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A study on attribute reduction by bayesian decision theoretic rough set models

Sharmistha bhattacharya Halder, Kalyani Debnath

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Abstract. Bayesian Decision theoretic rough set[1] has been invented by the author in 2012. In this paper the attribute reduction by the aid of Bayesian decision theoretic rough set has been studied. Also the concept of bayesian rough set based on coverage is introduced. By an example a comparative study is shown in the field of attribute reduction.

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Keywords: Bayesian decision theoretic rough set model, Decision theoretic rough set, Bayesian rough set, Bayesian rough set based on coverage.

Corresponding Author: Sharmistha Bhattacharya Halder
(halder_731@rediffmail.com)

1. Introduction

Pawlak[2] in 1982 defined the concept of Rough set. After the invention of Pawlak rough set, some more rough set concepts which are the generalized forms of Pawlak rough set were investigated. Such as probabilistic rough set, Decision theoretic rough set, Bayesian rough set etc. In 2002 the concept of Bayesian rough set model[3] was introduced and a detail study is done in 2012. Throughout this paper, the concepts of Bayesian decision theoretic rough set is studied. Also attribute reduction is done by Bayesian decision theoretic rough set model and a bayesian rough set model depending on coverage.

2. Preliminaries

In this section the various concepts of rough set defined by various Researchers which are necessary for further investigation were studied.

Definition 2.1 ([2])**.** Pawlak[2] in 1982 defined the concept of Rough set. According to Pawlak the rough set is defined as below:
Typically objective of rough set theory is to form an approximate definition of the target set X $\subseteq$ U in terms of some definable sets especially when the target set is indefinable or vague. The upper and lower approximation of X with respect to equivalence relation A are denoted as $\overline{A}X$ and $\underline{A}X$ respectively and defined as:
$\overline{A}$X = {E: P(X/E)>0, E$\in$ U/A}
$\underline{A}$X = {E: P(X/E)=1, E$\in$ U/A}

Definition 2.2 ([4])**.** In practical applications Pawlaks rough set model cannot deal with data sets which have some noisy data effectively. Lots of information in the boundary region will be abandoned which may provide latest useful knowledge. By applying the parameter the approximate regions can be adjusted and controlled in VPRSM. Given a parameter β, $0 \leq 1-\beta <$P(X) $< \beta \leq 1$, three kinds of approximation regions of concepts X$\subseteq$U with respect to equivalence relation A can be defined as follows:
positive region :
$POS_A^{\beta}(X)$=$\bigcup${E: P(X/E)$\geq \beta, E \in U/A$}
negative region :
$NEG_A^{\beta}(X)$=$\bigcup\{E : P(X/E) < 1-\beta, E \in U/A\}$
boundary region :
$BND_A^{\beta}(X)$=$\bigcup\{E : 1-\beta < P(X/E) < \beta, E \in U/A\}$

Definition 2.3 ([3])**.** Slezak and Ziarko[3] put forward BRSM in which the prior probability of the event under consideration is chosen as a benchmark value. BRSM is a hybrid product which connects rough set theory and Bayesian reasoning validity and reasonably. It is more appropriate to application problems concerned with achieving any certainty gain during the procedures of prediction or decision making rather than meeting a special certainty goal.
In BRSM three kinds of B approximation regions of concepts $X \subseteq U$ with respect to equivalence relation A can be defined as follows
B positive region :
$POS_A^{*}(X)$=$\bigcup\{E : P(X/E) > P(X), E \in U/A\}$
B negative region:
$NEG_A^{*}(X)$=$\bigcup\{E : P(X/E) < P(X), E \in U/A\}$
B boundary region:
$BND_A^{*}(X)$=$\bigcup\{E : P(X/E) = P(X), E \in U/A\}$

Definition 2.4 ([1])**.** In Bayesian Rough set model the parameter is considered as E but if we consider it as the decision then we get the concept of Bayesian decision theoretic rough set model which is defined as follows:
$apr_{D_i}^{*}([x]_C) = Pos_{D_i}^{*}([x]_C) = \cup\{[x]_C : |[x]_C \cap D_i|/[x]_C| > P(D_i\}$
$Neg_{D_i}^{*}([x]_C) = \cup\{[x]_C : |[x]_C \cap D_i|/[x]_C| < P(D_i\}$
$Bnd_{D_i}^{*}([x]_C) = \cup\{[x]_C : |[x]_C \cap D_i|/[x]_C| = P(D_i\}$
It is actually a Bayesian rough set model depending on decision.

