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On soft fuzzy G_δ pre continuity in soft fuzzy topological space

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Abstract. In this paper the concept of soft fuzzy G_δ pre open set and soft fuzzy G_δ-pre continuous functions are introduced. Also the concepts soft fuzzy G_δ pre kernel, soft fuzzy G_δ-pre connectedness, soft fuzzy G_δ-pre compactness and soft fuzzy G_δ-pre normal spaces are introduced and studied.

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1. Introduction

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [8]. Fuzzy sets have applications in many fields such as information [5] and control[6]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Chang [4] introduced and developed the concept of fuzzy topological spaces. The concept of soft fuzzy topological space is introduced by Ismail U.Tiryaki [7]. G.Balasubramanian [1] introduced the concept of fuzzy G_δ set. The concept of pre open set was introduced by A.S.Bin Shahana [3].

In this paper soft fuzzy G_δ pre open set is introduced and studied. Some of its properties are discussed. Several characterizations of soft fuzzy G_δ-pre continuous functions are established. In this connection, some of their interrelations are discussed and counter examples are provided wherever necessary.

A new characterizations of soft fuzzy G_δ-pre connected, soft fuzzy G_δ-pre compact and soft fuzzy G_δ-pre normal spaces are introduced and their properties are discussed.

2. Preliminaries

Definition 2.1 ([1])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$,where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ.

Definition 2.2 ([3])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy *pre open* set if $\lambda \leq int(cl(\lambda))$. The complement of a pre open set is *pre closed*.

Definition 2.3 ([3])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy α *open* set if $\lambda \leq int(cl(int(\lambda)))$. The complement of an α open set is α *closed*.

Definition 2.4 ([2])**.** Let (X, T) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy β *open* set if $(\lambda, N) \leq cl(int(cl(\lambda)))$. The complement of a β open set is β *closed*.

Definition 2.5 ([7])**.** Let X be a set, μ be a fuzzy subset of X and M $\subseteq$ X. Then,the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by *SF(X)*.

Definition 2.6 ([7])**.** The relation $\sqsubseteq$ on SF(X) is given by $(\mu, M) \sqsubseteq (\gamma, N) \Leftrightarrow (\mu(x) < \gamma(x)) or (\mu(x) = \gamma(x) and x \notin M/N), \forall\, x \in X$ and for all $(\mu, M), (\gamma, N) \in SF(X)$.

Proposition 2.7 ([7])**.** *If* $(\mu_j, M_j)_{j\in J} \in SF(X)$,*then the family* $\{(\mu_j, M_j)|\, j \in J\}$ *has a meet,that is greatest lower bound, in* $(SF(X), \sqsubseteq)$*, denoted by*

$$\sqcap_{j\in J}(\mu_j, M_j) \text{ such that } \sqcap_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where

$\mu(x) = \bigwedge_{j\in J} \mu_j(x), \forall\, x \in X.$
$M = \bigcap_{j\in J} M_j.$

Proposition 2.8 ([7])**.** *If* $(\mu_j, M_j)_{j\in J} \in SF(X)$*, then the family* $\{(\mu_j, M_j)|j \in J\}$ *has a join,that is least upper bound, in* $(SF(X), \sqsubseteq)$*, denoted by*

$$\sqcup_{j\in J}(\mu_j, M_j) \text{ such that } \sqcup_{j\in J}(\mu_j, M_j) = (\mu, M)$$

where

$\mu(x) = \bigvee_{j\in J} \mu_j(x), \forall\, x \in X.$
$M = \bigcup_{j\in J} M_j.$

Definition 2.9 ([7])**.** Let X be a non-empty set and the soft fuzzy sets A and B be in the form,

$$A = \{(\mu, M)/\mu(x) \in I^X, \forall\, x \in X, M \subseteq X\}$$
$$B = \{(\lambda, N)/\lambda(x) \in I^X, \forall\, x \in X, N \subseteq X\}$$

Then,

(1) $A \sqsubseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall\, x \in X, M \subseteq N.$
(2) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall\, x \in X, M = N.$
(3) $A' \Leftrightarrow 1 - \mu(x), \forall\, x \in X, X \mid M.$
(4) $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall\, x \in X, M \cap N.$

(5) $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X, M \cup N$.

Definition 2.10 ([7])**.** $(0, \phi)= \{(\lambda, N)/\lambda = 0, N = \phi\}$
$(1, X)= \{(\lambda, N)/\lambda = 1, N = X\}$

Definition 2.11 ([7])**.** For $(\mu, M) \in SF(X)$ the soft fuzzy set $(\mu, M)^{'} = (1 - \mu, X|M)$ is called the complement of (μ, M).

Definition 2.12 ([7])**.** Let $x \in X$ and $S \in I$ define$x_s : X \to I$ by,

$$x_s\,(z) = \begin{cases} s & \text{if } z = x; \\ 0 & \text{otherwise.} \end{cases}$$

Then, the soft fuzzy set $(x_s, \{x\})$ is called the point of SF(X)with base x and value s.

Proposition 2.13 ([7])**.** *Let $\varphi : X \to Y$ be a point function.*
(1) The mapping $\varphi^{\rightharpoonup}$ from SF(X) to SF(Y) corresponding to the image operator of the difunction (f,F) is given by

$\varphi^{\rightharpoonup}(\mu, M)$ $=(\gamma, N)$ *where* $\gamma(y) = sup\{\mu(x)/y = \varphi(x)\}$ *and* $N = \{\varphi(x)/x \in M\}$.

(2) The mapping $\varphi^{\leftharpoonup}$from SF(Y) to SF(X) corresponding to the inverse image of the difunction (f,F) is given by

$$\varphi^{\leftharpoonup}(\gamma, N) = (\gamma \circ \varphi, \varphi^{-1}[N]).$$

Note: $\varphi^{\rightharpoonup}(\mu, M) = \varphi(\mu, M)$ *and* $\varphi^{\leftharpoonup}(\gamma, N) = \varphi^{-1}(\gamma, N)$.

Definition 2.14 ([7])**.** A subset T$\subseteq$ SF(X) is called an SF-topology on X if
(1) $(0, \phi) and (1, X) \in T$.
(2) $(\mu_j, M_j) \in T, j = 1, 2, 3, ...n \Rightarrow \sqcap_{j=1}^{n}(\mu_j, M_j) \in T$.
(3) $(\mu_j, M_j), j \in J \Rightarrow \sqcup_{j\in J}(\mu_j, M_j) \in T$. the elements of T are called soft fuzzy open, and those of $T^{'} = \{(\mu, M)/(\mu, M)^{'} \in T\}$ soft fuzzy closed.
If T is a SF-topology on X we call the pair (X, T) an *SF-topological space*(in short, *SFTS*).

Definition 2.15 ([7])**.** The closure of a soft fuzzy set (μ, M) will be denoted by $\overline{(\mu, M)}$. It is given by
$\overline{(\mu, M)} = \sqcap\{(\gamma, N)/(\mu, M) \sqsubseteq (\gamma, N),\ (\gamma, N) \in T^{'}\}$
Likewise the interior is given by
$(\mu, M)^\circ = \sqcup\{(\gamma, N)/(\gamma, N) \in T, (\gamma, N) \sqsubseteq (\mu, M)\}$
Note: $\overline{(\mu, M)} = cl(\mu, M)$ and $(\mu, M)^\circ = int(\mu, M)$.

Definition 2.16 ([7])**.** A soft fuzzy topological space (X, T) is said to be a *soft fuzzy compact* if whenever $\sqcup_{i\in I}(\lambda_i, M_i)$ = (1,X), $(\lambda_i, M_i) \in$ T, $i \in I$, there is a finite subset J of I with $\sqcup_{j\in J}(\lambda_j, M_j)$ = (1,X).

3. Soft fuzzy G_δ pre open sets and its basic properties

Definition 3.1. Let (X, T) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be *soft fuzzy G_δ set* if $(\lambda, N) = \sqcap_{i=1}^{\infty}(\mu_i, M_i)$,where each (μ_i, M_i) is soft fuzzy open set. The complement of a soft fuzzy G_δ set is soft fuzzy F_σ.

Definition 3.2. Let (X, T) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be *soft fuzzy pre open set* if $(\lambda, N) \sqsubseteq int(cl(\lambda, N))$. The complement of a soft fuzzy pre open set is soft fuzzy pre closed.

Definition 3.3. Let (X, T) be a soft fuzzy topological space.Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be *soft fuzzy G_δ pre open set* if $(\lambda, N) = (\mu, M) \sqcap (\gamma, L)$,where (μ, M) is soft fuzzy G_δ set and $(\gamma, L)$is soft fuzzy pre open set. The complement of a soft fuzzy G_δ pre open set is soft fuzzy F_σ pre closed.

Remark 3.4.
(i) Arbitrary union of soft fuzzy G_δ sets is soft fuzzy G_δ.
(ii) Arbitrary union of soft fuzzy pre open sets is soft fuzzy pre open.

