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Interval-valued fuzzy prime and semiprime ideals of a hypersemiring

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Abstract. In this paper we introduce the notion of interval-valued (*i.v.*) fuzzy prime and semiprime ideals of a hypersemiring and study different properties of these two ideals. Finally, we examine the strongly irreducibility and irreducibility of an *i.v.* fuzzy ideal of a hypersemiring and characterize the equivalence of primeness, strongly irreducibility and irreducibility of an *i.v.* fuzzy ideal of a fully idempotent hypersemiring.

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1. Introduction

Hyperstructure theory was initiated by Marty [20] in 1934. He introduced the notion of hypergroup. Subsequently, hyperstructure theory has achieved manifold applications in various paths of Mathematics and computer science ([5],[7],[18],[25],[28]). Hyperrings were introduced by several researchers in different ways. M. Krasner [16] introduced the notion of hyperrings where addition is a hyperoperation and multiplication is a binary operation. Rota [21] studied the hyperring where addition is a binary operation and multiplication is a hyperoperation. These type of hyperrings are called multiplicative hyperrings. M. D. Salvo [22] introduced the hyperring where both the addition and multiplication are hyperoperations. The study of hypersemiring was commenced by Ameri and Hedayati in [2], where they have considered only the addition as hyperoperation. The general notion of hypersemiring (i.e. the hypersemiring where both the addition and multiplication are hyperoperation) was examined by Vougiouklis in [24] and by Davvaz in [9]. In this paper we have considered the general form of hypersemiring where both the addition and multiplication are hyperoperations.

In 1965, L. Zadeh first introduced the theory of fuzzy sets in his pioneer paper [26]. Interval-valued fuzzy sets were introduced independently by Zadeh [27], Grattan-Guiness [12], Jahn [13], Sambuc [23] in the same year 1975 as a generalization of ordinary fuzzy set. The success of the use of fuzzy set theory depends on the choice of the membership function. However, there are applications in which experts do not have precise knowledge of the membership function that should be taken. In these cases, it is appropriate to represent the membership degree of each element by means of an interval. From these considerations arises the extension of fuzzy sets called theory of Interval-valued Fuzzy Sets (IVFSs). In this case, the membership degree of each element is given by a closed subinterval of the interval [0, 1]. In 2012, we worked on i.v. fuzzy prime ideals of semirings in [10] and the interval-valued fuzzy semiprime ideals of a semirings in [11]. In 2013, S. Kar and P. Sarkar [15] worked on *i.v.* fuzzy completely regular subsemigroups of semigroups. Some rudimentary works on i.v. fuzzy subalgebras can be found in [14] and [17].

Now a days many researchers are interested in fuzzy hyperstructures because of nice connection between fuzzy sets and hyperstructures. Corsini introduced the notion of fuzzy hyperstructure in ([3],[4]). Also, Corsini and Leoreanu studied this notion further in ([6],[7]). The extension of fuzzy algebra to fuzzy hyperalgebra was established by Zahedi in [29]. Some fascinating results on fuzzy hyperrings can be found in [1] and [19].

In this paper, we establish the concept of *i.v.* fuzzy prime and semiprime ideal of a hypersemiring. We conclude this paper with the concept of strongly irreducibility and irreducibility of an *i.v.* fuzzy ideal of a hypersemiring. Finally, we characterize that the notion of primeness, strongly irreducibility and irreducibility of an *i.v.* fuzzy ideal are equivalent in a fully idempotent hypersemiring.

Now, we recall some basic notions and results of hypersemirings and fuzzy algebra which we shall use in this paper.

Definition 1.1 ([5])**.** Let H be a non-empty set and $\mathcal{P}^*(H)$ denote the set of all non-empty subsets of H. Then a mapping $\circ : H \times H \longrightarrow \mathcal{P}^*(H)$ is said to be a binary hyperoperation on H. The couple $(H, \circ)$ is called a hypergroupoid. For any two non-empty subsets A and B of H and $x \in H$, we define :
$A \circ B = \bigcup\limits_{a \in A, b \in B} a \circ b$, $A \circ x = A \circ \{x\}$ and $x \circ B = \{x\} \circ B$.

Definition 1.2 ([5])**.** A hypergroupoid $(H, \circ)$ is called a hypersemigroup if for all $a, b, c \in H$, we have $(a \circ b) \circ c = a \circ (b \circ c)$ i.e. $\bigcup\limits_{h_1 \in a \circ b} h_1 \circ c = \bigcup\limits_{h_2 \in b \circ c} a \circ h_2$.

Definition 1.3. A non-empty set H together with two binary hyperoperations '+' and '·' (called the hyperaddition and hypermultiplication respectively) is said to be a hypersemiring if

(1) $(H, +)$ is an abelian hypersemigroup;
(2) $(H, \cdot)$ is a hypersemigroup and
(3) $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all $a, b, c \in H$.

Let $(H, +, \cdot)$ be a hypersemiring. If there exists an element '0_H' $\in S$ such that $a + 0_H = a = 0_H + a$ and $a \cdot 0_H = 0_H = 0_H \cdot a$ for all $a \in H$; then '0_H' is called the zero element of H. If there exists an element '1_H' $\in H$ such that $a \cdot 1_H = a = 1_H \cdot a$ for all $a \in H$, then '1_H' is called the identity element of H.

• A hypersemiring may or may not have a zero and an identity element. Throughout this paper we consider a hypersemiring $(H, +, \cdot)$ with zero element '0_H'. Unless otherwise stated a hypersemiring $(H, +, \cdot)$ will be denoted simply by H and hypermultiplication '$\cdot$' will be denoted by juxtaposition.

Example 1.4. (1) Consider the semiring $(\mathbb{N}_0, +, \cdot)$ of non-negative integers with respect to usual addition and multiplication of non-negative integers. Define the hyperaddition and hypermultiplication '$\oplus$' and '$\odot$' as follows. $m \oplus n = \{m, n\}$ and $m \odot n = \{mn, kmn\}$, where $k \in \mathbb{N}_0$. Then the hyperringoid $(\mathbb{N}_0, \oplus, \odot)$ forms a hypersemiring.

(2) Consider the same semiring $(\mathbb{N}_0, +, \cdot)$ as above. Define the hyperaddition and hypermultiplication '$\oplus$' and '$\odot$' as follows. $m \oplus n = l.c.m.(a, b)\mathbb{N}_0$ and $m \odot n = (mn)\mathbb{N}_0$. Then $(\mathbb{N}_0, \oplus, \odot)$ forms a hypersemiring.

(3) Consider the same semiring $(\mathbb{N}_0, +, \cdot)$ as above. Define the hyperaddition and hypermultiplication '$\oplus$' and '$\odot$' as follows. $m \oplus n = \{m, n\}$ and $m \odot n = (mn)\mathbb{N}_0$. Then $(\mathbb{N}_0, \oplus, \odot)$ forms a hypersemiring.

Definition 1.5. Let I be a nonempty subset of a hypersemiring H. Then

(1) I is said to be a left hyperideal of H if $(I, +)$ is a subhypersemigroup of $(H, +)$ (i.e. $a + b \subseteq I$ for all $a, b \in I$)and $ha \subseteq H$ for all $h \in H$ and for all $a \in I$.

(2) I is said to be a right hyperideal of H if $(I, +)$ is a subhypersemigroup of $(H, +)$ and $ah \subseteq I$ for all $h \in H$ and for all $a \in I$.

(3) I is said to be a hyperideal of H if it is both a left hyperideal and a right hyperideal of H.

Definition 1.6 ([14])**.** An interval number on $[0, 1]$, denoted by $\widetilde{a}$, is defined as the closed subinterval of $[0, 1]$, where $\widetilde{a} = [a^-, a^+]$ satisfying $0 \leq a^- \leq a^+ \leq 1$.

• Suppose $\widetilde{a} = [a^-, a^+]$ and $\widetilde{b} = [b^-, b^+]$ be any two interval numbers. We define :

(1) $\widetilde{a} \leq \widetilde{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$.
(2) $\widetilde{a} = \widetilde{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$.
(3) $\widetilde{a} < \widetilde{b}$ if and only if $\widetilde{a} \neq \widetilde{b}$ and $\widetilde{a} \leq \widetilde{b}$.

Note 1.7. *We write* $\widetilde{a} \geq \widetilde{b}$ *whenever* $\widetilde{b} \leq \widetilde{a}$ *and* $\widetilde{a} > \widetilde{b}$ *whenever* $\widetilde{b} < \widetilde{a}$. *We denote the interval number* $[0, 0]$ *by* $\widetilde{0}$ *and* $[1, 1]$ *by* $\widetilde{1}$.

Definition 1.8 ([14])**.** Let $\{\widetilde{a_i} : i \in \Lambda\}$ be a family of interval numbers, where $\widetilde{a_i} = [a_i^-, a_i^+]$. Then we define $\sup\limits_{i\in\Lambda}\{\widetilde{a_i}\} = [\sup\limits_{i\in\Lambda} a_i^-, \sup\limits_{i\in\Lambda} a_i^+]$ and $\inf\limits_{i\in\Lambda}\{\widetilde{a_i}\} = [\inf\limits_{i\in\Lambda} a_i^-, \inf\limits_{i\in\Lambda} a_i^+]$.

• Suppose $D[0, 1]$ denotes the set of all interval numbers on $[0, 1]$.

Definition 1.9 ([27])**.** Let H be a non-empty set. A mapping $\widetilde{\mu} : H \longrightarrow D[0, 1]$ is called an interval-valued fuzzy subset of H.

