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Neutrosophic soft semirings

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Abstract. The purpose of this paper is to study semirings and its ideals by neutrosophic soft sets. After noting some preliminary ideas for subsequent use in Section 1 and 2, I have introduced and studied neutrosophic soft semiring, neutrosophic soft ideals, idealistic neutrosophic soft semiring, regular (intra-regular) neutrosophic soft semiring along with some of their characterizations in Section 3 and 4. In Section 5, I have illustrated all the necessary definitions and results by examples.

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1. Introduction

Most of the problems in economics, engineering and environment have various uncertainties. We cannot successfully use the classical methods because of various uncertainties typical for these problems. To solve this, the concept of fuzzy sets was introduced by Zadeh [22] in 1965 where each element have a degree of membership and has been extensively applied to many scientific fields. As a generalization of fuzzy sets, the intuitionistic fuzzy set was introduced by Atanassov [2] in 1986, where besides the degree of membership of each element there was considered a degree of non-membership with (membership value + non-membership value) ≤ 1.
There are also several well-known theories, for instances, rough sets, vague sets, interval-valued sets etc. which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their inherent difficulties. To overcome these difficulties, Molodtsov [17] introduced the soft sets which can be seen as a new mathematical tool for dealing with uncertainties. In the soft set theory, the problem of setting the membership function does not arise which makes the theory easily applies to many different fields. At present, works on soft set theory are progressing rapidly. Maji et al. [15] pointed out several directions for the applications of soft sets. They also studied several operations on the theory of soft sets. Aktas et

al. [1] studied the basic concepts of soft set theory and compared soft sets to fuzzy sets and rough sets providing some example to clarify the difficulties. Maji et al. [16], Feng et al. [10] also studied fuzzy soft sets based on fuzzy sets and soft sets. In 2010, Feng et al. [9] introduced an adjustable approach to fuzzy soft set based decision making.

In 2005, inspired from the sport games (winning/tie/ defeating), votes, from (yes /NA /no), from decision making(making a decision/ hesitating/not making), from (accepted /pending /rejected) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, F. Smarandache [20] combined the non-standard analysis [8, 18] with a tri-component logic/set/probability theory and with philosophy and introduced *Neutrosophic set* which represents the main distinction between *fuzzy* and *intuitionistic fuzzy* logic/set. Here he included the middle component, i.e., the neutral/ indeterminate/ unknown part (besides the truth/membership and falsehood/non-membership components that both appear in fuzzy logic/set) to distinguish between 'absolute membership and relative membership' or 'absolute non-membership and relative non-membership'(see, [13, 19]).

Motivated by this idea and combining it with the theory of soft sets, in 2013, P.K.Maji [14] introduced and studied 'Neutrosophic soft sets'. Broumi et al. [3, 4, 5, 6, 7] also enriched the theory of neutrosophic soft sets and used it in case of decision making problem. As a continuation of it, our main aim of this paper is to study 'Neutrosophic soft semirings', since semirings arise naturally in combinatorics, mathematical modelling, graph theory, automata theory, parallel computation system etc. [11, 12].

In this paper, I deal with the algebraic structure of the semirings by applying neutrosophic soft set theory. Here at first I recall some basic operations on 'Neutrosophic soft sets'. After that I introduce the concept of neutrosophic soft ideals of semirings and idealistic neutrosophic soft semirings and characterize these with the operations defined above. I also obtain some characterizations of neutrosophic soft regular semirings through idealistic neutrosophic soft semirings. At last, I illustrate the obtained results by proper examples.

2. Preliminaries

Definition 2.1. A semiring is a non-empty set S on which operations addition and multiplication have been defined such that $(S, +)$ is a semigroup, $(S, \cdot)$ is a semigroup and multiplication distributes over addition from either side.

We now recall following definitions from [14] for subsequent use.

Definition 2.2. A neutrosophic set A on the universe of discourse X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$, where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

Definition 2.3. A neutrosophic set A is contained in another neutrosophic set B i.e. $A \subseteq B$ if $\forall x \in X,\ T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x),\ F_A(x) \geq F_B(x)$.

Definition 2.4 ([17]). Let U be an initial universe set and E be a set of parameters. Let $\mathcal{P}(U)$ denotes the power set of U. Consider a nonempty set $A,\ A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

Definition 2.5. Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.6. Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if $A \subseteq B$, and $T_{F(e)}(x) \leq T_{G(e)}(x),\ I_{F(e)}(x) \leq I_{G(e)}(x),\ F_{F(e)}(x) \geq F_{G(e)}(x),\ \forall e \in A,\ x \in U$. We denote it by $(F, A) \subseteq (G, B)$; (G, B) is said to be neutrosophic soft super set of (F, A).

Definition 2.7. Two neutrosophic soft sets (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft subset of (G, B) and (G, B) is neutrosophic soft subset of (F, A). We denote it by $(F, A) = (G, B)$.

