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Some new soft sets and decompositions of some soft continuities

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Abstract. In this paper we introduce the concepts of R - closed, A_R-, αAN_1-, αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5- soft sets in the soft topological spaces and show the relationships between defined new soft sets. Also we investigate some properties of these soft sets. Additionally, we define the notions of R -, A_R-, αAN_1-, αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5- soft continuity. Consequently, we obtain decomposition of A_R - soft continuity, α - soft continuity and soft continuity using soft topological space.

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1. Introduction

Some set theories such as theory of fuzzy sets [15], intuitionistic fuzzy sets [2], vague sets [3], rough sets [13] etc. are dealt with unclear concepts. They are not sufficient to solve encountered difficulties. There are some vague problems in medical science, social science, economics etc. What is the reason of these problems? It is possible to solve the inadequacy using some parametrization tool in the theories. In 1999, Molodtsov [12] introduced the concept of soft set theory as a general mathematical tool for coping with these problems. In 2001, Maji et al. [9, 10] defined the notion of fuzzy soft set and intuitionistic fuzzy soft set. Again in 2003, Maji et al. [11] introduced the theoretical notions of the soft set theory and studied some properties of these notions. In 2009, Ali et al. [1] investigated several operations on soft set theory and defined some new notions such as the restricted intersection etc. In 2011, Naz et al. [14] introduced some concepts such as soft topological space, soft interior, soft closure etc. Also in 2011, Hussain et al. [4] studied some properties of soft topological spaces. In 2012, Zorlutuna et al. [16] introduced the image and inverse image of soft set under a function and soft continuity in the soft topological spaces. In 2012, Mahanta and Das [8] investigated soft topological space via semi - open and semi - closed soft sets. In 2013, Gnanambal and Mrudula [5] studied soft preopen sets. In 2014, Kandil et al. [7] defined α - open soft set, β - open soft sets, pre - open soft set, semi - open soft set, investigated some properties of these soft sets and obtained decompositions of some forms of soft continuity. In 2014, Gnanambal et al. [6] investigated some properties of soft α - open sets.

In this paper we introduce the concepts of R - closed, A_R-, αAN_1-, αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5- soft sets in the soft topological spaces and show the relationships between defined new soft sets. Also we investigate some properties of these soft sets. Additionally, we define the notions of R -, A_R-, αAN_1-, αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5- soft continuity. Consequently, we obtain decomposition of A_R - soft continuity, α - soft continuity and soft continuity in soft topological space.

2. Preliminaries

In this section we recall some known definitions and theorems.

Definition 2.1 ([12])**.** Let X be a universe and A be the set of parameters. Let $P(X)$ denote the power set of X. A pair (F, A), where F is mapping from A to $P(X)$ as follows

$$F : A \to P(X),$$

is called a soft set over X. The family of all soft sets on X denoted by $SS(X)_E$.

Definition 2.2 ([11])**.** Let (F, A) and (G, B) be two soft sets over a common universe X. Then (F, A) is said to be a soft subset of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. This relation is denoted by $(F, A)\widetilde{\subseteq}(G, B)$.
(F, A) is said to be soft equal to (G, B) if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. This relation is denoted by $(F, A) = (G, B)$.

Definition 2.3 ([1])**.** The complement of a soft set (F,A) is defined as $(F, A)^c = (F^c, A)$, where $F^c(e) = (F(e))^c = X - F(e)$, for all $e \in A$.

Definition 2.4 ([14])**.** The difference of two soft sets (F,A) and (G,A) is defined by $(F, A) - (G, A) = (F - G, A)$, where $(F - G)(e) = F(e) - G(e)$, for all $e \in A$.

Definition 2.5 ([11])**.** The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and $H(e) = F(e)$ if $e \in A - B$ or $H(e) = G(e)$ if $e \in B - A$ or $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$ for all $e \in C$.

Definition 2.6 ([11])**.** The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.7 ([11])**.** Let (F, A) be a soft set over X. Then (F, A) is said to be a null soft set if $F(e) = \varnothing$, for all $e \in A$. This denoted by $\widetilde{\varnothing}$.

Definition 2.8 ([11])**.** Let (F, A) be a soft set over X. Then (F, A) is said to be an absolute soft set if $F(e) = X$, for all $e \in A$. This denoted by $\widetilde{X}$.