Note 1.10. *We can write* $\widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$ *for all* $x \in X$, *for any i.v. fuzzy subset* $\widetilde{\mu}$ *of a non-empty set* X, *where* μ^- *and* μ^+ *are fuzzy subsets of* X.

Definition 1.11 ([10]). Let $X \neq \varnothing$ be a set and $A \subseteq X$. Then the interval-valued characteristic function $\widetilde{\chi}_A$ of A is an $i.v.$ fuzzy subset of X, defined as follows :

$$\widetilde{\chi}_A(x) = \begin{cases} \widetilde{1} & \text{when} \quad x \in A. \\ \widetilde{0} & \text{when} \quad x \notin A. \end{cases}$$

Definition 1.12 ([14]). Let $\widetilde{\mu_1}$ and $\widetilde{\mu_2}$ be two $i.v.$ fuzzy subsets of a non-empty set X. Then $\widetilde{\mu_1}$ is said to be subset of $\widetilde{\mu_2}$, denoted by $\widetilde{\mu_1} \subseteq \widetilde{\mu_2}$, if $\widetilde{\mu_1}(x) \leq \widetilde{\mu_2}(x)$ i.e. $\mu_1^-(x) \leq \mu_2^-(x)$ and $\mu_1^+(x) \leq \mu_2^+(x)$ for all $x \in X$, where $\widetilde{\mu_1}(x) = [\mu_1^-(x), \mu_1^+(x)]$ and $\widetilde{\mu_2}(x) = [\mu_2^-(x), \mu_2^+(x)]$.

Definition 1.13 ([14]). The interval Min-norm is a function $Min^i : D[0,1] \times D[0,1] \longrightarrow D[0,1]$, defined by :
$Min^i(\widetilde{a}, \widetilde{b}) = [min(a^-, b^-), min(a^+, b^+)]$ for all $\widetilde{a}, \widetilde{b} \in D[0,1]$, where $\widetilde{a} = [a^-, a^+]$ and $\widetilde{b} = [b^-, b^+]$.

Definition 1.14 ([14]). The interval Max-norm is a function $Max^i : D[0,1] \times D[0,1] \longrightarrow D[0,1]$, defined by :
$Max^i(\widetilde{a}, \widetilde{b}) = [max(a^-, b^-), max(a^+, b^+)]$ for all $\widetilde{a}, \widetilde{b} \in D[0,1]$, where $\widetilde{a} = [a^-, a^+]$ and $\widetilde{b} = [b^-, b^+]$.

Definition 1.15. Let H be a hypersemiring and $\widetilde{\mu_1}$, $\widetilde{\mu_2}$ be two $i.v.$ fuzzy subsets of H. Suppose $a, t \in H$. We define the $i.v.$ fuzzy subsets $a \circ \widetilde{\mu_1}$, $\widetilde{\mu_1} \circ a$, $\widetilde{\mu_1} \circ \widetilde{\mu_2}$, $\widetilde{\mu_1} + \widetilde{\mu_2}$ and $\widetilde{\mu_1}\widetilde{\mu_2}$ of H, as follows :

(1)

$$(a \circ \widetilde{\mu_1})(t) = \begin{cases} \sup\limits_{t \in ab}\{\widetilde{\mu_1}(b)\}, & \text{when } t \in ab \text{ for some } a, b \in H. \\ \widetilde{0}, & \text{otherwise.} \end{cases}$$

(2)

$$(\widetilde{\mu_1} \circ a)(t) = \begin{cases} \sup\limits_{t \in ba}\{\widetilde{\mu_1}(b)\}, & \text{when } t \in ba \text{ for some } a, b \in H. \\ \widetilde{0}, & \text{otherwise.} \end{cases}$$

(3)

$$(\widetilde{\mu_1} \circ \widetilde{\mu_2})(t) = \begin{cases} \sup\limits_{t \in uv} \Big\{ Min^i\Big(\widetilde{\mu_1}(u), \widetilde{\mu_2}(v)\Big)\Big\}, & \text{when } t \in uv \text{ for some } u, v \in H. \\ \widetilde{0}, & \text{otherwise.} \end{cases}$$

(4)

$$(\widetilde{\mu_1} + \widetilde{\mu_2})(t) = \begin{cases} \sup\limits_{t \in u+v} \Big\{ Min^i\Big(\widetilde{\mu_1}(u), \widetilde{\mu_2}(v)\Big)\Big\}, & \text{when } t \in u + v \text{ for some } u, v \in H. \\ \widetilde{0}, & \text{otherwise.} \end{cases}$$

(5)

$$(\widetilde{\mu_1}\widetilde{\mu_2})(t) = \begin{cases} \sup\Big\{ \inf\limits_{1 \leq i \leq m} Min^i\Big(\widetilde{\mu_1}(u_i), \widetilde{\mu_2}(v_i)\Big) : t \in \sum\limits_{i=1}^{m} u_i v_i \Big\}, & \text{when } t \in \sum\limits_{i=1}^{m} u_i v_i \\ \widetilde{0}, & \text{otherwise.} \end{cases}$$

• Throughout this paper we assume that any two interval numbers in $D[0,1]$ are comparable i.e. for any two interval numbers $\widetilde{a}$ and $\widetilde{b}$ in $D[0,1]$, we have either $\widetilde{a} \leq \widetilde{b}$ or $\widetilde{a} > \widetilde{b}$.

2. *i.v.* FUZZY PRIME IDEAL OF A HYPERSEMIRING

Definition 2.1. Let H be a hypersemiring. An *i.v.* fuzzy subset $\widetilde{\mu}$ of H is said to be an

(1) *i.v.* fuzzy left ideal of H if $\widetilde{\mu} + \widetilde{\mu} \subseteq \widetilde{\mu}$ and $h \circ \widetilde{\mu} \subseteq \widetilde{\mu}$ for all $h \in H$;
(2) an *i.v.* fuzzy right ideal of H, if $\widetilde{\mu} + \widetilde{\mu} \subseteq \widetilde{\mu}$ and $\widetilde{\mu} \circ h \subseteq \widetilde{\mu}$ for all $h \in H$;
(3) an *i.v.* fuzzy ideal of H, if it is both an *i.v.* fuzzy left ideal and an *i.v.* fuzzy right ideal of H.

Example 2.2. Consider the hypersemiring H as in the Example 1.4(1). Define an *i.v.* fuzzy subset $\widetilde{\mu}$ of H as follows :

$$\widetilde{\mu}(x) = \begin{cases} [0.8, 0.9] & when \quad x = 0; \\ [0.5, 0.6] & when \quad x \in 2\mathbb{N}_0 \setminus \{0\}; \\ [0.3, 0.4] & otherwise. \end{cases}$$

Then we can check that $\widetilde{\mu}$ is an *i.v.* fuzzy ideal of H.

The following Lemmas are easy to verify.

Lemma 2.3. *An i.v. fuzzy subset $\widetilde{\mu}$ of a hypersemiring H is an i.v. fuzzy left (resp. right) ideal of H if and only if $\widetilde{\mu}(t) \geq \widetilde{\mu}(h)$ for all $t \in ah$ (resp. $t \in ha$) and $a \in H$.*

Lemma 2.4. *Let $\widetilde{\mu}$ be an i.v. fuzzy left (or right) ideal of a hypersemiring H. Then $\widetilde{\mu}(0_H) \geq \widetilde{\mu}(h)$ for all $h \in H$.*

Lemma 2.5. *Let $[\alpha, \beta]$ and $[\gamma, \delta]$ be two interval numbers in $D[0,1]$ such that $[\alpha, \beta] \leq [\gamma, \delta] \neq \widetilde{0}$. Suppose, I be a hyperideal of a hypersemiring H. Then the i.v. fuzzy subset $\widetilde{\mu}$ of H, defined as :*

$$\widetilde{\mu}(x) = \begin{cases} [\gamma, \delta] & when \quad x \in I; \\ [\alpha, \beta] & otherwise, \end{cases}$$

is an i.v. fuzzy ideal of H.

Lemma 2.6. *Let $\widetilde{\mu}$ be an i.v. fuzzy ideal of a hypersemiring H. Then the set $\widetilde{\mu}_0 = \{x \in H : \widetilde{\mu}(x) = \widetilde{\mu}(0_H)\}$ is a hyperideal of H.*

Lemma 2.7. *If A and B be two subsets of a hypersemiring H, then $\widetilde{\chi}_A \widetilde{\chi}_B = \widetilde{\chi}_{AB}$.*

Definition 2.8. Let H be a hypersemiring. A proper hyperideal P of H is said to be a prime hyperideal of H if for any two hyperideals A, B of H; $AB \subseteq P \Longrightarrow A \subseteq P$ or $B \subseteq P$.

Theorem 2.9. *The following conditions are equivalent for a proper hyperideal P of a hypersemiring H :*

(1) *P is a prime hyperideal of H.*
(2) *For any $a, b \in H$, $\bigcup_{h \in H} ahb \subseteq P$ if and only if $a \in P$ or $b \in P$.*

Proof. $\underline{(1) \Longrightarrow (2)}$: Suppose P is a prime hyperideal of H. Let $a, b \in H$ and $\bigcup_{h\in H} ahb \subseteq P$. Now we consider $A = \bigcup_{\substack{x_i\in r_i a s_i,\\ r_i, s_i \in H,\\ m\in\mathbb{N}}} (\sum_{i=1}^{m} x_i)$ and $B = \bigcup_{\substack{y_j\in r'_j b s'_j,\\ r'_j, s'_j \in H,\\ n\in\mathbb{N}}} (\sum_{j=1}^{n} y_j)$.