Definition 2.8. A neutrosophic soft set (H, A) over the universe U is termed to be empty or null neutrosophic soft set with respect to the parameter A if $T_{H(e)}(m) = 0,\ F_{H(e)}(m) = 0$ and $I_{H(e)}(m) = 0, \forall m \in U,\ \forall e \in A$. In this case, the null neutrosophic soft set is denoted by ϕ_A .
For non-null neutrosophic soft set any one of $T_{H(e)}(m) \neq 0,\ F_{H(e)}(m) \neq 0$ or $I_{H(e)}(m) \neq 0$, for some $m \in U$, and $e \in A$. In this case, we say $e \in NSupp(H, A)$.

Definition 2.9. Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the union of (H, A) and (G, B) is denoted by '$(H, A) \cup (G, B)$' and is defined by $(H, A) \cup (G, B) = (K, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$\begin{aligned} T_{K(e)}(m) &= T_{H(e)}(m),\ \text{if } e \in A \setminus B \\ &= T_{G(e)}(m),\ \text{if } e \in B \setminus A \\ &= \max\{T_{H(e)}(m), T_{G(e)}(m)\},\ \text{if } e \in A \cap B \end{aligned}$$

$$\begin{aligned} I_{K(e)}(m) &= I_{H(e)}(m),\ \text{if } e \in A \setminus B \\ &= I_{G(e)}(m),\ \text{if } e \in B \setminus A \\ &= \tfrac{I_{H(e)}(m)+I_{G(e)}(m)}{2},\ \text{if } e \in A \cap B \end{aligned}$$

$$\begin{aligned} F_{K(e)}(m) &= F_{H(e)}(m),\ \text{if } e \in A \setminus B \\ &= F_{G(e)}(m),\ \text{if } e \in B \setminus A \\ &= \min\{F_{H(e)}(m), F_{G(e)}(m)\},\ \text{if } e \in A \cap B. \end{aligned}$$

Definition 2.10. Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the intersection of (H, A) and (G, B) is denoted by '$(H, A) \cap (G, B)$' and is defined by $(H, A) \cap (G, B) = (K, C)$, where $C = A \cap B$

and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$T_{K(e)}(m) = \min\{T_{H(e)}(m), T_{G(e)}(m)\},$$
$$I_{K(e)}(m) = \frac{I_{H(e)}(m)+I_{G(e)}(m)}{2},$$
$$F_{K(e)}(m) = \max\{F_{H(e)}(m), F_{G(e)}(m)\}$$

for all $e \in C$ and $m \in U$.

Definition 2.11. Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the 'AND' of (H, A) and (G, B) is denoted by '$(H, A) \wedge (G, B)$' and is defined by $(H, A) \wedge (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:

$$T_{K(\alpha,\beta)}(m) = \min\{T_{H(\alpha)}(m), T_{G(\beta)}(m)\},$$
$$I_{K(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m)+I_{G(\beta)}(m)}{2},$$
$$F_{K(\alpha,\beta)}(m) = \max\{F_{H(\alpha)}(m), F_{G(\beta)}(m)\}$$

$\forall \alpha \in A, \beta \in B$ and $m \in U$.

Definition 2.12. Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the 'OR' of (H, A) and (G, B) is denoted by '$(H, A) \vee (G, B)$' and is defined by $(H, A) \vee (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:

$$T_{K(\alpha,\beta)}(m) = \max\{T_{H(\alpha)}(m), T_{G(\beta)}(m)\},$$
$$I_{K(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m)+I_{G(\beta)}(m)}{2},$$
$$F_{K(\alpha,\beta)}(m) = \min\{F_{H(\alpha)}(m), F_{G(\beta)}(m)\}$$

$\forall \alpha \in A, \beta \in B$ and $m \in U$.

Example 2.13. For examples on neutrosophic soft set we refer to [14].

After this section, throughout this paper unless otherwise mentioned S denotes a semiring.

3. Neutrosophic soft semiring and its ideals

Let (η, A) be a non-null soft set over a semiring S. Then (η, A) is called a soft semiring over S if $\eta(x)$ is a subsemiring of S for all $x \in Supp(\eta, A)$ i.e., $\eta(x) \neq \phi$. Now, a non-null neutrosophic soft set (η, A) over a semiring S is called a neutrosophic soft semiring (NSS) over S if $\eta(x)$ is a neutrosophic subsemiring of S for all $x \in NSupp(\eta, A)$, i.e., $\{< y, T_{\eta(x)}(y), I_{\eta(x)}(y), F_{\eta(x)}(y) >: y \in \eta(x) \text{ and } x \in A\}$ forms a neutrosophic subsemiring of S .
Let (α, A) and (β, B) be NSS over S. Then the NSS (β, B) is called a neutrosophic soft subsemiring of (α, A) if it satisfies

(i) $B \subseteq A$
(ii) $\beta(x)$ is a subsemiring of $\alpha(x)$ for all $x \in NSupp(\beta, B)$.