Proposition 3.5. *Arbitrary union of soft fuzzy G_δ pre open sets is soft fuzzy G_δ pre open.*

Proof. Let (μ_i, M_i)be soft fuzzy G_δ sets and let (γ_i, N_i) be soft fuzzy pre open sets. Let (λ_i, P_i)be the family of soft fuzzy G_δ pre open sets.Therefore $(\lambda_i, P_i) = (\mu_i, M_i) \sqcap (\gamma_i, N_i)$. Now

$$\begin{aligned}\sqcup_{i=1}^{\infty}(\lambda_i, P_i) &= \sqcup_{i=1}^{\infty}[(\mu_i, M_i) \sqcap (\gamma_i, N_i)]\\ &= \sqcup_{i=1}^{\infty}(\mu_i, M_i) \sqcap \sqcup_{i=1}^{\infty}(\gamma_i, N_i)\end{aligned}$$

Since arbitrary union of soft fuzzy G_δ sets is soft fuzzy G_δ and arbitrary union of soft fuzzy pre open sets is soft fuzzy pre open. Therefore $\sqcup_{i=1}^{\infty}(\mu_i, M_i)$ is a soft fuzzy G_δ set and $\sqcup_{i=1}^{\infty}(\gamma_i, N_i)$ is a soft fuzzy pre open set. Thus $\sqcup_{i=1}^{\infty}(\mu_i, M_i) \sqcap \sqcup_{i=1}^{\infty}(\gamma_i, N_i)$ is soft fuzzy G_δ pre open set. Hence $\sqcup_{i=1}^{\infty}(\lambda_i, P_i)$ is soft fuzzy G_δ pre open. Hence the proof. □

Remark 3.6.
(i) Arbitrary intersection of soft fuzzy F_σ sets is soft fuzzy F_σ.
(ii) Arbitrary intersection of soft fuzzy pre closed sets is soft fuzzy pre closed.

Proposition 3.7. *Arbitrary intersection of soft fuzzy F_σ pre closed sets is soft fuzzy F_σ pre closed.*

Proof. Let (μ_i, M_i)be soft fuzzy F_σ sets and let (γ_i, N_i) be soft fuzzy pre closed sets. Let (λ_i, P_i)be the family of soft fuzzy F_σ pre closed sets. Therefore $(\lambda_i, P_i) = (\mu_i, M_i) \sqcup (\gamma_i, N_i)$. Now

$$\begin{aligned}\sqcap_{i=1}^{\infty}(\lambda_i, P_i) &= \sqcap_{i=1}^{\infty}[(\mu_i, M_i) \sqcup (\gamma_i, N_i)]\\ &= \sqcap_{i=1}^{\infty}(\mu_i, M_i) \sqcup \sqcap_{i=1}^{\infty}(\gamma_i, N_i)\end{aligned}$$

Since arbitrary intersection of soft fuzzy F_σ sets is soft fuzzy F_σ and arbitrary intersection of soft fuzzy pre closed sets is soft fuzzy pre closed. Therefore $\sqcap_{i=1}^{\infty}(\mu_i, M_i)$ is a soft fuzzy F_σ set and $\sqcap_{i=1}^{\infty}(\gamma_i, N_i)$ is a soft fuzzy pre closed set. Thus $\sqcap_{i=1}^{\infty}(\mu_i, M_i) \sqcup \sqcap_{i=1}^{\infty}(\gamma_i, N_i)$ is soft fuzzy F_σ pre closed set. Hence $\sqcap_{i=1}^{\infty}(\lambda_i, P_i)$ is soft fuzzy F_σ pre closed. □

Definition 3.8. Let (X, T) be any soft fuzzy topological space and let (λ, M) be any soft fuzzy set in (X, T). Then *soft fuzzy G_δ pre interior* of (λ, M) is defined as follows

SF G_δ pre int$(\lambda, M)= \sqcup\{(\mu, N)/(\mu, N)$ is SF G_δ pre open and $(\mu, N) \sqsubseteq (\lambda, M)\}$

Definition 3.9. Let (X, T) be any soft fuzzy topological space and let $(x_r, \{x\})$ be any soft fuzzy point in (X, T). Then soft fuzzy G_δ pre interior of $(x_r, \{x\})$ is defined as follows

$$\text{SF } G_\delta \text{ pre int}(x_r, \{x\})= \sqcup\{(\mu, N)/(\mu, N) \text{ is SF } G_\delta \text{ pre open and } (\mu, N) \sqsubseteq (x_r, \{x\})\}$$

Proposition 3.10. *Let (X, T) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Then SF G_δ pre int (λ, N) is a soft fuzzy G_δ pre open set in (X, T).*

Proof. It is easy to prove from the definition of SF G_δ pre interior of a soft fuzzy set. □

Proposition 3.11. *Let (X, T) be any soft fuzzy topological space and $(\lambda, M), (\mu, N)$ be soft fuzzy sets in (X, T). Then the following properties hold:*
(i) SF G_δ pre int$(\lambda, M) \sqsubseteq (\lambda, M)$.
(ii) $(\lambda, M) \sqsubseteq (\mu, N) \Rightarrow$ SF G_δ pre int$(\lambda, M) \sqsubseteq$ SF G_δ pre int(μ, N).
(iii) SF G_δ pre int(SF G_δ pre int$(\lambda, M))=$ SF G_δ pre int(λ, M).
(iv) SF G_δ pre int$((\lambda, M) \sqcap (\mu, N)) \sqsubseteq$ SF G_δ pre int$(\lambda, M)\sqcap$ SF G_δ pre int(μ, N).
(v) SF G_δ pre int$(1, X) = (1, X)$.

Proof. Proof is obvious. □

Definition 3.12. Let (X, T) be any soft fuzzy topological space and let (λ, M) be any soft fuzzy set in (X, T). Then *soft fuzzy F_σ pre closure* of (λ, M) is defined as follows

SF F_σ pre cl$(\lambda, M)= \sqcap\{(\mu, N)/(\mu, N)$ is SF F_σ pre closed and $(\mu, N) \sqsupseteq (\lambda, M)\}$

Definition 3.13. Let (X, T) be any soft fuzzy topological space and let $(x_r, \{x\})$ be any soft fuzzy point in (X, T). Then soft fuzzy F_σ pre closure of $(x_r, \{x\})$ is defined as follows

$$\text{SF } F_\sigma \text{ pre cl}(x_r, \{x\})= \sqcap\{(\mu, N)/(\mu, N) \text{ is SF } F_\sigma \text{ pre closed and } (\mu, N) \sqsupseteq (x_r, \{x\})\}$$

Proposition 3.14. *Let (X, T) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Then SF F_σ pre cl (λ, N) is a soft fuzzy F_σ pre closed set in (X, T).*

Proof. It is easy to prove from the definition of SF F_σ pre closure of a soft fuzzy set. □

Proposition 3.15. *Let (X, T) be any soft fuzzy topological space and $(\lambda, M), (\mu, N)$ be soft fuzzy sets in (X, T). Then the following properties hold:*
(i) $(\lambda, M) \sqsubseteq$ SF $F_\sigma precl(\lambda, M)$.
(ii) $(\lambda, M) \sqsubseteq (\mu, N) \Rightarrow$ SF $F_\sigma precl(\lambda, M) \sqsubseteq$ SF $F_\sigma precl(\mu, N)$.
(iii) SF $F_\sigma precl(SFF_\sigma precl(\lambda, M)) = SFF_\sigma precl(\lambda, M)$.
(iv) SF $F_\sigma precl((\lambda, M) \sqcup (\mu, N))=$ SF $F_\sigma precl(\lambda, M)\sqcup$ SF $F_\sigma precl(\mu, N)$.
(v) SF $F_\sigma precl(0, \phi) = (0, \phi)$.

Proof. Proof is obvious. □

Proposition 3.16. *For any soft fuzzy set (λ, M) in a soft fuzzy topological space (X,T) the following hold:*
(i) SF F_σ pre cl$((1,X)-(\lambda,M))$=$(1,X)$- SF G_δ pre int(λ,M).
(ii)SF G_δ pre cl$((1,X)-(\lambda,M))$=$(1,X)$- SF F_σ pre cl(λ,M).

Proof. Proof is simple. □

Definition 3.17. Let (X,T) be a soft fuzzy topological space. Let (λ,N) be a soft fuzzy set in (X,T). Then (λ,N) is said to be *soft fuzzy regular G_δ pre open* if (λ,N) = SF G_δ pre int (SF F_σ pre cl (λ,N)).

Definition 3.18. Let (X,T) be a soft fuzzy topological space. Let (λ,N) be a soft fuzzy set in (X,T). Then (λ,N) is said to be *soft fuzzy regular F_σ pre closed* if (λ,N) = SF F_σ pre cl (SF G_δ pre int (λ,N)).

Proposition 3.19.
(i) The soft fuzzy F_σ pre closure of a soft fuzzy G_δ pre open set is soft fuzzy regular F_σ pre closed.
(ii) The soft fuzzy G_δ pre interior of a soft fuzzy F_σ pre closed set is soft fuzzy regular G_δ pre open.