3. Attribute reduction by Bayesian decision theoretic rough set model

Definition 3.1. In Bayesian Decision Theoretic Rough Set Model the part positive decision is the set of decision classes with the precision higher than $P(D_i)$. A positive decision may lead to a definite and immediate action. The part of boundary decision is the set of decision classes with the precision equal to $P(D_i)$. A boundary decision may lead to a "wait-and-see" action. A decision with the precision lower than $P(D_i)$ is not strong enough to support any further action. The union of positive decision and boundary decision can be called the set of generalized that support actual decision making. Let D_{POS}, D_{BND}, D_{GEN} denote the positive, boundary and general decision sets, respectively. For an equivalence class $[x]_C \in \pi_A$
$D_{POS}([x]_C) = \{D_i \in \pi_D : P(D_i/[x]_C) > P(D_i\}$
$D_{BND}([x]_C) = \{D_i \in \pi_D : P(D_i/[x]_C) = P(D_i\}$
$D_{GEN}([x]_C) = D_{POS}([x]_C) \cup D_{BND}([x]_C)$
A reduct $R \subseteq C$ for positive decision preservation can be defined by requiring that the positive decisions of all objects are unchanged. Constructing a reduct for decision preservation can apply any traditional methods, for example the methods based on the discernibility matrix. Both rows and columns of the matrix correspond to the equivalence classes defined by C. An element of the matrix is the set of all attributes that distinguish the corresponding two equivalence classes. Namely, the matrix element consists of all attributes on which the corresponding two equivalence classes have distinct values and thus distinct decision making. A discernibility matrix is symmetric. The elements of a positive decision-based discernibility matrix $M_{D_{POS}}$ and a general decision-based discernibility matrix $M_{D_{GEN}}$ are defined as follows. For any equivalence classes $[x]_C$ and $[y]_C$,
$M_{D_{POS}}([x]_C, [y]_C) = \{a \in C : I_a(x) \neq I_a(y) \wedge D_{POS}([x]_C) \neq D_{POS}([y]_C)\}$
$M_{D_{GEN}}([x]_C, [y]_C) = \{a \in C : I_a(x) \neq I_a(y) \wedge D_{GEN}([x]_C) \neq D_{GEN}([y]_C)\}$

Skowron and Rauser showed that the set of attribute reducts are in fact the set of prime implicants of the reduced disjunctive form of the discernibility function. Thus, a positive decision reduct is a prime implicant of the reduced disjunctive form of the discernibility function
$f(M_{D_{POS}}) = \wedge\{\vee(M_{D_{POS}}([x]_C, [y]_C)) : x, y \in U(M_{D_{POS}}([x]_C, [y]_C) \neq \phi)\}$

The expression $\vee(M_{D_{POS}}([x]_C, [y]_C))$ is the disjunction of all attributes in $M_{D_{POS}}([x]_C, [y]_C)$, indicating that the pair of equivalence classes $[x]_C$ and $[y]_C$ can be distinguished by any attribute in M. The expression $\wedge\{\vee(M_{D_{POS}}([x]_C, [y]_C))$ is the conjunction of all $\vee(M_{D_{POS}}([x]_C, [y]_C))$, indicating that the family of discernible pairs of equivalence classes. In order to derive the reduced disjunctive form, the discernibility function $f(M_{D_{POS}})$ is transformed by using the absorption and distributive laws. Accordingly, finding the set of reducts can be modeled based on the manipulation of a Boolean function. Analogically, a general decision reduct is a prime implicant of the reduced disjunctive form of the discernibility function.

TABLE 1. Information Table

	C_1	C_2	C_3	C_4	C_5	C_6	D
O_1	1	1	1	1	1	1	M
O_2	1	1	0	0	1	1	M
O_3	1	1	1	1	1	1	M
O_4	1	1	0	0	1	1	Q
O_5	1	0	1	0	1	1	Q
O_6	1	0	1	0	1	1	F
O_7	1	1	1	0	0	0	F
O_8	1	1	1	0	0	0	F
O_9	1	0	1	0	1	1	F

