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Complementary nil domination in fuzzy graphs

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Abstract. In this paper, the complementary nil dominating set and its numbers in a fuzzy graph are defined. The bounds on this number is obtained for some standard fuzzy graphs. Theorems related to the above concepts are derived. Relation between complementary nil domination number and domination numbers are also derived. In this paper only connected fuzzy graphs which are not complete are considered.

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1. Introduction

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths, cycles and connectedness are introduced by Rosenfeld [9]. Bhattacharya [1] has established some connectivity concepts regarding fuzzy cut node and fuzzy bridges. A generalization of intersection graphs to fuzzy intersection graphs are presented by McAlester [3] in 1988. Mordeson [4] introduced the concept of fuzzy line graphs and the basic properties in the year 1993. The concept of Domination in fuzzy graphs are introduced by A. Somasundaram and S. Somasundaram [10] in 1998. In 2013, Antipodal interval-valued fuzzy graphs and Balanced interval valued fuzzy graphs are introduced by H. Rashmanlou and M. Pal [7, 8]. Complementary nil domination in a crisp graph was introduced by Tamizh Selvam and Robinson Chellathurai [11] in 2009. In this paper, we introduced the concept of complementary nil domination in fuzzy graphs. Theorems related to the above concepts are stated and proved. Relation between complementary nil domination number and domination numbers are also derived. Here, only connected fuzzy graphs which are not complete are considered.

2. Preliminaries

In this section, some preliminary definitions like dominating set, domination numbers and independent domination numbers are given.

Definition 2.1 ([10])**.** Let V be a finite non empty set and E be the collection of two element subsets of V. A *fuzzy graph* $G = (\sigma, \mu)$ is a set with two functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. The *scalar cardinality* of $S \subseteq V$ is defined by $\sum_{u \in S} \sigma(u)$. The *order* (denoted by p) and *size* (denoted by q) of a fuzzy graph G are the scalar cardinality of σ and μ respectively. An edge $e = (u, v)$ of a fuzzy graph is called an *effective edge* if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. If $e = (u, v)$ is an effective edge, then u and v are adjacent vertices and e is incident with u and v.

Definition 2.2 ([2])**.** A fuzzy graph $G = (\sigma, \mu)$ is said to be *M-strong fuzzy graph* if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $(u, v) \in E$.

Definition 2.3 ([6])**.** A fuzzy graph $G = (\sigma, \mu)$ is said to be *complete fuzzy graph* if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 2.4 ([10])**.** Let $u, v \in V$ and $e = (u, v) \in E$ in a fuzzy graph $G = (\sigma, \mu)$. Then $N(u) = \{v \in V : \mu(u, v) = \sigma(u) \wedge \sigma(v)\}$ is called *open neighbourhood* of u and $N[u] = N(u) \cup \{u\}$ is called *closed neighbourhood* of u. If $N(u) = \phi$ then u is said to be *isolated vertex*.

Definition 2.5 ([5])**.** A fuzzy graph $G = (\sigma, \mu)$ is said to be *bipartite* if the vertex set V can be partitioned into two sets σ_1 defined on V_1 and σ_2 defined on V_2 such that $\mu(v_1, v_2) = 0$ if $(v_1, v_2) \in V_1 \times V_1$ or $(v_1, v_2) \in V_2 X V_2$. A bipartite fuzzy graph $G = (\sigma, \mu)$ is said to be *complete bipartite* if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ and is denoted by K_{σ_1, σ_2}.

Definition 2.6 ([5])**.** A *path* in a fuzzy graph G is a sequence of distinct vertices $u_0, u_1, u_2, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) = \sigma(u_{i-1}) \wedge \sigma(u_i), 1 \leq i \leq n,\ n > 0$ is called the *length* of the path. The path in a fuzzy graph is called a *fuzzy cycle* if $u_0 = u_n, n \geq 3$. A fuzzy graph is said to be *cyclic* if it contains at least one cycle, otherwise it is called *acyclic*.

Definition 2.7 ([1])**.** A fuzzy graph is said to be *connected* if there exists at least one path between every pair of vertices. A connected acyclic fuzzy graph is said to be a *fuzzy tree*.