Proof.
(i) Let (λ,M) be a soft fuzzy G_δ pre open set of a soft fuzzy topological space (X,T).

$$\text{Therefore SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}(\lambda,M)) \sqsubseteq \text{SF } F_\sigma \text{ pre cl}(\lambda,M).$$

$$\begin{aligned} SFF_\sigma precl(SFG_\delta preint(SFF_\sigma precl(\lambda,M))) &\sqsubseteq SFF_\sigma precl(SFF_\sigma precl(\lambda,M)) \\ &\sqsubseteq SFF_\sigma precl(\lambda,M). \end{aligned}$$

Now (λ,M) is soft fuzzy G_δ pre open set.

$$\begin{aligned} (\lambda,M) &\sqsubseteq SFF_\sigma precl(\lambda,M) \\ SFG_\delta preint(\lambda,M) &\sqsubseteq SFG_\delta preint(SFF_\sigma precl(\lambda,M)) \\ \Rightarrow (\lambda,M) &\sqsubseteq SFG_\delta preint(SFF_\sigma precl(\lambda,M)) \\ SFF_\sigma precl(\lambda,M) &\sqsubseteq SFF_\sigma precl(SFG_\delta preint(SFF_\sigma precl(\lambda,M)) \end{aligned}$$

$$\text{Hence SF } F_\sigma \text{ pre cl}(\lambda,M)\text{= SF } F_\sigma \text{ pre cl(SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl}(\lambda,M))).$$

(ii) Let (λ,M) be a soft fuzzy F_σ pre closed set of a soft fuzzy topological space (X,T).

$$\text{Therefore SF } F_\sigma \text{ pre cl(SF } G_\delta \text{ pre int}(\lambda,M)) \sqsupseteq \text{SF}G_\delta \text{ pre int}(\lambda,M).$$

$$\text{SF } G_\delta \text{ pre int}(SFF_\sigma \text{ pre cl(SF } G_\delta \text{ pre int}(\lambda,M))) \sqsupseteq \text{SF } G_\delta \text{ pre int(SF } G_\delta \text{ pre int}(\lambda,M)).$$

$$\text{SF } G_\delta \text{ pre int(SF } F_\sigma \text{ pre cl(SF } G_\delta \text{ pre int}(\lambda,M))) \sqsupseteq SFG_\delta \text{ pre int}(\lambda,M).$$

Now (λ, M) is soft fuzzy F_σ pre closed set.

$$\begin{aligned}(\lambda, M) &\sqsupseteq SFG_\delta preint(\lambda, M)\\ SFF_\sigma precl(\lambda, M) &\sqsupseteq SFF_\sigma precl(SFG_\delta preint(\lambda, M))\\ (\lambda, M) &\sqsupseteq SFF_\sigma precl(SFG_\delta preint(\lambda, M))\\ SFG_\delta preint(\lambda, M) &\sqsupseteq SFG_\delta preint(SFF_\sigma precl(SFG_\delta preint(\lambda, M))).\end{aligned}$$

Hence SF G_δ pre int(λ, M)= SF G_δ pre int(SF $F_\sigma$pre cl(G_δ pre int(λ, M))). □

Definition 3.20. Let (X, T) be a Soft fuzzy topological space and (λ, M) be a Soft fuzzy set in (X, T). Then, the *kernel* (in short, Ker) of (λ, M) is defined as,

$$\text{Ker}(\lambda, M)=\sqcap\{(\mu, N)/(\mu, N) \text{ is a soft fuzzy open set and } (\lambda, M) \sqsubseteq (\mu, N)\}$$

Definition 3.21. A soft fuzzy set (λ, N) is called as a *Soft fuzzy quasicoincident* with a soft fuzzy set (μ, M) ,denoted by $(\lambda, N)q(\mu, M)$, if $\lambda(x) + \mu(x) \geq 1$,for some x∈ X and $M \cap N \neq \phi$.

Definition 3.22. A soft fuzzy set (λ, N) is called as a *Soft fuzzy non-quasicoincident* with a soft fuzzy set (μ, M) ,denoted by $(\lambda, N)\urcorner q(\mu, M)$, if $\lambda(x) + \mu(x) < 1$,for all x∈ X and $M \cap N = \phi$.

Proposition 3.23. *Let A,B be the soft fuzzy sets of X. Then* $A\urcorner qB \Leftrightarrow A \sqsubseteq (1-B)$.

Definition 3.24. Let (X, T) be any soft fuzzy topological space and let (λ, M) be any soft fuzzy set in (X, T). Then soft fuzzy G_δ pre kernel of (λ, M) is defined as follows

$$\text{SF } G_\delta \text{ pre ker}(\lambda, M)= \sqcap\{(\mu, N)/(\mu, N) \text{ is SF } G_\delta \text{ pre open and } (\mu, N) \sqsupseteq (\lambda, M)\}$$

Proposition 3.25. *Let* (λ, N) *be a soft fuzzy set in* (X, T)*. Let* (μ, M) *be a soft fuzzy* F_σ *pre closed set in* (X, T)*. Then* (μ, M) *q SF* G_δ *pre ker* $(\lambda, N) \Leftrightarrow (\mu, M)$ *q* (λ, N)*.*

Proof. Let $(\mu, M)\urcorner q(\lambda, N)$. Then $(\lambda, N) \sqsubseteq (1, X) - (\mu, M)$.

Now SF G_δ pre ker $(\lambda, N) \sqsubseteq$ SF G_δ pre ker $((1, X) - (\mu, M))$.
Since $((1, X) - (\mu, M))$ is soft fuzzy G_δ pre open set in (X, T).
Then SF G_δ pre ker $(\lambda, N) \sqsubseteq ((1, X) - (\mu, M))$.

Which implies SF G_δ pre ker $(\lambda, N)\urcorner q(\mu, M)$.
Conversely, SF G_δ pre ker $(\lambda, N)\urcorner q(\mu, M)$.

This implies that SF G_δ pre ker $(\lambda, N) \sqsubseteq ((1, X) - (\mu, M))$.
Now $(\lambda, N) \sqsubseteq$ SFG_δ pre ker $(\lambda, N) \sqsubseteq ((1, X) - (\mu, M))$.
It follows that $(\mu, M)\urcorner q(\lambda, N)$.

□

4. Interrelations of soft fuzzy G_δ pre open sets among various soft fuzzy sets

Definition 4.1. Let (X, T) be a soft fuzzy topological space. Let (λ, N) be a soft fuzzy set in (X, T). Then (λ, N) is said to be *Soft fuzzy α open (β open)* if $(\lambda, N) \sqsubseteq int(cl(int(\lambda, N)))[(\lambda, N) \sqsubseteq cl(int(cl(\lambda, N)))]$. The complement of a soft fuzzy α open (β open) set is soft fuzzy α closed(β closed).

Definition 4.2. Let (X, T) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Then (λ, N) is said to be *soft fuzzy $G_\delta{}^*$ open set* if (λ, N)=$(\mu, M)\sqcap(\gamma, L)$,where (μ, M) is soft fuzzy G_δ set and (γ, L) is soft fuzzy open set.

Definition 4.3. Let (X, T) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Then (λ, N) is said to be *soft fuzzy $G_\delta\alpha$ open set* if (λ, N)=$(\mu, M) \sqcap (\gamma, L)$,where (μ, M) is soft fuzzy G_δ set and $(\gamma, L)$is soft fuzzy α open set.

Definition 4.4. Let (X, T) be a soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Then (λ, N) is said to be *soft fuzzy $G_\delta\beta$ open set* if (λ, N)=$(\mu, M) \sqcap (\gamma, L)$,where (μ, M) is soft fuzzy G_δ set and (γ, L) is soft fuzzy β open set.

Remark 4.5. Every soft fuzzy open set is soft fuzzy α open set.

Proposition 4.6. *Every soft fuzzy $G_\delta{}^*$ set is soft fuzzy $G_\delta\alpha$ open.*

Proof. Let (X, T) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Assume that (λ, N) is soft fuzzy $G_\delta{}^*$. That is $(\lambda, N) = (\mu, M)\sqcap(\gamma, P)$, where (μ, M) is soft fuzzy G_δ set and (γ, P) is soft fuzzy open set. Since every soft fuzzy open set is soft fuzzy α open set. Thus (γ, P) is soft fuzzy α open set. Hence (λ, N) is soft fuzzy $G_\delta\alpha$ open set. □

Remark 4.7. The converse of the above property need not be true as shown in the following example.

Example 4.8. Let $X = \{a, b, c, d\}$, $T = \{(0, \phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0,1]$ for i = 1, 2, 3, 4, 5 and $M_i \subseteq X$, for i = 1, 2, 3, 4, 5 are defined as follows $\lambda_1(a) = 0$, $\lambda_1(b) = 0.4$, $\lambda_1(c) = 0$, $\lambda_1(d) = 0.3$; $\lambda_2(a) = 0.7$, $\lambda_2(b) = 0$, $\lambda_2(c) = 0.8$, $\lambda_2(d) = 0$; $\lambda_3(a) = 0.7$, $\lambda_3(b) = 0.4$, $\lambda_3(c) = 0.8$, $\lambda_3(d) = 0.3$; $\lambda_4(a) = 0.7$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.8$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.4$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.3$; $M_1 = \{b\}$, $M_2 = \{c\}$, $M_3 = \{b, c\}$, $M_4 = \{a, b, c\}$, $M_5 = \{b, c, d\}$. Then $(X,\ T)$ is a soft fuzzy topological space. Consider the soft fuzzy set (λ, M) where $\lambda : X \to [0,1]$ and $M \subset X$ are defined as $\lambda(a) = 1$, $\lambda(b) = 0.5$, $\lambda(c) = 1$, $\lambda(d) = 0.5$ and $M = \{a, b, c\}$.

$$\begin{aligned}\text{Now, } int(cl(int(\lambda, M))) &= int(cl(\lambda_3, M_3)) \\ &= int(1, X) \\ &= (1, X)\end{aligned}$$

Thus $int(cl(int(\lambda, M))) \sqsupseteq (\lambda, M)$. Therefore (λ, M) is a soft fuzzy α open set. Consider the soft fuzzy G_δ set (λ_5, M_5). Now, $(\lambda_5, M_5) \sqcap (\lambda, M) = (\delta, M_3)$. Where

(δ, M_3) is defined as $\delta(a) = 1$, $\delta(b) = 0.4$, $\delta(c) = 1$, $\delta(d) = 0.3$ and $M_3 = \{b, c\}$. Therefore (δ, M_3) *is a soft fuzzy* $G_\delta\alpha$ *open set.* But (δ, M_3) *is not a soft fuzzy* ${G_\delta}^*$ *set.*

Remark 4.9. Every soft fuzzy α open set is soft fuzzy pre open set.

