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A note on fuzzy σ-Baire spaces

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Abstract. In this paper we investigate several characterizations of fuzzy σ-Baire spaces and study the conditions under which a fuzzy topological space becomes a fuzzy σ-Baire space.

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1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. ZADEH in his classical paper [16] in the year 1965. This concept provides a natural foundation for treating mathematically the fuzzy phenomena, which exist pervasively in our real world and for new branches of fuzzy mathematics. Thereafter the paper of C.L.CHANG [4] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concepts of Baire spaces have been studied extensively in classical topology in [7], [8], [9] and [6]. The concept of Baire spaces in fuzzy setting was introduced and studied by G.THANGARAJ and S.ANJALMOSE in [11]. The concept of fuzzy σ-Baire spaces was introduced and studied by the authors in [13]. In this paper we investigate several characterizations of fuzzy σ-Baire spaces and study under what conditions a fuzzy topological space becomes a fuzzy σ-Baire space. The inter-relations between fuzzy σ-Baire spaces, fuzzy Baire spaces and fuzzy almost resolvable spaces are also studied in this paper.

2. Preliminaries

By a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X, T).

Definition 2.1. Let λ and μ be any two fuzzy sets in (X, T). Then we define $\lambda \vee \mu : X \to [0, 1]$ as follows : $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0, 1]$ as follows : $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$.

Definition 2.2. Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define

(i). $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$,
(ii). $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.1 ([1]). *For any fuzzy set λ in a fuzzy topological space (X, T),*

(a). $1 - cl(\lambda) = int(1 - \lambda)$,
(b). $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.3 ([10]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy dense* if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.

Definition 2.4 ([2]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy F_σ-set* in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.5 ([2]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy G_δ-set* in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in T$ for $i \in I$.

Definition 2.6 ([3]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy σ-nowhere dense set* if λ is a fuzzy F_σ-set in (X, T) such that $int(\lambda) = 0$.

Lemma 2.2 ([1]). For a family $\mathscr{A}$ of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space X, $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case $\mathscr{A}$ is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$.

Definition 2.7 ([3]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy σ-first category* if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of *fuzzy σ-second category.*

Definition 2.8 ([3]). Let λ be a fuzzy σ-first category set in a fuzzy topological space (X, T). Then $1 - \lambda$ is called a *fuzzy σ-residual set* in (X, T).

3. Fuzzy σ-Baire spaces

Motivated by the classical concept introduced in [3], the concept of fuzzy σ-Baire spaces was introduced and studied in [13].

Definition 3.1 ([13]). Let (X, T) be a fuzzy topological space. Then (X, T) is called a *fuzzy σ-Baire space* if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i's are fuzzy σ-nowhere dense sets in (X, T).

Theorem 3.1 ([13]). *Let (X, T) be a fuzzy topological space. Then the following are equivalent:*

(1) *(X, T) is a fuzzy σ-Baire space.*

(2) $int(\lambda) = 0$ *for every fuzzy* σ*-first category set* λ *in* (X, T).
(3) $cl(\mu) = 1$ *for every fuzzy* σ*-residual set* μ *in* (X, T).

Theorem 3.2 ([12])**.** *If* λ *is a fuzzy dense and fuzzy* G_δ*-set in a fuzzy topological space* (X, T)*, then* $1 - \lambda$ *is a fuzzy first category set in* (X, T).

Theorem 3.3 ([13])**.** *In a fuzzy topological space* (X, T)*, a fuzzy set* λ *is a fuzzy* σ*-nowhere dense set in* (X, T) *if and only if* $1 - \lambda$ *is a fuzzy dense and fuzzy* G_δ*-set in* (X, T).

Proposition 3.1. *If the fuzzy topological space* (X, T) *is a fuzzy* σ*-Baire space, then* $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$*, where the fuzzy sets* (λ_i)*'s* $(i = 1 \text{ to } \infty)$ *are fuzzy dense and fuzzy* G_δ*-sets in* (X, T).

Proof. Let (λ_i)'s $(i = 1 \text{ to } \infty)$ be fuzzy dense and fuzzy G_δ-sets in (X, T). By theorem 3.3, $(1 - \lambda_i)$'s are fuzzy σ-nowhere dense sets in (X, T). Then the fuzzy set $\lambda = \vee_{i=1}^{\infty}(1 - \lambda_i)$ is a fuzzy σ-first category set in (X, T). Now $int(\lambda) = int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = int(1 - [\wedge_{i=1}^{\infty}(\lambda_i)]) = 1 - cl[\wedge_{i=1}^{\infty}(\lambda_i)]$. Since (X, T) is a fuzzy σ-Baire space,by theorem 3.1, we have $int(\lambda) = 0$. Then $1 - cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 0$. This implies that $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$. □

Proposition 3.2. *If the fuzzy topological space* (X, T) *is a fuzzy* σ*-Baire space, then* $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$*, where the fuzzy sets* $(1 - \lambda_i)$*'s* $(i = 1 \text{ to } \infty)$ *are fuzzy first category sets formed from the fuzzy dense and fuzzy* G_δ*-sets* λ_i *in* (X, T).