Then, clearly, A and B are hyperideals of H. Let $t \in AB$.

$\Longrightarrow t \in \sum_{i=1}^{k} p_i q_i$ for some $p_i \in A, q_i \in B$ and $k \in \mathbb{N}$.

$\Longrightarrow t \in \bigcup_{u_i\in p_i q_i} (\sum_{i=1}^{k} u_i)$ or some $p_i \in A$ and $q_i \in B$.

$\Longrightarrow t \in \sum_{i=1}^{k} u_i$ for some $u_i \in p_i q_i$ where $p_i \in A$ and $q_i \in B$. Now, $p_i \in A \Longrightarrow p_i \in \bigcup_{x_i\in r_i a s_i} (\sum_{i=1}^{m} x_i)$ for some r_i, $s_i \in H$ and $m \in \mathbb{N}$. This shows that $p_i \in \sum_{i=1}^{m} x_i$ for some $x_i \in r_i a s_i$, where $r_i, s_i \in H$ and $m \in \mathbb{N}$. Similarly, $q_i \in \sum_{j=1}^{n} y_j$ for some $y_j \in r'_j b s'_j$, where $r'_j, s'_j \in H$ and $n \in \mathbb{N}$. Therefore, $u_i \in (\sum_{i=1}^{m} x_i)(\sum_{j=1}^{n} y_j) = \bigcup_{a_k\in x_i y_j} (\sum_{k=1}^{mn} a_k)$. Again, $a_k \in x_i y_j \Longrightarrow a_k \in (r_i a s_i)(r'_j b s'_j) \subseteq r_i(\bigcup_{h\in H} ahb)s'_j \subseteq r_i P s'_j \subseteq P$. So, $u_i \in P$ and hence $t \in P$. Therefore, $AB \subseteq P$. Since P is a prime hyperideal of H, either $A \subseteq P$ or $B \subseteq P$. $A \subseteq P \Longrightarrow \bigcup_{\substack{x_i\in r_i a s_i,\\ r_i, s_i \in H,\\ m\in\mathbb{N}}} (\sum_{i=1}^{m} x_i) \subseteq P \Longrightarrow \langle a \rangle^3 \subseteq \bigcup_{\substack{x_i\in r_i a s_i,\\ r_i, s_i \in H,\\ m\in\mathbb{N}}} (\sum_{i=1}^{m} x_i) \subseteq P$. Since, P is a prime hyperideal of H, it follows that $\langle a \rangle \subseteq P$. Thus we obtain that $a \in P$. Similarly, considering $B \subseteq P$, we can show that $b \in P$. Reverse implication is obvious, since P is a hyperideal of H.

$\underline{(2) \Longrightarrow (1)}$: Let A and B be two hyperideals of P such that $AB \subseteq P$ but $A \nsubseteq P$. Then there exists $x \in A$ such that $x \notin P$. Then for any $y \in B$ and for all $h \in H$, $xhy \subseteq AB \subseteq P$. This implies that $\bigcup_{h\in H} xhy \subseteq P \Longrightarrow y \in P$ (by assumption). Thus $B \subseteq P$. Hence P is a prime hyperideal of H. □

Now we present the definition of *i.v.* fuzzy prime ideal of a hypersemiring.

Definition 2.10. A non-constant *i.v.* fuzzy ideal $\widetilde{\mu}$ of a hypersemiring H is said to be an *i.v.* fuzzy prime ideal of H if for any two *i.v.* fuzzy ideals $\widetilde{\mu_1}$ and $\widetilde{\mu_2}$ of H; $\widetilde{\mu_1} \circ \widetilde{\mu_2} \subseteq \widetilde{\mu} \Longrightarrow \widetilde{\mu_1} \subseteq \widetilde{\mu}$ or $\widetilde{\mu_2} \subseteq \widetilde{\mu}$.

Theorem 2.11. *Let P be a prime hyperideal of a hypersemiring H and $[\alpha,\beta] \in D[0,1] \setminus \{\widetilde{1}\}$. Then the i.v. fuzzy subset $\widetilde{\mu}$ of H, defined by :*

$$\widetilde{\mu}(x) = \begin{cases} \widetilde{1} & \text{when} \quad x \in P, \\ [\alpha,\beta] & \text{otherwise}; \end{cases}$$

is an i.v. fuzzy prime ideal of H.

Proof. $\widetilde{\mu}$ is a non-constant $i.v.$ fuzzy ideal of H, by Lemma 2.5. Let $\widetilde{\mu_1}$ and $\widetilde{\mu_2}$ be two $i.v.$ fuzzy ideals of H such that $\widetilde{\mu_1} \nsubseteq \widetilde{\mu}$ and $\widetilde{\mu_2} \nsubseteq \widetilde{\mu}$. Since according to our assumption, any two interval numbers in $D[0,1]$ are comparable, there exist $x, y \in H$ such that $\widetilde{\mu_1}(x) > \widetilde{\mu}(x)$ and $\widetilde{\mu_2}(y) > \widetilde{\mu}(y)$. This shows that $\widetilde{\mu}(x) = [\alpha,\beta] = \widetilde{\mu}(y)$ i.e. $x \notin P$ and $y \notin P$. Since P is a prime hyperideal of H, it follows from Theorem 2.9 that there exists $h \in H$ such that $xhy \nsubseteq P$. Then there exists $h_1 \in xhy$ such that $h_1 \notin P$. So, $\widetilde{\mu}(h_1) = [\alpha,\beta]$. Now

$$\begin{aligned}(\widetilde{\mu_1} \circ \widetilde{\mu_2})(h_1) &= \sup\left\{Min^i(\widetilde{\mu_1}(a), \widetilde{\mu_2}(b)) : h_1 \in ab \text{ for some } a, b \in H\right\} \\ &\geq Min^i(\widetilde{\mu_1}(u), \widetilde{\mu_2}(y)) \\ &\qquad (\text{since } h_1 \in xhy = (xh)y \Longrightarrow h_1 \in uy \text{ for some } u \in xh) \\ &\geq Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) \text{ (by Lemma 2.3)} \\ &> Min^i(\widetilde{\mu}(x), \widetilde{\mu}(y)) \\ &= [\alpha,\beta] = \widetilde{\mu}(h_1).\end{aligned}$$

So, $\widetilde{\mu_1} \circ \widetilde{\mu_2} \nsubseteq \widetilde{\mu}$. Hence, $\widetilde{\mu}$ is an $i.v.$ fuzzy prime ideal of H. □

Theorem 2.11 helps us to produce examples of $i.v.$ fuzzy prime ideal of a hypersemiring easily. One such example is as follows.

Example 2.12. Consider the hypersemiring H as in the Example 1.4(3). Then $(p\mathbb{N}_0, \oplus, \odot)$ forms a prime hyperideal of H, where p is a prime number. Define an $i.v.$ fuzzy subset of H as follows :

$$\widetilde{\mu}(x) = \begin{cases} \widetilde{1} & \text{when} \quad x \in p\mathbb{N}_0; \\ [0.5, 0.6] & \text{otherwise}. \end{cases}$$

Then by Theorem 2.11, it follows that $\widetilde{\mu}$ forms an $i.v.$ fuzzy prime ideal of H.

Theorem 2.13. *Let $\widetilde{\mu}$ be an i.v. fuzzy prime ideal of a hypersemiring H. Then $\widetilde{\mu}(0_H) = \widetilde{1}$.*

Proof. If possible, let $\widetilde{\mu}(0_H) \neq \widetilde{1}$. Since according to our assumption, any two interval numbers in $D[0,1]$ are comparable, it follows that $\widetilde{\mu}(0_H) < \widetilde{1}$. Also, since $\widetilde{\mu}$ is an $i.v.$ fuzzy ideal of H and 0_H is an absorbing zero element of H, we have $\widetilde{\mu}(0_H) \geq \widetilde{\mu}(x)$ for all $x \in H$, by Lemma 2.4. Since, $\widetilde{\mu}$ is non-constant, there exists $h \in H$ such that $\widetilde{\mu}(0_H) > \widetilde{\mu}(h)$. Now we construct two $i.v.$ fuzzy ideals of H as follows :

$$\widetilde{\mu_1}(x) = \begin{cases} \widetilde{1} & \text{when} \quad x \in \widetilde{\mu}_0; \\ \widetilde{0} & \text{otherwise}; \end{cases}$$

and $\widetilde{\mu_2}(x) = \widetilde{\mu}(0_H)$ for all $x \in H$. $\widetilde{\mu_1}$ is an *i.v.* fuzzy ideal of H, by Lemma 2.5 and $\widetilde{\mu_2}$ is clearly an *i.v.* fuzzy ideal of H. Let $x \in \widetilde{\mu}_0$ i.e. $\widetilde{\mu}(x) = \widetilde{\mu}(0_H)$. This shows that $\widetilde{\mu_1}(x) = \widetilde{1}$. Then for any $y \in H$, $Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) = \widetilde{\mu_2}(y) = \widetilde{\mu}(0_H)$. Suppose, $t \in xy$. Since $\widetilde{\mu}$ is an *i.v.* fuzzy ideal of H, $\widetilde{\mu}(t) \geq \widetilde{\mu}(x)$, by Lemma 2.3. Therefore, $\widetilde{\mu}(t) \geq \widetilde{\mu}(0_H)$. Thus $\widetilde{\mu}(t) = \widetilde{\mu}(0_H)$. Since, t is arbitrary, we get that $Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) = \widetilde{\mu}(0_H) = \widetilde{\mu}(t)$ for any $t \in xy$. Suppose $x \notin \widetilde{\mu}_0$ i.e. $\widetilde{\mu_1}(x) = \widetilde{0}$. Therefore, $Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) = \widetilde{0} \leq \widetilde{\mu}(t)$ for any $t \in xy$. Thus we obtain that $\widetilde{\mu}(t) \geq \sup\Big\{Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) : t \in xy \text{ for some } x, y \in H\Big\} = (\widetilde{\mu_1} \circ \widetilde{\mu_2})(t)$. This implies that $\widetilde{\mu_1} \circ \widetilde{\mu_2} \subseteq \widetilde{\mu}$. Since, $\widetilde{\mu}$ is an *i.v.* fuzzy prime ideal of H, $\widetilde{\mu_1} \subseteq \widetilde{\mu}$ or $\widetilde{\mu_2} \subseteq \widetilde{\mu}$. But $\widetilde{\mu_1}(0_H) = \widetilde{1} > \widetilde{\mu}(0_H)$ and $\widetilde{\mu_2}(h) = \widetilde{\mu}(0_H) > \widetilde{\mu_2}(h)$. Thus, we arrive at a contradiction. Consequently, $\widetilde{\mu}(0_H) = \widetilde{1}$. □