Now by the definition of intersection $(\alpha, A) \cap (\beta, B) = (\gamma, C)$ where $C = A \cap B$ and $\gamma(x) = \alpha(x) \cap \beta(x)$ for all $x \in C$, is a non-null neutrosophic soft set over S. Since intersection of two neutrosophic subsemiring is also a neutrosophic subsemiring, $(\alpha, A) \cap (\beta, B)$ is a neutrosophic soft semiring over S.

If we define this type of intersection by bi-intersection then we can extend this to arbitrary intersection as follows:
Suppose $(\alpha_i, A_i)_{i\in Index}$ be a family of neutrosophic soft semirings over S. Then $\underset{i\in Index}{\cap}(\alpha_i, A_i) = (\beta, B)$ where $B = \cap A_i$ i.e., $T_{\beta(e)}(m) = \min_i T_{\alpha_i(e)}(m),\ I_{\beta(e)}(m) = \frac{\sum I_{\alpha_i}(m)}{\sum i},\ F_{\beta(e)}(m) = \max_i F_{\alpha_i(e)}(m)$, where $\sum i$ denotes the total number of respective family.
We can associate with intersection "the AND operation"of two NSS (η, A) and (γ, B) over the common universe S, defined as

$$(\eta, A) \wedge (\gamma, B) = (\nu, A \times B)$$

where $\nu(x, y) = \eta(x) \cap \gamma(y)$ for all $(x, y) \in A \times B$.
This operation can also be extended to $\underset{i\in Index}{\wedge}(\alpha_i, A_i)$ where $(\alpha_i, A_i)_{i\in Index}$ are the non-empty family of NSS over S. We can also easily verify that $\underset{i\in Index}{\wedge}(\alpha_i, A_i)$ is also a NSS over S by following:
From definition $\underset{i\in Index}{\wedge}(\alpha_i, A_i) = (\beta, B)$ where

$$B = \prod_{i\in Index} A_i \text{ and } \beta(x) = \underset{i\in Index}{\cap}\alpha_i(x_i)$$

for all $x = (x_i)_{i\in Index} \in B$. Suppose the neutrosophic soft set (β, B) is non-null. If $x = (x_i)_{i\in Index} \in NSupp(\beta, B)$, then $\beta(x) = \underset{i\in Index}{\cap}\alpha_i(x_i) \neq \phi$. Since for each $i \in Index$, (α_i, A_i) is a NSS over S, the non-empty set $\alpha_i(x_i)$ is a neutrosophic subsemiring of S and so $\beta(x)$ is a neutrosophic subsemiring of S for all $x \in NSupp(\beta, B)$. Therefore $\underset{i\in Index}{\wedge}(\alpha_i, A_i)$ is a NSS over S.
We can now check the case (i.e., NSS) for the operation 'union'. It can be easily shown that $\underset{i\in Index}{\cup}(\alpha_i, A_i)$ is a NSS over S only if $\{A_i|i \in Index\}$ are pairwise disjoint i.e,, $i \neq j$ implies $A_i \cap A_j = \phi$. Following is the way of proof:
Suppose $\underset{i\in Index}{\cup}(\alpha_i, A_i) = (\beta, B)$. Then $B = \underset{i\in Index}{\cup} A_i$ and for all $x \in B$, $\beta(x) = \underset{i\in In(x)}{\cup}\alpha_i(x)$ where $In(x) = \{i \in Index|x \in A_i\}$. First we note that (β, B) is non-null as $NSupp(\beta, B) = \underset{i\in Index}{\cup} NSupp(\alpha_i, A_i) \neq \phi$. Let $x \in NSupp(\beta, B)$. Then $\beta(x) = \underset{i\in In(x)}{\cup}\alpha_i(x) \neq \phi$ and so $\alpha_{i_0}(x) \neq \phi$ for some $i_0 \in In(x)$. But by the hypothesis, we know that $\{A_i|i \in Index\}$ are pairwise disjoint. Hence the above i_0 is indeed unique and so $\beta(x)$ coincides with $\alpha_{i_0}(x)$. Moreover, since (α_{i_0}, A_{i_0}) is a NSS over S we deduce that the non-empty set $\alpha_{i_0}(x)$ is a neutrosophic subsemiring of S; which implies $\beta(x)$ is a neutrosophic subsemiring of S for all $x \in NSupp(\beta, B)$. Therefore $\underset{i\in Index}{\cup}(\alpha_i, A_i)$ is a NSS over S.