Proposition 4.10. *Every soft fuzzy* $G_\delta\alpha$ *open set is soft fuzzy* G_δ *pre open set.*

Proof. Let (X, T) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Assume that (λ, N) is soft fuzzy $G_\delta\alpha$ open set. That is $(\lambda, N) = (\mu, M) \sqcap (\gamma, P)$, where (μ, M) is soft fuzzy G_δ set and (γ, P) is soft fuzzy α open set.

$$\begin{aligned} Now, (\gamma, P) &\sqsubseteq int(cl(int(\gamma, P))) \\ &\sqsubseteq int(cl(\gamma, P)) \end{aligned}$$

Therefore $(\gamma, P) \sqsubseteq \text{int}(\text{cl}(\gamma, P))$, which implies (γ, P) is soft fuzzy pre open set. Hence (λ, N) is soft fuzzy G_δ pre open set. □

Remark 4.11. The converse of the above property need not be true as shown in the following example.

Example 4.12. Let $X = \{a, b, c, d\}$, $T = \{(0, \phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0,1]$ for $i = 1, 2, 3, 4, 5$ and $M_i \subseteq X$, for $i = 1, 2, 3, 4, 5$ are defined as follows $\lambda_1(a) = 0$, $\lambda_1(b) = 0.4$, $\lambda_1(c) = 0$, $\lambda_1(d) = 0.3$; $\lambda_2(a) = 0.7$, $\lambda_2(b) = 0$, $\lambda_2(c) = 0.8$, $\lambda_2(d) = 0$; $\lambda_3(a) = 0.7$, $\lambda_3(b) = 0.4$, $\lambda_3(c) = 0.8$, $\lambda_3(d) = 0.3$; $\lambda_4(a) = 0.7$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.8$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.4$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.3$; $M_1 = \{b\}$, $M_2 = \{c\}$, $M_3 = \{b, c\}$, $M_4 = \{a, b, c\}$, $M_5 = \{b, c, d\}$. Then $(X,\ T)$ is a soft fuzzy topological space. Consider the soft fuzzy set (λ, M) where $\lambda : X \to [0,1]$ and $M \subset X$ are defined as $\lambda(a) = 0.4$, $\lambda(b) = 0$, $\lambda(c) = 0.7$, $\lambda(d) = 0$ and $M = \{c\}$.

$$\begin{aligned} \text{Now, } int(cl(\lambda, M)) &= int((\lambda_1, M_1)^{'}) \\ &= (\lambda_2, M_2) \\ &\sqsupseteq (\lambda, M) \\ \text{Therefore, } int(cl(\lambda, M)) &\sqsupseteq (\lambda, M). \end{aligned}$$

Thus (λ, M) is a soft fuzzy pre open set. Consider the soft fuzzy G_δ set (λ_2, M_2). Now $(\lambda_2, M_2) \sqcap (\lambda, M) = (\lambda, M)$ is a soft fuzzy $G_\delta$pre open set. But (λ, M) is not soft fuzzy α open.

$$\begin{aligned} int(cl(int(\lambda, M))) &= int(cl(0, \phi)) \\ &= int(0, \phi) \\ &= (0, \phi) \\ int(cl(int(\lambda, M))) &\not\sqsupseteq (\lambda, M) \end{aligned}$$

Therefore, (λ, M) is not a soft fuzzy α open set. *Hence* (λ, M) *is a soft fuzzy* G_δ*pre open set and it is not a soft fuzzy* $G_\delta\alpha$ *open set.*

Remark 4.13. Every soft fuzzy pre open set is soft fuzzy β open set.

Proposition 4.14. *Every soft fuzzy* G_δ *pre open set is soft fuzzy* $G_\delta\beta$ *open set.*

Proof. Let (X, T) be any soft fuzzy topological space. Let (λ, N) be any soft fuzzy set in (X, T). Assume that (λ, N) is soft fuzzy G_δ pre open set. That is $(\lambda, N) = (\mu, M) \sqcap (\gamma, P)$, where (μ, M) is soft fuzzy G_δ set and (γ, P) is soft fuzzy pre open set.

$$\begin{aligned} Now, (\gamma, P) &\sqsubseteq int(cl(\gamma, P)) \\ &\sqsubseteq cl(int(cl(\gamma, P))) \end{aligned}$$

Therefore (γ, P) is soft fuzzy β open set. Hence (λ, N) is soft fuzzy $G_\delta\beta$ open set. □

Remark 4.15. The converse of the above property need not be true as shown in the following example.

Example 4.16. Let $X = \{a, b, c, d\}$, $T = \{(0, \phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0,1]$ for i = 1, 2, 3, 4, 5 and $M_i \subseteq X$, for i = 1, 2, 3, 4, 5 are defined as follows $\lambda_1(a) = 0.6$, $\lambda_1(b) = 0$, $\lambda_1(c) = 0.2$, $\lambda_1(d) = 0$; $\lambda_2(a) = 0$, $\lambda_2(b) = 0.5$, $\lambda_2(c) = 0$, $\lambda_2(d) = 0.1$; $\lambda_3(a) = 0.6$, $\lambda_3(b) = 0.5$, $\lambda_3(c) = 0.2$, $\lambda_3(d) = 0.1$; $\lambda_4(a) = 0.6$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.2$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.5$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.1$; $M_1 = \{a\}$, $M_2 = \{c\}$, $M_3 = \{a, c\}$, $M_4 = \{a, d, c\}$, $M_5 = \{a, b, c\}$. Then $(X,\ T)$ is a soft fuzzy topological space. Consider the soft fuzzy set (λ, M) where $\lambda : X \to [0,1]$ and $M \subset X$ are defined as $\lambda(a) = 0.4$, $\lambda(b) = 0.5$, $\lambda(c) = 0.2$, $\lambda(d) = 0.3$ and $M = \{c\}$.

$$\begin{aligned} \text{Now, } cl(int(cl(\lambda, M))) &= cl(int(\lambda_1, M_1)^{'}) \\ &= cl(\lambda_2, M_2) \\ &= (\lambda_1, M_1)^{'} \\ \text{Therefore, } cl(int(cl(\lambda, M))) &\sqsupseteq (\lambda, M). \end{aligned}$$

Thus (λ, M) is soft fuzzy β open. Consider the soft fuzzy G_δ set (λ_4, M_4). Now $(\lambda_4, M_4) \sqcap (\lambda, M) = (\lambda, M)$ is a soft fuzzy $G_\delta\beta$ open set. But (λ, M) is not soft fuzzy pre open.

$$\begin{aligned} int(cl(\lambda, M)) &= int((\lambda_1, M_1)^{'}) \\ &= (\lambda_2, M_2) \\ int(cl(\lambda, M)) &\not\sqsupseteq (\lambda, M) \end{aligned}$$

Thus (λ, M) *is a soft fuzzy* $G_\delta\beta$ *open set and it is not a soft fuzzy* G_δ*pre open set.*

Remark 4.17. From the results obtained above the following implications are obtained.

$$\begin{array}{c} soft fuzzy G_\delta^* \\ \downarrow \\ soft fuzzy G_\delta \alpha open set \\ \downarrow \\ soft fuzzy G_\delta pre open set \\ \downarrow \\ soft fuzzy G_\delta \beta open set \end{array}$$

5. Soft fuzzy G_δ-pre continuous functions and their interrelations among various soft fuzzy continuous functions

Definition 5.1. Let (X,T) and (Y,S) be any two soft fuzzy topological spaces. A function f :(X,T)→ (Y,S) is said to be *soft fuzzy G_δ-pre continuous*, if the inverse image of every soft fuzzy open set in (Y,S) is soft fuzzy G_δ pre open in (X,T).

Proposition 5.2. *Let (X,T) and (Y,S) be any two soft fuzzy topological spaces. For a function f :(X,T)→ (Y,S), the following are equivalent.*
(i) f is soft fuzzy G_δ-pre continuous.
(ii) The inverse image of every soft fuzzy closed set in(Y,S) is soft fuzzy F_σ pre closed in (X,T).

Proposition 5.3. *Let (X,T) and (Y,S) be any two soft fuzzy topological spaces. For a function f :(X,T)→ (Y,S), the following are equivalent.*
(i) f is soft fuzzy G_δ-pre continuous.
(ii) For each $(\lambda,M) \in$ (Y,S), f^{-1} $(int(\lambda,M)) \sqsubseteq SFG_\delta$ pre $int(f^{-1}(\lambda,M))$.
(iii) For each $(\lambda,M) \in$(Y,S), SF F_σ pre $cl(f^{-1}(\lambda,M)) \sqsubseteq f^{-1}(cl(\lambda,M))$.

