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Coincidence point theorems through weak contractions in partially ordered fuzzy metric spaces

Ramesh Kumar Vats, Binayak S. Choudhury, Amit Kumar, P. Saha

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Abstract. In this paper we prove some coincidence point theorems and fixed point theorems by assuming weak contraction inequalities involving three control functions in a fuzzy metric space having a partial order defined on it. Two of the control functions are discontinuous. In the sequel we introduce a weak contraction mapping principle. Several existing results are extended by our theorems. Our results are supported with examples. The methodology is a blending of analytic and order theoretic approaches.

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Corresponding Author: P. Saha (parbati_saha@yahoo.co.in)

1. Introduction and preliminaries

The purpose of this paper is to generalize a fuzzy Banach contraction mapping principle to coincidence point and common fixed point results in a fuzzy metric space with a partial order. In the sequel we prove a weak contraction mapping theorem by way of weakening the contraction. Fuzzy metric space has been defined in a number of ways. It is the inherent flexibility of fuzzy concepts that makes possible the fuzzification of the notion of metric spaces in more than one inequivalent ways. The Banach's contraction mapping principle is also extended to fuzzy metric spaces in different ways for the same reason. References [12, 19, 22] are examples of that.

In particular, a notion of a fuzzy metric space was introduced by Kramosil and Michalek [20], which was later modified by George and Veeramani for topological reasons [10]. They proved that the topology induced by a fuzzy metric space, in their sense, is a Hausdorff topology. There are several fixed point results established over the years in this fuzzy metric space. Some instances of these works are in [2, 5, 25, 31, 33]. Fuzzy metric spaces have their own issues different from ordinary metric spaces, some of these issues appear in [13]. Attempts for generalizing the Banach's contraction mapping principle had been there for a long period of time. Still today it remains an active branch of fixed point theory. One such generalization is the weak contraction principle which was first introduced by Alber et al [1] in Hilbert spaces and later adopted to complete metric spaces by Rhoades [26]. A weak contraction mapping is intermediate to a contraction mapping and a nonexpansive mapping. Later on, several authors created a number of results using weak inequalities, that is, the inequalities of the type used in [8, 9, 23]. These results are fixed and coincidence point results, some of which further generalize the weak contraction while others are independent results. Fixed point theory in partially ordered metric spaces is of relatively recent origin. An early result in this direction is due to Turinici [29] in which fixed point problems were studied in partially ordered uniform spaces. More recently Ran et al [24] worked out some fixed point theorems in partially ordered metric spaces and made applications for solving matrix equations.

Later, this branch of fixed point theory has developed through a number of works. Weak contraction in partially ordered metric spaces was studied by Harjani et al [15] and later by Choudhury et al [4, 6, 7].

In this paper we prove certain coincidence point results in partially ordered fuzzy metric spaces for functions which satisfy a contraction inequality involving three control functions. Two of the control functions are discontinuous. Some illustrative examples are given. Fixed point problems in partially ordered fuzzy metric spaces have been studied recently as, for instance, in [5].
Below we describe a mathematical background for the discussion of the topics presented in this paper.

In 1965, the concept of fuzzy sets was initiated by Zadeh [32]. Zadeh provides a precise natural framework for mathematical modeling of those real world situations that are coupled with vagueness and uncertainty due to non-probabilistic reasons.

Definition 1.1 ([32])**.** A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

After its introduction the fuzzy concepts were quickly adopted in different branches of pure and applied mathematics. In particular, Kramosil et al introduced the following definition of fuzzy metric spaces.

Definition 1.2 ([14])**.** A t-norm is a binary operation T on $[0, 1]$ satisfying the following conditions:

(i) T is commutative and associative;
(ii) $T(a, 1) = a \ \forall \ \ a \in [0, 1]$;
(iii) T(a, b) $\leq T$(c, d) whenever a $\leq$ c and b $\leq$ d, $\forall$ a, b, c, d $\in [0, 1]$.

Examples of t-norms are $a * b = ab$, $a * b = \min\{a, b\}, etc.$

Definition 1.3 ([20])**.** A fuzzy metric space (in sense of Kramosil and Michalek) is a triplet $(X, M, *)$, where X is a nonempty set, $*$ is a t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ such that the following axioms hold:
(FM-1) $M(x, y, 0) = 0 \ \ \forall \ \ x, y \in X$

(FM-2) $M(x,y,t) = 1 \; \forall \;\; t > 0$ iff $x = y$;
(FM-3) $M(x,y,t) = M(y,x,t) \; \forall \; x,y \in X, \;\; t > 0$;
(FM-4) $M(x,z,t+s) \geq M(x,y,t) * M(y,z,s) \; \forall \; x,y,z \in X$ and $s,t > 0$;
(FM-5) $M(x,y,.) : [0,\infty) \to [0,1]$ is left- continuous $\forall \; x,y \in X$.

George and Veeramoni [10] modified the above definition for topological reasons. The modified definition is the following.

Definition 1.4 ([10])**.** A fuzzy metric space (in sense of George and Veeramani) is a triplet $(X,M,*)$, where X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ such that the following axioms hold :
(GV-1) $M(x,y,t) > 0 \;\; \forall \; x,y \in X, \;\; t > 0$;
(GV-2) $M(x,y,t) = 1$ iff $x = y$;
(GV-3) $M(x,y,t) = M(y,x,t) \; \forall \; x,y \in X, \;\; t > 0$;
(GV-4) $M(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous $\forall \; x,y \in X$;
(GV-5) $M(x,z,t+s) \geq M(x,y,t) * M(y,z,s) \; \forall \; x,y,z \in X$ and $s,t > 0$.
Notice that condition (GV-5) is a fuzzy version of triangular inequality. The interpretation of $M(x,y,t)$ is that its value can be thought of as degree of nearness between x and y with respect to t. A special feature of the above space is that its topology is a Hausdorff topology. We will work only with this space and henceforth refer it simply as fuzzy metric space. All the notions described in this paper refer to this definition.