Definition 2.8 ([5])**.** A vertex in a fuzzy graph having only one neighbour is called a *pendent vertex*. Otherwise it is called *non-pendent vertex*. An edge in a fuzzy graph incident with a pendent vertex is called a *pendent edge*. Otherwise it is called *non-pendent edge*. A vertex in a fuzzy graph adjacent to the pendent vertices is called a *support* of the pendent edges.

Definition 2.9 ([10])**.** Let $G = (\sigma, \mu)$ be a fuzzy graph and let $u, v \in V$. If $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ then u dominates v (or v is dominated by u) in G. A subset D of V is called a *dominating set* in G if for every $v \notin D$ there exist $u \in D$ such that u dominates v. The minimum scalar cardinality taken over all dominating set is called *domination number* and is denoted by the symbol γ. The maximum scalar cardinality of a minimal dominating set is called *upper domination number* and is denoted by the symbol Γ.

Definition 2.10 ([10]). A set $S \subset V$ in a fuzzy graph $G = (\sigma, \mu)$ is said to be *independent set* if no two vertices of S are adjacent.

3. Complementary Nil Dominating set in a fuzzy graph

In this section, Complementary nil dominating set and complementary nil domination numbers are defined with examples.

Definition 3.1. Let $G = (\sigma, \mu)$ be a fuzzy graph on V. A set $S \subset V$ is said to be a *complementary nil dominating set* (or simply called cnd-set) of a fuzzy graph G if S is a dominating set and its complement $V - S$ is not a dominating set.

Example 3.2. Consider the fuzzy graph given in FIGURE 1.
The complementary nil dominating sets are $\{v_1, v_3, v_4\}$ and $\{v_1, v_2, v_3\}$.

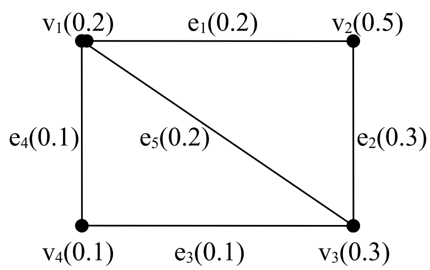

Figure 1. Complementary nil dominating set

Definition 3.3. A cnd-set S of a fuzzy graph $G = (\sigma, \mu)$ is called a *minimal cnd-set* if there is no cnd-set S' such that $S' \subset S$.

Definition 3.4. A cnd-set S of a fuzzy graph $G = (\sigma, \mu)$ is called a *minimum cnd-set* if there is no cnd-set S' such that $|S'| < |S|$. The minimum scalar cardinality taken over all cnd-set is called a *complementary nil domination number* and is denoted by the symbol γ_{cnd}, the corresponding minimum cnd-set is denoted by γ_{cnd}-set.

Observation 3.5. For any fuzzy graph $G = (\sigma, \mu)$,

1. Every super set of a cnd-set is also a cnd-set.
2. Complement of a cnd-set is not a cnd-set.
3. Complement of a γ-set is not a cnd-set.
4. γ_{cnd}-set need not be unique.

Definition 3.6. The maximum scalar cardinality taken over all minimal cnd-set is called a *Upper complementary nil dominating number* and is denoted by the symbol Γ_{cnd}.

Note 3.7. *For any fuzzy graph $G = (\sigma, \mu)$, $\gamma_{cnd} \leq \Gamma_{cnd}$.*

Example 3.8. Consider the fuzzy graph $G = (\sigma, \mu)$ given in FIGURE 1. Here, $S_1 = \{\mathrm{v}_1, \mathrm{v}_3, \mathrm{v}_4\}$ and $S_2 = \{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ are cnd-sets. S_1 is a minimal as well as minimum cnd-set whereas S_2 is a minimal but not minimum cnd-set. The complementary nil domination number of G is $\gamma_{cnd} = 0.6$. The Upper complementary nil domination number of G is $\Gamma_{cnd} = 1.0$.