Proof. Let the fuzzy topological space (X, T) is a fuzzy σ-Baire space and the fuzzy sets (λ_i)'s $(i = 1 \text{ to } \infty)$ be fuzzy dense and fuzzy G_δ-sets in (X, T). By proposition 3.1, $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$. Then $1 - cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 0$. This implies that $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$. Also by theorem 3.2, $(1 - \lambda_i)$'s are fuzzy first category sets in (X, T). Hence $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where the fuzzy sets $(1 - \lambda_i)$'s $(i = 1 \text{ to } \infty)$ are fuzzy first category sets formed from the fuzzy dense and fuzzy G_δ-sets λ_i in (X, T). □

Theorem 3.4 ([11])**.** *Let* (X, T) *be a fuzzy topological space. Then the following are equivalent:*

(1) (X, T) *is a fuzzy Baire space.*
(2) $int(\lambda) = 0$ *for every fuzzy first category set* λ *in* (X, T).
(3) $cl(\mu) = 1$ *for every fuzzy residual set* μ *in* (X, T).

Proposition 3.3. *If the fuzzy first category sets are formed from the fuzzy dense and fuzzy* G_δ*-sets in a fuzzy* σ*-Baire space* (X, T)*, then* (X, T) *is a fuzzy Baire space.*

Proof. Let the fuzzy topological space (X, T) be a fuzzy σ-Baire space. By proposition 3.2, $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where the fuzzy sets $(1 - \lambda_i)$'s $(i = 1 \text{ to } \infty)$ are fuzzy first category sets formed from the fuzzy dense and fuzzy G_δ-sets λ_i in (X, T). Now $\vee_{i=1}^{\infty}[int(1 - \lambda_i)] \leq int(\vee_{i=1}^{\infty}[1 - \lambda_i])$. Then we have $\vee_{i=1}^{\infty}[int(1 - \lambda_i)] = 0$. This implies that $int(1 - \lambda_i) = 0$, where $(1 - \lambda_i)$ is a fuzzy first category set in (X, T). By theorem 3.4, (X, T) is a fuzzy Baire space. □

Definition 3.2 ([13])**.** A fuzzy topological space (X, T) is called *fuzzy* σ*-first category* if the fuzzy set 1_X is a fuzzy σ-first category set in (X, T). That is, $1_X =$

$\vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Otherwise, (X, T) will be called a *fuzzy σ-second category space.*

Proposition 3.4. *If the fuzzy topological space (X, T) is a fuzzy σ-first category space, then (X, T) is not a fuzzy σ-Baire space.*

Proof. Let the fuzzy topological space (X, T) is a fuzzy σ-first category space. Then $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Now $int(\vee_{i=1}^{\infty}(\lambda_i)) = int(1_X) = 1 \neq 0$. Hence by definition, (X, T) is not a fuzzy σ-Baire space. □

Proposition 3.5. *If $\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$, where the fuzzy sets (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy σ-second category space.*

Proof. Let (λ_i)'s ($i = 1$ *to* ∞) are fuzzy dense and fuzzy G_δ-sets in (X, T). By theorem 3.3, $(1 - \lambda_i)$'s are fuzzy σ-nowhere dense sets in (X, T). Now $\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$ implies that $1 - \wedge_{i=1}^{\infty}(\lambda_i) \neq 1$. Then $\vee_{i=1}^{\infty}(1 - \lambda_i) \neq 1$. Hence (X, T) is not a fuzzy σ-first category space and therefore (X, T) is a fuzzy σ-second category space. □

Proposition 3.6. *If λ is a fuzzy σ-first category set in (X, T), then there is a fuzzy F_σ-set δ in (X, T) such that $\lambda \leq \delta$.*

Proof. Let λ be a fuzzy σ-first category set in (X, T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Now $[1 - cl(\lambda_i)]$'s are fuzzy open sets in (X, T). Then $\mu = \wedge_{i=1}^{\infty}(1 - cl(\lambda_i))$ is a fuzzy G_δ-set in (X, T) and $1 - \mu = 1 - [\wedge_{i=1}^{\infty}(1 - cl(\lambda_i))] = \vee_{i=1}^{\infty}(cl(\lambda_i))$. Now $\lambda = \vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty}(cl(\lambda_i)) = 1 - \mu$. That is, $\lambda \leq 1 - \mu$ and $1 - \mu$ is a fuzzy F_σ-set in (X, T). Let $\delta = 1 - \mu$. Hence, if λ is a fuzzy σ-first category set in (X, T), then there is a fuzzy F_σ-set δ in (X, T) such that $\lambda \leq \delta$. □