Theorem 1.5 ([21])**.** *The function M is continuous in all its three variables.*

Theorem 1.6 ([11])**.** *$M(x,y,.)$ is monotone increasing function for fixed $x,y \in X$.*

A sequence $\{x_n\}$ in $(X,M,*)$ is said to converge to a point $x \in X$ if $\lim_{n\to\infty} M(x_n,x,t) = 1 \;\;$ for $\;$ all $\; t > 0$.

A sequence $\{x_n\}$ in $(X,M,*)$ is said to be a Cauchy sequence if given $\epsilon > 0$, $1 > \lambda > 0$ there exists N such that $M(x_m,x_n,\epsilon) > 1 - \lambda$ for all $m,n > N$.
A fuzzy metric space $(X,M,*)$ is said to complete if every Cauchy sequence is convergent.

Several examples of fuzzy metric spaces can be found in [10]. In the last section of the present work we also describe some examples.
Let X be any nonempty set. A point $x \in X$ is said to be a fixed point of $f : X \to X$ if $fx = x$. A point $x \in X$ is said to be a coincidence point of $f,g : X \to X$ if $fx = gx$. A point $x \in X$ is said to be a common fixed point of $f,g : X \to X$ if $x = fx = gx$. Compatibility between two mappings was defined by Jungck [17, 18]. It is a generalization of commuting mappings in that it can be described as a commuting condition in the limit.

Several years later, Singh and Chouhan [27] introduced the concept of compatible mappings in the fuzzy metric spaces and proved two common fixed point theorems.

Definition 1.7 ([27])**.** Two self maps f and g on a fuzzy metric space $M(x,y,*)$ are said to be compatible if for all $t > 0$, $lim_{n\to\infty} \; M(fgx_n, gfx_n, t) = 1$, whenever

$\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$ for some $z \in X$. In particular it follows that whenever z is a coincidence point of f and g, if (f, g) is a compatible pair, then $fgz = gfz$.
A pair of mappings (f, g) with the above property, that is, $fgz = gfz$ whenever $fz = gz$, is said to be a weakly compatible pair [28]. This notion was studied in a number of papers.
Another notion is compatible pairs of type -A which was introduced by Jungck et al. [18]. In fuzzy metric space the definition was given by Cho et al. [2] which is the following.

Two self maps f and g on a fuzzy metric space $(X, M, *)$ are said to be compatible of type-A if $M(fgx_n, ggx_n, t) \to 1$ and $M(gfx_n, ffx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $gx_n,\ fx_n \to p$ for some p in X, as $n \to \infty$.
Both the concepts of weak compatibility and compatibility of type-A are weaker than compatibility [16].
A partial order is a binary relation " $\preceq$ " over a nonempty set which is reflexive, antisymmetric, and transitive. A set with a partial order is called partially ordered set.

Definition 1.8 ([3])**.** (g-non-decreasing Mapping [3]) Suppose $(X, \preceq)$ is partially ordered set and $f, g : X \to$ X are mappings of X to itself, f is said to be g-non-decreasing if for $x, y \in$ X, $gx \preceq gy$ implies $fx \preceq fy$.
Particularly when $g = I$, we have a non-decreasing function, that is, $f : X \longrightarrow X$ is non-decreasing whenever $x \preceq y$ implies $fx \preceq fy$.
In our results in the following sections, we use the following classes of functions.
We denote by Ψ the set of all functions $\psi : [0, \infty) \to [0, \infty)$ satisfying

(i_ψ) ψ is continuous and monotone non-decreasing,
(ii_ψ) $\psi(t) = 0$ if and only if $t = 0$;

and by Θ we denote the set of all functions $\alpha : [0, \infty) \to [0, \infty)$ such that

(i_α) α is bounded on any bounded interval in $[0, \infty)$,
(ii_α) α is continuous at 0 and $\alpha(0) = 0$

2. Major section

Theorem 2.1. *Let $(X, \preceq)$ be a partially ordered set and $(X, M, *)$ is a complete fuzzy metric space. Let $f, g : X \to X$ be two self mappings on X such that $f(X) \subseteq g(X)$, f is g-non-decreasing, g is continuous and $g(X)$ is closed, and that the following inequality*

$$\psi\left(\frac{1}{M(fx,fy,t)} - 1\right) \le \alpha\left(\frac{1}{M(gx,gy,t)} - 1\right) - \beta\left(\frac{1}{M(gx,gy,t)} - 1\right) \tag{2.1}$$

holds for all $t > 0$ and for all $x,\ y \in X$ such that $gx \preceq gy$, where $\psi, \alpha, \beta : [0, \infty) \to [0, \infty)$ are such that
$\psi \in \Psi$ and $\alpha, \beta \in \Theta$. Further that for all $s, t \ge 0$

$$\psi(s) \le \alpha(t) \Rightarrow s \le t \tag{2.2}$$

and for any sequence $\{t_n\}$ *in* $[0,\infty)$ *with* $t_n \to t > 0$,

$$\psi(t) - \overline{lim}_{n\to\infty}\alpha(t_n) + \underline{lim}_{n\to\infty}\beta(t_n) > 0 \qquad (2.3)$$

Also assume if any non-decreasing sequence $\{x_n\}$ *in* X *converges to* z*, then*

$$x_n \preceq z \quad \text{for all} n \geq 0. \qquad (2.4)$$

If there is a point $x_0 \in X$ *such that* $gx_0 \preceq fx_0$*, then* f *and* g *have a coincidence point.*

Proof. Using the assumptions of our theorem, there exists $x_0 \in$ X such that $gx_0 \preceq fx_0$ and as $f(X) \subseteq g(X)$, we can find $x_1 \in$ X such that $gx_1 = fx_0$. Then $gx_0 \preceq fx_0 = gx_1$. Since f is g-nondecreasing, we have $fx_0 \preceq fx_1$. Following this way we can construct the sequence $\{x_n\}$ as

$$fx_n = gx_{n+1} \text{for all } n \geq 0 \qquad (2.5)$$

for which

$$\begin{aligned} gx_0 \preceq fx_0 = gx_1 \preceq fx_1 = gx_2 \preceq fx_2 \\ = gx_3 \preceq fx_3 \quad . \quad . \quad . \preceq fx_{n-1} = gx_n \preceq fx_n = gx_{n+1} \preceq ... \end{aligned} \qquad (2.6)$$