Definition 3.9. Let $G = (\sigma, \mu)$ be a fuzzy graph and $S \subseteq V$. A vertex $u \in S$ is said to be an *enclave* of S if $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V - S$ that is $N[u] \subseteq S$

Example 3.10. In the fuzzy graph given in FIGURE 1, v_4 and v_2 are enclaves of the cnd-sets S_1 and S_2 respectively.

4. Theorems related to complementary nil dominating set

In this section, theorems related to complementary nil dominating sets are stated and proved.

Theorem 4.1. *A dominating set S is a cnd-set if and only if it contains at least one enclave.*

Proof. Let S be a cnd-set of a fuzzy graph $G = (\sigma, \mu)$. Then $V - S$ is not a dominating set which implies that there exists a vertex $u \in S$ such that $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V - S$. Therefore u is an enclave of S. Hence S contains at least one enclave.

Conversely, Suppose the dominating set S contains enclaves. Without loss of generality let us take u be the enclave of S. That is $\mu(u, v) < \sigma(u) \wedge \sigma(v)$, for all $v \in V - S$. Hence $V - S$ is not a dominating set. Hence the dominating set S is a cnd-set. □

Theorem 4.2. *If S is a cnd-set of a fuzzy graph $G = (\sigma, \mu)$, then there is a vertex $u \in S$ such that $S - \{u\}$ is a dominating set.*

Proof. Let S be a cnd-set of a fuzzy graph $G = (\sigma, \mu)$. By theorem 4.1, every cnd-set contains at least one enclave in S. Let $u \in S$ be an enclave of S. Then $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V - S$. Since G is a connected fuzzy graph, there exists at least a vertex $w \in S$ such that $\mu(u, w) = \sigma(u) \wedge \sigma(w)$. Hence $S - \{u\}$ is a dominating set. □

Theorem 4.3. *A cnd-set in a fuzzy graph $G = (\sigma, \mu)$ is not singleton.*

Proof. Let S be a cnd-set of a fuzzy graph $G = (\sigma, \mu)$. By theorem 4.1, every cnd-set contains at least one enclave in S. Let $u \in S$ be an enclave of S. Then $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V - S$ ......(1)
Suppose S contains only one vertex u, (1) shows it must be isolated in G, which is a contradiction to connectedness. Hence cnd-set contains more than one vertex. □

Example 4.4. For a fuzzy graph $G = (\sigma, \mu)$ given in FIGURE 2,
where $\sigma = \{\mathrm{u}_1|0.3, \mathrm{u}_2|0.4, \mathrm{u}_3|0.6, \mathrm{u}_4|0.2, \mathrm{u}_5|0.8, \mathrm{u}_6|0.7\}$ and $\mu = \{(\mathrm{u}_1, \mathrm{u}_2)|0.3,$
$(\mathrm{u}_2, \mathrm{u}_3)|0.4, (\mathrm{u}_3, \mathrm{u}_4)|0.2, (\mathrm{u}_4, \mathrm{u}_5)|0.2, (\mathrm{u}_5, \mathrm{u}_6)|0.7, (\mathrm{u}_6, \mathrm{u}_1)|0.3, (\mathrm{u}_3, \mathrm{u}_6)|0.4, \}$,
$S = \{\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3, \mathrm{u}_4\}$ is a γ_{cnd}-set and $\mathrm{u}_2, \mathrm{u}_3$ are two enclaves of S.

Theorem 4.5. *Let $G = (\sigma, \mu)$ be a fuzzy graph and S be a γ_{cnd}-set of G. If u and v are two enclaves of S, then*

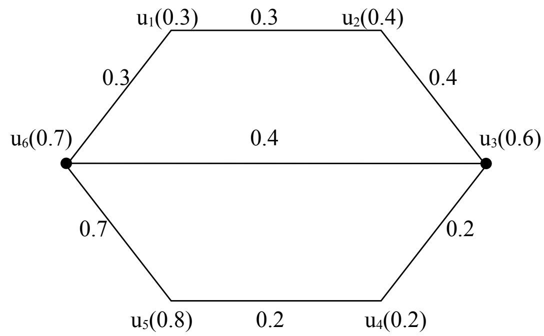

FIGURE 2. Enclaves in a cnd-set

(1) *$N[u] \cap N[v] \neq \phi$ and*
(2) *$\mu(u, v) = \sigma(u) \wedge \sigma(v)$, that is u and v are adjacent.*

Proof. Let S be a minimum cnd-set of a fuzzy graph $G = (\sigma, \mu)$ and let u and v are two enclaves of S.