TABLE 2. D Pos, D Bnd, D Gen

	C_1	C_2	C_3	C_4	C_5	C_6	D_{POS}	D_{BND}	D_{GEN}
$[O_1]_C$	1	1	1	1	1	1	{M}	ϕ	{M}
$[O_2]_C$	1	1	0	0	1	1	{M,Q}	ϕ	{M,Q}
$[O_3]_C$	1	1	1	1	1	1	{M}	ϕ	{M}
$[O_4]_C$	1	1	0	0	1	1	{M,Q}	ϕ	{M,Q}
$[O_5]_C$	1	0	1	0	1	1	{Q,F}	ϕ	{Q,F}
$[O_6]_C$	1	0	1	0	1	1	{Q,F}	ϕ	{Q,F}
$[O_7]_C$	1	1	1	0	0	0	{F}	ϕ	{F}
$[O_8]_C$	1	1	1	0	0	0	{F}	ϕ	{F}
$[O_9]_C$	1	0	1	0	1	1	{Q,F}	ϕ	{Q,F}

$f(M_{D_{GEN}}) = \wedge\{\vee(M_{D_{GEN}}([x]_C, [y]_C)) : x, y \in U(M_{D_{GEN}}([x]_C, [y]_C) \neq \phi)\}$
Let us now consider an examples to show the attribute reduction by Pawlaks rough set model , Variable precision rough set model and Bayesian decision theoretic rough set model.

Example 3.2. Let us consider the information table(table1).

According to the definition of Bayesian decision theoretic rough set model and Pawlak rough set model a reformation of table 1 indicating the decision associated with each equivalence class $[x]_C$ is shown in table2

Table 3. Discernibility Matrix

	$[O_1]_C$	p	$[O_2]_C$	p	$[O_3]_C$	p
$[O_1]_C$	ϕ	ϕ				
$[O_2]_C$	ϕ	ϕ	ϕ	ϕ		
$[O_3]_C$	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
$[O_4]_C$	{3,4}	{3,4}	ϕ	ϕ	{3,4}	{3,4}
$[O_5]_C$	{2,4}	{2,4}	{2,3}	ϕ	{2,4}	{2,4}
$[O_6]_C$	{2,4}	{2,4}	{2,3}	ϕ	{2,4}	{2,4}
$[O_7]_C$	{4,5,6}	{4,5,6}	{3,5,6}	{3,5,6}	{4,5,6}	{4,5,6}
$[O_8]_C$	{4,5,6}	{4,5,6}	{3,5,6}	{3,5,6}	{4,5,6}	{4,5,6}
$[O_9]_C$	{2,4}	{2,4}	{2,3}	ϕ	{2,4}	{2,4}

	$[O_4]_C$	p	$[O_5]_C$	p	$[O_6]_C$	p
$[O_1]_C$						
$[O_2]_C$						
$[O_3]_C$						
$[O_4]_C$	ϕ	ϕ				
$[O_5]_C$	ϕ	ϕ	ϕ	ϕ		
$[O_6]_C$	{2,3}	ϕ	ϕ	ϕ	ϕ	ϕ
$[O_7]_C$	{3,5,6}	{3,5,6}	{2,5,6}	{2,5,6}	ϕ	ϕ
$[O_8]_C$	{3,5,6}	{3,5,6}	{2,5,6}	{2,5,6}	ϕ	ϕ
$[O_9]_C$	{2,3}	ϕ	ϕ	ϕ	ϕ	ϕ

	$[O_7]_C$	p	$[O_8]_C$	p	$[O_9]_C$	p
$[O_1]_C$						
$[O_2]_C$						
$[O_3]_C$						
$[O_4]_C$						
$[O_5]_C$						
$[O_6]_C$						
$[O_7]_C$	ϕ	ϕ				
$[O_8]_C$	ϕ	ϕ	ϕ	ϕ		
$[O_9]_C$	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

According to this discernibility matrix in Bayesian decision theoretic rough set model, we can find and verify the following reducts positive decision preservation: {C_2, C_3, C_4}, {C_2, C_3, C_5}, {C_2, C_3, C_6}, {C_2, C_4, C_5}, {C_2, C_4, C_6}, {C_3, C_4, C_5}, {C_3, C_4, C_6}. According to Pawlak we can find and verify the following reducts positive decision preservation: {C_4, C_5}, {C_4, C_6}, {C_2, C_3, C_4},{C_2, C_3, C_5}, {C_2, C_3, C_6}.

N. ow similarly as above if we reduce the attribute using variable precision rough set model than we get

{C_3,C_4}, {C_2, C_3, C_5}, {C_2, C_3, C_6}, {C_2, C_4, C_5}, {C_2, C_4, C_6} as the reduced attributes.