Definition 3.1. Let (η, A) be a NSS over a semiring S. A non-null neutrosophic soft set (γ, I) over S is called a neutrosophic soft left ideal of (η, A) if

(i) $I \subseteq A$
(ii) $\gamma(x)$ is an left ideal of $\eta(x)$ for all $x \in NSupp(\gamma, I)$, i.e.,

$$\begin{aligned}
&T_{\gamma(x)}(y_1+y_2) \geq \min\{T_{\gamma(x)}(y_1), T_{\gamma(x)}(y_2)\}\\
&I_{\gamma(x)}(y_1+y_2) \geq \tfrac{I_{\gamma(x)}(y_1)+I_{\gamma(x)}(y_2)}{2}\\
&F_{\gamma(x)}(y_1+y_2) \leq \max\{F_{\gamma(x)}(y_1), F_{\gamma(x)}(y_2)\}\\
&T_{\gamma(x)}(ay_1) \geq T_{\gamma(x)}(y_1)\\
&I_{\gamma(x)}(ay_1) \geq I_{\gamma(x)}(y_1)\\
&F_{\gamma(x)}(ay_1) \leq F_{\gamma(x)}(y_1)
\end{aligned}$$

for all $y_1, y_2 \in \gamma(x)$ and $a \in \eta(x)$.
Similarly we can define right ideal also.

Proposition 3.2. *Let (γ_1, I_1) and (γ_2, I_2) be neutrosophic soft ideals of a NSS (η, A) over a semiring S. Then $(\gamma_1, I_1) \cap (\gamma_2, I_2)$ is a neutrosophic soft ideal of (η, A) if it is non-null.*

Proof. Assume that (γ_1, I_1) and (γ_2, I_2) be neutrosophic soft ideals of a NSS (η, A) over a semiring S. Then $(\gamma_1, I_1) \cap (\gamma_2, I_2) = (\gamma, I)$ where $I = I_1 \cap I_2$ and $\gamma(x) = \gamma_1(x) \cap \gamma_2(x)$ for all $x \in I$. Obviously, $I \subseteq A$. Suppose that the neutrosophic soft set (γ, I) is non-null. Then for $x \in NSupp(\gamma, I)$, $\gamma(x) = \gamma_1(x) \cap \gamma_2(x) \neq \phi$. Since (γ_1, I_1) and (γ_2, I_2) are neutrosophic soft ideals of (η, A), $\gamma_1(x)$ and $\gamma_2(x)$ are ideals of $\eta(x)$ and so $\gamma(x)$ is an ideal of $\eta(x)$ for all $x \in NSupp(\gamma, I)$. Therefore $(\gamma_1, I_1) \cap (\gamma_2, I_2)$ is a neutrosophic soft ideal of (η, A). □

Proposition 3.3. *Let (γ_1, I_1) and (γ_2, I_2) be neutrosophic soft ideals of a NSS (η, A) over a semiring S. If I_1 and I_2 are disjoint then $(\gamma_1, I_1) \cup (\gamma_2, I_2)$ are neutrosophic soft ideal of (η, A) when it is non-null.*

Proof. Assume that (γ_1, I_1) and (γ_2, I_2) be neutrosophic soft ideals of a NSS (η, A) over a semiring S. Then by definition $(\gamma_1, I_1) \cup (\gamma_2, I_2) = (\gamma, I)$ where $I = I_1 \cup I_2$ and for every $x \in I$, $\gamma(x) = \begin{cases} \gamma_1(x) & \text{if } x \in I_1 \setminus I_2 \\ \gamma_2(x) & \text{if } x \in I_2 \setminus I_1 \\ \gamma_1(x) \cup \gamma_2(x) & \text{if } x \in I_1 \cap I_2 \end{cases}$. Suppose $I_1,\ I_2$ are disjoint. Then for every $x \in NSupp(\gamma, I)$, we know that either $x \in I_1 \setminus I_2$ or $x \in I_2 \setminus I_1$. If $x \in I_1 \setminus I_2$ then $\gamma(x) = \gamma_1(x) \neq \phi$ is an ideal of $\eta(x)$ and if $x \in I_2 \setminus I_1$ then $\gamma(x) = \gamma_2(x) \neq \phi$ is also an ideal of $\eta(x)$. Therefore $(\gamma_1, I_1) \cup (\gamma_2, I_2) = (\gamma, I)$ is a neutrosophic soft ideal of (η, A). □

We can combine all the above results to the following Theorem:

Theorem 3.4. *Let (η, A) be a NSS over S and $(\alpha_i, A_i)_{i \in Index}$ are the family of non-empty neutrosophic soft ideals of (η, A). Then we have*