(1) Suppose $N[u] \cap N[v] = \phi$. Then u is an enclave of $S - N(v)$ which implies that $V - (S - N(v))$ is not a dominating set. Therefore $S - N(v)$ is a cnd-set of G and $|S - N(v)| < |S| = \gamma_{cnd}(G)$. Which is a contradiction to the minimality of S. Hence $N[u] \cap N[v] \neq \phi$.

(2) Suppose $\mu(u, v) < \sigma(u) \wedge \sigma(v)$, that is u and v are non-adjacent. Then $u \notin N(v)$ and so u is an enclave of $S - \{v\}$ which implies that $V - (S - \{v\})$ is not a dominating set. Hence $S - \{v\}$ is a cnd-set, which is a contradiction to minimality of S. Hence u and v are adjacent. □

Theorem 4.6. *A cnd-set of a fuzzy graph $G = (\sigma, \mu)$ is not independent.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph. Suppose a cnd-set S of G is independent. Then S is a minimal dominating set which implies that V-S is a dominating set. Hence S is not a cnd-set, which is a contradiction. □

Theorem 4.7. *A cnd-set S of a fuzzy graph $G = (\sigma, \mu)$ is minimal if and only if for each $u \in S$ at least one of the following condition is satisfied*

(i) *there exists $v \in V - S$ such that $N(v) \cap S = \{u\}$.*
(ii) *$V - (S - \{u\})$ is a dominating set of G.*

Proof. If S is a minimal cnd-set of a fuzzy graph $G = (\sigma, \mu)$. Suppose, if there exists $u \in S$ such that u does not satisfy both the given conditions (i) and (ii). Then by a theorem 4.1, S is not a minimal dominating set. Hence the proper subset $S_1 = S - \{u\}$ is not a dominating set. By our assumption on (ii) $V - (S - \{u\})$ is a dominating set. Hence $S_1 = S - \{u\}$ is a cnd-set. Which is a contradiction to the minimality of the cnd-set S.

Conversely, let S is a cnd-set and for each $u \in S$ at least one of the two conditions holds. Now we show that S is minimal cnd-set of G. Suppose S is not minimal, then there exists a vertex $u \in S$ such that $S - \{u\}$ is a cnd-set. As $S - \{u\}$ is a cnd-set, u is adjacent to at least one vertex $v \in S - \{u\}$. Also $S - \{u\}$ is a dominating set, every vertex $v \in V - S$ is adjacent to at least one vertex in $S - \{u\}$. That is $N(v) \cap S \neq \{u\}$, condition (i) does not hold. As $S - \{u\}$ is a cnd-set, $V - (S - \{u\})$ not a dominating set, condition (ii) does not hold, which is a contradiction to our assumption. Hence S is a minimal end-set of G. □

Theorem 4.8. *For any fuzzy graph $G = (\sigma, \mu)$ every γ_{cnd}-set of G intersects with every γ-set of G.*

Proof. Let S be a γ_{cnd}-set and D be a γ-set of $G = (\sigma, \mu)$. Suppose $S \cap D = \phi$, then $D \subseteq V - S$, V-S contains a dominating set D. Therefore V-S, a super set of D, is a dominating set. Which is a contradiction to our assumption. Hence $S \cap D \neq \phi$. □

Corollary 4.9. *For any fuzzy graph $G = (\sigma, \mu)$ any two γ_{cnd}-sets are intersects .*

5. Bounds for complementary nil domination number of a fuzzy graph

In this section, bounds for complementary nil domination numbers are determined for standard fuzzy graphs. Theorems related to complementary nil domination numbers are stated and proved. Relation between complementary nil domination number and domination numbers are also derived.