If any two consecutive terms in the sequence $\{x_n\}$ are equal then there is a coincidence point of f and g. So we assume that $x_{n-1} \neq x_n$ for all $n \geq 1$, which implies that

$$M(fx_{n-1}, fx_n, t) \neq 1 \text{for all } n \geq 1, \text{for all } t > 0. \qquad (2.7)$$

Taking $x = x_n$, and y $= x_{n+1}$ in (2.1), using (2.5) and (2.6), for all $n \geq 1, t > 0$, we have

$$\begin{aligned} \psi(\tfrac{1}{M(fx_n,fx_{n+1},t)} - 1) &\leq \alpha(\tfrac{1}{M(gx_n,gx_{n+1},t)} - 1) - \beta(\tfrac{1}{M(gx_n,gx_{n+1},t)} - 1) \\ &= \alpha(\tfrac{1}{M(fx_{n-1},fx_n,t)} - 1) - \beta(\tfrac{1}{M(fx_{n-1},fx_n,t)} - 1) \end{aligned} \qquad (2.8)$$

Inequality (2.8) further implies that for all $n \geq 1$, and $t > 0$,

$$\psi(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1) \leq \alpha(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1)$$

which, by (2.2), implies that for all $n \geq 1$, and $t > 0$,

$$\frac{1}{M(fx_n, fx_{n+1}, t)} - 1 \leq \frac{1}{M(fx_{n-1}, fx_n, t)} - 1.$$

We write

$$a_n(t) = \frac{1}{M(fx_n, fx_{n+1}, t)} - 1$$

Then $\{a_n(t)\}$, for all $t > 0$, is a monotonic decreasing sequence of non-negative real numbers and, consequently, there exists $r(t) \geq 0$ such that

$$\lim_{n\to\infty} a_n(t) = \lim_{n\to\infty}(\tfrac{1}{M(fx_n,fx_{n+1},t)} - 1) = r(t) \qquad (2.9)$$

Taking limit supremum on both sides of (2.8), for all $t > 0$, we obtain

$$\psi(r(t)) \leq \overline{\lim_{n\to\infty}}\alpha(a_n(t)) + \overline{\lim_{n\to\infty}}(-\beta(a_n(t))) = \overline{\lim_{n\to\infty}}\alpha(a_n(t)) - \underline{\lim_{n\to\infty}}(\beta(a_n(t)))$$

$$(\text{since } \overline{\lim_{n\to\infty}}(-\beta(a_n(t))) = -\underline{\lim_{n\to\infty}}(\beta(a_n(t)), \beta(a_n(t)), \textit{being non-negative.})$$

The above inequality along with (2.3) and (2.9) implies that $r(t) = 0$ for all $t > 0$. Then, from (2.9), we conclude that

$$\lim_{n\to\infty} M(fx_n, fx_{n+1}, t) = 1 \text{ for all } t > 0. \qquad (2.10)$$

From (2.10) it follows that for any λ with $0 < \lambda < 1$, we can find $N = N(\lambda)$ such that for all $n \geq N(\lambda)$,

$$M(fx_{n-1}, fx_n, \lambda) > (1 - \lambda). \tag{2.11}$$

Now we prove that $\{fx_n\}$ is a Cauchy sequence. If not, then there exist some $s > 0$, and some ϵ with $0 < \epsilon < 1$, for which we can find two sequences, $\{fx_{m(k)}\}$ and $\{fx_{n(k)}\}$ of $\{fx_n\}$ such that for all $k > 0$,

$$n(k) > m(k) > k \tag{2.12}$$

and

$$M(fx_{m(k)}, fx_{n(k)}, s) \leq (1 - \epsilon) \tag{2.13}$$

By taking $n(k)$ to be the smallest integer corresponding to $m(k)$ for which (2.13) is satisfied, we have that for all $k > 0$,

$$M(fx_{m(k)}, fx_{n(k)-1}, s) > (1 - \epsilon), \tag{2.14}$$

With any choice of $0 < \lambda < s$, from (2.11), (2.12) and (2.13), it follows that for all $k > N(\lambda)$,

$$\begin{aligned} (1-\epsilon) &\geq M(fx_{m(k)}, fx_{n(k)}, s) \\ &\geq M(fx_{m(k)}, fx_{n(k)-1}, s-\lambda) * M(fx_{n(k)-1}, fx_{n(k)}, \lambda) \\ &\geq M(fx_{m(k)}, fx_{n(k)-1}, s-\lambda) * (1-\lambda), \qquad \text{(by (2.11) and (2.12))} \end{aligned}$$

that is, $$(1-\epsilon) \geq \inf_{k\geq 1} M(fx_{m(k)}, fx_{n(k)-1}, s-\lambda) * (1-\lambda) \tag{2.15}$$

We construct the function $h(t) = \inf_{k\geq 1} M(fx_{m(k)}, fx_{n(k)}, t)$.

Since, by (2.14), $h(s) = \inf_{k\geq 1} M(fx_{m(k)}, fx_{n(k)-1}, s) \geq (1-\epsilon)$ and that $M(x, y, .)$ is continuous and monotone increasing in the third variable, it follows that $h(t)$ is continuous and monotone increasing. Then,

$$h(s-\lambda) = \inf_{k\geq 1} M(fx_{m(k)}, fx_{n(k)-1}, s-\lambda) \geq 1 - \epsilon - g(\lambda) \tag{2.16}$$

$$where \ \ g(\lambda) \to 0, \qquad as \ \ \lambda \to 0. \tag{2.17}$$

Combining (2.15) and (2.16), we obtain

$$\begin{aligned} (1-\epsilon) &\geq M(fx_{m(k)}, fx_{n(k)}, s) \geq h(s-\lambda) * (1-\lambda) \\ &\geq (1-\epsilon-g(\lambda)) * (1-\lambda). \end{aligned}$$

