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On intuitionistic dimension functions

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Abstract. In this paper, the concepts of intuitionistic small inductive dimension, intuitionistic $\mathcal{AB}$ open set, intuitionistic partitions are introduced and studied.The concepts of intuitionistic large inductive dimension, intuitionistic addition theorem and intuitionistic product theorem are introduced and studied.

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1. Introduction

The concept of intuitionistic sets was introduced by çoker [1] . In 1998,J.Donchev [4] introduced the concept of $\mathcal{AB}$ - sets and decomposition of continuity. In this paper,the concepts of intuitionistic small inductive dimension, intuitionistic $\mathcal{AB}$ open sets,intuitionistic partitions are introduced and studied.The concepts of intuitionistic large inductive dimension, intuitionistic addition theorem and intuitionistic product theorem are introduced and studied.

2. Preliminaries

Definition 2.1 ([1]). Let X be a non empty set. An *intuitionistic set* (IS for short) A is an object having the form $A = \langle x, A^1, A^2\rangle$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \varnothing$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A. Every crisp set A on a nonempty set X is obviously an intuitionistic set having the form $\langle x, A, A^c\rangle$.

Definition 2.2 ([1]). Let X be a non empty set, $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, B^1, B^2\rangle$ be intuitionistic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A^i = \langle x, A_i^1, A_i^2\rangle$.

(i) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $A^2 \supseteq B^2$.
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(iii) $\overline{A} = \langle x, A^2, A^1 \rangle$.
(iv) $\bigcup A_i = \langle x, \cup A_i^1, \cap A_i^2 \rangle$.
(v) $\bigcap A_i = \langle x, \cap A_i^1, \cup A_i^2 \rangle$.
(vi) $\phi_\sim = \langle x, \phi, X \rangle$; $X_\sim = \langle x, X, \phi \rangle$.

Definition 2.3 ([2])**.** An intuitionistic topology (IT for short) on a nonempty set X is a family T of intuitionistic set in X satisfying the following axioms:

(i) $\phi_\sim$, $X_\sim \in T$.
(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$.
(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i : i \in J\} \subseteq T$.

In this case the pair (X, T) is called an *intuitionistic topological space* (ITS for short) and any intuitionistic set in T is called an *intuitionistic open set*(IOS for short) in X. The complement $\overline{A}$ of an intuitionistic open set A is called an *intuitionistic closed set* (ICS for short) in X.

Definition 2.4 ([2])**.** Let (X, T) be an intuitionistic topological space and $A = \langle X, A^1, A^2 \rangle$ be an intuitionistic set in X. Then the *closure* and *interior* of A are defined by

$cl(A) = \cap\{K : K$ is an intuitionistic closed set in X and $A \subseteq K\}$,
$int(A) = \cup\{G : G$ is an intuitionistic open set in X and $G \subseteq A\}$.

It can be also shown that $cl(A)$ is an intuitionistic closed set and $int(A)$ is an intuitionistic open set in X, and A is an intuitionistic closed set in X iff $cl(A) = A$; and A is an intuitionistic open set in X iff $int(A) = A$.

Definition 2.5 ([7])**.** Let (X, T) be a topological space. A subset A in X is said to be *semi - open* if $A \subseteq cl(int(A))$.

Definition 2.6 ([7])**.** Let (X, T) be a topological space. A subset A in X is said to be *semi - closed* if $int(cl(A)) \subseteq A$

Definition 2.7 ([3])**.** Let (X, T) be a topological space. A subset S in X is said to be *semi - regular* if both semi - open and semi - closed.

Definition 2.8 ([4])**.** Let (X, T) be a topological space. A subset A in X is said to be an *AB - set* if $A = U \cap V$, where U $\in T$ and V is semi - regular.The family of all AB - sets of a space X will be denoted by AB(X).

Definition 2.9 ([5])**.** Let (X, T) be a topological spaces. A *boundary* of a subset V in X is denoted and defined as $\partial V = cl(V) \cap cl(\overline{V})$.

3. Intuitionistic small inductive dimension

Notation 3.1. Let (X, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X.

(i) cl(A) denotes Icl(A).
(ii) int(A) denotes Iint(A).

Definition 3.2. Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2 \rangle$ in X is said to be *intuitionistic semi - open* if $A \subseteq Icl(Iint(A))$.

Definition 3.3. Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2 \rangle$ in X is said to be *intuitionistic semi - closed* if $Iint(Icl(A)) \subseteq A$.

Definition 3.4. Let (X, T) be an intuitionistic topological space. An intuitionistic set $S = \langle x, S^1, S^2 \rangle$ in X is said to be *intuitionistic semi - regular* if both intuitionistic semi - open and intuitionistic semi - closed.