Observation 5.1.
(1) For any fuzzy graph $G = (\sigma, \mu)$, $\gamma < \gamma_{cnd} < p$.
(2) $\gamma_{cnd}(K_\sigma - e) \leq p - \sigma_0$, where $\sigma_0 = min_{u \in V} \sigma(u)$.
(3) $\gamma_{cnd}(K_\sigma - e) = p - \sigma(u)$,
where $\sigma(u)$ is obtained from $\mu(e) = \sigma(u) \wedge \sigma(v) = \sigma(v)$.
(4) For any fuzzy graph $G = (\sigma, \mu)$, $2.\sigma_0 \leq \gamma_{cnd} \leq p - \sigma_0$.
(5) For a complete bipartite fuzzy graph, $\gamma_{cnd}(K_{\sigma_1, \sigma_2}) \leq min(|\sigma_1|, |\sigma_2|) + \sigma_n$, where $\sigma_n = max_{u \in V} \sigma(u)$.
(6) For a complete bipartite fuzzy graph

$$\gamma_{cnd}(K_{\sigma_1, \sigma_2}) = \begin{cases} |\sigma_1| + \sigma_{20} & , if |\sigma_1| < |\sigma_2| \\ |\sigma_2| + \sigma_{10} & , if |\sigma_1| > |\sigma_2| \\ |\sigma_1| + \sigma_0 & , if |\sigma_1| = |\sigma_2| \end{cases}$$

where σ_{10} and σ_{20} are the minimum membership grade of a vertex in σ_1-set and σ_2-set respectively.
(7) Let T_σ be tree in a fuzzy graph $G = (\sigma, \mu)$, Then $\gamma_{cnd}(T_\sigma) \leq \gamma(T_\sigma) + \sigma_n$.

Theorem 5.2. *For any fuzzy graph $G = (\sigma, \mu)$,*
$\delta_N + \sigma_0 \leq \gamma_{cnd} \leq \gamma + \delta_N + \sigma_n - \sigma_0$.

Proof. Let S be a cnd-set of a fuzzy graph $G = (\sigma, \mu)$. Since V-S is not a dominating set, there exists $u \in S$ such that $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V - S$. Then $N[u] \subseteq S$ which implies $|N[u]| \leq |S|$. Hence $\delta_N + \sigma_0 \leq \gamma_{cnd}$.

Let S_1 be a γ-set of G and let $u \in V$ such that $d_N(u) = \delta_N$. Then u is either in D or in V-D.

Case(i): If $u \in D$, then $D \cup N(u)$ contains an enclave. Therefore $D \cup N(u)$ is a cnd-set. Hence $\gamma_{cnd} \leq \gamma + \delta_N$.

Case(ii): If $u \in V - D$, then at least a vertex $v \in D$ such that $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. Then $D \cup N[u]$ contains an enclave. Therefore $D \cup N[u]$ is a cnd-set and $D \cap N[u]$ is a non empty set because $v \in D \cap N[u]$. Hence $\gamma_{cnd} \leq |D \cup N[u]| = |D| + |N[u]| -$

$|D \cap N[u]| = \gamma + \delta_N + \sigma_n - \sigma_0$.
In both the cases, $\gamma_{cnd} \leq |D \cup N[u]| = |D| + |N[u]| - |D \cap N[u]| = \gamma + \delta_N + \sigma_n - \sigma_0$. □

Example 5.3. From the fuzzy graph $G = (\sigma, \mu)$ given in FIGURE 3, where $\sigma = \{\mathrm{u}_1|0.5, \mathrm{u}_2|0.2, \mathrm{u}_3|0.5\}$ and $\mu = \{(\mathrm{u}_1, \mathrm{u}_2)|0.2, (\mathrm{u}_2, \mathrm{u}_3)|0.2, (\mathrm{u}_3, \mathrm{u}_1)|0.2\}$, $\gamma_{cnd} = 0.7, \gamma = 0.2, \delta_N = 0.2, \sigma_n = 0.5, \sigma_0 = 0.2$. Hence $\delta_N + \sigma_0 \leq \gamma_{cnd} \leq \gamma + \delta_N + \sigma_n - \sigma_0$