(i) $\underset{i \in Index}{\cap} (\alpha_i, A_i)$ *is a neutrosophic soft ideal of (η, A), if it is non-null.*
(ii) *If both $\underset{i \in Index}{\wedge} (\eta, A)$ and $\underset{i \in Index}{\wedge} (\alpha_i, A_i)$ are non-null, then $\underset{i \in Index}{\wedge} (\alpha_i, A_i)$ is a neutrosophic soft ideal of $\underset{i \in Index}{\wedge} (\eta, A)$.*
(iii) *If $\{A_i | i \in Index\}$ are pairwise disjoint, then $\underset{i \in Index}{\cup} (\alpha_i, A_i)$ is a neutrosophic soft ideal of (η, A).*

4. Idealistic neutrosophic soft semirings

Definition 4.1. Let (η, A) be a nonempty neutrosophic soft set over S. Then (η, A) is called an idealistic neutrosophic soft semiring over S, if $\eta(x)$ is an ideal of $N(S)$ for all $x \in NSupp(\eta, A)$, where $N(S)$ is the neutrosophic set on the universe S.

Proposition 4.2. *Let (α, A) and (β, B) be idealistic NSS over S. Then $(\alpha, A) \cap (\beta, B)$ is an idealistic NSS over S.*

Proof. By definition of Intersection, we can write $(\alpha, A) \cap (\beta, B) = (\gamma, C)$ where $C = A \cap B$ and $\gamma(x) = \alpha(x) \cap \beta(x)$ for all $x \in C$ i.e., for $x \in NSupp(\gamma, C)$, $\gamma(x) = \alpha(x) \cap \beta(x) \neq \phi$. Now since $\alpha(x)$ and $\beta(x)$ both are ideals of $N(S)$, $\gamma(x)$ is an ideal of $N(S)$ for all $x \in NSupp(\gamma, C)$. Hence (γ, C) is an idealistic neutrosophic soft semiring over S. □

Proposition 4.3. *Let (α, A) and (β, B) be idealistic NSS over S. Then $(\alpha, A) \wedge (\beta, B)$ is an idealistic NSS over S.*

Proof. By definition of AND, we can write $(\alpha, A) \wedge (\beta, B) = (\gamma, C)$ where $C = A \times B$ and $\gamma(x, y) = \alpha(x) \cap \beta(y)$ for all $(x, y) \in C$ i.e., for $(x, y) \in NSupp(\gamma, C)$, $\gamma(x) = \alpha(x) \cap \beta(y) \neq \phi$. Now since $\alpha(x)$ and $\beta(x)$ both are ideals of $N(S)$, $\gamma(x, y)$ is an ideal of $N(S)$ for all $(x, y) \in NSupp(\gamma, C)$. Therefore (γ, C) is an idealistic neutrosophic soft semiring over S. □

Proposition 4.4. *Let (α, A) and (β, B) be idealistic NSS over S. Then $(\alpha, A) \cup (\beta, B)$ is an idealistic NSS over S, provided A and B are disjoint.*

Proof. By definition of Union, we can write $(\alpha, A) \cup (\beta, B) = (\gamma, C)$ where $C = A \cup B$ and for every $x \in C$,

$$\gamma(x) = \begin{cases} \alpha(x) & \text{if } x \in A \setminus B \\ \beta(x) & \text{if } x \in B \setminus A \\ \alpha(x) \cup \beta(x) & \text{if } x \in A \cap B. \end{cases}$$

Suppose $A \cap B = \phi$. Then for every $x \in NSupp(\gamma, C)$, either $x \in A \setminus B$ or $x \in B \setminus A$ so as $\gamma(x) = \alpha(x)$ or $\gamma(x) = \beta(x)$ - is an ideal of $N(S)$. Therefore (γ, C) is an idealistic NSS over S. □

Combining the above three propositions we can deduce the following theorem:

Theorem 4.5. *Let $(\alpha_i, A_i)_{i \in Index}$ a non-empty family of idealistic neutrosophic soft semirings over S. Then we have*

(i) $\underset{i \in Index}{\cap} (\alpha_i, A_i)$ *is an idealistic neutrosophic soft semirings over S.*
(ii) $\underset{i \in Index}{\wedge} (\alpha_i, A_i)$ *is an idealistic neutrosophic soft semirings over S.*
(iii) $\underset{i \in Index}{\cup} (\alpha_i, A_i)$ *is an idealistic neutrosophic soft semirings over S, provided $\{A_i | i \in Index\}$ are pairwise disjoint.*

Definition 4.6. Let (η, A) be a neutrosophic soft set over a semiring S. Then

(i) (η, A) is called a right idealistic NSS if $\eta(x)$ is a right ideal of $N(S)$ for all $x \in A$.

(ii) (η, A) is called a left idealistic NSS if $\eta(x)$ is a left ideal of $N(S)$ for all $x \in A$.
(iii) (η, A) is called a bi-idealistic NSS if $\eta(x)$ is a bi-ideal of $N(S)$ for all $x \in A$.
(iv) (η, A) is called a quasi-idealistic NSS if $\eta(x)$ is a quasi-ideal of $N(S)$ for all $x \in A$.