Proof.
(i)⇒(ii) Assume that f is soft fuzzy G_δ-pre continuous. Let (λ,M) be any soft fuzzy set in (Y,S). int(λ,M) is soft fuzzy open set in (Y,S). Since f is soft fuzzy G_δ-pre continuous, $f^{-1}(int(\lambda,M))$ is soft G_δ pre open in (X,T).
Since $f^{-1}(int(\lambda,M)) \sqsubseteq f^{-1}(\lambda,M)$.

$$\text{SF } G_\delta \text{ pre int}(f^{-1}(int(\lambda,M))) \sqsubseteq \text{SF}G_\delta \text{ pre int}(f^{-1}(\lambda,M)).$$

Which implies $f^{-1}(int(\lambda,M)) \sqsubseteq$ SFG_δ pre int$(f^{-1}(\lambda,M))$.
(ii)⇒(iii) For each soft fuzzy set $(\lambda,M) \in$(Y,S),
f^{-1} (int$(\lambda,M)) \sqsubseteq$ SFG_δ pre int$(f^{-1}(\lambda,M))$.

$$(1,X) - f^{-1}(int(\lambda,M)) \sqsupseteq (1,X) - SFG_\delta preint(f^{-1}(\lambda,M))$$
$$f^{-1}(1,Y) - f^{-1}(int(\lambda,M)) \sqsupseteq SFF_\sigma precl((1,X) - f^{-1}(\lambda,M))$$
$$f^{-1}((1,Y) - int(\lambda,M)) \sqsupseteq SFF_\sigma precl(f^{-1}(1,Y) - f^{-1}(\lambda,M))$$

$f^{-1}(cl((1,Y) - (\lambda,M))) \sqsupseteq SFF_\sigma precl(f^{-1}((1,Y) - (\lambda,M)))$ for each soft fuzzy set $((1,Y) - (\lambda,M)) \in (Y,S)$.

(iii)⇒(i) Assume that for each soft fuzzy set (λ,M) in (Y,S)
SF F_σ pre cl$(f^{-1}(\lambda,M)) \sqsubseteq f^{-1}(cl(\lambda,M))$. Let (λ,M) be any soft fuzzy closed set in(Y,S). That is $cl(\lambda,M) = (\lambda,M)$.Now SF F_σ pre cl$(f^{-1}(\lambda,M)) \sqsubseteq f^{-1}(\lambda,M)$.
But $f^{-1}(\lambda,M) \sqsubseteq SFF_\sigma$ pre cl$(f^{-1}(\lambda,M))$.

$$\text{Therefore } f^{-1}(\lambda,M) = F_\sigma \text{ pre cl}(f^{-1}(\lambda,M)).$$

Hence f is soft fuzzy G_δ-pre continuous. □

Proposition 5.4. *Let (X,T) and (Y,S) be any two soft fuzzy topological spaces. For a bijective function f :(X,T)→ (Y,S), the following are equivalent.*
(i) f is soft fuzzy G_δ-pre continuous.
(ii)For each $(\lambda,M) \in (X,T)$, $f(SFG_\delta$ pre $int(\lambda,M)) \sqsupseteq int(f(\lambda,M))$.

(iii) For each $(\lambda, M) \in (X, T)$,$f(SFF_\sigma$ *pre* $cl(\lambda, M)) \sqsubseteq cl(f(\lambda, M))$.

Proof.
(i)⇒(ii) Assume that f is soft fuzzy G_δ-pre continuous. Let (λ, M) be any soft fuzzy closed set in (X, T). Then $f(\lambda, M)$ is a soft fuzzy set in (Y,S). Now int$(f(\lambda, M))$ is a soft fuzzy open set in (Y,S). By assumption $f^{-1}(\text{int}(f(\lambda, M)))$ is a soft fuzzy G_δ pre open set in (X, T). We know that $\text{int}(f(\lambda, M)) \sqsubseteq f(\lambda, M)$. Since f is bijective, $f^{-1}(\text{int}(f(\lambda, M))) \sqsubseteq f^{-1}(f(\lambda, M)) = (\lambda, M)$.

$$\Rightarrow SFG_\delta \text{ pre int}(f^{-1}(int(f(\lambda, M)))) \sqsubseteq SFG_\delta \text{ pre int}(\lambda, M).$$

By assumption, $f^{-1}(int(f(\lambda, M))) = SFG_\delta$ pre int$(f^{-1}(int(f(\lambda, M))))$.

$$\text{Therefore } f^{-1}(int(f(\lambda, M))) \sqsubseteq SFG_\delta \text{ pre int } (\lambda, M).$$

Thus $int(f(\lambda, M)) \sqsubseteq f(SFG_\delta$ pre int $(\lambda, M))$.
(ii)⇒(iii) For each soft fuzzy set $(\lambda, M) \in$(X,T), $f(SFG_\delta$ pre int$(\lambda, M)) \sqsupseteq \text{int}(f(\lambda, M))$.

$$(1, Y) - f(SFG_\delta preint(\lambda, M)) \sqsubseteq (1, Y) - int(f(\lambda, M))$$
$$f(1, X) - f(SFG_\delta preint(\lambda, M)) \sqsubseteq cl((1, Y) - f(\lambda, M))$$
$$f((1, X) - SFG_\delta preint(\lambda, M)) \sqsubseteq cl(f((1, X) - (\lambda, M)))$$

$$f(SFF_\sigma \text{ pre cl}((1, X) - (\lambda, M))) \sqsubseteq cl(f((1, X) - (\lambda, M))) \text{ for each soft fuzzy set } ((1, X) - (\lambda, M)) \in (X, T).$$

(iii)⇒(i) Assume that for each soft fuzzy set (λ, M) in (X, T),
$f(SFF_\sigma$ pre cl$(\lambda, M)) \sqsubseteq cl(f(\lambda, M))$. Let (λ, M) be any soft fuzzy closed set in(Y,S). That is $cl(\lambda, M) = (\lambda, M)$. By assumption $f(SFF_\sigma$ pre cl$(f^{-1}(\lambda, M))) \sqsubseteq cl(f(f^{-1}(\lambda, M)))$. Thus SF F_σ pre cl$(f^{-1}(\lambda, M)) \sqsubseteq f^{-1}(\lambda, M)$. But $f^{-1}(\lambda, M) \sqsubseteq SFF_\sigma$ pre cl$(f^{-1}(\lambda, M))$.

$$\text{Therefore } f^{-1}(\lambda, M) = SFF_\sigma precl(f^{-1}(\lambda, M)).$$

Hence f is soft fuzzy G_δ-pre continuous. □

Theorem 5.5. *Let* (X, T),*(Y,S) and(Z,R) be any three soft fuzzy topological spaces. A function* f :*(X,T)*→ (Y, S) *be soft fuzzy* G_δ*-pre continuous and* $g : (Y, S) \to (Z, R)$ *be soft fuzzy continuous function. Then* gof :*(X,T)*→ (Z, R) *is soft fuzzy* G_δ*-pre continuous.*

Proposition 5.6. *Let* (X, T) *and (Y,S) be any two soft fuzzy topological spaces. Let* f : (X, T)→ *(Y,S) be soft fuzzy* G_δ*-pre continuous function then the following hold.*
(i) f*(SF* G_δ *pre int* $(\lambda, N)) \sqsubseteq ker(f(\lambda, N))$, *for every soft fuzzy subset* (λ, N) *of* (X, T).
(ii) SF G_δ *pre int* $(f^{-1}(\mu, M)) \sqsubseteq f^{-1}(ker(\mu, M))$,*for every soft fuzzy subset* (μ, M) *of (Y,S).*

Proof.

(i) Let (λ, N) be any soft fuzzy set in (X, T). Then ker$(f(\lambda, N))$ is a soft fuzzy open set in (Y,S). By assumption, f^{-1} (ker$(f(\lambda, N))$) is a soft fuzzy G_δ pre open set in (X, T).

$$
\begin{aligned}
Now SFG_\delta preint(\lambda, N) &\sqsubseteq SFG_\delta preint(f^{-1}(f(\lambda, N))) \\
&\sqsubseteq SFG_\delta preint(f^{-1}(ker(f(\lambda, N)))) \\
&\sqsubseteq f^{-1}(ker(f(\lambda, N)))
\end{aligned}
$$

Hence $f(SFG_\delta$ pre int $(\lambda, N)) \sqsubseteq$ ker$(f(\lambda, N))$, for any soft fuzzy set (λ, N) in (X, T).
(ii) Let (μ, M) be any soft fuzzy set in (Y,S). Then $ker(\mu, M))$ is a soft fuzzy open set in (Y,S). By assumption,f^{-1}(ker(μ, M))) is a soft fuzzy G_δ pre open set in (X, T).

Now $f^{-1}(\mu, M) \sqsubseteq f^{-1}$(ker$(\mu, M)$).
Thus SF G_δ pre int $(f^{-1}(\mu, M)) \sqsubseteq SFG_\delta$ pre int $(f^{-1}$(ker(μ, M))).
$\Rightarrow SFG_\delta preint(f^{-1}(\mu, M)) \sqsubseteq f^{-1}(ker(\mu, M))$, for any soft fuzzy set (μ, M) in (Y,S).

□

Remark 5.7. The converse of the above property need not be true as shown in the following examples.

Example 5.8. Let X={a,b,c,d}, T={$(0, \phi)$, (1,X), (λ_1, M_1),(λ_2, M_2), (λ_3, M_3), (λ_4, M_4), (λ_5, M_5)} where λ_i : X → [0,1] for i= 1,2,3,4,5 and M_i fori=1,2,3,4,5 are defined as follows $\lambda_1(a) = 0.6$, $\lambda_1(b) = 0$, $\lambda_1(c) = 0.2$, $\lambda_1(d) = 0$; $\lambda_2(a) = 0$, $\lambda_2(b) = 0.5$, $\lambda_2(c) = 0$, $\lambda_2(d) = 0.1$; $\lambda_3(a) = 0.6$, $\lambda_3(b) = 0.5$, $\lambda_3(c) = 0.2$, $\lambda_3(d) = 0.1$; $\lambda_4(a) = 0.6$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.2$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.5$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.1$; $M_1 = \{a\}$, $M_2 = \{c\}$, $M_3 = \{a, c\}$, $M_4 = \{a, d, c\}$, $M_5 = \{a, b, c\}$ and Y=$\{p, q, r, s\}$, S={($(0, \phi)$, (1,Y), (μ, N)} where μ : X → [0,1] and N is defined by $\mu(p) = 0.4$, $\mu(q) = 0.5$, $\mu(r) = 0.2$, $\mu(s) = 0.3$; $N = \{r\}$. Let f : (X, T) → (Y,S) be the identity function. For the soft fuzzy set (λ_2, M_2), $f(SFG_\delta$ pre int $(\lambda_2, M_2)) \sqsubseteq$ ker$(f(\lambda_2, M_2))$. But f is not soft fuzzy G_δ-pre continuous.