Example 3.5. Let (X, T) be an intuitionistic topological space. Let $X = \{a, b, c\}$ and $T = \{X_\sim, \phi_\sim, \langle\{a\}, \{b, c\}\rangle, \langle\{c\}, \{a, b\}\rangle, \langle\{a, b\}, \{c\}\rangle, \langle\{a, c\}, \{b\}\rangle\}$. Let $A = \langle\{a, b\}, \{c\}\rangle$ be an intuitionistic set now A is *intuitionistic semi - regular*.

Definition 3.6. Let (X, T) be an intuitionistic topological space. A subset $A = \langle x, A^1, A^2 \rangle$ in X is said to be an *intuitionistic $\mathcal{AB}$ open-set* if $A = U \cap V$, where $U = \langle x, U^1, U^2 \rangle \in T$ and $V = \langle x, V^1, V^2 \rangle$ is an intuitionistic semi - regular. The complement $\overline{A}$ of an intuitionistic $\mathcal{AB}$ open-set A is called an intuitionistic $\mathcal{AB}$ closed-set.

Example 3.7. Let (X, T) be an intuitionistic topological space. Let $X = \{a, b, c\}$ and $T = \{X_\sim, \phi_\sim, \langle\{a\}, \{b, c\}\rangle, \langle\{b\}, \{a\}\rangle, \langle\{a, b\}, \phi\rangle\}$. Let $A = \langle\{a\}, \{b, c\}\rangle$ be an intuitionistic set now A is intuitionistic $\mathcal{AB}$ - open set.

Definition 3.8. Let (X, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X. Then the *intuitionistic $\mathcal{AB}$ interior* (*$I\mathcal{AB}int$* for short) of A are defined by

$$I\mathcal{AB}int(A) = \cup\{G = \langle x, G^1, G^2 \rangle : G \text{ is an } \textit{intuitionistic } \mathcal{AB} \textit{ open set} \text{ in } X \text{ and } G \subseteq A\}.$$

Definition 3.9. Let (X, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X. Then the *intuitionistic $\mathcal{AB}$ closure* (*$I\mathcal{AB}cl$* for short) of A are defined by

$$I\mathcal{AB}cl(A) = \cap\{K = \langle x, K^1, K^2 \rangle : K \text{ is an intuitionistic } \mathcal{AB} \text{ closed set in } X \text{ and } A \subseteq K\}.$$

Notation 3.10. Let (X, T) be an intuitionistic topological space and $U = \langle x, U^1, U^2 \rangle$ be an intuitionistic set, $x \in U$ denotes $x \in U^1$ and $x \notin U^2$.

Definition 3.11. Let (X, T) be an intuitionistic topological space. An *intuitionistic $\mathcal{AB}$ boundary* of an intuitionistic set $V = \langle x, V^1, V^2 \rangle$ in X is denoted and defined as $I\mathcal{AB}\partial V = I\mathcal{AB}cl(V) \cap I\mathcal{AB}cl(\overline{V})$.

Definition 3.12. Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2 \rangle$ in X is said to be an *intuitionistic $\mathcal{AB}$ neighborhood* of x if there exists an intuitionistic $\mathcal{AB}$ open set $U = \langle x, U^1, U^2 \rangle$ such that $x \in U \subseteq A$.

Definition 3.13. Let (X, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set of X is said to be an *intuitionistic dense* in X, if $I\mathcal{AB}cl(A) = X_\sim$.

Definition 3.14. An intuitionistic small inductive dimension of X is denoted *I-ind(X)*, and is defined as follows:

(i) We say that the intuitionistic dimension of a space X, *(I-ind(X))* is -1 iff $X = \phi_{\sim}$.
(ii) *I-ind(X)* $\leq n$ if for every point $x \in X$ and for every intuitionistic $\mathcal{AB}$ - open set $U = \langle x, U^1, U^2 \rangle$ there exists an intuitionistic $\mathcal{AB}$ - open set $V = \langle x, V^1, V^2 \rangle$, $x \in V$ such that $I\mathcal{AB}cl(V) \subseteq U$ and *I-ind*$(I\mathcal{AB}\partial V) \leq n-1$. Where $I\mathcal{AB}\partial V$ is an intuitionistic $\mathcal{AB}$ boundary of V.
(iii) *I-ind(X)* $= n$ if (ii) is true for n, but false for $n-1$.
(iv) *I-ind(X)* $= \infty$ if for every n, *I-ind(X)*$\leq n$ is false.

Definition 3.15. Let R^I be a real line. An interval $I = \langle (a,b), (c,d) \rangle$ is said to be an intuitionistic interval if (a,b) and (c,d) are disjoint.

Example 3.16. Let's show that *I-ind*(R^I) is 1.