Definition 2.9 ([14])**.** Let τ be a collection of soft sets over X. Then τ is said to be a soft topology on X if

(1) $\widetilde{\varnothing}$, $\widetilde{X} \in \tau$;
(2) the intersection of any two soft sets in τ belongs to τ;
(3) the union of any number of soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. Here E is the set of parameters. The members of τ are said to be $\tau-$ soft open sets or soft open sets in X. A soft set over X is said to be soft closed in X if its complement belongs to τ. The set of all soft open sets over X denoted by $OS(X, \tau, E)$ or $OS(X)$ and the set of all soft closed sets denoted by $CS(X, \tau, E)$ or $CS(X)$.

Definition 2.10. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X. Then

[16] the soft interior of (F, E) is the soft set
$int(F, E) = \cup\{(G, E) : (G, E)$ is soft open and $(G, E)\widetilde{\subseteq}(F, E)\}$;
[14] the soft closure of (F, E) is the soft set
$cl(F, E) = \widetilde{\cap}\{(H, E) : (H, E)$ is soft closed and $(F, E)\widetilde{\subseteq}(H, E)\}$.

Theorem 2.11 ([4])**.** *Let (X, τ, E) be a soft topological space over X, (F, E) and (G, E) are soft sets over X. Then*

(1) $cl(\widetilde{\varnothing}) = \widetilde{\varnothing}$ *and* $cl(\widetilde{X}) = \widetilde{X}$.
(2) $(F, E)\widetilde{\subseteq}cl(F, E)$.
(3) (F, E) *is a closed set if and only if* $(F, E) = cl(F, E)$.
(4) $cl(cl(F, E)) = cl(F, E)$.
(5) $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $cl(F, E)\widetilde{\subseteq}cl(G, E)$.
(6) $cl((F, E)\widetilde{\cup}(G, E)) = cl(F, E)\widetilde{\cup}cl(G, E)$.
(7) $cl((F, E)\widetilde{\cap}(G, E))\widetilde{\subseteq}cl(F, E)\widetilde{\cap}cl(G, E)$.

Theorem 2.12 ([4])**.** *Let (X, τ, E) be a soft topological space over X and (F, E) and (G, E) are soft sets over X. Then*

(1) $int\widetilde{\varnothing} = \widetilde{\varnothing}$ *and* $int\widetilde{X} = \widetilde{X}$.
(2) $int(F, E)\widetilde{\subseteq}(F, E)$.
(3) $int(int(F, E)) = int(F, E)$.
(4) (F, E) *is a soft open set if and only if* $int(F, E) = (F, E)$.
(5) $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $int(F, E)\widetilde{\subseteq}int(G, E)$.
(6) $int(F, E)\widetilde{\cap}int(G, E) = int((F, E)\widetilde{\cap}(G, E))$.
(7) $int(F, E)\widetilde{\cup}int(G, E)\widetilde{\subseteq}int((F, E)\widetilde{\cup}(G, E))$.

Definition 2.13 ([16])**.** Let $SS(X)_A$ and $SS(Y)_B$ be two families, $u : X \to Y$ and $p : A \to B$ be mappings. Then the mapping $f_{pu} : SS(X)_A \to SS(Y)_B$ is defined as:

(1) Let $(F, A) \in SS(X)_A$. The image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that
$f_{pu}(F)(y) = \bigcup_{x \in p^{-1}(y) \cap A} u(F(x))$ if $p^{-1}(y) \cap A \neq \varnothing$ and
$f_{pu}(F)(y) = \varnothing$ if $p^{-1}(y) \cap A = \varnothing$ for all $y \in B$.
(2) Let $(G, B) \in SS(Y)_B$. The inverse image of (G, B) under f_{pu}, written as $f^{-1}_{pu}(G, B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$f_{pu}^{-1}(G)(x) = u^{-1}(G(p(x)))$ if $p(x) \in B$ and
$f_{pu}^{-1}(G)(x) = \varnothing$ if $p(x) \notin B$ for all $x \in A$.

Definition 2.14 ([16]). Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces, $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the function f_{pu} is called soft continuous (soft - cts) if $f_{pu}^{-1}(G, B) \in \tau$ for all $(G, B) \in \tau*$.

Theorem 2.15 ([7]). *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. If (G, E) is a soft open set, then $(G, E) \mathbin{\widetilde{\cap}} cl(F, E) \mathbin{\widetilde{\subseteq}} cl((G, E)\widetilde{\cap}(F, E))$.*

3. R - closed, A_R-, αAN_1-, αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5- soft sets

In this section we define R - closed, A_R-, αAN_1-, αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5- soft sets and investigate some properties of these soft sets.