Taking $\lambda \to 0$ in the above inequality, using (2.17) and the continuity of $*$, we obtain

$$\lim_{k\to\infty} M(fx_{m(k)}, fx_{n(k)}, s) = 1 - \epsilon. \tag{2.18}$$

Again with any choice of λ with $0 < \lambda < \frac{s}{2}$, for all $k \geq N(\lambda)$,

$$\begin{aligned} (1-\epsilon) &\geq M(fx_{m(k)}, fx_{n(k)}, s) \qquad \text{(by (2.13))} \\ &\geq M(fx_{m(k)}, fx_{m(k)-1}, \lambda) * M(fx_{m(k)-1}, fx_{n(k)-1}, s-2\lambda) \\ &\quad * M(fx_{n(k)-1}, fx_{n(k)}, \lambda) \\ &\geq M(fx_{m(k)-1}, fx_{n(k)-1}, s-2\lambda) * (1-\lambda) * (1-\lambda) \qquad (2.19) \\ &\quad \text{(by (2.11) and (2.12))} \end{aligned}$$

$$\text{Let } h_1(t) = \overline{\lim_{k\to\infty}} M(fx_{m(k)-1}, fx_{n(k)-1}, t), t > 0.$$

Then, by the continuity property of $M, h_1(t)$ is a continuous function. Then, from (2.19),

$$\begin{aligned} (1-\lambda) * (1-\lambda) * h_1(s-2\lambda) &= (1-\lambda) * (1-\lambda) * \overline{\lim_{k\to\infty}} M(fx_{m(k)-1}, fx_{n(k)-1}, s-2\lambda) \\ &\leq (1-\epsilon). \end{aligned} \tag{2.20}$$

Letting $\lambda \to 0$ in the above inequality, and using the continuity of h_1, we obtain

$$\overline{\lim_{k\to\infty}} M(fx_{m(k)-1}, fx_{n(k)-1}, s) \leq (1-\epsilon). \tag{2.21}$$

Let

$$h_2(t) = \varliminf_{k\to\infty} M(fx_{m(k)-1}, fx_{n(k)-1}, t), t > 0 \tag{2.22}$$

Then, for the same reason, that is, by the continuity of M, we conclude that $h_2(t)$ is a continuous function.
Again, for all $k \geq N(\lambda)$

$$\begin{aligned} M(fx_{m(k)-1}, fx_{n(k)-1}, s+\lambda) &\geq M(fx_{m(k)-1}, fx_{m(k)}, \lambda) * M(fx_{m(k)}, fx_{n(k)-1}, s) \\ &\geq (1-\lambda) * (1-\epsilon). \qquad (2.23) \\ &\text{(by (2.11), (2.12) and (2.14))} \end{aligned}$$

Then

$$h_2(s+\lambda) = \varliminf_{k\to\infty} M(fx_{m(k)-1}, fx_{n(k)-1}, s+\lambda) \geq (1-\lambda) * (1-\epsilon)$$

Taking $\lambda \to 0$ in the above inequality, and using the continuities of h_2 and $*$, we obtain

$$\varliminf_{k\to\infty} M(fx_{m(k)-1}, fx_{n(k)-1}, s) \geq (1-\epsilon). \tag{2.24}$$

The inequalities (2.21) and (2.24) jointly imply that

$$\lim_{k\to\infty} M(fx_{m(k)-1}, fx_{n(k)-1}, s) = (1-\epsilon). \tag{2.25}$$

Let

$$s_k = \frac{1}{M(fx_{m(k)-1}, fx_{n(k)-1}, s)} - 1 \tag{2.26}$$

Then, from (2.25),

$$\lim_{k\to\infty} s_k = \frac{\epsilon}{1-\epsilon} \tag{2.27}$$

Again, by (2.6) and (2.12), we have that $gx_{m(k)} \preceq gx_{n(k)}$. Putting $x = x_{m(k)}$, $y = x_{n(k)}$, in (2.1), for all $k \geq 1$, we have

$$\begin{aligned} &\psi(\frac{1}{M(fx_{m(k)}, fx_{n(k)}, s)} - 1) \\ &\leq \alpha(\frac{1}{M(gx_{m(k)}, gx_{n(k)}, s)} - 1) - \beta(\frac{1}{M(gx_{m(k)}, gx_{n(k)}, s)} - 1) \\ &= \alpha(\frac{1}{M(fx_{m(k)-1}, fx_{n(k)-1}, s)} - 1) - \beta(\frac{1}{M(fx_{m(k)-1}, fx_{n(k)-1}, s)} - 1) \\ &= \alpha(s_k) - \beta(s_k) \qquad (by\ (2.26)) \qquad (2.28) \end{aligned}$$

Taking limit supremum as $k \to \infty$ in (2.28), by the continuity of ψ, and using (2.18), we obtain

$$\begin{aligned} \psi(\frac{\epsilon}{1-\epsilon}) &\leq \varlimsup_{k\to\infty} \alpha(s_k) + \varlimsup_{k\to\infty}(-\beta(s_k)) \\ &= \varlimsup_{k\to\infty} \alpha(s_k) - \varliminf_{k\to\infty}(\beta(s_k)) \quad \text{[since } \beta(s_k)\text{'s are positive]} \qquad (2.29) \end{aligned}$$

Combining (2.3), (2.27) and (2.29) we conclude that $\epsilon = 0$, which is a contradiction. It then follows that $\{fx_n\}$ is Cauchy sequence and hence $\{fx_n\}$ is convergent in the complete fuzzy metric space $(X, M, *)$. Since g(X) is closed and, by (2.5), $fx_n = gx_{n+1}$ for all $n \geq 0$, we have that there exists $z \in X$ for which

$$\lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_n = gz. \tag{2.30}$$

Finally, we prove that z is a coincidence point of f and g. From (2.6), we have $\{gx_n\}$ is a non-decreasing sequence in X. From (2.4), and (2.30), for all $n \geq 0$, we get

$$gx_n \preceq gz. \tag{2.31}$$

Putting $x = x_n$ and $y = z$ in (2.1), by virtue of (2.5) and (2.31), we get

$$\psi(\frac{1}{M(gx_{n+1}, fz, t)} - 1) = \psi(\frac{1}{M(fx_n, fz, t)} - 1)$$
$$\leq \alpha(\frac{1}{M(gx_n, gz, t)} - 1) - \beta(\frac{1}{M(gx_n, gz, t)} - 1).$$