Next theorem tells us about the cardinality of the image of an *i.v.* fuzzy prime ideal of a hypersemiring.

Theorem 2.14. *If $\widetilde{\mu}$ is an i.v. fuzzy prime ideal of a hypersemiring H, then $|Im\widetilde{\mu}| = 2$.*

Proof. Since, $\widetilde{\mu}$ is non-constant, $|Im\widetilde{\mu}| \geq 2$. If possible, let $|Im\widetilde{\mu}| > 2$. We choose $a, b \in H$ such that $\widetilde{1} > \widetilde{\mu}(a) > \widetilde{\mu}(b)$. Now, we construct two *i.v.* fuzzy subsets $\widetilde{\mu_1}$ and $\widetilde{\mu_2}$ of H as follows :

$$\widetilde{\mu_1}(x) = \begin{cases} \widetilde{1} & \text{when} \quad x \in <a>; \\ \widetilde{0} & \text{otherwise;} \end{cases}$$

and $\widetilde{\mu_2}(x) = \widetilde{\mu}(a)$ for all $x \in H$. Then $\widetilde{\mu_1}$ is an *i.v.* fuzzy ideal of H, by Lemma 2.5 and $\widetilde{\mu_2}$ is an *i.v.* fuzzy ideal of H clearly. Suppose $x \in <a>$. Then for any $y \in H$; $Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) = \widetilde{\mu_2}(y) = \widetilde{\mu}(a)$. Again, since $\widetilde{\mu}$ is an *i.v.* fuzzy ideal of H, for any $t \in xy$, $\widetilde{\mu}(t) \geq \widetilde{\mu}(x)$. Now, $x \in <a> \Longrightarrow x \in ra + as + \sum_{i=1}^{m} r_i a s_i + na$ for some $r, s, r_i, s_i \in H$ and $m, n \in \mathbf{N}$. This implies that $x \in \bigcup_{\substack{t_1 \in ra, t_2 \in as, \\ t_3 \in \Sigma_{i=1}^m r_i a s_i, \\ t_4 \in na}} (t_1 + t_2 + t_3 + t_4)$. This shows that $x \in t_1 + t_2 + t_3 + t_4$ for some $t_1 \in ra, t_2 \in as, t_3 \in \sum_{i=1}^{m} r_i a s_i$ and $t_4 \in na$. Since, $\widetilde{\mu}$ is an *i.v.* fuzzy ideal of H, we have $\widetilde{\mu}(t_1) \geq \widetilde{\mu}(a)$ for all $t_1 \in ra$, $\widetilde{\mu}(t_2) \geq \widetilde{\mu}(a)$ for all $t_2 \in as$, $\widetilde{\mu}(t_3) \geq \widetilde{\mu}(a)$ for all $t_3 \in \sum_{i=1}^{m} r_i a s_i$ and $\widetilde{\mu}(t_4) \geq \widetilde{\mu}(a)$ for all $t_4 \in na$. Again, since $\widetilde{\mu}$ is an *i.v.* fuzzy ideal of H, we get that $\widetilde{\mu}(x) \geq \inf_{1 \leq i \leq 4} \widetilde{\mu}(t_i) \geq \widetilde{\mu}(a)$. Thus we obtain that $\widetilde{\mu}(t) \geq \widetilde{\mu}(x) \geq \widetilde{\mu}(a)$. Then $Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) = \widetilde{\mu}(a) \leq \widetilde{\mu}(t)$. Suppose $x \notin <a>$. Then $\widetilde{\mu_1}(x) = \widetilde{0}$. This shows that $Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) = \widetilde{0}$ for any $y \in H$. Therefore, $Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) \leq \widetilde{\mu}(t)$. Thus $\widetilde{\mu}(t) \geq Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y))$ for all $t \in xy$. It demonstrates that $\widetilde{\mu}(t) \geq \sup\Big\{Min^i(\widetilde{\mu_1}(x), \widetilde{\mu_2}(y)) : t \in xy$ for some $x, y \in H\Big\} = (\widetilde{\mu_1} \circ \widetilde{\mu_2})(t)$. Then it follows that $\widetilde{\mu_1} \circ \widetilde{\mu_2} \subseteq \widetilde{\mu}$. Since, $\widetilde{\mu}$ is an *i.v.*

fuzzy prime ideal of H, we obtain that $\widetilde{\mu_1} \subseteq \widetilde{\mu}$ or $\widetilde{\mu_2} \subseteq \widetilde{\mu}$. But $\widetilde{\mu_1}(a) = \widetilde{1} > \widetilde{\mu}(a)$ and $\widetilde{\mu_2}(b) = \widetilde{\mu}(a) > \widetilde{\mu}(b)$. Thus we arrive at a contradiction. Hence, $|Im\widetilde{\mu}| = 2$. □

Theorem 2.15. *If $\widetilde{\mu}$ is an i.v. fuzzy prime ideal of a hypersemiring H, then $\widetilde{\mu}_0 = \{x \in H : \widetilde{\mu}(x) = \widetilde{\mu}(0_H)\}$ is a prime hyperideal of H.*

Proof. Since, $\widetilde{\mu}$ is non-constant, $\widetilde{\mu}_0$ is a proper hyperideal of H, by Lemma 2.6. Let A, B be two hyperideals of H such that $AB \subseteq \widetilde{\mu}_0$ i.e. $\widetilde{\chi}_{AB} \subseteq \widetilde{\chi}_{\widetilde{\mu}_0}$. Then it follows that $\widetilde{\chi}_A \circ \widetilde{\chi}_B \subseteq \widetilde{\chi}_A\ \widetilde{\chi}_B = \widetilde{\chi}_{AB}$ (by Lemma 2.7) $\subseteq \widetilde{\chi}_{\widetilde{\mu}_0}$. Now let $h \in H$. If $\widetilde{\chi}_{\widetilde{\mu}_0}(h) = \widetilde{0}$, then $\widetilde{\chi}_{\widetilde{\mu}_0}(h) \leq \widetilde{\mu}(h)$. If $\widetilde{\chi}_{\widetilde{\mu}_0}(h) = \widetilde{1}$, then $h \in \widetilde{\mu}_0 \Longrightarrow \widetilde{\mu}(h) = \widetilde{\mu}(0_H) = \widetilde{1}$. Thus $\widetilde{\chi}_{\widetilde{\mu}_0}(h) \leq \widetilde{\mu}(h)$. So we obtain that $\widetilde{\chi}_{\widetilde{\mu}_0} \subseteq \widetilde{\mu}$. Consequently, it follows that $\widetilde{\chi}_A \circ \widetilde{\chi}_B \subseteq \widetilde{\mu}$. Since, $\widetilde{\mu}$ is an $i.v.$ fuzzy prime ideal of H, we get that $\widetilde{\chi}_A \subseteq \widetilde{\mu}$ or $\widetilde{\chi}_B \subseteq \widetilde{\mu}$. Suppose, $\widetilde{\chi}_A \subseteq \widetilde{\mu}$. Then $t \in A \Longrightarrow \widetilde{\chi}_A(t) = \widetilde{1} \Longrightarrow \widetilde{\mu}(t) = \widetilde{1} = \widetilde{\mu}(0_H)$ (by Theorem 2.13) $\Longrightarrow t \in \widetilde{\mu}_0$. This shows that $A \subseteq \widetilde{\mu}_0$. Similarly, considering $\widetilde{\chi}_B \subseteq \widetilde{\mu}$, we can obtain that $B \subseteq \widetilde{\mu}_0$. Hence, $\widetilde{\mu}_0$ is a prime hyperideal of H. □

From the Theorems 2.11, 2.13, 2.14, and 2.15; we obtain the following characterization theorem for $i.v.$ fuzzy prime ideal of a hypersemiring.