For each $x \in R^I$, lets select an intuitionistic neighborhood V and an intuitionistic set $U = \langle (a,b), (c,d) \rangle$, *I-ind*$(I\mathcal{AB}\partial U)$= *I-ind* $\langle \{a,b\}, \{c,d\} \rangle = 0$. This implies that *I-ind*$(R^I) \leq 1$.

so it is enough to prove that *I-ind*(R^I) is not 0. But that is easy. If *I-ind*(R^I)=0, that means that an intuitionistic set U exists such that *I-ind*$(I\mathcal{AB}\partial U) = -1 \Leftrightarrow I\mathcal{AB}\partial U = \phi_{\sim} \Leftrightarrow U$ is intuitionistic clopen, but if U is intuitionistic clopen, R^I is intuitionistic disconnected. Because R^I is intuitionistic connected we get that *I-ind*$(R^I) > 0$. So finally, *I-ind*(R^I)= 1.

Proposition 3.17. *If* $Y \subseteq X$ *then* I-ind(Y)$\leq$ I-ind(X).

Proof. By induction, it is true for *I-ind(X)* $= -1$. If *I-ind(X)* $= n$,for every point $y \in X$ there is an intuitionistic $\mathcal{AB}$ neighborhood $V = \langle x, V^1, V^2 \rangle$, with an intuitionistic $\mathcal{AB}$ open set $U = \langle x, U^1, U^2 \rangle$, $I\mathcal{AB}cl(V) \subseteq U$ such that I -ind($I\mathcal{AB}\partial V$) $\leq n-1$.

Note that $V_Y = Y \cap V$ is an intuitionistic $\mathcal{AB}$ neighborhood in Y and U_Y is an intuitionistic $\mathcal{AB}$ open in Y. By induction,it is enough to show that $I\mathcal{AB}\partial U_Y \subseteq I\mathcal{AB}\partial U$.

Because then *I-ind*$(\partial U_Y) \leq n-1$, and therefore *I-ind(Y)*$\leq n = I$ - *ind(X)*(By definition 3.13). And indeed $I\mathcal{AB}cl(U_y) \subseteq IABcl(U)$,

$$\begin{aligned} I\mathcal{AB}\partial U_y &= I\mathcal{AB}cl(U_y) \cap I\mathcal{AB}cl(\overline{U_y}) \\ &\subseteq I\mathcal{AB}cl(U) \cap I\mathcal{AB}cl(\overline{U}) \\ &\subseteq I\mathcal{AB}\partial U. \end{aligned}$$

Hence proved. □

Definition 3.18. Let (X,T) be an intuitionistic topological space. Let $A = \langle x, A^1, A^2 \rangle$ and $B = \langle x, B^1, B^2 \rangle$ a pair of disjoint intuitionistic sets of the space (X,T), we say that an intuitionistic set $L = \langle x, L^1, L^2 \rangle \subseteq X$ is an intuitionistic partition between A and B if there exist intuitionistic $\mathcal{AB}$ open sets $U = \langle x, U^1, U^2 \rangle$ and $W = \langle x, W^1, W^2 \rangle$ in (X,T) satisfying $A \subseteq U$, $B \subseteq W$ such that $W \cap U$=$\phi_{\sim}$ and $L = \overline{U} \cap \overline{W}$.

Proposition 3.19. I-ind(X)$\leq n \Leftrightarrow$ *For every point* x *and every intuitionistic* $\mathcal{AB}$ *closed set* B *there is an intuitionistic partition* L*, such that* I-ind(L)$\leq n-1$.

Proof. If $x \in X$ and $B =< x, B^1, B^2 >$ is an intuitionistic $\mathcal{AB}$ closed set, then there is an intuitionistic set $V =< x, V^1, V^2 >$ such that $I\mathcal{AB}cl(V) \cap B = \phi_\sim$. By definition of *I-ind*, there is an $I\mathcal{AB}cl(U) \subseteq V$, where $U = \langle x, U^1, U^2 \rangle$, $x \in U$, *I - ind(*$I\mathcal{AB}\partial U$*)*$\leq n-1$. Now, let $W = I\mathcal{AB}cl(U)$ and $L = I\mathcal{AB}\partial U$. Hence $I - ind(L) \leq n-1$.

