresponding to each $a \in A$ and $x, y \in S\alpha \in \Gamma$. Suppose $f_a(x+y) < min\{f_a(x), f_a(y)\} = t_1$(say). Then $x, y \in (f_a)_{t_1}$, but $x+y \notin (f_a)_{t_1}$ which is a contradiction. So $f_a(x+y) \geq min\{f_a(x), f_a(y)\}$. Suppose $f_a(x\alpha y) < max\{f_a(x), f_a(y)\} = t_2$(say). Then $x, y \in (f_a)_{t_2}$, but $x\alpha y \notin (f_a)_{t_2}$ which is a contradiction. Therefore $f_a(x\alpha y) \geq max\{f_a(x), f_a(y)\}$. Let $x, y \in S$ and $a \in A$, $min\{f_a(x+y), f_a(y)\} = t_1$(say). Then $y, x+y \in (f_a)_{t_1}$ implies that $x \in (f_a)_{t_1}$ so that $f_a(x) \geq t_1 = min\{f_a(x+y), f_a(y)\}$. Hence f_a is a fuzzy $k-$ideal of S. Therefore (f, A) is fuzzy soft $k-$ideal over S. □

Definition 4.3. A fuzzy ideal f of a $\Gamma-$semiring S is said to be a $k-$fuzzy ideal of S if $(f(x+y)) = f(0)$ and $f(y) = f(0)$ then $f(x) = f(0)$ for all $x, y \in S$.

Definition 4.4. Let (f, A) be fuzzy soft ideal over $\Gamma-$semiring S. Then (f, A) is said to be $k-$fuzzy soft ideal if f_a is $k-$ fuzzy ideal of $\Gamma-$semiring S, for all $a \in A$.

Theorem 4.6. *Let (f, A) be a fuzzy soft $k-$ideal over $\Gamma-$semiring S then (f, A) is a $k-$fuzzy soft ideal.*

Proof. Let $a \in A$. Then f_a is a fuzzy $k-$ideal since (f, A) is fuzzy soft $k-$ideal. Let $x, y \in S$ such that $f_a(x+y) = f_a(0)$ and $f_a(y) = f_a(0)$. Then

$$\begin{aligned} f_a(x) &\geq min\{f_a(x+y), f_a(y)\} \\ &\geq min\{f_a(0), f_a(0)\} \\ &= f_a(0) \end{aligned}$$

Therefore $f_a(x) \geq f_a(0)$. Also, we have $f_a(0) \geq f_a(x)$. Hence $f_a(x) = f_a(0)$. Therefore f_a is a $k-$fuzzy ideal of S.
Hence (f, A) is a $k-$fuzzy soft ideal over $\Gamma-$semiring S. □

5. Fuzzy soft ideal of a fuzzy soft $\Gamma-$semiring

In this section the concept of fuzzy soft ideal of a fuzzy soft $\Gamma-$semiring is introduced and studied the properties related to this notion.

Definition 5.1. Let (f, A) be a fuzzy soft $\Gamma-$semiring over $\Gamma-$semiring S. A non null fuzzy soft set (g, B) over S is called a fuzzy soft ideal of (f, A) if it satisfies the following conditions
(i) (g, B) is a fuzzy soft subset of (f, A)
(ii) (g, B) is a fuzzy soft ideal over S.

Theorem 5.1. *Let (f, A) and (g, B) be two fuzzy soft ideals of a fuzzy soft $\Gamma-$semiring (h, C) over $\Gamma-$semiring S. Then $(f, A) \cap (g, B)$ is a fuzzy soft ideal of (h, C) if it is non null.*

Proof. By theorem 3.21, $(f, A) \cap (g, B)$ is a fuzzy soft ideal over S. By definition 5.1, (f, A) and (g, B) are fuzzy soft subsets of (h, C). Hence $(f, A) \cap (g, B)$ is a fuzzy soft ideal of $(h, C.)$ □

The following theorem can be proved easily.

Theorem 5.2. *Let (f, A) and (g, B) be two fuzzy soft ideals of a fuzzy soft $\Gamma-$semiring (h, C) over $\Gamma-$semiring S. Then $(f, A) \cup (g, B)$ is a fuzzy soft ideal of (h, C) if it is non null.*

Theorem 5.3. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over $\Gamma-$semiring S. Let (f_1, C) and (g_1, D) be fuzzy soft ideals of (f, A) and (g, B) respectively. Then $(f_1, C) \cap (g_1, D)$ is a fuzzy soft ideal of $(f, A) \cap (g, B)$ if it is non null.*

Proof. Since (f_1, C) and (g_1, D) are fuzzy soft ideals of (f, A) and (g, B) respectively, we have (f_1, C) and (g_1, D) are fuzzy soft ideals over S. By theorem 3.21, $(f_1, C) \cap (g_1, D)$ is a fuzzy soft ideal over S and by theorem 3.9, $(f, A) \cap (g, B)$ is a fuzzy soft $\Gamma-$semiring over S. Hence $(f_1, C) \cap (g_1, D)$ is a fuzzy soft ideal of $(f, A) \cap (g, B)$. □

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References

[1] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Comput. Math. Appl. 59 (2010) 3458–3463.
[2] T. K. Dutta, S. K. Sardar and S. Goswami, Operations on fuzzy ideals of Γ-semirings, arXiv:1101.4791 (math.RA), 25 Jan 2011.
[3] F. Feng, Y. B. Jun and X. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008) 2621–2628.
[4] J. Ghosh, B. Dinda and T. K. Samanta, Fuzzy soft rings and fuzzy soft ideals, International Journal of Pure and Applied Sciences and Technology 2(2) (2011) 66–74.
[5] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 589–602.
[6] D. Molodtsov, Soft set theory-First results, Comput. Math. Appl. 37 (1999) 19–31.
[7] M. Murali Krishna Rao, $\Gamma-$semirings-I, Southeast Asian Bull. Math. 19(1) (1995) 49–54.
[8] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512–517.
[9] H. S. Vandiver, Note on a simple type of algebra in which cancellatin law of addition does not hold, Bull. Amer. Math. Soc.(N.S.) 40 (1934) 914–920.
[10] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338–353.

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