Taking $n \to \infty$ in the above inequality, using (2.30), the continuities of ψ and M, continuities of α, β at zero, and the fact that $\alpha(0) = 0 = \beta(0)$, for all $t > 0$, we obtain

$$\psi(\frac{1}{M(gz, fz, t)} - 1) = \alpha(0) - \beta(0) = 0,$$

which in turn implies that

$$\frac{1}{M(gz, fz, t)} - 1 = 0, \quad \textit{for all } t > 0,$$

that is,

$$M(gz, fz, t) = 1, \quad \textit{for all } t > 0,$$

that is,

$$fz = gz \tag{2.32}$$

Hence z is the coincidence point of f and g. This completes the proof. □

Theorem 2.2. *If in the Theorem* 2.1 *it is additionally assumed that*

$$gz \preceq ggz, \tag{2.33}$$

whenever z is a coincidence point of f and g and (f, g) is a compatible pair, then f and g have a common fixed point in X.

Proof. From the condition of the theorem, we have $gz \preceq ggz$ where z is obtained in (2.32). Since f and g are compatible, we have that $fgz = gfz$.
Now, we set

$$w = gz = fz \tag{2.34}$$

Therefore, by (2.33),

$$gz \preceq ggz = gw \tag{2.35}$$

Then, by (2.34), since (f, g) is a compatible pair,

$$fw = fgz = gfz = gw. \tag{2.36}$$

If $z = w$, then, by (2.34), z is a common fixed point. If $z \neq w$ then, by (2.1), (2.34), (2.35) and (2.36) we have

$$\psi(\tfrac{1}{M(gz,gw,t)} - 1) = \psi(\tfrac{1}{M(fz,fw,t)} - 1)$$
$$\leq \alpha(\tfrac{1}{M(gz,gw,t)} - 1) - \beta(\tfrac{1}{M(gz,gw,t)} - 1). \tag{2.37}$$

For given $t > 0$ we consider the constant sequence $\{t_n\}$ with

$$t_n = \tfrac{1}{M(gz,gw,t)} - 1, \text{for all } n \geq 1, \tag{2.38}$$

Then ,

$$t_n \to (\tfrac{1}{M(gz,gw,t)} - 1), \text{as } n \to \infty \tag{2.39}$$

Then

$$\overline{\lim_{n\to\infty}} \alpha(t_n) = \alpha(\tfrac{1}{M(gz,gw,t)} - 1) \tag{2.40}$$

and

$$\lim_{n\to\infty} \beta(t_n) = \beta(\tfrac{1}{M(gz,gw,t)} - 1). \tag{2.41}$$

Then, from (2.3), (2.37), (2.40) and (2.41) we obtain a contradiction unless

$$\tfrac{1}{M(gz,gw,t)} - 1 = 0. \tag{2.42}$$

Thus we conclude, for all $t > 0$,

$$M(gz, gw, t) = 1,$$

that is ,

$$gz = gw \tag{2.43}$$

From (2.34), (2.36) and (2.43) we conclude that

$$w = gw = fw \tag{2.44}$$

This completes the proof. □

Remark 2.3. The condition in theorem 2.2, that is, given through (2.33) has been used in many papers as, for example in [5] in problems of fixed point theory.

Remark 2.4. The compatibility condition in theorem 2.2 can be replaced by the assumption of weak compatibility or compatibility of type-A between the pair (f, g), as discussed in the lines following definition 1.7. The proof essentially remains the same.

In our next theorem we omit the order condition (2.4) of theorem 2.1 in whose place the continuity of f and the compatibility of the pair (f, g) are assumed.

Theorem 2.5. *Let $(X, \preceq)$ be a partially ordered set and $(X, M, *)$ is a complete fuzzy metric space. Let $f, g : X \to X$ be two self mappings on X such that $f(X) \subseteq g(X)$, f is g-non-decreasing and continuous, g is continuous and $g(X)$ is closed, (f, g) is a compatible pair of mappings and that the inequality* (2.1) *holds for all $t > 0$ and for all x, $y \in X$ such that $gx \preceq gy$, where $\psi, \alpha, \beta : [0, \infty) \to [0, \infty)$ are such that $\psi \in \Psi$ and $\alpha, \beta \in \Theta$. Let further that for all $s, t \geq 0$*

$$\psi(s) \leq \alpha(t) \Rightarrow s \leq t \tag{2.45}$$

and for any sequence $\{t_n\}$ in $[0, \infty)$ with $t_n \to t > 0$.

$$\psi(t) - \overline{lim}\alpha(t_n) + \underline{lim}\beta(t_n) > 0. \tag{2.46}$$

If there is a point $x_0 \in X$ such that $gx_0 \preceq fx_0$, then f and g have a coincidence point.

Proof. Proceeding as in theorem 2.1 we arrive at (2.30), that is, we have

$$\lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_n = gz = \overline{z} \ \ (say) \tag{2.47}$$

Then for $t > 0$,

$$M(g\overline{z}, fgx_n, t) \geq M(g\overline{z}, gfx_n, \frac{t}{2}) * M(gfx_n, fgx_n, \frac{t}{2})$$

Then from (2.47), by compatibility of the pair (f, g), and the continuities of f, g, M and $*$, we have, for all $t > 0$

$$M(g\overline{z}, f\overline{z}, t) \geq 1 * 1 = 1,$$

that is,

$$g\overline{z} = f\overline{z}, \tag{2.48}$$

that is, $\overline{z}$ is a coincidence point of f and g. This completes the proof of the theorem. □

Remark 2.6. If in the theorem 2.5 it is additionally assumed that
$$gz \preceq ggz, \tag{2.49}$$
whenever z is a coincidence point of f and g, then f and g have a common fixed point in X.
The proof is identical with that of theorem 2.2 except that we have now the compatibility of the pair (f, g) from the assumption of the theorem 2.5.
Combining theorems 2.1, 2.2, 2.5 and remark 2.6 we obtain the following theorem. The purpose is to study the case where the control functions α and β are continuous.