Conversely, let $x \in X$ and V be an intuitionistic $\mathcal{AB}$ neighborhood x. An intuitionistic set $B = \overline{V}$ is intuitionistic $\mathcal{AB}$ closed and therefore there are $U = \langle x, U^1, U^2 \rangle$, $W = \langle x, W^1, W^2 \rangle$ such that $x \in U$, $B \subseteq W$. Note that by definition of B, $U \subseteq V$. Now $\overline{W}$ is intuitionistic $\mathcal{AB}$ closed, therefore $I\mathcal{AB}cl(U) \subseteq \overline{W}$ and obviously

$$I\mathcal{AB}\partial U \subseteq I\mathcal{AB}cl(U) \subseteq \overline{W}$$

and

$$I\mathcal{AB}\partial U = I\mathcal{AB}cl(U) \cap I\mathcal{AB}cl(\overline{U}) \subseteq \overline{U}$$

Therefore $I\mathcal{AB}\partial U \subseteq \overline{U} \cap \overline{W} = L$. So that *I-ind(*$I\mathcal{AB}\partial U$*)* $\leq n-1$. Hence *I-ind(X)* $\leq n$. □

Definition 3.20. An intuitionistic topological space is called *intuitionistic separable* if it contains a countable, intuitionistic dense subset.

Definition 3.21. An *intuitionistic metrizable space* is an intuitionistic topological space that is homeomorphic to an intuitionistic metric space.

Proposition 3.22. *If X is an intuitionistic zero-dimensional separable metric space, then for every pair A, B of disjoint intuitionistic closed subsets of X the empty set is an intuitionistic partition between A and B, that is there exists an intuitionistic open and intuitionistic closed set $U \subset X$ such that $A \subset U$ and $B \subset X \setminus U$.*

Proof. The proof is similar to Theorem 1.2.6[5] with suitable modification. □

Proposition 3.23. *Let M be an intuitionistic subspace of a metric space X and A, B a pair of disjoint intuitionistic closed sets of X. For every intuitionistic partition L' in the space M between $M \cap \overline{V_1}$ and $M \cap \overline{V_2}$, where V_1, V_2 are intuitionistic open sets of X such that $A \subset V_1$, $B \subset V_2$ and $\overline{V_1} \cap \overline{V_2} = \phi_\sim$, there exists an intuitionistic partition L in the space X between A and B which satisfies the inclusion $M \cap L \subset L'$.*

Proof. The proof is similar to Lemma 1.2.9[5] with suitable modification. □

Proposition 3.24. *If X is an arbitrary intuitionistic metric space and Z is an intuitionistic zero-dimensional separable subspace of X, then for all intuitionistic closed set A, B of X, $A \cap B = \phi_\sim$ there is an intuitionistic partition between them L, such that $L \cap Z = \phi_\sim$.*

Proof. The proof is similar to Theorem 1.2.11[5] with suitable modification. □

Proposition 3.25. *X is an intuitionistic separable metrizable space, then* I-ind(X)$\leq n \Leftrightarrow$ *X is the union of two intuitionistic sub spaces, Y,Z such that* I-ind(Y)$\leq n-1$, I-ind(Z)≤ 0.

Proof. The proof is similar to Theorem 3.9[8] with suitable modification. □

Proposition 3.26. *For every intuitionistic $\mathcal{AB}$ closed sets A and B of an intuitionistic separable metric space X,* I-ind(X)$\leq n$, *there is an intuitionistic partition L, such that* I-ind(L) $\leq n-1$.

Proof. We can decompose X into two intuitionistic spaces Y, Z by Proposition 3.24 *I - ind(Y)* $\leq n-1$, *I-ind (Z)*= 0. By Proposition 3.23, There is an intuitionistic partition L between an intuitionistic sets $A = \langle x, A^1, A^2\rangle$, $B = \langle x, B^1, B^2\rangle$ and $L \cap Z = \phi_\sim \Rightarrow L \subseteq Y$ and *I- ind(L)* $\leq n-1$ as an intuitionistic subspace of Y. □

Proposition 3.27. *Let X be an intuitionistic separable metric spaces,* I - ind(X)$\leq$ $n \iff$ *Then for every sequence of* (A_1, B_1).........(A_{n+1}, B_{n+1}) *of an intuitionistic* $\mathcal{AB}$ *closed disjoint sets. An intuitionistic partitions* L_1...........L_{n+1} *exist such that they have an empty intersection.*

Proof. Let us take $A_1 = \langle x, A_1^1, A_1^2\rangle$, $B_1 = \langle x, B_1^1, B_1^2\rangle$ by Proposition 3.25, we can find an intuitionistic partition L_1, *I - ind(*L_1*)* $\leq n-1$. Let us take $A_2 = \langle x, A_2^1, A_2^2\rangle$, $B_2 = \langle x, B_2^1, B_2^2\rangle$ and $M = L_1$. we can find an intuitionistic partition L_2 such that *I - ind(*$M \cap L_2$*)* $\leq n-2$.