Theorem 2.16. *A non-constant i.v. fuzzy ideal $\widetilde{\mu}$ of a hypersemiring H is an i.v. fuzzy prime ideal of H if and only if $Im\widetilde{\mu} = \{\widetilde{1}, [\alpha, \beta]\}$; where $[\alpha, \beta] \in D[0,1] \setminus \{\widetilde{1}\}$ and $\widetilde{\mu}_0$ is a prime hyperideal of H.*

Definition 2.17. Let $\widetilde{a}$ and $\widetilde{b}$ be any two interval numbers, where $\widetilde{a} = [a^-, a^+]$ and $\widetilde{b} = [b^-, b^+]$. Then the difference of these two interval numbers is defined by $\widetilde{a} - \widetilde{b} = [a^- - b^-, a^+ - b^+]$ when $a^- - b^- \leq a^+ - b^+$ and $\widetilde{a} - \widetilde{b} = [a^+ - b^+, a^- - b^-]$ when $a^- - b^- > a^+ - b^+$.

Definition 2.18. A non-empty subset M of a hypersemiring H is said to be an m-system of H if for any two elements $a, b \in M$, there exists an element $x \in H$, such that $axb \subseteq M$.

Definition 2.19. A non-empty $i.v.$ fuzzy subset $\widetilde{\mu}$ of a hypersemiring H is called an $i.v.$ fuzzy m-system of H if for any two interval numbers $\widetilde{a}$, $\widetilde{b} \in D[0,1] \setminus \{\widetilde{1}\}$ and $x, y \in H$; $\widetilde{\mu}(x) > \widetilde{a}$ and $\widetilde{\mu}(y) > \widetilde{b} \Longrightarrow$ there exists an element $z \in H$ such that $\sup\{\widetilde{\mu}(t) : t \in xzy\} > Max^i(\widetilde{a}, \widetilde{b})$.

Theorem 2.20. *A non-empty subset M of a hypersemiring H is an m-system of H if and only if $\widetilde{\chi}_M$ is an i.v. fuzzy m-system of H.*

Definition 2.21. Let $\widetilde{\mu}$ be an $i.v.$ fuzzy subset of a hypersemiring H. Then the complement of $\widetilde{\mu}$, denoted by $\widetilde{\mu}^c$, is an $i.v.$ fuzzy subset of H defined by $\widetilde{\mu}^c(x) = \widetilde{1} - \widetilde{\mu}(x)$ for all $x \in H$.

Lemma 2.22. *Let $x_{\widetilde{a}}$ and $y_{\widetilde{b}}$ be two i.v. fuzzy points of a hypersemiring H. Then $x_{\widetilde{a}} \circ y_{\widetilde{b}} = u_{Min^i(\widetilde{a}, \widetilde{b})}$, where $u \in xy$.*

Now we produce two more characterizations for $i.v.$ fuzzy prime ideal of a hypersemiring.

Theorem 2.23. *The following statements are equivalent for an i.v. fuzzy ideal $\widetilde{\mu}$ of a hypersemiring H.*

(1) $\widetilde{\mu}$ *is an i.v. fuzzy prime ideal of* H.
(2) *For any two i.v. fuzzy points* $x_{\widetilde{a}}$ *and* $y_{\widetilde{b}}$ *of* H, $x_{\widetilde{a}} \circ \widetilde{\chi}_H \circ y_{\widetilde{b}} \subseteq \widetilde{\mu}$ *if and only if* $x_{\widetilde{a}} \in \widetilde{\mu}$ *or* $y_{\widetilde{b}} \in \widetilde{\mu}$.

Theorem 2.24. *Suppose* $\widetilde{\mu}$ *be a non-constant i.v. fuzzy ideal of a hypersemiring* H. *Then* $\widetilde{\mu}$ *is an i.v. fuzzy prime ideal of* H *if and only if* $\widetilde{\mu}^c$ *is an i.v. fuzzy m-system of* H.

Proof. Let $\widetilde{\mu}$ be an *i.v.* fuzzy prime ideal of a hypersemiring H. Suppose, $x, y \in H$. Consider two interval numbers $\widetilde{a}, \widetilde{b} \in D[0,1] \setminus \{1\}$ such that $\widetilde{\mu}^c(x) > \widetilde{a}$ and $\widetilde{\mu}^c(y) > \widetilde{b}$. Then it follows that $\widetilde{1} - \widetilde{\mu}(x) > \widetilde{a}$ and $\widetilde{1} - \widetilde{\mu}(y) > \widetilde{b}$. This implies that $[1-\mu^+(x), 1-\mu^-(x)] > \widetilde{a}$ and $[1-\mu^+(y), 1-\mu^-(y)] > \widetilde{b}$. Now
$[1-\mu^+(x), 1-\mu^-(x)] > \widetilde{a}$
$\Longrightarrow [1-\mu^+(x), 1-\mu^-(x)] \geq \widetilde{a}$ and $[1-\mu^+(x), 1-\mu^-(x)] \neq \widetilde{a}$.
$\Longrightarrow (1-\mu^+(x) \geq a^-$ and $1-\mu^-(x) \geq a^+)$ and $(1-\mu^+(x) \neq a^-$ or $1-\mu^-(x) \neq a^+)$
$\Longrightarrow (\mu^+(x) \leq 1-a^-$ and $\mu^-(x) \leq 1-a^+)$ and $(\mu^+(x) \neq 1-a^-$ or $\mu^-(x) \neq 1-a^+)$
$\Longrightarrow [\mu^-(x), \mu^+(x)] < [1-a^+, 1-a^-]$.
$\Longrightarrow \widetilde{\mu}(x) < 1-\widetilde{a}$. Similarly, from the inequality $[1-\mu^-(y), 1-\mu^-(y)] > \widetilde{b}$, we can obtain that $\widetilde{\mu}(y) < \widetilde{1}-\widetilde{b}$. Then $x_{\widetilde{1}-\widetilde{a}} \notin \widetilde{\mu}$ and $y_{\widetilde{1}-\widetilde{b}} \notin \widetilde{\mu}$. Since, $\widetilde{\mu}$ is an *i.v.* fuzzy prime ideal of H, we have $x_{\widetilde{1}-\widetilde{a}} \circ \widetilde{\chi}_H \circ y_{\widetilde{1}-\widetilde{b}} \nsubseteq \widetilde{\mu}$, by Theorem 2.23. So, there exists one *i.v.* fuzzy point $z_{\widetilde{c}}$ of H such that $x_{\widetilde{1}-\widetilde{a}} \circ z_{\widetilde{c}} \circ y_{\widetilde{1}-\widetilde{b}} \notin \widetilde{\mu}$. Now $x_{\widetilde{1}-\widetilde{a}} \circ z_{\widetilde{c}} \circ y_{\widetilde{1}-\widetilde{b}} \notin \widetilde{\mu} \Longrightarrow x_{\widetilde{1}-\widetilde{a}} \circ u_{Min^i(\widetilde{c}, \widetilde{1}-\widetilde{b})} \notin \widetilde{\mu}$; where $u \in zy$, by Lemma 2.22. $\Longrightarrow v_{Min^i\left(\widetilde{1}-\widetilde{a}, Min^i(\widetilde{c}, \widetilde{1}-\widetilde{b})\right)} \notin \widetilde{\mu}$; where $v \in xu \subseteq x(zy)$. $\Longrightarrow Min^i\Big(\widetilde{1}-\widetilde{a}, Min^i(\widetilde{c}, \widetilde{1}-\widetilde{b})\Big) \nleq \widetilde{\mu}(v)$. Since, by our assumption, any two interval numbers are comparable, we have $\widetilde{\mu}(v) < Min^i\Big(\widetilde{1}-\widetilde{a}, Min^i(\widetilde{c}, \widetilde{1}-\widetilde{b})\Big) < Min^i(\widetilde{1}-\widetilde{a}, \widetilde{1}-\widetilde{b}) = \widetilde{1} - Max^i(\widetilde{a}, \widetilde{b})$, where $v \in xzy$. $\Longrightarrow \widetilde{1} - \widetilde{\mu}(v) > Max^i(\widetilde{a}, \widetilde{b}) \Longrightarrow (\widetilde{\mu}^c)(v) > Max^i(\widetilde{a}, \widetilde{b})$; where $v \in xzy \Longrightarrow \sup\{(\widetilde{\mu}^c)(v) : v \in xzy\} > Max^i(\widetilde{a}, \widetilde{b})$. Consequently, $\widetilde{\mu}^c$ is an *i.v.* fuzzy m-system of H. Conversely, suppose that $\widetilde{\mu}^c$ is an *i.v.* fuzzy m-system of H. Let $\widetilde{\mu}_1$ and $\widetilde{\mu}_2$ be two *i.v.* fuzzy ideals of H such that $\widetilde{\mu_1} \circ \widetilde{\mu}_2 \subseteq \widetilde{\mu}$. If possible, let $\widetilde{\mu}_1 \nsubseteq \widetilde{\mu}$ and $\widetilde{\mu}_2 \nsubseteq \widetilde{\mu}$. Then there exist $x, y \in H$ such that $\widetilde{\mu}_1(x) \nleq \widetilde{\mu}(x)$ and $\widetilde{\mu}_2(y) \nleq \widetilde{\mu}(y)$. Since, any two interval numbers are comparable, we find that $\widetilde{\mu}_1(x) > \widetilde{\mu}(x)$ and $\widetilde{\mu}_2(y) > \widetilde{\mu}(y)$. So, we can choose two interval numbers $\widetilde{a}$ and $\widetilde{b}$ in such a way that $\widetilde{\mu}_1(x) > \widetilde{1} - \widetilde{a} > \widetilde{\mu}(x)$ and $\widetilde{\mu}_2(y) > \widetilde{1} - \widetilde{b} > \widetilde{\mu}(y)$. Now, $\widetilde{1} - \widetilde{a} > \widetilde{\mu}(x) \Longrightarrow \widetilde{\mu}^c(x) > \widetilde{a}$ and $\widetilde{1} - \widetilde{b} > \widetilde{\mu}(y) \Longrightarrow \widetilde{\mu}^c(y) > \widetilde{b}$. Since, $\widetilde{\mu}^c$ is an *i.v.* fuzzy m-system of H, there exists $z_1 \in H$ such that $\sup\{\widetilde{\mu}^c(t) : t \in xz_1y\} > Max^i(\widetilde{a}, \widetilde{b})$. Again, for $t \in xz_1y$; $\widetilde{\mu}^c(t) > Max^i(\widetilde{a}, \widetilde{b}) \Longrightarrow \widetilde{1} - \widetilde{\mu}(t) > Max^i(\widetilde{a}, \widetilde{b})$. $\Longrightarrow \widetilde{\mu}(t) < \widetilde{1} - Max^i(\widetilde{a}, \widetilde{b}) = Min^i(\widetilde{a}, \widetilde{b})$.