Theorem 2.7. *Let $(X, \preceq)$ be a partially ordered set and $(X, M, *)$ is a complete fuzzy metric space. Let f, g be two self maps on X such that $f(X) \subseteq g(X)$, f is g-non-decreasing, (f, g) is a compatible pair, g is continuous and $g(X)$ is closed, and that the inequality (2.1) holds for all $t > 0$ and for all x, $y \in X$ such that $gx \preceq gy$, where $\psi, \alpha, \beta : [0, \infty) \to [0, \infty)$ are such that*
$\psi \in \Psi$ and α, β are continuous with $\alpha(0) = 0 = \beta(0)$. Let further that for all $s, t \geq 0$
$$\psi(s) \leq \alpha(t) \Rightarrow s \leq t, \tag{2.50}$$
and for all $t > 0$,
$$\psi(t) - \alpha(t) + \beta(t) > 0 \tag{2.51}$$
Also assume either
(i) f is continuous, or
(ii) X has the property that if any non-decreasing sequence $\{x_n\}$ in X converges to z,
then
$$x_n \preceq z \quad for\ all\ n \geq 0. \tag{2.52}$$
If there is a point $x_0 \in X$ such that $gx_0 \preceq fx_0$, then f and g have a coincidence point. If it is additionally assumed that
$$gz \preceq ggz$$
where z is a coincidence point of f and g, then f and g have a common fixed point in X.

Proof. Since α, β are continuous with $\alpha(0) = 0 = \beta(0)$, they belong to class Θ. Also the inequality (2.3) is reduced to (2.46) in the case where α, β are continuous. The theorem then follows by applications of theorem 2.5 and remark 2.6 in the case of condition (i) and by applications of the theorems 2.1 and 2.2 in the case of condition (ii). □

Remark 2.8. The compatibility condition in theorem 2.2, theorem 2.5, or theorem 2.7 can be replaced by the assumption of weak compatibility or compatibility of type-A of the pair (f, g), which are discussed in the lines following definition1.7. The proof essentially remains the same.

Next we deal with the uniqueness of the fixed point. The common fixed point in theorem 2.2 and remark 2.6 are not in general unique. In the next theorem we put some additional assumptions in theorems 2.1 and 2.5 which ensure the existence of a unique fixed point.

Theorem 2.9. *In addition to the hypothesis of theorem* 2.1 *it is assumed that for every $x, y \in X$ there exists $u \in X$ such that $fu \preceq fx$ and $fu \preceq fy$, then there exists a unique common fixed point.*

Proof. From theorem 2.1, the set of coincidence points of f and g is non-empty. Suppose x and y are coincidence points of f and g, that is, $fx = gx$ and $fy = gy$. By the assumption, there exists $u \in X$ such that $fu \preceq fx = gx$ and $fu \preceq fy = gy$. Put $u_0 = u$ and choose $u_1 \in X$ so that $gu_1 = fu_0$. Then, as in the proof of the theorem 2.1, we can inductively define the sequence $\{gu_n\}$ by $gu_{n+1} = fu_n$ for all $n \geq 0$. Here $fx(= gx)$ and $fu(= fu_0 = gu_1)$ are comparable by our assumption.
From our assumption we have that $gu_1 \preceq gx$ (the proof is similar to that in the other case).
We claim that $gu_n \preceq gx$ for each $n \in N$.
In fact, we will use mathematical induction. Since $gu_1 \preceq gx$, our claim is true for $n = 1$.
We presume that $gu_n \preceq gx$ holds for some $n > 1$. Since f is $g-$ non-decreasing with respect to $\preceq$, we get $fu_n \preceq fx$. Then $gu_{n+1} = fu_n \preceq fx = gx$.
Thus $gu_n \preceq gx$ for all $n \geq 1$.
Let, for $t > 0$, $R_n(t) = \frac{1}{M(gx,gu_n,t)} - 1$.
Since $gu_n \preceq gx$, using the contractive condition (2.1), for all $n \geq 1$, $t \geq 0$, we have

$$\psi(\tfrac{1}{M(gx,gu_{n+1},t)} - 1) = \psi(\tfrac{1}{M(fx,fu_n,t)} - 1) \leq \alpha(\tfrac{1}{M(gx,gu_n,t)} - 1) - \beta(\tfrac{1}{M(gx,gu_n,t)} - 1),$$

that is,

$$\psi(R_{n+1}(t)) \leq \alpha(R_{n+1}(t)) - \beta(R_{n+1}(t)), \tag{2.53}$$

which, in view of the fact that $\beta \geq 0$, yields $\psi(R_{n+1}(t)) \leq \alpha(R_n(t))$, which by (2.2) implies that $R_{n+1}(t) \leq R_n(t)$ for all positive integer n, and $t > 0$, that is, $\{R_n(t)\}$ is a monotone decreasing sequence for each $t > 0$. Hence $\lim_{n\to\infty} R_n(t) = R(t) \geq 0$.
Again, taking limit supremum as $n \longrightarrow \infty$ in (2.53), for all $t > 0$, we obtain

$$\psi(R(t)) \leq \overline{\lim_{n\to\infty}}\, \alpha(R_n(t)) + \overline{\lim_{n\to\infty}}\, (-\beta(R_n(t)) = \overline{\lim_{n\to\infty}}\, \alpha(R_n(t)) - \underline{\lim_{n\to\infty}}\, (\beta(R_n(t)).$$

Then (2.3) implies that $R(t) = 0$, that is,

$$\lim_{n\to\infty} (\frac{1}{M(gx, gu_n, t)} - 1) = 0,$$

that is,

$$\lim_{n\to\infty} (M(gx, gu_n, t)) = 1.$$

Similarly, we show that

$$\lim_{n\to\infty} (\frac{1}{M(gy, gu_n, t)} - 1) = 0,$$

that is,

$$\lim_{n\to\infty} (M(gy, gu_n, t)) = 1.$$

Then, for $t > 0$, using the continuity of $*$, we have

$$M(gx, gy, t) \geq M(gx, gu_n, \frac{t}{2}) * M(gu_n, gy, \frac{t}{2}) \to 1 * 1 = 1 \quad as \;\; n \;\to\; \infty.$$