Let us take $A_i = \langle x, A_i^1, A_i^2\rangle$, $B_i = \langle x, B_i^1, B_1^i\rangle$ and let $M = L_1 \cap L_2 \cap$........L_{i-1}. we can find an intuitionistic partition L_i such that *I - ind(*$M \cap L_i$*)* $= n-i$. When we go to do $A_{n+1} = \langle x, A_{n+1}^1, A_{n+1}^2\rangle$, $B_{n+1} = \langle x, B_{n+1}^1, B_{n+1}^2\rangle$,we have *I - ind*$(L_1 \cap L_2 \cap$........$L_{n+1}) \leq -1 \iff L_1 \cap L_2 \cap$........$\cap L_{n+1} = \phi_\sim$. □

4. Intuitionistic large inductive dimension

Definition 4.1. (X, T) is an intuitionistic $\mathcal{AB}$ normal space if, given any intuitionistic disjoint $\mathcal{AB}$ closed sets $E = \langle x, E^1, E^2\rangle$ and $F = \langle x, F^1, F^2\rangle$, there are an intuitionistic $\mathcal{AB}$ neighborhood $U = \langle x, U^1, U^2\rangle$ of E and an intuitionistic $\mathcal{AB}$ neighborhood $V = \langle x, V^1, V^2\rangle$ of F that are also intuitionistic disjoint.

Definition 4.2. An intuitionistic large inductive dimension of intuitionistic $\mathcal{AB}$ normal space X is denoted *I-* $\mathcal{I}nd(X)$ and is defined as follows:

- (i) We say that the intuitionistic dimension of a space X, *I-*$\mathcal{I}nd(X)$ is -1 iff $X = \phi_\sim$.
- (ii) *I -* $\mathcal{I}nd(X) \leq n$ if for every intuitionistic $\mathcal{AB}$ closed set $C = \langle x, C^1, C^2\rangle \subseteq X$ and for every intuitionistic $\mathcal{AB}$ open set $U = \langle x, U^1, U^2\rangle$ there exists an intuitionistic $\mathcal{AB}$ open $V = \langle x, V^1, V^2\rangle$, $C \subseteq V$ such that $I\mathcal{AB}cl(V) \subseteq U$ and *I-*$\mathcal{I}nd(\mathcal{AB}\partial V) \leq n-1$. Where $I\mathcal{AB}\partial V$ is an intuitionistic boundary of V.
- (iii) *I-*$\mathcal{I}nd(X)$ = n if (ii) is true for n, but false for n-1.
- (iv) *I-*$\mathcal{I}nd(X) = \infty$ if for every n , *I-*$\mathcal{I}nd(X) \leq n$ is false.

Proposition 4.3. *An intuitionistic* $\mathcal{AB}$ *normal space X satisfies the inequality* *I-*$\mathcal{I}nd(X) \leq n$ *iff for every pair E and F of disjoint intuitionistic* $\mathcal{AB}$ *closed set of X exists an intuitionistic partition L between* $A = \langle x, A^1, A^2\rangle$ *and* $B = \langle x, B^1, B^2\rangle$ *such that I-*$\mathcal{I}nd(L) \leq n-1$.

Proof. Proof is obvious. □

Proposition 4.4. *For every intuitionistic separable space X we have* I-ind(X)= I-$\mathcal{I}$nd(X)

Proof. For every intuitionistic $\mathcal{AB}$ normal space X we have *I - ind(X)*$\leq$ *I -* $\mathcal{I}nd(X)$by definition. To show that *I -* $\mathcal{I}nd(X) \leq$ *I - ind(X)* we will use induction with respect to *I -ind(X)*, clearly one can suppose that *I -ind(X)* $< \infty$.

If *I- ind(X)* = -1 $\Rightarrow$ *I -ind(X)* $\leq$ *I-$\mathcal{I}nd$*(X). Assume that the inequality is proven for all intuitionistic separable metric space X of *I- ind(X)* $< n$ and consider an intuitionistic separable metric space X such that *I -ind(X)*= n.

Let $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, B^1, B^2\rangle$ be a pair of intuitionistic disjoint $\mathcal{AB}$ closed subset of X, by Proposition 3.26, there exists an intuitionistic partition L between A and B such that *I - ind(L)* $\leq n-1$,by the inductive assumption for every $k < n$, *I-$\mathcal{I}nd$(L)*$\leq n-1$ and according to the Proposition 4.4, *I - $\mathcal{I}nd$(X)* $\leq n$ and finally we get *I - $\mathcal{I}nd$(X)* $\leq$ *I - ind(X)* $\Rightarrow$ *$\mathcal{I}$ - Ind(X)* = *I - ind(X)*. □

5. Intuitionistic Addition theorem

Proposition 5.1. *Let X be an intuitionistic $\mathcal{AB}$ normal space and Y be an intuitionistic dense subset of X. Let also A and B be disjoint intuitionistic subset of X;$M = \langle x, M^1, M^2\rangle$ and $N = \langle x, N^1, N^2\rangle$ are intuitionistic $\mathcal{AB}$ open sets of X and $A \subset M$, $B \subset N$, $I\mathcal{AB}cl_X M \cap I\mathcal{AB}cl_X N = \phi_\sim$. Assume that C_Y be an intuitionistic partition between the intuitionistic sets $I\mathcal{AB}cl_X M \cap Y$ and $I\mathcal{AB}cl_X N \cap Y$ in Y.Then there exists an intuitionistic partition C in the intuitionistic space X between A and B such that $C \cap Y = C_Y$.*