Definition 3.1. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called

1. α - open soft set [7] if $(F, E)\widetilde{\subseteq} int(cl(int(F, E)))$,
2. semi - open soft set [7] if $(F, E)\widetilde{\subseteq} cl(int(F, E))$,
3. pre - open soft set [7] if $(F, E)\widetilde{\subseteq} int(cl(F, E))$,
4. t - soft set if $int(F, E) = int(cl(F, E))$,
5. $t*$ - soft set if $int(F, E) = int(cl(int(F, E)))$.

Definition 3.2. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called

1. a weakly soft locally - closed set (briefly, WLC - soft set) if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is soft open and (H, E) is τ - soft closed,
2. a B - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is soft open and (H, E) is t - soft set,
3. a C - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is soft open and (H, E) is $t*$ - soft set.

The family of all α - open soft (resp. semi - open soft, pre - open soft, weakly soft locally - closed, B - soft and C - soft) sets in a soft topological space (X, τ, E) is denoted by $\alpha OS(X)$ (resp. $SOS(X)$, $POS(X)$, $WLCS(X)$, $BS(X)$ and $CS(X)$).

Definition 3.3. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called a R - closed soft if $(F, E) = cl(int(F, E))$.

Definition 3.4. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called an A_R - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is soft open and (H, E) is a R - closed soft.

By $A_RS(X)$ (resp. $RS(X)$) we denote the family of all A_R - (R - closed) soft sets of (X, τ, E).

Definition 3.5. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called

(1) an αAN_1 - soft set if $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where $(G,E) \in \alpha OS(X)$ and $cl(int(H,E)) = \widetilde{X}$,
(2) an αAN_2 - soft set if $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where $(G,E) \in \alpha OS(X)$ and $cl(H,E) = \widetilde{X}$,
(3) an αNA_1 - soft set if $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where $(G,E) \in \alpha OS(X)$ and (H,E) is pre - closed soft set,
(4) an αNA_2 - soft set if $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where $(G,E) \in \alpha OS(X)$ and (H,E) is regular closed soft set,
(5) an αNA_3 - soft set if $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where $(G,E) \in \alpha OS(X)$ and (H,E) is t - soft set,
(6) an αNA_4 - soft set if $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where $(G,E) \in \alpha OS(X)$ and (H,E) is β - soft closed set,
(7) an αNA_5 - soft set if $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where $(G,E) \in \alpha OS(X)$ and (H,E) is τ - soft closed set.

We denote the family of all αAN_1- (resp. αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5-) sets in the soft topological space (X,τ,E) by $\alpha AN_1S(X)$ (resp. $\alpha AN_2S(X)$, $\alpha NA_1S(X)$, $\alpha NA_2S(X)$, $\alpha NA_3S(X)$, $\alpha NA_4S(X)$ and $\alpha NA_5S(X)$).

Proposition 3.6. *Every αAN_1 - soft set is αAN_2 - soft set.*

Proof. It is obvious from Definition 3.5. □

Remark 3.7. The converse of Proposition 3.6 need not be true as shown in the following example.

Example 3.8. Let $X = \{a,b,c\}$, $E = \{e\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1,E), (F_2,E)\}$, where (F_1,E), (F_2,E) are soft sets over X defined as follows:
$(F_1,E) = \{(e,\{c\})\}$,
$(F_2,E) = \{(e,\{a,b\})\}$.
The soft set $(G,E) = \{(e,\{a,c\})\}$ is an αAN_2 - soft set, but it is not an αAN_1 - soft set.

Proposition 3.9. *Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. Then the following hold:*

(1) *If (F,E) is an αNA_2 - soft set, then (F,E) is an αNA_5 - soft set.*
(2) *If (F,E) is an αNA_5 - soft set, then (F,E) is an αNA_3 - soft set.*
(3) *If (F,E) is an αNA_3 - soft set, then (F,E) is an αNA_4 - soft set.*

Proof. (1) Let $(F,E) = (G,E)\widetilde{\cap}(H,E) \in \alpha NA_2S(X)$, where $(G,E) \in \alpha OS(X)$ and $cl(int(H,E)) = (H,E)$. Since $cl(int(H,E)) = cl(H,E) = (H,E)$, we obtain $(F,E) \in \alpha NA_5S(X)$.
(2) It is obvious from Definition 3.5.
(3) It is clear from Definition 3.5. □

Remark 3.10. The converses of Proposition 3.9 need not be true as shown in the following examples.

Example 3.11. Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F, E)\}$, where (F, E) is a soft set over X defined as follows:
$(F, E) = \{(e_1, \{a\}), (e_2, \{b\})\}$.
The soft set $(G, E) = \{(e_1, \{b\}), (e_2, \{a\})\}$ is an αNA_5 - soft set, but it is not an αNA_2 - soft set.