Proposition 3.7. *If δ is a fuzzy σ-residual set in a fuzzy topological space (X, T) such that $\eta \leq \delta$, where η is a fuzzy dense and fuzzy G_δ-set in (X, T), then (X, T) is a fuzzy σ-Baire space.*

Proof. Let δ be a fuzzy σ-residual set in a fuzzy topological space (X, T). Then $1 - \delta$ is a fuzzy σ-first category set in (X, T). Now by proposition 3.6, there is a fuzzy F_σ-set μ in (X, T) such that $1 - \delta \leq \mu$. This implies that $1 - \mu \leq \delta$. Let $\eta = 1 - \mu$. Then η is a fuzzy G_δ-set in (X, T) and $\eta \leq \delta$ implies that $cl(\eta) \leq cl(\delta)$. If $cl(\eta) = 1$, then we have $cl(\delta) = 1$. Hence, by theorem 3.1, (X, T) is a fuzzy σ-Baire space. □

Proposition 3.8. *If the fuzzy topological space (X, T) is a fuzzy σ-Baire space and if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, then there exists atleast one F_σ-set λ_i such that $int(\lambda_i) \neq 0$.*

Proof. Suppose that $int(\lambda_i) = 0$, for $i = 1$ *to* ∞, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$. Implies that $int[\vee_{i=1}^{\infty}(\lambda_i)] = int[1] = 1 \neq 0$, a contradiction to (X, T) being a fuzzy σ-Baire space. Hence $int(\lambda_i) \neq 0$, for atleast one F_σ-set λ_i in (X, T). □

Proposition 3.9. *If the fuzzy topological space (X, T) is a fuzzy σ-Baire space, then no non-zero fuzzy open set is a fuzzy σ-first category set in (X, T).*

Proof. Let λ be a non-zero fuzzy open set in a fuzzy σ-Baire space (X, T). Suppose that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where the fuzzy sets (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Then $int(\lambda) = int(\vee_{i=1}^{\infty}(\lambda_i))$. Since (X, T) is a fuzzy σ-Baire space, $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$. This implies that $int(\lambda) = 0$. Then we will have $\lambda = 0$, which is a contradiction, since $\lambda \in T$ implies that $int(\lambda) = \lambda \neq 0$. Hence no non-zero fuzzy open set is a fuzzy σ-first category set in (X, T). □

Definition 3.3 ([2])**.** A fuzzy topological space (X, T) is called a *fuzzy submaximal space* if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, then $\lambda \in T$ in (X, T).

Proposition 3.10. *If the fuzzy topological space* (X, T) *is a fuzzy submaximal space and if* λ *is a fuzzy* σ*-first category set in* (X, T)*, then* λ *is a fuzzy first category set in* (X, T)*.*

Proof. Let $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ be a fuzzy σ-first category set in (X, T), where the fuzzy sets (λ_i)'s $(i = 1\ to\ \infty)$ are fuzzy σ-nowhere dense sets in (X, T). Then we have $int(\lambda_i) = 0$ and (λ_i)'s $(i = 1\ to\ \infty)$ are fuzzy F_σ-sets in (X, T). Now $int(\lambda_i) = 0$, implies that $1 - int(\lambda_i) = 1 - 0 = 1$ and hence $cl(1 - \lambda_i) = 1$. Since (X, T) is a fuzzy submaximal space, the fuzzy dense sets $(1-\lambda_i)$'s are fuzzy open sets in (X, T) and hence (λ_i)'s are fuzzy closed sets in (X, T). Then $cl(\lambda_i) = (\lambda_i)$ and $int(\lambda_i) = 0$ implies that $intcl(\lambda_i) = int(\lambda_i) = 0$. That is, (λ_i)'s are fuzzy nowhere dense sets in (X, T). Therefore $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy first category set in (X, T). □

Proposition 3.11. *If the fuzzy topological space* (X, T) *is a fuzzy* σ*-Baire space and fuzzy submaximal space, then* (X, T) *is a fuzzy Baire space.*

Proof. Let μ be a fuzzy σ-first category set in (X, T). Since (X, T) is a fuzzy submaximal space, by proposition 3.10, μ is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy σ-Baire space, by theorem 3.1, $int(\mu) = 0$. Hence, for the fuzzy first category set μ in (X, T), we have $int(\mu) = 0$. Therefore, by theorem 3.4, (X, T) is a fuzzy Baire space. □

Definition 3.4 ([14])**.** A fuzzy topological space (X, T) is called a *fuzzy P-space* if countable intersection of fuzzy open sets in (X, T) is fuzzy open. That is, every non-zero fuzzy G_δ-set in (X, T) is fuzzy open in (X, T).