Proof. Let $U = \langle x, U^1, U^2\rangle$ and $V = \langle x, V^1, V^2\rangle$ be any two disjoint intuitionistic $\mathcal{AB}$ - open subsets of Y such that $Y \setminus C_Y = U \cup V$, $I\mathcal{AB}cl_X M \cap Y \subset U$ and $I\mathcal{AB}cl_X N \cap Y \subset V$. Put $Y_1 = U \cup C_Y$,$Y_2 = V \cup C_Y$ and $X_i = I\mathcal{AB}cl_X Y_i$ for each i = 1,2.
It is easy to see that

(i) $Y_1 = \langle y, Y_1^1, Y_1^2\rangle$ and $Y_2 = \langle y, Y_2^1, Y_2^2\rangle$ are intuitionistic $\mathcal{AB}$ - closed subset of Y and $Y = Y_1 \cup Y_2$, $C_Y = Y_1 \cap Y_2$.

(ii) $X_1 = \langle x, X_1^1, X_1^2\rangle$ and $X_2 = \langle x, X_2^1, X_2^2\rangle$ are intuitionistic $\mathcal{AB}$ - closed subset of X and $X = I\mathcal{AB}cl_X Y = I\mathcal{AB}cl_X(Y_1 \cup Y_2)$= $I\mathcal{AB}cl_X(Y_1) \cup I\mathcal{AB}cl_X(Y_2)$ $= X_1 \cup X_2$.

(iii) $X_i \cap Y$ =Y_i for each $i = 1, 2$.

Moreover, we have $X_1 \cap B = \langle x, B^1, B^2\rangle$ =$\phi_\sim$ (because $Y_1 \cap (I\mathcal{AB}cl_X N \cap Y) = \phi_\sim$, so $Y_1 \cap N = \phi_\sim$ and consequently, $I\mathcal{AB}cl_X Y_1 \cap N = \phi_\sim$).

Analogously, we get $X_2 \cap A = \langle x, A^1, A^2\rangle = \phi_\sim$. Put $U = \langle x, U^1, U^2\rangle = X \setminus X_2$, $V = \langle x, V^1, V^2\rangle = X \setminus X_1$ *and* $C = X_1 \cap X_2$. Observe that U and V are disjoint intuitionistic $\mathcal{AB}$ open sets of the intuitionistic space X and $A \subset U$,$B \subset V$ *and* C $= X \setminus (U \cup V)$. In addition we have $C \cap Y = (X_1 \cap Y) \cap (X_2 \cap Y) = Y_1 \cap Y_2 = C_Y$. □

Remark 5.2. Let X be an intuitionistic space, x be a point of X,$B = \langle x, B^1, B^2\rangle$ is an intuitionistic $\mathcal{AB}$ closed set of X and $x \notin B$. Put $U = \langle x, U^1, U^2\rangle = X \setminus B$. In the intuitionistic space X let us consider intuitionistic $\mathcal{AB}$ open subsets $M = \langle x, M^1, M^2\rangle$, $V = \langle x, V^1, V^2\rangle$ and $W = \langle x, W^1, W^2\rangle$ such that

$$x \in M \subset I\mathcal{AB}cl(M) \subset V \subset I\mathcal{AB}cl(V) \subset W \subset I\mathcal{AB}cl(W) \subset U$$

.
put $N = \langle x, N^1, N^2\rangle = X \setminus I\mathcal{AB}cl(W)$. It is evident that $B \subset N$ and $I\mathcal{AB}cl(M) \cap I\mathcal{AB}cl(N) = \phi_\sim$. Let Y be an intuitionistic subset of X. In Y let us consider an intuitionistic $\mathcal{AB}$ open subset $P = \langle x, P^1, P^2\rangle$ such that $I\mathcal{AB}cl(M) \cap Y \subset P \subset V \cap Y$. It is evident that $I\mathcal{AB}cl_Y P \subset I\mathcal{AB}cl_Y(V \cap Y) \subset I\mathcal{AB}cl_Y(V) \cap Y \subset W \cap$

$Y \subset Y \setminus I\mathcal{AB}clN$. So the $I\mathcal{AB}\partial_Y P$ is an intuitionistic partition between the sets $I\mathcal{AB}clM \cap Y$, $I\mathcal{AB}clN \cap Y$ in Y. By *trI-ind*, *trI-$\mathcal{I}$nd* we will denote the natural transfinite extension of *I-ind trI-$\mathcal{I}$nd*.