Example 5.9. Consider the soft fuzzy topology and function defined in the above example. Let (μ_1, N_1) be the soft fuzzy set in (X, T), μ_1 : X → [0,1] and N_1 is defined by $\mu_1(p) = 0, \mu_1(q) = 0.5, \mu_1(r) = 0, \mu_1(s) = 0.1$; $N_1 = \{c\}$,such that G_δ pre int $(f^{-1}(\mu_1, N_1)) \sqsubseteq f^{-1}$(ker$(\mu_1, N_1)$). But f is not soft fuzzy G_δ-pre continuous.

Definition 5.10. Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. A function f :(X,T)→ (Y, S) is said to be *soft fuzzy G^*_δ continuous($G_\delta\alpha$-continuous, $G_\delta\beta$-continuous)* if the inverse image of every soft fuzzy open set in (Y,S) is soft fuzzy G^*_δ open ($G_\delta\alpha$ open, $G_\delta\beta$ open) in (X, T).

Proposition 5.11. *Every soft fuzzy G^*_δ-continuous function is soft fuzzy $G_\delta\alpha$-continuous.*

Proof. Proof is obvious □

Remark 5.12. The converse of the above property need not be true as shown in the following example.

Example 5.13. Let $X = \{a, b, c, d\}$, $T = \{(0, \phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0, 1]$ for i= 1, 2, 3, 4, 5 and $M_i \subseteq X$, for i = 1, 2, 3, 4, 5 are defined as follows $\lambda_1(a) = 0$, $\lambda_1(b) = 0.4$, $\lambda_1(c) = 0$, $\lambda_1(d) = 0.3$; $\lambda_2(a) = 0.7$, $\lambda_2(b) = 0$, $\lambda_2(c) = 0.8$, $\lambda_2(d) = 0$; $\lambda_3(a) = 0.7$, $\lambda_3(b) = 0.4$, $\lambda_3(c) = 0.8$, $\lambda_3(d) = 0.3$; $\lambda_4(a) = 0.7$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.8$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.4$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.3$; $M_1 = \{b\}$, $M_2 = \{c\}$, $M_3 = \{b, c\}$, $M_4 = \{a, b, c\}$, $M_5 = \{d, b, c\}$. Then (X, T) is a soft fuzzy topological space. Let $Y = \{p, q, r, s\}$, S = $\{((0, \phi), (1, Y), (\mu_1, N_1), (\mu_2, N_2)\}$ where $\mu_i : Y \to [0,1]$ for i = 1, 2 and $N_i \subseteq Y$, for i = 1, 2 are defined as follows $\mu_1(p) = 1$, $\mu_1(q) = 0.4$, $\mu_1(r) = 1$, $\mu_1(s) = 0.3$; $\mu_2(p) = 0$, $\mu_2(q) = 0.4$, $\mu_2(r) = 0$, $\mu_2(s) = 0.3$; $N_1 = \{q, r\}$, $N_2 = \{q\}$. Then (Y, S) is a soft fuzzy topological space. Let $f : (X, T) \to (Y, S)$ be a function defined as $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$. Then f *is soft fuzzy* $G_\delta\alpha$ *continuous but not soft fuzzy* G^*_δ *continuous.* Consider the soft fuzzy set (μ_1, N_1) in (Y, S), $f^{-1}(\mu_1, N_1)$ is not soft fuzzy G^*_δ open in (X, T).

Proposition 5.14. *Every soft fuzzy* $G_\delta\alpha$*-continuous function is soft fuzzy* G_δ*pre-continuous.*

Proof. Proof is obvious. □

Remark 5.15. The converse of the above property need not be true as shown in the following example.

Example 5.16. Let $X = \{a, b, c, d\}$, $T = \{(0, \phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where $\lambda_i : X \to [0,1]$ for i = 1, 2, 3, 4, 5 and $M_i \subseteq X$, for i = 1, 2, 3, 4, 5 are defined as follows $\lambda_1(a) = 0$, $\lambda_1(b) = 0.4$, $\lambda_1(c) = 0$, $\lambda_1(d) = 0.3$; $\lambda_2(a) = 0.7$, $\lambda_2(b) = 0$, $\lambda_2(c) = 0.8$, $\lambda_2(d) = 0$; $\lambda_3(a) = 0.7$, $\lambda_3(b) = 0.4$, $\lambda_3(c) = 0.8$, $\lambda_3(d) = 0.3$; $\lambda_4(a) = 0.7$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.8$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.4$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.3$; $M_1 = \{b\}$, $M_2 = \{c\}$, $M_3 = \{b, c\}$, $M_4 = \{a, b, c\}$, $M_5 = \{d, b, c\}$. Then (X, T) is a soft fuzzy topological space. Let $Y = \{p, q, r\}$, S = $\{(0, \phi), (1, Y), (\mu_1, N_1), (\mu_2, N_2)\}$ where $\mu_i : Y \to [0,1]$ for i=1,2 and $N_i \subseteq Y$, for i = 1, 2 are defined as follows $\mu_1(p) = 0.4$, $\mu_1(q) = 0$, $\mu_1(r) = 0.7$; $\mu_2(p) = 0.4$, $\mu_2(q) = 0.3$, $\mu_2(r) = 0.7$; $N_1 = \{r\}$, $N_2 = \{q, r\}$. Then (Y, S) is a soft fuzzy topological space. Let $f : (X, T) \to (Y, S)$ be a function defined as f(a)=p, f(b)=q, f(c)=r, f(d)=q. Then f *is soft fuzzy* G_δ*pre continuous but not soft fuzzy* $G_\delta\alpha$ *continuous.* Consider the soft fuzzy set (μ_1, N_1) in (Y, S), $f^{-1}(\mu_1, N_1)$ is not soft fuzzy $G_\delta\alpha$ open in (X, T).

Proposition 5.17. *Every soft fuzzy* G_δ*-pre continuous function is soft fuzzy* $G_\delta\beta$*-continuous.*

Proof. Proof is obvious. □

Remark 5.18. The converse of the above property need not be true as shown in the following example.

Example 5.19. Let $X = \{a, b, c, d\}$, T =$\{(0, \phi), (1, X), (\lambda_1, M_1), (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4), (\lambda_5, M_5)\}$ where λ_i :$X \to [0,1]$ for i = 1, 2, 3, 4, 5 and $M_i \subseteq X$, for i = 1, 2, 3, 4, 5 are defined as follows $\lambda_1(a) = 0.6$, $\lambda_1(b) = 0$, $\lambda_1(c) = 0.2$, $\lambda_1(d) = 0$; $\lambda_2(a) = 0$, $\lambda_2(b) = 0.5$, $\lambda_2(c) = 0$, $\lambda_2(d) = 0.1$; $\lambda_3(a) = 0.6$, $\lambda_3(b) = 0.5$, $\lambda_3(c) = 0.2$,

$\lambda_3(d) = 0.1$; $\lambda_4(a) = 0.6$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.2$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.5$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.1$; $M_1 = \{a\}$, $M_2 = \{c\}$, $M_3 = \{a, c\}$, $M_4 = \{a, d, c\}$, $M_5 = \{a, b, c\}$. Then $(X,\ T)$ is a soft fuzzy topological space. Let $Y = \{p, q, r, s\}$, $S = \{((0, \phi),\ (1, Y),\ (\mu, N)\}$ where $\mu : Y \to$ [0,1] and $N \subset Y$ are defined as $\mu(p) = 0.4$, $\mu(q) = 0.5$, $\mu(r) = 0.2$; $\mu(s) = 0.3$, $N = \{r\}$. Then $(Y,\ S)$ is a soft fuzzy topological space. Let $f : (X,\ T) \to (Y,\ S)$ be a function defined as f(a)=p, f(b)=q, f(c)=r, f(d)=s. Then f is soft fuzzy $G_\delta\beta$ continuous but not soft fuzzy $G_\delta$pre continuous. Consider the soft fuzzy set (μ, N) in $(Y,\ S)$, $f^{-1}(\mu, N)$ is not soft fuzzy $G_\delta$pre open in $(X,\ T)$.

6. Soft fuzzy G_δ-pre irresolute function and its properties

Definition 6.1. Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. A function f :(X,T)$\to$ (Y, S) is said to be *soft fuzzy G_δ-pre irresolute*, if the inverse image of every soft fuzzy G_δ pre open set in (Y,S) is soft fuzzy G_δ pre open in (X, T).

Theorem 6.2. *Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. For a function f :(X,T)$\to$ (Y, S), the following are equivalent.*
(i) f is soft fuzzy G_δ-pre irresolute.
(ii) The inverse image of every soft fuzzy F_σ pre closed set in(Y,S) is soft fuzzy F_σ pre closed in (X, T).