Example 3.12. Let $X = \{a, b, c\}$, $E = \{e\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E) are soft sets over X defined as follows:
$(F_1, E) = \{(e, \{a\})\}$,
$(F_2, E) = \{(e, \{b\})\}$,
$(F_3, E) = \{(e, \{a, b\})\}$.
The soft set $(G, E) = \{(e, \{a\})\}$ is an αNA_3 - soft set, but it is not an αNA_5 - soft set.

Example 3.13. Let $X = \{a, b, c, d\}$, $E = \{e\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E) are soft sets over X defined as follows:
$(F_1, E) = \{(e, \{a, b\})\}$,
$(F_2, E) = \{(e, \{d\})\}$,
$(F_3, E) = \{(e, \{a, b, d\})\}$.
The soft set $(G, E) = \{(e, \{b, c, d\})\}$ is an αNA_4 - soft set, but it is not an αNA_3 - soft set.

Proposition 3.14. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then the following hold:*

1. *If (F, E) is an αNA_5 - soft set, then (F, E) is an αNA_1 - soft set.*
2. *If (F, E) is an αNA_1 - soft set, then (F, E) is an αNA_4 - soft set.*

Proof. This proof is obvious from Definition 3.5. □

Remark 3.15. The converses of Proposition 3.14 need not be true as shown in the following examples.

Example 3.16. Let $X = \{a, b, c\}$, $E = \{e\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F, E)\}$, where (F, E) is a soft set over X defined as follows:
$(F, E) = \{(e, \{b, c\})\}$.
The soft set $(G, E) = \{(e, \{a, c\})\}$ is an αNA_1 - soft set, but it is not an αNA_5 - soft set.

Example 3.17. Let $X = \{a, b, c\}$, $E = \{e\}$ and $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E) are soft sets over X defined as follows:
$(F_1, E) = \{(e, \{a\})\}$,
$(F_2, E) = \{(e, \{b\})\}$,
$(F_3, E) = \{(e, \{a, b\})\}$.
The soft set $(G, E) = \{(e, \{a\})\}$ is an αNA_4 - soft set, but it is not an αNA_1 - soft set.

Definition 3.18. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then

1. (F, E) is said to be soft - dense if $cl(F, E) = \widetilde{X}$.

(2) (X, τ, E) is said to be soft submaximal if each soft - dense (F, E) is a soft open set.

Definition 3.19. A soft topological space (X, τ, E) is said to be extremally soft disconnected if the soft closure of every soft open set is soft open.

Theorem 3.20. *If (X, τ, E) is an extremally soft disconnected space, then $\alpha NA_4S(X)$ $= \alpha NA_1S(X)$.*

Proof. Let $(F, E) = (G, E)\widetilde{\cap}(H, E) \in \alpha NA_4S(X)$, where $(G, E) \in \alpha OS(X)$ and $int(cl(int(H, E))) \widetilde{\subseteq} (H, E)$. Since the soft topological space (X, τ, E) is soft extremally disconnected, we obtain

$$int(cl(int(H, E))) = cl(int(H, E))$$

and so $cl(int(H, E)) \widetilde{\subseteq} (H, E)$. As a consequence, we obtain

$$\alpha NA_4S(X) \widetilde{\subseteq} \alpha NA_1S(X) \text{ and } \alpha NA_4S(X) = \alpha NA_1S(X)$$

by Proposition 3.14. □

Proposition 3.21. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then the following hold:*

(1) *If (F, E) is an A_R - soft set, then (F, E) is a weakly soft locally - closed.*
(2) *If (F, E) is a weakly soft locally - closed, then (F, E) is a B - soft set.*
(3) *If (F, E) is a B - soft set, then (F, E) is a C - soft set.*

Proof. (1) Let $(F, E) = (G, E)\widetilde{\cap}(H, E) \in A_RS(X)$, where (G, E) is soft open and $cl(int(H, E)) = (H, E)$. Then we obtain $cl(cl(int(H, E))) = cl(H, E) = cl(int(H, E))$ and so $(H, E) = cl(H, E)$. As a consequence, (F, E) is a weakly soft locally - closed.
(2) It is obvious. For, every τ - soft closed set is a t - soft set.
(3) It is clear. For, every t - soft set is a $t*$ - soft set. □

Proposition 3.22. *Let (X, τ, E) be a soft topological space and*

$$(F, E) = (G, E)\widetilde{\cap}(H, E) \in SS(X)_E.$$

Then the following hold:

(1) *If (F, E) is an A_R - soft set, then (F, E) is an αNA_2 - soft set.*
(2) *If (F, E) is a weakly soft locally - closed, then (F, E) is a αNA_5 - soft set.*
(3) *If (F, E) is a B - soft set, then (F, E) is a αNA_3 - soft set.*
(4) *If (F, E) is a C - soft set, then (F, E) is a αNA_4 - soft set.*

Proof. It is obvious since every soft open set is α - open soft set. □

Theorem 3.23. *Let (X, τ, E) be a soft submaximal space. Then the following hold:*

(1) $A_RS(X) = \alpha NA_2S(X)$.
(2) $WLCS(X) = \alpha NA_5S(X)$.
(3) $BS(X) = \alpha NA_3S(X)$.
(4) $CS(X) = \alpha NA_4S(X)$.