Also,

$$\begin{aligned}\widetilde{\mu}(t) &\geq (\widetilde{\mu}_1 \circ \widetilde{\mu}_2)(t)\\ &\geq Min^i\Big(\widetilde{\mu}_1(p), \widetilde{\mu}_2(y)\Big) \quad \text{(where } p \in xz)\\ &\geq Min^i\Big(\widetilde{\mu}_1(x), \widetilde{\mu}_2(y)\Big) \quad \text{(since } \widetilde{\mu}_1 \text{ is an } i.v. \text{ fuzzy ideal of } H)\\ &> Min^i(\widetilde{1} - \widetilde{a}, \widetilde{1} - \widetilde{b}).\end{aligned}$$

Thus we arrive at a contradiction. So, $\widetilde{\mu}_1 \subseteq \widetilde{\mu}$ or $\widetilde{\mu_2} \subseteq \widetilde{\mu}$. Hence, $\widetilde{\mu}$ is an $i.v.$ fuzzy prime ideal of H. □

3. $i.v.$ fuzzy semiprime ideal of a hypersemiring :

Definition 3.1. A proper hyperideal I of a hypersemiring H is said to be semiprime if for any hyperideal H of S, $H^2 \subseteq I \Longrightarrow H \subseteq I$.

Theorem 3.2. *The following statements on a hyperideal of I of a hypersemiring H are equivalent:*

(1) *I is a semiprime hyperideal of H.*
(2) *For any $a \in H$, $\bigcup\limits_{h \in H} aha \subseteq I \Leftrightarrow a \in I$.*

Proof. The proof is similar to the proof of the Theorem 2.9. □

Definition 3.3. An $i.v.$ fuzzy ideal $\widetilde{\mu}$ of a hypersemiring H is said to be an $i.v.$ fuzzy semiprime ideal of H if $\widetilde{\mu}$ is non-constant and for any $i.v.$ fuzzy ideal $\widetilde{\theta}$ of H, $\widetilde{\theta} \circ \widetilde{\theta} \subseteq \widetilde{\mu} \Longrightarrow \widetilde{\theta} \subseteq \widetilde{\mu}$.

Theorem 3.4. *Let I be a semiprime hyperideal of a hypersemiring H and $[\alpha, \beta] \in D[0,1] \setminus \{\widetilde{1}\}$. Then the i.v. fuzzy subset $\widetilde{\mu}$ of H, defined by :*

$$\widetilde{\mu}(x) = \begin{cases} \widetilde{1} & \textit{when} \quad x \in I, \\ [\alpha, \beta] & \textit{otherwise}; \end{cases}$$

is an $i.v.$ fuzzy semiprime ideal of H.

Proof. The proof is similar to the proof of the Theorem 2.11. □

Example 3.5. Consider the hypersemiring $H = (\mathbb{N}_0, \oplus, \odot)$, where $\oplus$ and $\odot$ are defined as follows : $m \oplus n = \{m, n\}$ and $m \odot n = (mn)I$, where $I = \mathbb{N} \setminus \{6\}$. Then $J = (6\mathbb{N}_0, \oplus, \odot)$ forms a semiprime hyperideal of H. Define an $i.v.$ fuzzy ideal of H as follows :

$$\widetilde{\mu}(x) = \begin{cases} [0.8, 0.9] & \textit{when} \quad x \in 6\mathbb{N}_0 \\ [0.3, 0.4] & \textit{otherwise}. \end{cases}$$

Then we can check that $\widetilde{\mu}$ is an $i.v.$ fuzzy semiprime ideal of H. But it is clear that $\widetilde{\mu}$ is not an $i.v.$ fuzzy prime ideal of H, since $\widetilde{\mu}(0_H) \neq \widetilde{1}$. Also from definition, it clear that every $i.v.$ fuzzy prime ideal of a hypersemiring H is also an $i.v.$ fuzzy semiprime ideal of H. But this example shows that converse is not true in general.

Now we produce a characterization theorem for $i.v.$ fuzzy semiprime ideal of a hypersemiring.

Theorem 3.6. *A non-constant i.v. fuzzy ideal* $\widetilde{\mu}$ *of a hypersemiring* H *(with identity) is an i.v. fuzzy semiprime ideal of* H *if and only if for any* $a \in H$, $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in aha\} : h \in H\Big\} = \widetilde{\mu}(a)$.

Proof. Let $\widetilde{\mu}$ be an *i.v.* fuzzy semiprime ideal of H and $a, h \in H$. Then $\widetilde{\mu}(t) \geq \widetilde{\mu}(a)$ for all $t \in aha$ by Lemma 2.3. This implies that $\sup\{\widetilde{\mu}(t) : t \in aha\} \geq \widetilde{\mu}(a)$. Then it follows that $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in aha : h \in H\} : h \in H\Big\} \geq \widetilde{\mu}(a)$. If possible, let $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in aha\} : h \in H\Big\} > \widetilde{\mu}(a)$. Let us choose an interval number $[\alpha, \beta]$ in such a way that $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in aha\} : h \in H\Big\} > [\alpha, \beta] > \widetilde{\mu}(a)$. Now we construct an *i.v.* fuzzy ideal $\widetilde{\theta}$ of H as follows :

$$\widetilde{\theta}(x) = \begin{cases} [\alpha, \beta] & \text{when} \quad x \in < a >; \\ \widetilde{0} & \text{otherwise.} \end{cases}$$

Let $x \in H$. If there does not exist any $u, v \in H$ such that $x \in uv$, where $u \in < a >$ and $v \in < a >$; then $(\widetilde{\theta} \circ \widetilde{\theta})(x) = \sup\Big\{Min^i(\widetilde{\theta}(u), \widetilde{\theta}(v)) : x \in uv$ for some $u, v \in H\Big\} = \widetilde{0} \leq \widetilde{\mu}(x)$. Suppose, there exist $u \in < a >$ and $v \in < a >$ such that $x \in uv$. Then $(\widetilde{\theta} \circ \widetilde{\theta})(x) = \sup\Big\{Min^i(\widetilde{\theta}(a), \widetilde{\theta}(v)) : x \in uv$ for some $u, v \in H\Big\} = [\alpha, \beta]$. Since, we have taken the hypersemiring H with identity, $u \in < a > \Longrightarrow u \in \bigcup_{x_i \in r_i a s_i} (\sum_{i=1}^{m} x_i)$ for some $r_i, s_i \in H$ and $m \in \mathbb{N}$. This shows that $u \in \sum_{i=1}^{m} x_i$ for some $x_i \in r_i a s_i$, where $r_i, s_i \in H$ and $m \in \mathbb{N}$. Similarly, $v \in < a > \Longrightarrow v \in \sum_{j=1}^{n} y_j$ for some $y_j \in r'_j a s'_j$, where $r'_j, s'_j \in H$ and $n \in \mathbb{N}$. Now $x \in uv \Longrightarrow x \in (\sum_{i=1}^{m} x_i)(\sum_{j=1}^{n} y_j) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_i y_j = \bigcup_{u_k \in x_i y_j} (\sum_{k=1}^{mn} u_k)$. Then it follows that $x \in \sum_{k=1}^{mn} u_k$ for some $u_k \in x_i y_j$. Again, $u_k \in x_i y_j \Longrightarrow u_k \in (r_i a s_i)(r'_j a s'_j)$. Then we obtain that $\widetilde{\mu}(u_k) \geq \widetilde{\mu}(t)$ for $t \in a s_i r'_j a$, since $\widetilde{\mu}$ is an *i.v.* fuzzy ideal of H. It demonstrates that $\widetilde{\mu}(u_k) \geq \widetilde{\mu}(t)$ for $t \in aha$, where $h \in s_i r'_j$. Moreover, since $\widetilde{\mu}$ is an *i.v.* fuzzy ideal of H, we have $\widetilde{\mu}(x) \geq \inf_{1 \leq k \leq mn} \widetilde{\mu}(u_k) \geq \widetilde{\mu}(t)$ for $t \in aha$, where $h \in H$. Thus, $\widetilde{\mu}(x) \geq \widetilde{\mu}(t)$ for all $t \in aha$. So, $\widetilde{\mu}(x) \geq \sup\{\widetilde{\mu}(t) : t \in aha\}$. Consequently, $\widetilde{\mu}(x) \geq \inf\Big\{\sup\{\widetilde{\mu}(t) : t \in aha\} : h \in H\Big\} > [\alpha, \beta] = (\widetilde{\theta} \circ \widetilde{\theta})(x)$. So, we get that $\widetilde{\theta} \circ \widetilde{\theta} \subseteq \widetilde{\mu}$. Since, $\widetilde{\mu}$ is an *i.v.* fuzzy semiprime ideal of H, it follows that $\widetilde{\theta} \subseteq \widetilde{\mu}$. But $\widetilde{\theta}(a) = [\alpha, \beta] > \widetilde{\mu}(a)$. So, we arrive at a contradiction. Hence, $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in aha\} : h \in H\Big\} = \widetilde{\mu}(a)$.