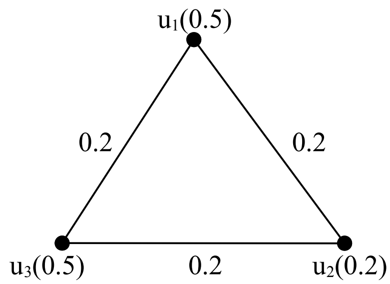

FIGURE 3. Fuzzy graph

Theorem 5.4. *Let T_σ be a fuzzy tree , then $\gamma_{cnd}(T_\sigma) \leq p - r + \sigma_{p_0}$, where r is the scalar cardinality of set of all pendent vertices in T_σ and σ_{p_0} is a minimum membership grade of a pendent vertex.*

Proof. Let T_σ be a fuzzy tree ,then the set of all non-pendent vertices together with a pendent vertex form a cnd-set. Hence $\gamma_{cnd}(T_\sigma) \leq r + \sigma_{p_0}$, where r is the scalar cardinality of set of all pendent vertices in T_σ and σ_{p_0} is a minimum membership grade of a pendent vertex. □

Theorem 5.5. *Let $G = (\sigma, \mu)$ be a fuzzy graph . If diam(G^*)=2 then $\gamma_{cnd} = \Delta_N + \sigma_n$.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph and diam(G^*)=2. Then there exists $u, v \in V$ and $u \neq v$ such that $v \notin N[u]$. Therefore $N[u]$ is a cnd-set of G. Hence $\gamma_{cnd} = \Delta_N + \sigma_n$. □

Theorem 5.6. *For any fuzzy graph $G = (\sigma, \mu)$ if $\gamma = \frac{p}{2}$, then $\gamma_{cnd}(G) = \frac{p}{2} + \sigma_0$.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph and let S be γ-set of G with $\gamma(G) = \frac{p}{2}$. Hence V-S is a γ-set with $|V - S| = \frac{p}{2}$. Choose a vertex $u \in V$, such that $\sigma(u) = \sigma_0$. Now either $u \in S$ or $u \in V - S$. Hence $[(v - S) \cup \{u\}]$ or $S - \{u\}$ is a cnd-set. Then either $D \cup \{u\}$ or $(V - S) - \{u\}$ is not a dominating set. Therefore either $D \cup \{u\}$ or $[(V - S) - \{u\}]$ is a cnd-set. Hence $\gamma_{cnd} = \frac{p}{2} + \sigma_0$. □

Theorem 5.7. *For any fuzzy graph $G = (\sigma, \mu)$, $\Gamma + \gamma_{cnd} \leq p + \sigma_n$.*

Proof. Let S be a Γ-set of G, then there exists a $u \in S$ such that $S - \{v\}$ is not a dominating set of G. Since S is a minimal dominating set. Then $V - S$ is a dominating set and $[(V - s) \cup \{u\}]$ is also a dominating set. Complement of $[(V - s) \cup \{u\}]$ is $S - \{u\}$, but $S - \{u\}$ is not a dominating set. Therefore $(V - S) \cup \{u\}$ is a cnd-set. Hence $\gamma_{cnd} \leq |(V - S) \cup \{u\}| = p - \Gamma + \sigma_n$. Thus $\Gamma + \gamma_{cnd} \leq p + \sigma_n$. □

Theorem 5.8. *Let $G = (\sigma, \mu)$ be a fuzzy graph with $diam(G^*) \geq 3$, $\gamma_{cnd} \leq p - \delta_N$.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph and $diam(G^*) \geq 3$, then there exists $u, v \in V$ such that u is not adjacent to any vertex in $N(v)$, that is $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in N(u)$. Now $V - N(v)$ is a dominating set but $N(v)$ is not a dominating set . Therefore $V - N(v)$ is a cnd-set. Hence $\gamma_{cnd} \leq |V - N(v)| = p - \delta_N$. □

6. Conclusion

The complementary nil dominating set and complementary nil dominating numbers in a fuzzy graph are defined. The bounds on this number is obtained for some standard fuzzy graphs. Theorems related to this concept are derived. Finally, the relation between complementary nil domination number and domination numbers are established.

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