Definition 4.7. The semiring S is called regular (resp. intra-regular) if for each element x of S there exists $a \in S$ (resp. $a_i, a'_i \in S,\ i \in N$) such that $x = xax$ (resp. $x = \sum_i a_i x^2 a'_i,\ i \in N$).

Definition 4.8. A NSS (η, A) over a semiring S is called regular (resp. intra-regular) NSS if for all $x \in A$, $\eta(x)$ is regular (resp. intra-regular).

Definition 4.9. The semiring S is called regular (resp. intra-regular) NSS if every NSS (η, A) over S is regular (resp. intra-regular).

Theorem 4.10. *Let S be a neutrosophic soft regular (resp. intra-regular) semiring and (α, A), (β, B) are NSS over S. Then*
(i) $(\alpha, A) \wedge (\beta, B)$ *is a neutrosophic soft regular (resp. intra-regular) semiring.*
(ii) $(\alpha, A) \cap (\beta, B)$ *is a neutrosophic soft regular (resp. intra-regular) semiring.*
(iii) $(\alpha, A) \cup (\beta, B)$ *is a neutrosophic soft regular (resp. intra-regular) semiring, provided A and B are disjoint.*

Proof. Let $(\gamma, C) =(\alpha, A) \wedge (\beta, B)$. Then $\gamma(x, y) = \alpha(x) \cap \beta(y)$ for any $(x, y) \in A \times B$. Since (α, A) and (β, B) are NSS over S, $\alpha(x)$ and $\beta(y)$ are neutrosophic subsemirings of S for all $x \in A$, $y \in B$ and so $\alpha(x) \cap \beta(y)$ is a neutrosophic subsemirings of S. Therefore (γ, C) is a NSS over S. Since S is neutrosophic soft regular (resp. intra-regular), we have (γ, C) is a neutrosophic soft regular (resp. intra-regular) semiring. The proof of (ii) and (iii) follows similarly. □

Definition 4.11 ([17])**.** The product of two soft sets (α, A) and (β, B) over S is the soft set (γ, C) where $\gamma(x, y) = \alpha(x)\beta(y)$, $(x, y) \in C = A \times B$ and denoted by $(\alpha, A) \star (\beta, B) = (\gamma, A \times B)$.

Theorem 4.12 ([21])**.** *A semiring S is regular if and only if for any right ideal R and any left ideal L of S we have $RL = R \cap L$.*

Theorem 4.13. *The semiring S is neutrosophic soft regular if and only if $(\alpha, A) \wedge (\beta, B) = (\alpha, A) \star (\beta, B)$ for every right idealistic NSS (α, A) and left idealistic NSS (β, B) over S.*

Proof. Suppose S is neutrosophic soft regular and (α, A), (β, B) are respectively a right idealistic NSS and a left idealistic NSS over S. Then $\alpha(x)$ and $\beta(y)$ are respectively, the right ideal and the left ideal of $N(S)$ for all $x \in A$, $y \in B$. Since S is neutrosophic soft regular $\alpha(x) \cap \beta(y) = \alpha(x)\beta(y)$. Therefore $(\alpha, A) \wedge (\beta, B) = (\alpha, A) \star (\beta, B)$.
Conversely, suppose (η, X) is a NSS over S and $x \in X$. Let P and Q be any right ideal and any left ideal of $\eta(x)$. Then their exist neutrosophic soft sets (α, A) and (β, B) over S such that $\alpha(z) = P$ and $\beta(y) = Q$ for all $z \in A$ and $y \in B$. Then (α, A) and (β, B) are respectively the right idealistic NSS and the left idealistic NSS.

By assumption, $(\alpha, A) \wedge (\beta, B) = (\alpha, A) \star (\beta, B)$ and so, $\alpha(z) \cap \beta(y) = \alpha(z) \star \beta(y)$ i.e., $P \cap Q = PQ$ which implies S is neutrosophic soft regular (by Theorem 4.12). □

Theorem 4.14 ([21]). *Let S be a semiring. Then the following conditions are equivalent.*

(i) *S is intra-regular.*
(ii) *$L \cap R \subseteq LR$ for every left ideal L and every right ideal R of S.*

Theorem 4.15. *The semiring S is neutrosophic soft intra-regular if and only if $(\alpha, A) \wedge (\beta, B) \subseteq (\alpha, A) \star (\beta, B)$ for every left idealistic NSS (α, A) and every right idealistic NSS (β, B) over S.*