Proposition 5.3. *Let X be an intuitionistic space and Y be an intuitionistic dense subset of X with $trI\text{-}\mathcal{I}ndY = \alpha$,where α is an intuitionistic ordinal number ≥ 0.Then for each point $x \in X$and every intuitionistic $\mathcal{AB}$ closed set $B \subset X$ such that $x \notin B$ there exist an intuitionistic partition C in X between the intuitionistic point x and the intuitionistic set B such that $trI\text{-}\mathcal{I}nd(C \cap Y) < \alpha$. In particular,if $trI\text{-}\mathcal{I}nd(Y)=0$ then $C \subset X \setminus Y$.*

Proof. Let us choose intuitionistic $\mathcal{AB}$ open subsets $M = \langle x, M^1, M^2\rangle$, $N = \langle x, N^1, N^2\rangle$ and $V = \langle x, V^1, V^2\rangle$ of the intuitionistic space (X, T) as in Remark 5.2. In the intuitionistic space (Y, T) by our assumption there exists an intuitionistic $\mathcal{AB}$ open set $P = \langle x, P^1, P^2\rangle$ such that $I\mathcal{AB}clM \cap Y \subset P \subset V \cap Y$ and $trI\text{-}\mathcal{I}nd\mathcal{AB}\partial_Y P < \alpha$.

Moreover,the intuitionistic set $I\mathcal{AB}\partial_Y P$ is an intuitionistic partition between the sets $I\mathcal{AB}clM \cap Y$,$I\mathcal{AB}clN \cap Y$ in Y. By proposition 5.2 there is an intuitionistic partition C in X between the intuitionistic point x and the intuitionistic set $B = \langle x, B^1, B^2\rangle$ such that $C \cap Y = I\mathcal{AB}\partial_Y P$. □

6. Intuitionistic product theorem

Definition 6.1. Let d be an intuitionistic dimension function which is monotone with respect to intuitionistic $\mathcal{AB}$ - closed subsets. Then the *intuitionistic finite sum dimension function* for d holds in an intuitionistic $\mathcal{AB}$ normal space (X, T) (in intuitionistic dimension $k \geq 0$ ($I\mathcal{F}SDF(d)$ in short) (respectively ($I\mathcal{F}SDF(d, k)$, if $d\ (A \cup B) = max\{dA, dB\}$for every intuitionistic $\mathcal{AB}$- closed in X intuitionistic sets $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, B^1, B^2\rangle$ (such that $dA, dB \leq k$).
For any intuitionistic space X,let us define

$$I\mathcal{F}SDF(d, X) = \begin{cases} \infty, & \text{if } I\mathcal{F}\text{SDF(d)hold in X;} \\ \min\{k \geq 0 : I\mathcal{F}SDF(d, k) & \\ \text{does not hold in X}\}, & \text{otherwise.} \end{cases}$$

It is evident that either $0 \leq I\mathcal{F}SDF(d, k) \leq dX - 1$ or $I\mathcal{F}SDF(d, k) = \infty$.

Moreover, for every intuitionistic $\mathcal{AB}$ closed set A in X we have $I\mathcal{F}SDF(d, A) \geq I\mathcal{F}SDF(d, X)$, and if $dA \leq I\mathcal{F}SDF(d, X)$ then $I\mathcal{F}SDF(d)$ holds in A.

Remark 6.2. Let(X, T) and (Y, T) be any two intuitionistic $\mathcal{AB}$ normal space, $I\text{-}ind(X \times Y) \leq I\text{-}ind(X) + I\text{-}ind(Y)$ if $I\mathcal{F}SDF(ind)$ holds in the intuitionistic space X and Y.

The proof is similar to (problem 2.4 H[6]) with suitable modification.

Remark 6.3. Let (X, T) and (Y, T) be any two intuitionistic $\mathcal{AB}$ normal spaces with $I\text{-}indX = m \geq 0$ and $I\text{-}indY = n \geq 0$ then

$$I - ind(X \times Y) = \begin{cases} m + n, & \text{if m=0 or n=0;} \\ 2(m + n) - 1, & \text{otherwise.} \end{cases}$$

Remark 6.4. Let (X, T) be an intuitionistic $\mathcal{AB}$ normal space. Let A and B be any two intuitionistic $\mathcal{AB}$ closed subsets such that $A \cup B = X_\sim$ then I - $indX$ $\leq max\{$I - $indA$,I - $indB\}$+1.