H. ence the attribute reduction by

(1) Pawlak method is {C_4, C_5}, {C_4, C_6}, {C_2, C_3, C_4},{C_2, C_3, C_5}, {C_2, C_3, C_6}
(2) Variable Precision rough set method is {C_3,C_4}, {C_2, C_3, C_5}, {C_2, C_3, C_6}, {C_2, C_4, C_5}, {C_2, C_4, C_6}
(3) Bayesian decision theoretic rough set model is {C_2, C_3, C_4}, {C_2, C_3, C_5}, {C_2, C_3, C_6}, {C_2, C_4, C_5}, {C_2, C_4, C_6}, {C_3, C_4, C_5}, {C_3, C_4, C_6}.

I. n first and second there are five alternative reductions but in third there are seven alternative reductions.

4. Bayesian rough set depending on coverage

In this section another rough set model is introduced and by the previous example positive region general region and boundary region were studied. Also considering a second example a comparative study is shown between other methods and this new method.

Definition 4.1. $apr^*_{D_i}([x]_C) = Pos = \cup\{[x]_C : |[x]_C \cap D_i|/D_i| > P([x]_c\}$
$Neg = \cup\{[x]_C : |[x]_C \cap D_i|/D_i| < P([x]_c\}$
$Bnd = \cup\{[x]_C : |[x]_C \cap D_i|/D_i| = P([x]_c\}$

The positive, boundary and general decision sets are defined as

Definition 4.2. $D_{POS}([x]_C) = \{D_i \in \pi_D : P([x]_c/D_i) > P([x]_c\}$
$D_{BND}([x]_C) = \{D_i \in \pi_D : P([x]_c/D_i) = P([x]_c\}$
$D_{GEN}([x]_C) = D_{POS}([x]_C) \cup D_{BND}([x]_C)$

Considering the previous example(table1) we get all the region associated with each equivalence classes in Bayesian rough set depending on coverage which is shown in table4.

This table is similar as table 2. Hence attribute reduction will give the same result as Bayesian decision theoretic rough set model.

Now let us consider another example

Example 4.3. In this table(table5) the significance of attributes C_1, C_2, C_3, C_5, C_6 are zero but the significance of the attribute C_4 is 1/3 . So if we remove attribute C_4 then it will effect the consistent decision rule. Now after finding the discernibility matrix if we reduce the attribute then we get the following results

(1) By pawlak method the reduction is {C_3, C_4}, {C_4, C_5, C_6},{C_2, C_4, C_6}
(2) By Variable precision rough set method the reduction is {C_3, C_4}, {C_4, C_5, C_6},{C_2, C_4, C_6}
(3) By Bayesian decision theoretic rough set method the reduction is {C_3, C_4}, {C_4, C_5, C_6},{C_2, C_4, C_6}
(4) By Bayesian rough set method depending on coverage the reduction is { C_4}, {C_3, C_6},{C_2, C_3, C_5}

Hence we get a comparative study between the various methods of attribute reduction.

Table 4. $D_{POS}, D_{BND}, D_{GEN}$ in Bayesian rough set depending on coverage

	C_1	C_2	C_3	C_4	C_5	C_6	D_{POS}	D_{BND}	D_{GEN}
$[O_1]_C$	1	1	1	1	1	1	{M}	ϕ	{M}
$[O_2]_C$	1	1	0	0	1	1	{M,Q}	ϕ	{M,Q}
$[O_3]_C$	1	1	1	1	1	1	{M}	ϕ	{M}
$[O_4]_C$	1	1	0	0	1	1	{M,Q}	ϕ	{M,Q}
$[O_5]_C$	1	0	1	0	1	1	{Q,F}	ϕ	{Q,F}
$[O_6]_C$	1	0	1	0	1	1	{Q,F}	ϕ	{Q,F}
$[O_7]_C$	1	1	1	0	0	0	{F}	ϕ	{F}
$[O_8]_C$	1	1	1	0	0	0	{F}	ϕ	{F}
$[O_9]_C$	1	0	1	0	1	1	{Q,F}	ϕ	{Q,F}

Table 5. Information Table

	C_1	C_2	C_3	C_4	C_5	C_6	D
O_1	1	0	0	1	1	1	M
O_2	1	0	0	1	1	1	M
O_3	1	0	1	0	1	0	M
O_4	1	0	1	0	1	0	M
O_5	1	0	0	0	1	1	Q
O_6	1	0	1	0	1	0	Q
O_7	1	1	1	1	0	1	F
O_8	1	0	0	1	1	1	F
O_9	1	1	1	1	0	1	F

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Sharmistha bhattacharya Halder (halder_731@rediffmail.com)
Department of Mathematics, Tripura University-799022, Tripura, India

Kalyani Debnath (mou9900@live.in)
Department of Mathematics, Tripura University-799022, Tripura, India