Proposition 3.12. *If the fuzzy topological space* (X, T) *is a fuzzy* σ*-Baire space and fuzzy* P*-space, then* (X, T) *is a fuzzy Baire space.*

Proof. Let the fuzzy topological space (X, T) be a fuzzy σ-Baire space. Then, by proposition 3.1, $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, where the fuzzy sets (λ_i)'s $(i = 1\ to\ \infty)$ are fuzzy dense and fuzzy G_δ-sets in (X, T). Now from, $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, we have $1 - cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 0$. This implies that $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$. Since the fuzzy sets (λ_i)'s are fuzzy dense in (X, T), $cl(\lambda_i) = 1$. Then we have $1 - cl(\lambda_i) = 0$. This implies that $int(1 - \lambda_i) = 0$. Also, since (X, T) is a fuzzy P-space, the non-zero fuzzy G_δ-sets (λ_i)'s in (X, T), are fuzzy open in (X, T). Then $(1 - \lambda_i)$'s are fuzzy closed sets in (X, T). Then $cl(1 - \lambda_i) = 1 - \lambda_i$ and $int(1 - \lambda_i) = 0$ implies that $intcl(1 - \lambda_i) = int(1 - \lambda_i) = 0$. That is, $(1 - \lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Therefore we have $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where $(1 - \lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Hence, by theorem 3.4, (X, T) is a fuzzy Baire space. □

Definition 3.5 ([15]). A fuzzy topological space (X, T) is called a *fuzzy almost resolvable space* if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets λ_i's in (X, T) are such that $int(\lambda_i) = 0$. Otherwise (X, T) is called a *fuzzy almost irresolvable space*.

Proposition 3.13. *If the fuzzy topological space (X, T) is a fuzzy almost resolvable space, then (X, T) is a fuzzy σ-second category space.*

Proof. Let (λ_i)'s ($i = 1$ *to* ∞) be the fuzzy dense and fuzzy G_δ-sets in (X, T). Now $cl(\lambda_i) = 1$ implies that $1 - cl(\lambda_i) = 0$. That is, $int(1-\lambda_i) = 0$. Since (X, T) is a fuzzy almost irresolvable space, $\vee_{i=1}^{\infty}(1 - \lambda_i) \neq 1$, where the fuzzy sets (λ_i)'s in (X, T) are such that $int(1 - \lambda_i) = 0$. Now $\vee_{i=1}^{\infty}(1 - \lambda_i) \neq 1$ implies that $1 - \vee_{i=1}^{\infty}(1 - \lambda_i) \neq 0$. Hence we have $\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$, where the fuzzy sets λ_i's are fuzzy dense and fuzzy G_δ-sets in a fuzzy topological space (X, T). Thus, by proposition 3.5, (X, T) is a fuzzy σ-second category space. □

Definition 3.6 ([5]). A fuzzy topological space (X, T) is called a *fuzzy hyperconnected space* if every fuzzy open set λ is fuzzy dense in (X, T). That is, $cl(\lambda) = 1$ for all $0 \neq \lambda \in T$.

Theorem 3.5 ([13]). *If $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where λ_i's are fuzzy dense and fuzzy G_δ-sets in (X, T), then (X, T) is a fuzzy σ-Baire space.*

Proposition 3.14. *If $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where the fuzzy sets (λ_i)'s are fuzzy G_δ-sets in a fuzzy hyperconnected and fuzzy P-space (X, T), then (X, T) is a fuzzy σ-Baire space.*

Proof. Let (λ_i)'s ($i = 1$ *to* ∞) be the fuzzy G_δ-sets in (X, T) such that $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$. Since (X, T) is a fuzzy P-space, the fuzzy G_δ-sets (λ_i)'s in (X, T), are fuzzy open in (X, T). Also since (X, T) is a fuzzy hyperconnected space, the fuzzy open sets λ_i's in (X, T), are fuzzy dense sets in (X, T). Hence the fuzzy sets λ_i's ($i = 1$ *to* ∞) are fuzzy dense and fuzzy G_δ-sets in (X, T) and $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$. Hence, by theorem 3.5, (X, T) is a fuzzy σ-Baire space. □

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