Proof. It is clear. For, in a soft submaximal space $\tau = \alpha OS(X)$. □

Proposition 3.24 ([7]). *Let (X, τ, E) be a soft topological space.*

(1) *If* $(F, E) \in SOS(X)$ *and* $(G, E) \in \alpha OS(X)$, *then* $(F, E)\widetilde{\cap}(G, E) \in SOS(X)$.
(2) *If* $(F, E) \in POS(X)$ *and* $(G, E) \in \alpha OS(X)$, *then* $(F, E)\widetilde{\cap}(G, E) \in POS(X)$.

Proposition 3.25. *Every* αNA_2 *- soft set is semi - open soft set.*

Proof. Let $(F, E) = (G, E)\widetilde{\cap}(H, E) \in \alpha NA_2S(X)$, where $(F, E) \in \alpha OS(X)$ and $cl(int(H, E)) = (H, E)$. Thus (H, E) is semi - open soft set. By Proposition 3.24 we obtain that (F, E) is semi - open soft set. □

Theorem 3.26. *Let* $(F, E) \in SS(X)_E$. (F, E) *is an* A_R *- soft set if and only if* (F, E) *is a semi - open soft set and a weakly soft locally - closed set.*

Proof. Let $(F, E) = (G, E)\widetilde{\cap}(H, E) \in A_RS(X)$, where (G, E) is soft open and $cl(int(H, E)) = (H, E)$. Then we obtain $cl(cl(int(H, E))) = cl(int(H, E)) = cl(H, E) = (H, E)$. Also $int(F, E) = (G, E) \widetilde{\cap}\ int(H, E)$ and by Theorem 2.15 $(F, E) = (G, E)\widetilde{\cap}\ cl(int(H, E)) = cl((G, E)\widetilde{\cap}int(H, E))$. Hence $(F, E) \widetilde{\subseteq} cl(int(F, E))$. As a consequence, (F, E) is semi - open soft set.

Conversely, let (F, E) be semi - open soft set and weakly soft locally - closed. Then $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is soft open and $cl(H, E) = (H, E)$. Since (F, E) is semi - open soft, we have $(F, E) \widetilde{\subseteq} cl(int(F, E)) \widetilde{\subseteq} cl(F, E)$ and so $cl(F, E) \widetilde{\subseteq} cl(int(F, E)) \widetilde{\subseteq} cl(F, E)$. Thus we obtain $cl(F, E) = cl(int(F, E))$. Also since $(F, E) = (G, E)\widetilde{\cap}(H, E)$ and $(F, E) \widetilde{\subseteq} (G, E)\widetilde{\cup}cl(F, E)$, we obtain $(F, E) \widetilde{\subseteq} (G, E)\widetilde{\cap}cl(F, E) \widetilde{\subseteq} (G, E)\widetilde{\cap}\ cl((G, E)\widetilde{\cap}(H, E)) \widetilde{\subseteq} (G, E)\widetilde{\cap}\ [cl(G, E)\widetilde{\cap}\ cl(H, E)] = [(G, E)\widetilde{\cap}cl(G, E)]\ \widetilde{\cap}cl(H, E) = (G, E)\widetilde{\cap}cl(H, E) = (G, E)\widetilde{\cap}\ (H, E) = (F, E)$. As a consequence, $(F, E) = (G, E)\widetilde{\cap}cl(F, E) = (G, E)\widetilde{\cap}\ cl(int(F, E))$ and so (F, E) is an A_R - soft set. □