Proposition 6.3. *Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. For a bijective function f :(X,T)$\to$ (Y, S) the following are equivalent.*
(i) f is soft fuzzy G_δ-pre irresolute.
(ii)For each $(\lambda, M) \in (X, T)$, $f(SFF_\sigma$ pre cl$(\lambda, M)) \sqsubseteq SFF_\sigma$ pre cl$(f(\lambda, M))$.
(iii) For each $(\mu, N) \in$ (Y,S),SF F_σ pre cl$(f^{-1}(\mu, N)) \sqsubseteq f^{-1}(SFF_\sigma$ pre cl$(\mu, N))$.

Proof.
(i)$\Rightarrow$(ii) Assume that f is soft fuzzy G_δ-pre irresolute. Let (λ, M) be any soft fuzzy set in (X, T). Then SF $F_\sigma$pre cl$(f(\lambda, M))$ is a soft fuzzy F_σ pre closed set in (Y,S). By assumption $f^{-1}(SFF_\sigma$ pre cl$(f(\lambda, M)))$ is a soft fuzzy F_σ pre closed set in (X, T). Hence $(\lambda, M) \sqsubseteq f^{-1}(f(\lambda, M)) \sqsubseteq f^{-1}(SFF_\sigma$ pre cl$(f(\lambda, M)))$. $\Rightarrow (\lambda, M) \sqsubseteq f^{-1}(SFF_\sigma$ pre cl$(f(\lambda, M)))$. Which implies SF F_σ pre cl$(\lambda, M) \sqsubseteq f^{-1}(SFF_\sigma$ pre cl$(f(\lambda, M)))$. Therefore $f(SFF_\sigma$ pre cl$(\lambda, M)) \sqsubseteq SFF_\sigma$ pre cl$(f(\lambda, M))$.
(ii)$\Rightarrow$(iii) For each soft fuzzy set $(\lambda, M) \in (X, T)$,$f(SFF_\sigma$ pre cl$(\lambda, M)) \sqsubseteq SFF_\sigma$ pre cl$(f(\lambda, M))$. Let (μ, N) be a soft fuzzy set in (Y,S). Therefore, $f^{-1}(\mu, N)$ is a soft fuzzy set in (X, T).

$$Byassumption, f(SFF_\sigma precl(f^{-1}(\mu, N))) \sqsubseteq SFF_\sigma precl(f(f^{-1}(\mu, N))).$$
$$f(SFF_\sigma precl(f^{-1}(\mu, N))) \sqsubseteq SFF_\sigma precl(\mu, N).$$

$$\text{Hence SF } F_\sigma \text{ pre cl}(f^{-1}(\mu, N)) \sqsubseteq f^{-1}(SFF_\sigma \text{ pre cl}(\mu, N)).$$

(iii)$\Rightarrow$(i) Assume that for each soft fuzzy set (μ, N) in (Y,S) $SFF_\sigma precl(f^{-1}(\mu, N)) \sqsubseteq f^{-1}(SFF_\sigma precl(\mu, N))$. Let (γ, P) be a soft fuzzy F_σ pre closed set in (Y,S). That is SF F_σ pre cl$(\gamma, P) = (\gamma, P)$.By assumption SF F_σ pre cl$(f^{-1}(\gamma, P)) \sqsubseteq f^{-1}(SFF_\sigma$ pre cl$(\gamma, P))$. Thus SF F_σ pre cl$(f^{-1}(\gamma, P)) \sqsubseteq f^{-1}(\gamma, P)$. But $f^{-1}(\gamma, P) \sqsubseteq SFF_\sigma$ pre cl$(f^{-1}(\gamma, P))$. Therefore $f^{-1}(\gamma, P) = SFF_\sigma$ pre cl$(f^{-1}(\gamma, P))$. Hence f is soft fuzzy G_δ-pre irresolute. □

Proposition 6.4. *Let (X,T) and (Y,S) be any two soft fuzzy topological spaces. Let f : $(X,T)\to$ (Y,S) be soft fuzzy G_δ-pre irresolute function then the following hold.*
(i) f(SF G_δ pre int (λ, N)) $\sqsubseteq$ SF G_δ pre ker($f(\lambda, N)$), for every soft fuzzy subset (λ, N) of (X,T).
(ii) SF G_δ pre int $(f^{-1}(\mu, M)) \sqsubseteq f^{-1}$(SF G_δ pre ker(μ, M)),for every soft fuzzy subset (μ, M) of (Y,S).

Proof.
(i) Let (λ, N) be any soft fuzzy set in (X,T). Then SF G_δ pre ker($f(\lambda, N)$) is a soft fuzzy G_δ pre open set in (Y,S). By assumption, f^{-1} (SF G_δ pre ker($f(\lambda, N)$)) is a soft fuzzy G_δ pre open set in (X,T).

Now SF G_δ pre int $(\lambda, N) \sqsubseteq SFG_\delta$ pre int $(f^{-1}(f(\lambda, N))) \sqsubseteq SFG_\delta$ pre int (f^{-1}(SF G_δ pre ker($f(\lambda, N)$))).
$\Rightarrow SFG_\delta preint(\lambda, N) \sqsubseteq f^{-1}(SFG_\delta preker(f(\lambda, N)))$.
Hence $f(SFG_\delta$ pre int $(\lambda, N)) \sqsubseteq SFG_\delta$ pre ker($f(\lambda, N)$), for any soft fuzzy set (λ, N) in (X,T).
(ii) Let (μ, M) be any soft fuzzy set in (Y,S). Then SF G_δ pre ker(μ, M) is a soft fuzzy G_δ pre open set in (Y,S). By assumption,f^{-1} (SF G_δ pre ker(μ, M))) is a soft fuzzy G_δ pre open set in (X,T).

Now $f^{-1}(\mu, M) \sqsubseteq f^{-1}$(SF G_δ pre ker(μ, M)).
Thus SF G_δ pre int $(f^{-1}(\mu, M)) \sqsubseteq SFG_\delta$ pre int $(f^{-1}(SFG_\delta$ pre ker$(\mu, M)))$.
$\Rightarrow SFG_\delta$ pre int $(f^{-1}(\mu, M)) \sqsubseteq f^{-1}(SFG_\delta$ pre ker$(\mu, M)))$, for any soft fuzzy set (μ, M) in (Y,S).

□

Remark 6.5. The converse of the above property need not be true as shown in the following examples.

Example 6.6. Let X={a,b,c,d}, T={(0, ϕ), (1,X), (λ_1, M_1),(λ_2, M_2), (λ_3, M_3), (λ_4, M_4), (λ_5, M_5)} where λ_i : X $\to$ [0,1] for i= 1,2,3,4,5 and M_i fori=1,2,3,4,5 are defined as follows $\lambda_1(a) = 0.6$, $\lambda_1(b) = 0$, $\lambda_1(c) = 0.2$, $\lambda_1(d) = 0$; $\lambda_2(a) = 0$, $\lambda_2(b) = 0.5$, $\lambda_2(c) = 0$, $\lambda_2(d) = 0.1$; $\lambda_3(a) = 0.6$, $\lambda_3(b) = 0.5$, $\lambda_3(c) = 0.2$, $\lambda_3(d) = 0.1$; $\lambda_4(a) = 0.6$, $\lambda_4(b) = 1$, $\lambda_4(c) = 0.2$, $\lambda_4(d) = 1$; $\lambda_5(a) = 1$, $\lambda_5(b) = 0.5$, $\lambda_5(c) = 1$, $\lambda_5(d) = 0.1$; $M_1 = \{a\}$, $M_2 = \{c\}$, $M_3 = \{a, c\}$, $M_4 = \{a, d, c\}$, $M_5 = \{a, b, c\}$ and Y=$\{p, q, r, s\}$, S={((0, ϕ), (1,Y), (μ, N)} where μ : X $\to$ [0,1] and N is defined by $\mu(p) = 0.4$, $\mu(q) = 0.5$, $\mu(r) = 0.2$, $\mu(s) = 0.3$; $N = \{r\}$. Let f : $(X,T) \to$ (Y,S) be the identity function. For the soft fuzzy set (λ_2, M_2), $f(SFG_\delta$pre int $(\lambda_2, M_2)) \sqsubseteq SFG_\delta$ pre ker($f(\lambda_2, M_2)$). But f is not soft fuzzy G_δ-pre irresolute.

Example 6.7. Consider the soft fuzzy topology and function defined in the above example. Let (μ_1, N_1) be the soft fuzzy set in (X,T),μ_1 : X $\to$ [0,1] and N_1 is defined by $\mu_1(p) = 0, \mu_1(q) = 0.5, \mu_1(r) = 0, \mu_1(s) = 0.1$; $N_1 = \{c\}$,such that SF $G_\delta$pre int $(f^{-1}(\mu_1, N_1)) \sqsubseteq f^{-1}$(SF G_δ pre ker(μ_1, N_1)). But f is not soft fuzzy G_δ-pre irresolute.

Proposition 6.8. *Let (X,T),(Y,S) and(Z,R) be any three soft fuzzy topological spaces. A function f :$(X,T)\to (Y,S)$ be soft fuzzy G_δ-pre irresolute and $g : (Y,S) \to$*

(Z, R) be soft fuzzy G_δ-pre continuous function. Then gof :$(X,T) \to (Z, R)$ is soft fuzzy G_δ-pre continuous.

7. Soft fuzzy G_δ-pre connected space, soft fuzzy G_δ-pre compact space and soft fuzzy G_δ-pre normal space

Definition 7.1. A soft fuzzy topological space (X, T) is said to be a *soft fuzzy Connected space* if it has no proper soft fuzzy set which is both soft fuzzy open and soft fuzzy closed set.