Proof. Suppose S is neutrosophic soft intra-regular and (α, A), (β, B) be a left idealistic NSS and a right idealistic NSS over S, respectively. Then $\alpha(x)$ and $\beta(y)$ are respectively, left ideal and right ideal of $N(S)$ for all $x \in A$, $y \in B$ and since S is neutrosophic soft intra-regular $\alpha(x) \cap \beta(y) \subseteq \alpha(x)\beta(y)$. Therefore $(\alpha, A) \wedge (\beta, B \subseteq (\alpha, A) \star (\beta, B)$.
Conversely, suppose (η, X) is a NSS over S and $x \in X$. Let P and Q be any left ideal and any right ideal of $\eta(x)$. Then their exist neutrosophic soft sets (α, A) and (β, B) over S such that $\alpha(z) = P$ and $\beta(y) = Q$ for all $z \in A$ and $y \in B$. Then (α, A) and (β, B) are respectively the left idealistic NSS and the right idealistic NSS. By assumption, $(\alpha, A) \wedge (\beta, B) \subseteq (\alpha, A) \star (\beta, B)$ and so, $\alpha(z) \cap \beta(y) \subseteq \alpha(z) \star \beta(y)$ i.e., $P \cap Q \subseteq PQ$ which implies S is neutrosophic soft intra-regular (by Theorem 4.14). □

Theorem 4.16 ([21]). *Let S be a semiring. Then the following conditions are equivalent.*

(i) *S is both regular and intra-regular.*
(ii) *$B = B^2$ for every bi-ideal B of S.*
(iii) *$Q = Q^2$ for every quasi-ideal Q of S.*

Theorem 4.17. *If one of the following conditions holds for S, then S is both neutrosophic soft regular and neutrosophic soft intra-regular.*

(i) *$(\alpha, A) \cap (\beta, B) = (\alpha, A) \star (\beta, B)$ for any two bi-idealistic NSS (α, A) and (β, B) over S.*
(ii) *$(\alpha, A) \cap (\beta, B) = (\alpha, A) \star (\beta, B)$ for any two quasi-idealistic NSS (α, A) and (β, B) over S.*
(iii) *$(\alpha, A) \cap (\beta, B) = (\alpha, A) \star (\beta, B)$ for any bi-idealistic NSS (α, A) and quasi-idealistic NSS (β, B) over S.*
(iv) *$(\alpha, A) \wedge (\beta, B) = (\alpha, A) \star (\beta, B)$ for any quasi-idealistic NSS (α, A) and bi-idealistic NSS (β, B) over S.*

Proof. We only show (i); others follows similarly.
Let P be a bi-ideal of $N(S)$ and (α, A) is a neutrosophic soft set over S such that $\alpha(x) = P$ for all $x \in A$. Then (α, A) is a bi-idealistic NSS over S. By the assumption $(\alpha, A) = (\alpha, A) \star (\alpha, A)$ i.e., $P = P^2$. Therefore by Theorem 4.16, S is both neutrosophic soft regular and neutrosophic soft intra-regular. □

5. Illustration by examples

Throughout the examples, I have used the following notational convention: Let (α, P) is a neutrosophic soft set over $U = \{e, f, g, ...\}$ and $q \in P$. Then $\alpha(q) = \{e\}$ is equivalent to $\alpha(q) = \{< e; r, s, t >\}$, where $r, s, t \in [0, 1]$ are the corresponding neutrosophic values.

Example 5.1. Let $S = \{0, a, b, c\}$. Then S forms a semiring with the following operations:

$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	c	c	c	c

and

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	a	a
c	0	a	a	a

Let (η, A) and (γ, I) are two neutrosophic soft sets over S with $A = \{x, y, z\}$, $I = \{x, y\}$ and defined as:

$S\backslash A$	x	y	z
0	(1,0,0)	(1,0,0)	(1,0,0)
a	(0.5,0.3, 0.5)	(0.6,0.3, 0.3)	(0.4,0.4,0.4)
b	(0.7,0.2,0.3)	(0.7,0.3,0.2)	(0.7,0.2,0.2)
c	(0.8,0.1,0.2)	(0.9,0.1,0.1)	(0.8,0.2,0.2)

$S\backslash I$	x	y
0	(1,0,0)	(1,0,0)
a	(0.4,0.2, 0.7)	(0.5,0.3, 0.4)
b	(0.7,0.15,0.5)	(0.6,0.2,0.4)
c	(0.7,0.1,0.3)	(0.8,0.1,0.2)

Obviously $I \subseteq A$. Now if we define $\eta(x) = \{0, a, b, c\}$, $\eta(y) = \{0, b, c\}$, $\eta(z) = \{0, c\}$, $\gamma(x) = \{0, a, b\}$, $\gamma(y) = \{0\}$ then $\gamma(t)$ is an ideal of $\eta(t)$ for all $t \in I$. Therefore (γ, I) is an neutrosophic soft ideal of (η, A).

Example 5.2. Let S, (η, A) and (γ, I) are as in Example 5.1. Define another neutrosophic soft set (α, J) over S as follows, where $J = \{x\}$ and $\alpha(x) = \{0, a\}$.