Proposition 6.5. *Let X and Y be an intuitionistic space and* $A = \langle x, A^1, A^2 \rangle$ *and* $B = \langle x, B^1, B^2 \rangle$ *are intuitionistic* $\mathcal{AB}$ *closed sets of X and Y,respectively. Assume that* I-ind$A \leq$ $I\mathcal{F}SDF$(I-ind,X) *and I-indB* $\leq I\mathcal{F}SDF$*(I-ind,Y). Then I-ind*$(A \times B) \leq$ *I-indA + I-indB.*

Proof. Observe that $I\mathcal{F}SDF(ind)$ holds in the subspaces $A = \langle x, A^1, A^2 \rangle$ and $B = \langle x, B^1, B^2 \rangle$. Hence by Remark 6.2, *I-ind*$(A \times B) \leq$ *I-indA*+*I-indB*. Hence proved. □

Proposition 6.6. *Let* (X, T) *be an intuitionistic space and trI-indX = 0. Then trI-ind*$(X \times Y)$ *= trI-indY for any intuitionistic space Y.*

Proof. Proof is obvious. □

Proposition 6.7. *Let* (X, T) *and* (Y, T) *be any two intuitionistic spaces with I-indX* $\leq m \geq 0$ *and I-indY* $\leq n \geq 0$*.Assume that* $I\mathcal{F}SDF$*(I-ind, X),* $I\mathcal{F}SDF$*(I-ind, Y)*$\geq k$ *for some* $k = 0, 1, ...or\infty$*. Then*

$$I - ind(X \times Y) \leq \begin{cases} m + n & \textit{if } n = 0, m = 0, or\ m,n \leq k, ; \\ 2(m + n) - k - 1 & \textit{otherwise.} \end{cases}$$

Proof. Observe that the proposition is valid for k = 0(by Remark 6.3) and $k = \infty$ (by Remark 6.2).Assume that k is an integer ≥ 1. Note that the proposition holds if

either $m, n \leq k$(by proposition 6.5) or m = 0,or n= 0 (by proposition 6.6).

Let us prove the statement for the remained part of the set $\{m, n \geq 0\}$.Put $s = m + n$. It would be enough if we prove that *I-ind*$(X \times Y) \leq 2s - k - 1$,for $s \geq k + 1$. Apply induction.Observe that for $s = k + 1$ the inequality evidently holds.Suppose that the inequality is valid for all $s : k + 1 < s < r$.

Let now $s = r$ and $m, n \geq 1$ for each point $p \in X \times Y$ and every intuitionistic $\mathcal{AB}$ open neighborhood W of p let us choose a rectangular intuitionistic $\mathcal{AB}$ open neighborhood $U \times V \subset W$ of this point such that

$$\textit{I-ind}(\mathrm{I}\mathcal{AB}\partial U) \leq m - 1 \text{ and } \textit{I-ind}(I\mathcal{AB}\partial V) \leq n - 1$$

Observe that $I\mathcal{AB}\partial(U \times V) = (I\mathcal{AB}\partial U \times I\mathcal{AB}clV) \cup (I\mathcal{AB}clU \times I\mathcal{AB}\partial V)$and *I-ind* $I\mathcal{AB}\partial(U \times V) \leq max\{$*I-ind*$\mathrm{I}(\mathcal{AB}\partial U \times I\mathcal{AB}clV)$,*I-ind*$(I\mathcal{AB}clU \times I\mathcal{AB}\partial V)\} + 1$ (By Definition 6.4). Moreover,we have

$I\mathcal{F}SDF$(*I-ind*, $I\mathcal{AB}\partial U$),$I\mathcal{F}SDF$(*I-ind*, $I\mathcal{AB}clU$),$I\mathcal{F}SDF$(*I-ind*, $I\mathcal{AB}\partial V$), $I\mathcal{F}SDF$(*I-ind*, $I\mathcal{AB}clV$)$\geq k$. (by the proposition condition

$$I\mathcal{F}SDF(\textit{I-ind}, X) \geq k \text{ and } I\mathcal{F}SDF(\textit{I-ind}, Y) \geq k)$$

This allow us to apply induction.By induction assumption,

$$max\{\textit{I-ind}(\ I\mathcal{AB}\partial U\ \times\ I\mathcal{AB}clV), \textit{I-ind}(I\mathcal{AB}clU \times I\mathcal{AB}\partial V)\} \leq 2(r - 1) - 1 - k.$$

$$max\{\textit{I-ind}(\ I\mathcal{AB}\partial U\ \times\ I\mathcal{AB}clV), \textit{I-ind}(I\mathcal{AB}clU \times I\mathcal{AB}\partial V)\} \leq 2r - 3 - k.$$

So by Remark 6.4 we get *I-ind*$(\ I\mathcal{AB}\partial U\ \times V\ )\ \leq\ 2r - 3 - k + 1 = 2r - 2 - k$. Hence the inequality *I-ind*$(X \times Y) \leq 2r - 1 - k$ holds. □

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