Theorem 3.27. *Let* (X, τ, E) *be a soft topological space. Then*

$$A_RS(X) = \alpha NA_2S(X) \cap WLCS(X).$$

Proof. By Proposition 3.25 we have $\alpha NA_2S(X) \subseteq SOS(X)$ and by Theorem 3.26 $A_RS(X) = SOS(X) \cap WLCS(X)$. Then

$$\alpha NA_2S(X) \cap WLCS(X) \subseteq SOS(X) \cap WLCS(X) = A_RS(X).$$

Hence, we obtain $\alpha NA_2S(X) \cap WLCS(X) \subseteq A_RS(X)$. By Proposition 3.22 we have $A_RS(X) \subseteq \alpha NA_2S(X)$. Since $A_RS(X) = SOS(X) \cap WLCS(X)$, $A_RS(X) \subseteq WLCS(X)$ and so $A_RS(X) \subseteq \alpha NA_2S(X) \cap WLCS(X)$. As a consequence, we obtain $A_RS(X) = \alpha NA_2S(X) \cap WLCS(X)$. □

Proposition 3.28. *Every* αAN_2 *- soft set is pre - open soft set.*

Proof. Let $(F, E) = (G, E)\ \widetilde{\cap}\ (H, E) \in \alpha NA_2S(X)$, where $(G, E) \in \alpha OS(X)$ and $cl(H, E) = \widetilde{X}$. Since $int(cl(H, E)) = \widetilde{X}$, then $(H, E) \widetilde{\subseteq} int(cl(H, E))$ and so (H, E) is pre - open soft. By Proposition 3.24 we obtain that (F, E) is pre - open soft. □

Theorem 3.29. *For a soft topological space* (X, τ, E), $\alpha OS(X) = POS(X) \cap \alpha NA_3S(X)$.

Proof. Let (F, E) be an α - open soft set. Put $(G, E) = (F, E)$ and $(H, E) = \widetilde{X} \in \tau$. Then $(F, E) = (G, E) \widetilde{\cap} (H, E)$, where $(G, E) \in \alpha OS(X)$ and $int(cl(H, E)) = int(H, E)$. Since (F, E) is pre - open soft, $\alpha OS(X) \subseteq POS(X) \cap \alpha NA_3S(X)$.
Let (F, E) be pre - open soft set and $\alpha NA_3S(X)$ - soft set. Since (F, E) is a pre - open soft set, we have $(F, E) \widetilde{\subseteq} int(cl(F, E))$. Also we have $(F, E) = (G, E) \widetilde{\cap} (H, E)$, where (G, E) is α - open soft set and $int(cl(H, E)) = int(H, E)$ since (F, E) is an αNA_3 - soft set. Then $(F, E) \widetilde{\subseteq} int(cl(F, E)) = int(cl((G, E)\widetilde{\cap}(H, E))) \widetilde{\subseteq} int(cl(G, E)\widetilde{\cap} cl(H, E)) \widetilde{\subseteq} int(cl(int(cl(int(G, E))))\widetilde{\cap} cl(H, E)) = int(cl(int(G, E)) \widetilde{\cap} cl(H, E)) = int(cl((int(G, E))) \widetilde{\cap} int(cl(H, E)) = int(cl(int(G, E))) \widetilde{\cap} int(H, E)$ and so $(F, E) \widetilde{\subseteq} int[(cl(int(G, E))) \widetilde{\cap} int(H, E)] \widetilde{\subseteq} int(cl[int\ (G, E) \widetilde{\cap} int(H, E)] = int(cl(int((G, E) \widetilde{\cap} (H, E)))) = int(cl(int(F, E)))$. As a consequence, $(F, E) \in \alpha OS(X)$. □

Lemma 3.30 ([7]). *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. (F, E) is α - open soft if and only if it is semi - open soft and pre - open soft.*

Theorem 3.31. *For a soft topological space (X, τ, E), the following hold:*

(1) $\alpha OS(X) = POS(X) \cap \alpha NA_5S(X)$.
(2) $\alpha OS(X) = \alpha AN_2S(X) \cap \alpha NA_2S(X)$.

Proof. (1) By Lemma 3.30 $\alpha OS(X) \subseteq POS(X)$ since

$$\alpha OS(X) = POS(X) \cap SOS(X).$$

Also since $\alpha OS(X) \subseteq \alpha NA_5S(X)$, we obtain that $\alpha OS(X) \subseteq POS(X) \cap \alpha NA_5S(X)$. By Theorem 3.29 we have $\alpha OS(X) = POS(X) \cap \alpha NA_3S(X)$. Also since $\alpha NA_5S(X) \subseteq \alpha NA_3S(X)$, we obtain that $POS(X) \cap \alpha NA_5S(X) \subseteq POS(X) \cap \alpha NA_3S(X) = \alpha OS(X)$.
(2) By Proposition 3.28 we have $\alpha AN_2S(X) \subseteq POS(X)$ and by Proposition 3.25 $\alpha NA_2S(X) \subseteq SOS(X)$. Since $\alpha OS(X) = POS(X) \cap SOS(X)$, then $\alpha AN_2S(X) \cap \alpha NA_2S(X) \subseteq POS(X) \cap SOS(X) = \alpha OS(X)$. Also since $\alpha OS(X) \subseteq \alpha AN_2S(X) \cap \alpha NA_2S(X)$, we obtain that $\alpha OS(X) = \alpha AN_2S(X) \cap \alpha NA_2S(X)$. □