The above inequality implies that for any two coincidence points x and y,

$$gx = gy. \tag{2.54}$$

Since $gx = fx$, by compatibility of g and f, we have

$$ggx = gfx = fgx. \tag{2.55}$$

Denote

$$gx = z. \tag{2.56}$$

Then, from (2.55), we have

$$gz = fz.$$

Thus z is a coincidence point of g and f. Then form (2.54), since y is any coincidence point of f and g, with $y = z$ it follows that

$$gx = gz$$

By (2.56), it follows that

$$z = gz \tag{2.57}$$

From (2.56) and (2.57), we get $z = gz = fz$.
Therefore, z is common fixed point of g and f.
To prove the uniqueness, assume that r is another common fixed point of g and f. Then by (2.54) we have $r = gr = gz = z$. Hence the common fixed point of g and f is unique. □

Remark 2.10. If in addition to the hypothesis of theorem 2.5 it is assumed that for every $x, y \in X$, there exists $u \in X$ such that $fu \preceq fx$ and $fu \preceq fy$, and that (f, g) is a pair of compatible mappings, then f and g have a unique common fixed point.
Also if in the above or in theorem 2.9 the condition of the compatibility between f and g is replaced by the commuting condition between f and g, then the conclusions of either of the theorems are valid.
The proofs of the above statements follow from the observation that commuting condition implies compatibility condition between two mappings.

3. Weak fuzzy contraction mapping principle

In this section we apply the results of the previous section to establish in partially ordered fuzzy metric spaces a weak version of the fuzzy contraction mapping principle established originally by Gregori et al. [12].

Theorem 3.1. *Let $(X, \preceq)$ be a partially ordered set and $(X, M, *)$ is a complete fuzzy metric space. Let $f : X \to X$ be a self mapping such that the following inequality*

$$\psi(\tfrac{1}{M(fx,fy,t)} - 1) \leq \alpha(\tfrac{1}{M(x,y,t)} - 1) - \beta(\tfrac{1}{M(x,y,t)} - 1) \tag{3.1}$$

holds for all $t > 0$ and for all $x, y \in X$ such that $x \preceq y$, where $\psi, \alpha, \beta : [0, \infty) \to [0, \infty)$ are such that $\psi \in \Psi$ and $\alpha, \beta \in \Theta$. Further that for all $s, t \geq 0$

$$\psi(s) \leq \alpha(t) \Rightarrow s \leq t, \tag{3.2}$$

and for any sequence $\{t_n\}$ in $[0, \infty)$ with $t_n \to t > 0$,

$$\psi(t) - \overline{\lim_{n\to\infty}} \alpha(t_n) + \underline{\lim_{n\to\infty}} \beta(t_n) > 0. \tag{3.3}$$

Also assume either
(i) f is continuous, or
(ii) if any non-decreasing sequence $\{x_n\}$ in X converges to z,
then

$$x_n \preceq z \quad \text{for all } n \geq 0. \tag{3.4}$$

Then f has a fixed point.
If further it is assumed that for every $x, y \in X$ there exists $u \in X$ such that $fu \preceq fx$ and $fu \preceq fy$, then the fixed point is unique.

Proof. Taking $g = I$, we see that (f, g) is a compatible pair. The existence of a fixed point then follows from theorem 2.2 and remark 2.6 respectively for the cases i) and ii). The uniqueness part follows from a joint application of theorem 2.9 and remak 2.10. □

For the unordered case we have the following result.

Theorem 3.2. *Let $(X, M, *)$ be a complete metric space. Let $f : X \to X$ be a selfmapping such that the inequality (3.1) is satisfied for all $t > 0$ and $x, y \in X$ where $\psi, \alpha, \beta : [0, \infty) \to [0, \infty)$ are such that $\psi \in \Psi$ and $\alpha, \beta \in \Theta$. Further that for all $s, t \geq 0$*

$$\psi(s) \leq \alpha(t) \Rightarrow s \leq t,$$

and for any sequence $\{t_n\}$ in $[0, \infty)$ with $t_n \to t > 0$,

$$\psi(t) - \overline{\lim_{n\to\infty}} \alpha(t_n) + \underline{\lim_{n\to\infty}} \beta(t_n) > 0.$$

Then f has a unique fixed point.

Proof. Inequality (3.1) is now valid for any pair of points $x, y \in X$. It then follows from (3.1) that for all $x, y \in X, t > 0$,

$$\psi(\tfrac{1}{M(fx,fy,t)} - 1) \leq \alpha(\tfrac{1}{M(x,y,t)} - 1) \tag{3.5}$$

which, by (3.2), implies that

$$M(fx, fy, t) \geq M(x, y, t),$$

which in turn implies continuity of f.
To prove the uniqueness , let us assume that x and y are two fixed points of f, that is, $fx = x$ and $fy = y$. Then from (3.1), for $t > 0$,

$$\psi(\frac{1}{M(fx, fy, t)} - 1) \leq \alpha(\frac{1}{M(x, y, t)} - 1) - \beta(\frac{1}{M(x, y, t)} - 1)$$

which, by (3.3), implies that

$$\tfrac{1}{M(fx,fy,t)} - 1 = 0 \tag{3.6}$$

that is, $M(fx, fy, t) = 1$, that is, $x = y$. This proves the theorem. □

Remark 3.3. Theorem 3.2 contains as a special case a result of [30] which is obtained by putting $\psi = \alpha = I$ and β to be positive continuous and monotone increasing in $G-$ complete fuzzy metric spaces. Since complete fuzzy metric spaces are more general that G- complete fuzzy metric spaces, theorem (3.2) contains the main result of [30] as a special case.
The result of theorem 3.2 is a generalization of the fuzzy contraction principle established by Gregori et al [12] which is obtained by putting $\psi(t) = \alpha(t) = t$, for all $t > 0$, and $\beta(t) = (1 - k)t$ where $0 < k < 1$, for all $t > 0$.