$S\backslash J$	x
0	(1,0,0)
a	(0.3,0.2, 0.8)
b	(0.5,0.1,0.6)
c	(0.6,0.1,0.3)

Then (α, J) is an neutrosophic soft ideal of (η, A). Now

$S\backslash I \cap J$	x
0	(1,0,0)
a	(0.3,0.2, 0.8)
b	(0.5,0.125,0.6)
c	(0.6,0.1,0.3)

Then $(\beta, I \cap J)$ is also an neutrosophic soft ideal of (η, A) where $\beta(x) = \gamma(x) \cap \alpha(x) = \{0, a\}$.

Example 5.3. Let S, (η, A) and (γ, I) are as in Example 5.1. Define another neutrosophic soft set (δ, K) over S as follows, where $K = \{y, z\}$

$S \backslash K$	y	z
0	(1,0,0)	(1,0,0)
a	(0.6,0.2, 0.5)	(0.3,0.3, 0.7)
b	(0.6,0.1,0.3)	(0.5,0.2,0.3)
c	(0.7,0.1,0.1)	(0.7,0.15,0.2)

Then

$S \backslash I \cup K$	x	y	z
0	(1,0,0)	(1,0,0)	(1,0,0)
a	(0.4,0.2, 0.7)	(0.6,0.25, 0.4)	(0.3,0.3,0.7)
b	(0.7,0.15,0.5)	(0.6,0.15,0.3)	(0.5,0.2,0.3)
c	(0.7,0.1,0.3)	(0.8,0.1,0.1)	(0.7,0.15,0.2)

Now if we define $\eta(x) = \{0, a, b, c\}$, $\eta(y) = \{0, a, b\}$, $\eta(z) = \{0, a, c\}$, $\gamma(x) = \{0, a, b\}$, $\gamma(y) = \{0\}$, $\delta(y) = \{0, a\}$, $\delta(z) = \{a\}$ then $(\gamma, I) \cup (\delta, K)$ will be a neutrosophic soft ideal of (η, A), though $I \cap K \neq \phi$.

Example 5.4. Let S, (γ, I) and (δ, K) are as in Example 5.1 and Example 5.3, respectively. Now define $(\gamma, I) \wedge (\delta, K)$ as follows

$S \backslash I \times K$	(x, y)	(x, z)	(y, y)	(y, z)
0	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
a	(0.4,0.2, 0.7)	(0.3,0.25, 0.7)	(0.5,0.25,0.5)	(0.3,0.3,0.7)
b	(0.6,0.125,0.5)	(0.5,0.175,0.5)	(0.6,0.15,0.4)	(0.5,0.2,0.4)
c	(0.7,0.1,0.3)	(0.7,0.125,0.3)	(0.7,0.1,0.2)	(0.7,0.125,0.2)

Then $(\gamma, I) \wedge (\delta, K)$ is an neutrosophic soft ideal of (η, A)

Example 5.5. The neutrosophic soft set (η, A) defined in Example 5.1 is a left ideal of $N(S) := \{< 0; 1, 0.4, 0 >, < a; 0.9, 0.4, 0.2 >, < b; 0.9, 0.4, 0.2 >, < c; 1, 0.2, 0.1 > \}$.

Example 5.6. Let $S = \{0, a, b, c\}$. Then S forms a semiring with the following operations:

$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	c	c	c	c

and

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Define $N(S)$ and neutrosophic soft set (γ, B) over S, where $B = \{x, y\}$, as follows:

S	$N(S)$
0	(1,0,0)
a	(0.6,0.4, 0.3)
b	(0.7,0.4,0.2)
c	(0.8,0.3,0.1)

$S\backslash B$	x	y
0	(1,0,0)	(1,0,0)
a	(0.5,0.3, 0.4)	(0.6,0.3,0.3)
b	(0.6,0.4,0.3)	(0.6,0.3,0.3)
c	(0.8,0.2,0.2)	(0.7,0.2,0.2)

Then if $\gamma(x) = \{0, c\}$ and $\gamma(y) = \{0, a, b\}$, (γ, B) is a neutrosophic soft bi-ideal of $N(S)$.

Example 5.7. Let $S = \{0, a, b\}$. Define addition $(+)$ and multiplication $(\cdot)$ on S as follows:

$+$	0	a	b
0	0	a	b
a	a	a	b
b	b	b	b

$\cdot$	0	a	b
0	0	0	0
a	0	a	b
b	0	a	b

Then $(S, +, \cdot)$ is a regular semiring.
Now if we define $N(S)$ as $\{< 0; 1, 0, 0 >, \ < a; 0.6, 0.8, 0.5 >, \ < b; 0.9, 0.5, 0.4 >\}$, then S will be neutrosophic soft regular semiring [$\because$ every neutrosophic subsemiring $\{0\}$, $\{0, a\}$, $\{0, a, b\}$ are regular].

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