Conversely, let $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in aha\} : h \in H\Big\} = \widetilde{\mu}(a)$ for any $a \in H$. Suppose, $\widetilde{\theta}$ be an *i.v.* fuzzy ideal of H such that $\widetilde{\theta}\circ\widetilde{\theta} \subseteq \widetilde{\mu}$. If possible, let $\widetilde{\theta} \nsubseteq \widetilde{\mu}$. Since, according to our assumption, any two interval numbers in $D[0,1]$ are comparable, there exists $b \in H$ such that $\widetilde{\theta}(b) > \widetilde{\mu}(b)$. Again $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in bhb\} : h \in H\Big\} = \widetilde{\mu}(b)$. Let $h \in H$ and $t \in bhb$. Then

$$\begin{aligned}(\widetilde{\theta} \circ \widetilde{\theta})(t) &\geq Min^i(\widetilde{\theta}(h_1), \widetilde{\theta}(b)) \text{ where } h_1 \in bh\\ &\geq Min^i(\widetilde{\theta}(b), \widetilde{\theta}(b)) \text{ (since } \widetilde{\theta} \text{ is an } i.v. \text{ fuzzy ideal of } H)\\ &= \widetilde{\theta}(b).\end{aligned}$$

Thus we obtain that $\widetilde{\mu}(t) \geq (\widetilde{\theta} \circ \widetilde{\theta})(t) \geq \widetilde{\theta}(b)$ for all $h \in H$ and $t \in bhb$. This implies that $\inf\Big\{\sup\{\widetilde{\mu}(t) : t \in bhb\} : h \in H\Big\} \geq \widetilde{\theta}(b)$. Consequently, $\widetilde{\mu}(b) = \inf\Big\{\sup\{\widetilde{\mu}(t) : t \in bhb\} : h \in H\Big\} \geq \widetilde{\theta}(b) > \widetilde{\mu}(b)$. So, we arrive at a contradiction. Therefore, $\widetilde{\theta} \subseteq \widetilde{\mu}$. Hence, $\widetilde{\mu}$ is an *i.v.* fuzzy semiprime ideal of H. □

Definition 3.7. Let H_1 and H_2 be two hypersemirings. A mapping $f : H_1 \longrightarrow H_2$ is said to be a homomorphism if

(1) $f(0_{H_1}) = 0_{H_2}$,
(2) $f(1_{H_1}) = 1_{H_2}$ and
(3) $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in H_1$.

Definition 3.8. Let A and B be two non-empty sets and $f : A \longrightarrow B$ be a function. Let $\widetilde{\mu}$ be an *i.v.* fuzzy subset of A and $\widetilde{\theta}$ be an *i.v.* fuzzy subset of B. Then the image of $\widetilde{\mu}$ under the function f, denoted by $f(\widetilde{\mu})$, is an *i.v.* fuzzy subset of B, defined by :

$$f(\widetilde{\mu})(y) = \begin{cases} \sup\limits_{z\in f^{-1}(y)} \widetilde{\mu}(z) & \text{when} \quad f^{-1}(y) = \{x \in A : f(x) = y\} \neq \varnothing, \\ \widetilde{0} & \text{otherwise}; \quad y \in B. \end{cases}$$

The pre-image of $\widetilde{\theta}$ under the function f, denoted by $f^{-1}(\widetilde{\theta})$, is an *i.v.* fuzzy subset of A, defined as : $f^{-1}(\widetilde{\theta})(x) = \widetilde{\theta}(f(x))$ for all $x \in A$.

Proposition 3.9. *Let $f : H_1 \longrightarrow H_2$ be an epimorphism of hypersemirings. Suppose $\widetilde{\mu}$ be an i.v. fuzzy left (or right) ideal of H_1 and $\widetilde{\theta}$ be an i.v. fuzzy left (or right) ideal of H_2. Then*

(1) *$f^{-1}(\widetilde{\theta})$ is an i.v. fuzzy left (or right) ideal of H_1.*
(2) *$f(\widetilde{\mu})$ is an i.v. fuzzy left (or right) ideal of H_2.*

Theorem 3.10. *Let $f : H_1 \longrightarrow H_2$ be an epimorphism of hypersemirings. Let $\widetilde{\mu}$ and $\widetilde{\theta}$ be two i.v. fuzzy semiprime ideals of H_1 and H_2 respectively. Let $\widetilde{\mu}$ be f-invariant i.e. $f(x) = f(y) \Longrightarrow \widetilde{\mu}(x) = \widetilde{\mu}(y)$, for any $x, y \in H_1$. Then*

(1) *$f(\widetilde{\mu})$ is an i.v. fuzzy semiprime ideal of H_2.*
(2) *$f^{-1}(\widetilde{\theta})$ is an i.v. fuzzy semiprime ideal of H_1.*
(3) *There is a one to one correspondence between the f-invariant i.v. fuzzy semiprime ideals of H_1 and the i.v. fuzzy semiprime ideals of H_2.*

Proof. (1) Since, $\widetilde{\mu}$ is an *i.v.* fuzzy semiprime ideal of H_1, $f(\widetilde{\mu})$ is a non-constant *i.v.* fuzzy ideal of H_2, by Proposition 3.9. Let $b \in H_2$. Then $\inf\Big\{\sup\{f(\widetilde{\mu})(t_1) : t_1 \in bh_1b\} : h_1 \in H_2\Big\} = \inf\Big\{\sup\{\sup\limits_{f(z)=t_1} \widetilde{\mu}(z) : t_1 \in bh_1b\} : h_1 \in H_2\Big\}$.
Again, $t_1 \in bh_1b = f(b')f(h_1')f(b') = f(b'h_1'b')$ for some $b', h_1' \in H_1$. So, we obtain that

$$\inf\Big\{\sup\{f(\widetilde{\mu})(t_1) : t_1 \in bh_1b\} : h_1 \in H_2\Big\}$$
$$= \inf\Big\{\sup\{\sup_{f(z)=t_1} \widetilde{\mu}(z) : t_1 \in bh_1b\} : h_1 \in H_2\Big\}$$
$$= \sup_{f(z)=t_1}\Big\{\inf\{\sup \widetilde{\mu}(z) : t_1 \in bh_1b\} : h_1 \in H_2\Big\} \quad \text{(since } \widetilde{\mu} \text{ is } f\text{-invariant)}$$
$$= \sup_{f(z)=t_1 \in bh_1b = f(b'h_1'b')}\Big\{\inf\{\sup \widetilde{\mu}(t_1') : t_1' \in b'h_1'b'\} : h_1' \in H_1\Big\}$$
$$\text{(since } \widetilde{\mu} \text{ is } f\text{-invariant)}$$
$$= \sup_{f(b')=b} \widetilde{\mu}(b')$$
$$= f(\widetilde{\mu})(b).$$

Consequently, $f(\widetilde{\mu})$ is an *i.v.* fuzzy semiprime ideal of H_2.

(2) Since, $\widetilde{\theta}$ is an *i.v.* fuzzy semiprime ideal of H_2, $f^{-1}(\widetilde{\theta})$ is a non-constant *i.v.* fuzzy ideal of H_1, by Proposition 3.9. Let $a \in H_1$. Then $\inf\Big\{\sup\{f^{-1}(\theta)(t) : t \in aha\} : h \in H_1\Big\} = \inf\Big\{\sup\{\widetilde{\theta}(f(t)) : t \in aha\} : h \in H_1\Big\}$. Again, $t \in aha \Longrightarrow f(t) \in f(aha) = f(a)f(h)f(a)$. Consequently,

$$\inf\Big\{\sup\{f^{-1}(\theta)(t) : t \in aha\} : h \in H_1\Big\}$$
$$= \inf\Big\{\sup\{\widetilde{\theta}(f(t)) : f(t) \in f(a)f(h)f(a)\} : f(h) \in H_2\Big\}$$
$$= \widetilde{\theta}(f(a)) \qquad \text{(by Theorem 3.6)}$$
$$= f^{-1}(\widetilde{\theta})(a).$$

Hence, $f^{-1}(\widetilde{\theta})$ is an *i.v.* fuzzy semiprime ideal of H_1.

(3) Let FSI_{H_1} denote the set of all f-invariant *i.v.* fuzzy semiprime ideals of H_1 and FSI_{H_2} denote the set of all *i.v.* fuzzy semiprime ideals of H_2. If we define a map $\Psi : FSI_{H_1} \longrightarrow FSI_{H_2}$ by $\Psi(\widetilde{\mu}) = f(\widetilde{\mu})$, we can easily prove that Ψ is a one to one correspondence between FSI_{H_1} and FSI_{H_2}. □

Definition 3.11. A proper hyperideal I of a hypersemiring H is said to be an irreducible hyperideal of H if for any two hyperideals J, K of H, $J \cap K = I \Longrightarrow J = I$ or $K = I$.
I is called strongly irreducible if for any two hyperideals J, K of H, $J \cap K \subseteq I \Longrightarrow J \subseteq I$ or $K \subseteq I$.

The following lemma is easy to check.