Proposition 3.32. *Let (X, τ, E) be a soft topological space. For $(F, E) \in SS(X)_E$, the following equivalent:*

(1) *(F, E) is a soft open set;*
(2) *(F, E) is an α - open soft set and A_R - soft set;*
(3) *(F, E) is an pre - open soft set and A_R - soft set.*

Proof. (1) → (2). Let (F, E) be a soft open set. Then (F, E) is an α - open soft set. Also $(F, E) = (F, E) \widetilde{\cap} \widetilde{X}$, where (F, E) is a soft open and $\widetilde{X}$ is a R - closed soft. As a consequence, (F, E) is an A_R - soft set.
(2) → (3). It is clear that every α - open soft set is a pre - open soft.
(3) → (1). Let (F, E) be pre - open soft and A_R - soft set. We have that $(F, E) \widetilde{\subseteq} int(cl(F, E))$ and $(F, E) = (G, E) \widetilde{\cap} (H, E)$, where (G, E) is a soft open and $cl(int(H, E)) = (H, E)$. Hence we have

$$(F, E) \widetilde{\subseteq} int(cl(F, E)) = int(cl((G, E)\widetilde{\cap}(H, E))) \widetilde{\subseteq} int(cl(G, E)\widetilde{\cap} cl(H, E)).$$

Every R -closed soft set is soft closed, (H, E) is soft closed and $cl(H, E) = (H, E)$. Thus we have $int(cl(G, E) \widetilde{\cap} (H, E)) = int(cl(G, E)) \widetilde{\cap} int(H, E)$ and $(G, E) \widetilde{\cap} (H, E) \widetilde{\subseteq} (G, E) \widetilde{\cap} int(cl((G, E) \widetilde{\cap} (H, E))) \widetilde{\subseteq} (G, E) \widetilde{\cap} int(cl(G, E)) \widetilde{\cap} int(cl(H, E))$ $= (G, E) \widetilde{\cap} int(cl(G, E)) \widetilde{\cap} int(H, E) = int((G, E) \widetilde{\cap} cl(G, E) \widetilde{\cap} (H, E)) = int((G, E) \widetilde{\cap} (H, E))$. As a consequence, we obtain that $(G, E) \widetilde{\cap} (H, E) \widetilde{\subseteq} int((G, E) \widetilde{\cap} (H, E))$ and (F, E) is soft open. □

Theorem 3.33. *$\tau = \alpha AN_2S(X) \cap \alpha NA_2S(X) \cap WLCS(X)$ for a soft topological space (X, τ, E).*

Proof. By Proposition 3.32 we have $\tau = \alpha OS(X) \cap A_RS(X)$ and by Theorem 3.27

$$A_RS(X) = \alpha NA_2S(X) \cap WLCS(X).$$

Also by Theorem 3.31 we have $\alpha OS(X) = \alpha AN_2S(X) \cap \alpha NA_2S(X)$. As a consequence, we obtain $\tau = \alpha AN_2S(X) \cap \alpha NA_2S(X) \cap WLCS(X)$. □

$$\begin{tikzcd}
A_RS(X) \arrow[r] \arrow[d] & WLCS(X) \arrow[rr] \arrow[d] & & BS(X) \arrow[r] \arrow[d] & CS(X) \arrow[dr] & \\
\alpha NA_2S(X) \arrow[r] & \alpha NA_5S(X) \arrow[rr] \arrow[dr] & & \alpha NA_3S(X) \arrow[rr] & & \alpha NA_4S(X) \\
 & & \alpha NA_1S(X) \arrow[ur] & & &
\end{tikzcd}$$

Diagram 1. The relationships between the soft sets defined above.

4. Decompositions of some soft continuities

In this section we define some new soft continuities and obtain some decompositions.

Definition 4.1. Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the function is said to be α - soft continuous [7] (resp. semi - soft continuous [7], pre - soft continuous [7], WLC - soft continuous, B - soft continuous, C - soft continuous) if $f_{pu}^{-1}(G, B) \in \alpha OS(X)$ (resp. $SOS(X)$, $POS(X)$, $WLCS(X)$, $BS(X)$, $CS(X)$) for all $(G, B) \in OS(Y)$.