4. Examples

In this section we present two examples to illustrate the results obtained in this paper.

Example 4.1. Let $X = [0, \infty)$. Then $(X, \preceq)$ is a partially ordered set with the partial ordering defined by, $x \preceq y$ if and only if $x \geq y$. Let $a * b = min\{a, b\}$ and $M(x, y, t) = e^{-\frac{|x-y|}{t}}$ for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a complete fuzzy metric space.
Let $f, g : X \to X$ be given respectively by the formulae $fx = \frac{1}{3}x^2$ and $gx = x^2$ for all $x \in X$.
Let $\psi, \alpha, \beta : [0, \infty) \to [0, \infty)$ be given respectively by the formulas

$$\psi(s) = s, \text{ for all } s \geq 0,\ \alpha(s) = \left\{ \begin{array}{cc} (s+1)^{\frac{1}{3}} - 1, & if \quad 0 \leq s < 1, \\ s^2 + 1, & if \quad s \geq 1, \end{array} \right\},$$

$$\beta(s) = \left\{ \begin{array}{cc} 0, & if \quad 0 \leq s < 1, \\ 1, & if \quad s \geq 1. \end{array} \right\}.$$

Then $\psi \in \Psi$ and $\alpha, \beta \in \Theta$. Then (f, g) is a compatible pair of mappings.
Without loss of generality we assume that $x > y$. The inequality (2.1) in this case reduces to

$$\psi(e^{\frac{(x^2-y^2)}{3t}} - 1) \leq \alpha(e^{\frac{(x^2-y^2)}{t}} - 1) - \beta(e^{\frac{(x^2-y^2)}{t}} - 1)$$

$$\text{for all } x, y \in [0, \infty) \text{ and } t > 0$$

For fixed x, y and t, putting $e^{\frac{(x^2-y^2)}{t}} = c$, we write the above inequality as

$$\psi(c^{\frac{1}{3}} - 1) \leq \alpha(c - 1) - \beta(c - 1)$$

It can be verified that the above inequality is satisfied with the aforesaid choices of ψ, α and β.
Also f is a continuous function. The function f is g- non-decreasing and for $x_0 \in (0, 1)$, $gx_0 \preceq fx_0$. Then all the conditions of theorem 2.2 and remark 2.6 are satisfied. Then f and g have a common fixed point by applications of either theorem 2.2 or remark 2.6. Also the assumption of theorem 2.9 and remark 2.10 are satisfied. Thus the common fixed point is also unique by application of any of the two theorems. Here '0' is the unique fixed point.

Example 4.2. Let X $= [0, 1]$. We define a partial order $'\preceq'$ on X as $x \preceq y$ if and only if $x \geq y$ for all $x, y \in X$. We take the usual metric $d(x, y) = |x - y|$ for $x, y \in X$ and a fuzzy metric defined by $M(x, y, t) = \frac{t}{t+d(x,y)}$. Let $a * b = $ min $\{a, b\}$. Then $(X, M, *)$ is a complete metric space.
Let $f, g : X \to X$ be defined as $fx = \frac{5}{6}x - \frac{1}{3}x^2$ and $gx = x - \frac{1}{3}x^2$ for all $x \in [0, 1]$.
Let ψ, α, β :$[0, \infty) \to [0, \infty)$ be defined as $\psi(s) = s$ for $s \in [0, 1]$,
$\alpha(s) = s$ for $s \in [0, 1]$ and $\beta(s) = \frac{s}{6}$ for $s \in [0, 1]$.
Without loss of generality, we assume that $x > y$ and verify the inequality (2.1). For all $x, y \in [0, 1]$ with $x > y$, $d(fx, fy) = \frac{5}{6}(x - y)$ - $\frac{1}{3}(x^2 - y^2)$ and

$$d(gx, gy) = (\text{x - y}) - \tfrac{1}{3}(x^2 - y^2)$$

Now, $\alpha(\frac{1}{M(gx,gy,t)} - 1) - \beta(\frac{1}{M(gx,gy,t)} - 1) = \alpha(\frac{d(gx,gy)}{t}) - \beta(\frac{d(gx,gy)}{t})$

$$= [\frac{(x-y)-\frac{1}{3}(x^2-y^2)}{t}] - \frac{1}{6}[\frac{(x-y)-\frac{1}{3}(x^2-y^2)}{t}].$$

therefore, $\alpha(\frac{1}{M(gx,gy,t)}-1)-\beta(\frac{1}{M(gx,gy,t)}-1) \geq \frac{1}{t}[(x-y)]-\frac{1}{3t}(x^2-y^2)-\frac{1}{6t}(x-y)$
$$\geq \frac{5}{6t}(x-y)-\frac{1}{3t}(x^2-y^2)=\psi(\frac{1}{M(fx,fy,t)}-1).$$
Therefore, the inequality (2.1) is satisfied. Then, with any choice of x_0 in $(0,1)$, $gx_0 \preceq fx_0$ the conditions of theorem 2.1 are satisfied. It also follows that all the conditions of theorem 2.7 are satisfied in this case. Then, by an application of theorem 2.7, there exists a common fixed point of f and g. Here $'0'$ is a common fixed point. Also both the theorem 2.9 and the fixed point of remark 2.10 are applicable in the case of this example. The common fixed point is thus proved to be unique.

Remark 4.3. In the above example (f,g) is not a compatible pair. Hence the first part of remark 2.10 cannot be applied to the case of the example 4.2.

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Ramesh Kumar Vats (ramesh-vats@rediffmail.com)
Department of Mathematics, National Institute of Technology Hamirpur (H.P.) India

Binayak S. Choudhury (binayak12@yahoo.co.in)
Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, P.O.: B. Garden, Shibpur, Howrah - 711103, West Bengal, India

Amit Kumar (amitsu48@gmail.com)
Department of Mathematics, National Institute of Technology Hamirpur (H.P.) India

P. Saha (parbati_saha@yahoo.co.in)
Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, P.O.: B. Garden, Shibpur, Howrah - 711103, West Bengal, India