Lemma 3.12. *Every prime hyperideal of a hypersemiring H is also an strongly irreducible hyperideal of H and every strongly irreducible hyperideal of H is also an irreducible hyperideal of H.*

The following example shows that every strongly irreducible hyperideal of a hypersemiring H is not a prime hyperideal of H in general.

Example 3.13. Let us consider the hypersemiring $H = (\mathbb{N}_0, \oplus, \odot)$, where $\oplus$ and $\odot$ are defined as follows. $m \oplus n = \{m, n\}$ and $m \odot n = (mn)\mathbb{N}_0$. If we choose the hyperideal $I = (4\mathbb{N}_0, \oplus, \odot)$ of H, we can verify that I is a strongly irreducible hyperideal of H but not a prime hyperideal of H.

Theorem 3.14. *A proper hyperideal I of a hypersemiring H is prime if and only if it is strongly irreducible and semiprime.*

Proof. Suppose I be a prime hyperideal of the hypersemiring H. Then it follows from Lemma 3.12 that it is also a strongly irreducible ideal of H. From the definition of prime hyperideal and the semiprime hyperideal of H we find that every prime hyperideal of H is also a semiprime hyperideal of H.

Conversely, let I be a strongly irreducible and semiprime hyperideal of H. Consider two hyperideals A and B of H such that $AB \subseteq I$. Then it demonstrates that $(A \cap B)(A \cap B) \subseteq AB \subseteq I$. Since I is semiprime, we obtain that $A \cap B \subseteq I$. Then we get that $A \subseteq I$ or $B \subseteq I$, since I is a strongly irreducible ideal of H. Consequently, I is a prime hyperideal of H. □

Definition 3.15. A non-constant $i.v.$ fuzzy ideal $\widetilde{\mu}$ of a hypersemiring H is said to be $i.v.$ fuzzy irreducible ideal of H if for any two $i.v.$ fuzzy ideals $\widetilde{\theta}$ and $\widetilde{\eta}$ of H, $\widetilde{\mu} = \widetilde{\theta} \cap \widetilde{\eta} \Longrightarrow \widetilde{\mu} = \widetilde{\theta}$ or $\widetilde{\mu} = \widetilde{\eta}$. $\widetilde{\mu}$ is said to be strongly irreducible if for any two $i.v.$ fuzzy ideals $\widetilde{\theta}$ and $\widetilde{\eta}$ of H, $\widetilde{\mu} \subseteq \widetilde{\theta} \cap \widetilde{\eta} \Longrightarrow \widetilde{\mu} \subseteq \widetilde{\theta}$ or $\widetilde{\mu} \subseteq \widetilde{\eta}$.

Lemma 3.16. *Any $i.v.$ fuzzy prime ideal of a hypersemiring H is also an $i.v.$ fuzzy strongly irreducible ideal of H. Again, any $i.v.$ fuzzy strongly irreducible ideal of a hypersemiring H is also an $i.v.$ fuzzy irreducible ideal of H.*

The following theorem is another characterization theorem for $i.v.$ fuzzy prime ideal of a hypersemiring.

Theorem 3.17. *A non-constant $i.v.$ fuzzy ideal $\widetilde{\mu}$ of a hypersemiring H is an $i.v.$ fuzzy prime ideal of H if and only if $\widetilde{\mu}$ is $i.v.$ fuzzy semiprime ideal of H and strongly irreducible.*

Proof. The proof is straightforward. □

Now we show that in fully idempotent hypersemiring the notion of primeness, strongly irreducibility and irreducibility coincide. In [8], U. Dasgupta proved the equivalence of these three kind of hyperideals in fully idempotent multiplicative hypersemiring. We like to study the equivalence since we have taken a general hypersemiring in spite of the multiplicative hypersemiring.

Definition 3.18. A hyperideal I of a hypersemiring H is said to be idempotent if $I = I^2$. A hypersemiring H is said to be fully idempotent if each of its hyperideal is idempotent.

Theorem 3.19. *Let H be a fully idempotent hypersemiring. Then the following statements are equivalent for a proper hyperideal I of H.*

(1) *I is a prime hyperideal of H.*
(2) *I is strongly irreducible.*
(3) *I is irreducible.*

Proof. The implication (1) $\Longrightarrow$ (3) is clear.
(3) $\Longrightarrow$ (2) : Let I be an irreducible hyperideal of the fully idempotent hypersemiring H. Suppose J and K be two hyperideals of H such that $J \cap K \subseteq I$. Consider two hyperideals $J+I$ and $K+I$ of H. Then $(J+I)\cap(K+I) = (J+I)(K+I)$, (since H is fully idempotent, $A \cap B = AB$ for any two hyperideals A and B of H). Again,

$$\begin{aligned}(J+I)(K+I) &= JK + JI + IK + I^2 \\ &= (J\cap K) + JI + IK + I \quad \text{(since } H \text{ is fully idempotent)} \\ &= (J \cap K) + I \quad \Big(\text{since } JI + IK = (J\cap I) + (I \cap K) \subseteq I\Big).\end{aligned}$$

So we obtain that $(J+I)\cap(K+I) = (J\cap K)+I = I$, since $J\cap K \subseteq I$. Therefore, $J+I = I$ or $K+I = I$, as I is irreducible. This implies that $J \subseteq I$ or $K \subseteq I$. Consequently, I is strongly irreducible.
(2) $\Longrightarrow$ (1) : Suppose I is a strongly irreducible hyperideal of H. Consider two hyperideals A and B of H such that $AB \subseteq I$. Again, $(A\cap B) = (A\cap B)(A\cap B) \subseteq AB \subseteq I$. Since, I is strongly irreducible hyperideal of H, we have $A \subseteq I$ or $B \subseteq I$. Hence, I is a prime hyperideal of H. □

Now we represent the fuzzy version of the above theorem.

Theorem 3.20. *In a fully idempotent hypersemiring H the following statements are equivalent for a non-constant i.v. fuzzy ideal $\widetilde{\mu}$ of H.*

(1) *$\widetilde{\mu}$ is prime.*
(2) *$\widetilde{\mu}$ is strongly irreducible.*
(3) *$\widetilde{\mu}$ is irreducible.*

Proof. The implication (1) $\Longrightarrow$ (3) is clear.
(3) $\Longrightarrow$ (2) : Let $\widetilde{\mu}$ be an *i.v.* fuzzy irreducible ideal of H. Suppose $\widetilde{\theta}$ and $\widetilde{\eta}$ be two *i.v.* fuzzy ideals of H such that $\widetilde{\theta}\cap\widetilde{\eta} \subseteq \widetilde{\mu}$ for any two *i.v.* fuzzy ideals of H. Consider the the two *i.v.* fuzzy ideals $\widetilde{\theta}+\widetilde{\mu}$ and $\widetilde{\eta}+\widetilde{\mu}$ of H. Then

$$\begin{aligned}(\widetilde{\theta}+\widetilde{\mu})\cap(\widetilde{\eta}+\widetilde{\mu}) &= (\widetilde{\theta}+\widetilde{\mu})(\widetilde{\eta}+\widetilde{\mu}) \quad \text{(since } H \text{ is fully idempotent)} \\ &= \widetilde{\theta}\widetilde{\eta} + \widetilde{\theta}\widetilde{\mu} + \widetilde{\mu}\widetilde{\eta} + \widetilde{\mu}^2 \\ &= (\widetilde{\theta}\cap\widetilde{\eta}) + \widetilde{\theta}\widetilde{\mu} + \widetilde{\mu}\widetilde{\eta} + \widetilde{\mu} \\ &= (\widetilde{\theta}\cap\widetilde{\eta}) + \widetilde{\mu} \quad (\text{since } \widetilde{\theta}\widetilde{\mu} + \widetilde{\mu}\widetilde{\eta} = (\widetilde{\theta}\cap\widetilde{\mu}) + (\widetilde{\mu}\cap\widetilde{\eta}) \subseteq \widetilde{\mu}).\end{aligned}$$

Consequently, we get that $(\widetilde{\theta}+\widetilde{\mu})\cap(\widetilde{\mu}+\widetilde{\eta}) = (\widetilde{\theta}\cap\widetilde{\eta})+\widetilde{\mu} = \widetilde{\mu}$, as $\widetilde{\theta}\cap\widetilde{\eta} \subseteq \widetilde{\mu}$. This shows that $\widetilde{\theta}+\widetilde{\mu} = \widetilde{\mu}$ or $\widetilde{\eta}+\widetilde{\mu} = \widetilde{\mu}$, since $\widetilde{\mu}$ is irreducible. Therefore, $\widetilde{\theta} \subseteq \widetilde{\mu}$ or $\widetilde{\eta} \subseteq \widetilde{\mu}$. Consequently, $\widetilde{\mu}$ is strongly irreducible.

The implication (2) $\Longrightarrow$ (1) follows from the fact that $\widetilde{\mu_1} \cap \widetilde{\mu_2} = \widetilde{\mu_1}\widetilde{\mu_2}$ for any two *i.v.* fuzzy ideals of a fully idempotent hypersemiring H. □

4. Conclusions

We have characterized regular and intra-regular semiring in terms of i-v fuzzy quasi-ideals and i-v fuzzy bi-ideals of a semiring. So this paper helps us to realize that we can study different properties of semirings and even some other algebraic structures from the view of i-v fuzzy set theory. For example, as a continuation of this paper we shall study the k-regularity and k-intra-regularity of a semiring in terms of i-v fuzzy k-quasi ideal and i-v fuzzy k-bi-ideal of semirings.

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