Definition 4.2. Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the function is said to be R - soft continuous (A_R - soft continuous) if $f_{pu}^{-1}(G, B) \in RS(X)$ ($A_RS(X)$) for all $(G, B) \in OS(Y)$.

Definition 4.3. Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the function is said to be αAN_1 - soft continuous (resp. αAN_2 - soft continuous, αNA_1 - soft continuous, αNA_2 - soft continuous, αNA_3 - soft continuous, αNA_4 - soft continuous, αNA_5 - soft continuous) if $f_{pu}^{-1}(G, B) \in \alpha AN_1S(X)$ (resp. $\alpha AN_2S(X)$, $\alpha NA_1S(X)$, $\alpha NA_2S(X)$, $\alpha NA_3S(X)$, $\alpha NA_4S(X)$, $\alpha NA_5S(X)$) for all $(G, B) \in OS(Y)$.

Theorem 4.4. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. If a function f_{pu} is αAN_1 - soft continuous (resp. αNA_2 - soft continuous, αNA_5 - soft continuous, αNA_3 - soft continuous), then f_{pu} is αAN_2 - soft continuous (resp. αNA_5 - soft continuous, αNA_3 - soft continuous, αNA_4 - soft continuous).*

Proof. It is clear from Proposition 3.6 and 3.9. □

Theorem 4.5. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. If a function f_{pu} is αNA_5 - soft continuous (αNA_1 - soft continuous), then f_{pu} is αNA_1 - soft continuous (αNA_4 - soft continuous).*

Proof. The proof is obvious from Proposition 3.14. □

Theorem 4.6. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. If a function f_{pu} is A_R - soft continuous (resp. WLC - soft continuous, B - soft continuous), then f_{pu} is WLC - soft continuous (resp. B - soft continuous, C - soft continuous).*

Proof. This is clear from Proposition 3.21. □

Theorem 4.7. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. If a function f_{pu} is A_R - soft continuous (resp. WLC - soft continuous, B - soft continuous, C - soft continuous), then f_{pu} is αNA_2 - soft continuous (resp. αNA_5 - soft continuous, αNA_3 - soft continuous, αNA_4 - soft continuous).*

Proof. It is a direct consequence of Proposition 3.22. □

Theorem 4.8. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. If a function f_{pu} is αNA_2 - soft continuous, then f_{pu} is semi - soft continuous.*

Proof. The proof is clear from Proposition 3.25. □

Theorem 4.9. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. A function f_{pu} is A_R - soft continuous if and only if it is both semi - soft continuous and WLC - soft continuous.*

Proof. This is an immediate consequence of Theorem 3.26. □

Theorem 4.10. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. A function f_{pu} is A_R - soft continuous if and only if it is both αNA_2 - soft continuous and WLC - soft continuous.*

Proof. This is a direct consequence of Theorem 3.27. □

Theorem 4.11. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. If a function f_{pu} is αAN_2 - soft continuous, then f_{pu} is pre - soft continuous.*

Proof. It is obvious from Proposition 3.28. □

Theorem 4.12. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. A function f_{pu} is α - soft continuous if and only if it is both pre - soft continuous and αNA_3 - soft continuous.*

Proof. It is an immediate consequence of Theorem 3.29. □

Theorem 4.13. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. The following properties are equivalent:*

(1) *f_{pu} is pre - soft continuous and αNA_5 - soft continuous;*
(2) *f_{pu} is α - soft continuous;*
(3) *f_{pu} is αAN_2 - soft continuous and αNA_2 - soft continuous.*

Proof. This is obvious from Theorem 3.31. □

Theorem 4.14. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. The following properties are equivalent:*

(1) *f_{pu} is soft continuous;*
(2) *f_{pu} is α - soft continuous and A_R - soft continuous;*
(3) *f_{pu} is pre - soft continuous and A_R - soft continuous.*

Proof. It follows immediately from Proposition 3.32. □

Theorem 4.15. *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $SS(X)_A$ and $SS(Y)_B$ be two families and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. A function f_{pu} is soft continuous if and only if it is αAN_2 - soft continuous, αNA_2 - soft continuous and WLC - soft continuous.*

Proof. Clear from Theorem 3.33. □

5. Conclusions

The concepts of R - closed, A_R-, αAN_1-, αAN_2-, αNA_1-, αNA_2-, αNA_3-, αNA_4- and αNA_5- soft sets have been introduced and the notions of some soft continuities have been defined. Also, some properties of these new soft sets and these continuities has been investigated. Finally, decompositions of some soft continuities have been obtained. These concepts may be used in other topological spaces and can be defined in different forms.

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