Definition 7.2. A soft fuzzy topological space (X, T) is said to be *soft fuzzy G_δ-pre connected* if it has no proper soft fuzzy set which is both soft fuzzy G_δ pre open and soft fuzzy F_σ pre closed set.[A soft fuzzy set (λ, M) in a soft fuzzy topological space (X, T) is proper if $(\lambda, M) \neq (0, \phi)$ and $(\lambda, M) \neq (1, X)$.]

Proposition 7.3. *A soft fuzzy topological space (X, T) is soft fuzzy G_δ-pre connected if and only if it has no proper soft fuzzy G_δ pre open sets (λ, M) and (μ, N) such that*
$(\lambda, M) + (\mu, N) = (1, X)$.

Proof. Suppose that (X, T) is soft fuzzy G_δ-pre connected. Assume that (X, T) has proper soft fuzzy G_δ pre open sets (λ, M) and (μ, N) such that $(\lambda, M) + (\mu, N) = (1, X)$.

$$Now (\lambda, M) + (\mu, N) = (1, X).$$
$$\Rightarrow (\lambda, M) = (1, X) - (\mu, N).$$

$$\Rightarrow (\lambda, M) \text{ is soft fuzzy } F_\sigma \text{ pre closed and soft fuzzy } G_\delta \text{ pre open set in } (X, T).$$

Thus (X, T) is not soft fuzzy G_δ-pre connected. Which is a contradiction.
Conversely,
(X, T) has no proper soft fuzzy G_δ pre open sets $(\lambda, M) and (\mu, N)$ such that
(λ, M) + (μ, N)= (1,X). Assume that (X, T) is not soft fuzzy G_δ-pre connected. Then there exists a proper soft fuzzy set (λ, M) which is both soft fuzzy F_σ pre closed and soft fuzzy G_δ pre open set in (X, T).

$$\text{Thus } (\mu, N) = (1, X) - (\lambda, M).$$

$$\text{Since } (\lambda, M) \neq (0, \phi) and (1, X); (\mu, N) \neq (0, \phi) and (1, X).$$

Thus there exists a proper soft fuzzy set (μ, N) which is both soft fuzzy F_σ pre closed and soft fuzzy G_δ pre open set in (X, T) such that $(\lambda, M) + (\mu, N) = (1, X)$. Which is a contradiction. □

Proposition 7.4. *The following statements are equivalent for a soft fuzzy topological space (X, T).*
(i) (X, T) is soft fuzzy G_δ-pre connected.
(ii) There exist no soft fuzzy G_δ pre open sets $(\lambda, M) \neq (0, \phi)$ and $(\mu, N) \neq (0, \phi)$ such that $(\lambda, M) + (\mu, N) = (1, X)$.
(iii) There exist no soft fuzzy F_σ pre closed sets $(\lambda, M) \neq (1, X) and (\mu, N) \neq (1, X)$ such that $(\lambda, M) + (\mu, N) = (1, X)$.

Proof.
(i) $\Rightarrow$ (ii) Assume that (X, T) is soft fuzzy G_δ-pre connected. Then by the above property, it has no proper soft fuzzy G_δ pre open sets $(\lambda, M) and (\mu, N)$ such that $(\lambda, M) + (\mu, N) = (1, X)$.
(ii) $\Rightarrow$ (iii) Assume that there exist no soft fuzzy G_δ pre open sets $(\lambda, M) \neq (0, \phi)$ and $(\mu, N) \neq (0, \phi)$ such that $(\lambda, M) + (\mu, N) = (1, X)$. Suppose that there exist soft fuzzy F_σ pre closed sets $(\lambda, M) \neq (1, X)$ and $(\mu, N) \neq (1, X)$ such that $(\lambda, M) + (\mu, N) = (1, X)$.Then $(1, X) - (\lambda, M) \neq (0, \phi)$ is a non-zero soft fuzzy $G_\delta$pre open set. Similarly $(1, X) - (\mu, N) \neq (0, \phi)$ is a non zero soft fuzzy G_δ pre open set.

$$\begin{aligned} Now (1, X) - (\lambda, M) + (1, X) - (\mu, N) &= (1, X) + (1, X) - [(\lambda, M) + (\mu, N)]. \\ &= (1, X) + (1, X) - (1, X). \\ &= (1, X). \end{aligned}$$

Which is a contradiction.
(iii)$\Rightarrow$ (i) Assume that there exist no soft fuzzy F_σ pre closed sets $(\lambda, M) \neq (1, X)$ and $(\mu, N) \neq (1, X)$ such that $(\lambda, M) + (\mu, N) = (1, X)$. Suppose that (X, T) is not soft fuzzy G_δ-pre connected. There exists a proper soft fuzzy set (λ, M) which is both soft fuzzy F_σ pre closed and soft fuzzy G_δ pre open set in (X, T). Then$(1, X) - (\lambda, M)$ is a proper soft fuzzy F_σ pre closed set. Also by assumption (λ, M) is soft fuzzy F_σ pre closed. Now $(\lambda, M) + (1, X) - (\lambda, M) = (1, X)$. Which is a contradiction. □

Proposition 7.5. *Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. If f :(X,T)$\rightarrow$ (Y, S) is soft fuzzy G_δ-pre continuous surjection and (X, T) is soft fuzzy G_δ-pre connected, then (Y,S) is soft fuzzy connected.*

Proposition 7.6. *Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. If f :(X,T)$\rightarrow$ (Y, S) is soft fuzzy G_δ-pre irresolute surjection and (X, T) is soft fuzzy G_δ-pre connected, then (Y,S) is soft fuzzy G_δ-pre connected.*

Definition 7.7. A soft fuzzy topological space (X, T) is said to be a *soft fuzzy G_δ pre compact* if whenever $\sqcup_{i \in I}(\lambda_i, M_i)$ = (1,X), (λ_i, M_i) is soft fuzzy G_δ pre open, $i \in I$, there is a finite subset J of I with $\sqcup_{j \in J}(\lambda_i, M_i) = (1, X)$.

Proposition 7.8. *Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. If f :(X,T)$\rightarrow$ (Y, S) is soft fuzzy G_δ-pre continuous bijection and (X, T) is soft fuzzy G_δ-pre compact, then (Y,S) is soft fuzzy compact.*

Proposition 7.9. *Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. If f :(X,T)$\rightarrow$ (Y, S) is soft fuzzy G_δ-pre irresolute bijection and (X, T) is soft fuzzy G_δ-pre compact, then (Y,S) is soft fuzzy G_δ-pre compact.*

Definition 7.10. Let (X, T) and (Y,S) be any two soft fuzzy topological spaces. A function $f : (X, T) \rightarrow$ (Y,S) is said to be *soft fuzzy closed function*, if the image of every soft fuzzy closed set in (X, T) is soft fuzzy closed in (Y,S).

Definition 7.11. A soft fuzzy topological space (X, T) is said to be a *soft fuzzy normal space* , if for each pair of disjoint soft fuzzy closed sets (λ, N) and (μ, M) of (X, T), there exist open sets (δ, L) and (γ, K) with $(\lambda, N) \sqsubseteq (\delta, L)$ and $(\mu, M) \sqsubseteq (\gamma, K)$ such that $(\delta, L) \sqcap (\gamma, K) = (0, \phi)$.

Definition 7.12. A soft fuzzy topological space (X, T) is said to be a *soft fuzzy G_δ-pre normal space*, if for each pair of disjoint soft fuzzy closed sets (λ, N) and (μ, M) of (X, T), there exists G_δ pre open sets (δ, L) and (γ, K) with $(\lambda, N) \sqsubseteq (\delta, L)$ and $(\mu, M) \sqsubseteq (\gamma, K)$ such that $(\delta, L) \sqcap (\gamma, K) = (0, \phi)$.

Proposition 7.13. *If f is soft fuzzy G_δ- pre continuous, soft fuzzy closed ,injective function and (Y,S) is soft fuzzy Normal space, then (X, T) is soft fuzzy G_δ-pre normal space.*

Proof. Let (λ, N) and (μ, M) be any two disjoint soft fuzzy closed sets of (X, T). Since f is a soft fuzzy closed function and injective, $f(\lambda, N)$ and $f(\mu, M)$ are disjoint closed sets of (Y,S). Since (Y,S) is soft fuzzy Normal, there exist disjoint soft fuzzy open sets (δ, L) and (γ, K) of (Y,S) with $f(\lambda, N) \sqsubseteq (\delta, L)$ and $f(\mu, M) \sqsubseteq (\gamma, K)$ such that $(\delta, L) \sqcap (\gamma, K) = (0, \phi)$. Now,$(\lambda, N) \sqsubseteq f^{-1}(f(\lambda, N)) \sqsubseteq f^{-1}(\delta, L)$ and $(\mu, M) \sqsubseteq f^{-1}(f(\mu, M)) \sqsubseteq f^{-1}(\gamma, K)$. Since f is a soft fuzzy G_δ-pre continuous function, $f^{-1}(\delta, L)$ and $f^{-1}(\gamma, K)$ are soft fuzzy G_δ pre open sets of (X, T). Now, $f^{-1}(\delta, L) \sqcap f^{-1}(\gamma, K) = f^{-1}((\delta, L) \sqcap (\gamma, K)) = f^{-1}(0, \phi) = (0, \phi)$.Hence, (X, T) is a soft fuzzy G_δ-pre normal space. □

Proposition 7.14. *If f is soft fuzzy G_δ-pre irresolute, soft fuzzy closed,injective function and (Y,S) is soft fuzzy G_δ-pre normal space, then (X, T) is soft fuzzy G_δ-pre normal space.*

Proof. Proof is similar to the above property. □

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