Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 8, No. 5, (November 2014), pp. 753–768
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

A novel algorithm for solving intuitionistic fuzzy transportation problem via new ranking method

Shashi Aggarwal, Chavi Gupta

Received 27 February 2014; Revised 17 April 2014; Accepted 16 May 2014

Abstract. In this paper, a new ranking method for generalized trapezoidal intuitionistic fuzzy number (GTRIFN) is introduced to overcome the limitations of the existing methods. Further, we have considered a transportation problem in intuitionistic fuzzy environment. In this problem, costs are represented by GTRIFNs. An algorithm is proposed to evaluate the initial basic feasible and optimal solution of intuitionistic fuzzy transportation problem. An illustrative numerical example is solved to demonstrate the efficiency of the proposed methods.

2010 AMS Classification: 90C70, 90C05, 90C08

Keywords: Intuitionistic fuzzy number, Centroid point, Transportation problem.

Corresponding Author: Chavi Gupta (chavi.gupta85@gmail.com)

1. Introduction

In fuzzy environment, ranking of fuzzy numbers play a vital role in decision making problems. In literature, numerous approaches for ranking fuzzy numbers have been extensively studied. Several authors namely Abbasbandy and Hajjari [1], Chen and Chen [7], Wang and Lee [23] rank fuzzy numbers by different approaches. The concept of fuzzy set theory introduced by Zadeh [25] was extended to intuitionistic fuzzy sets (IFS) by Atanassov [3, 4]. In IFS, degree of non - membership (rejection) and degree of membership function (acceptance) are defined simultaneously such that sum of both values is less than one [2]. It is not always possible to define membership and non - membership function up to decision maker's (DMs) satisfaction due to insufficient available information. As a result, there remains an indeterministic part in which reluctance perseveres. Therefore, intuitionistic fuzzy set theory seems to be more consistent to deal with ambiguity and vagueness. In recent past, ranking intuitionistic fuzzy numbers (IFNs) draws the attention of several researchers. Nehi [19] ranked IFNs based on characteristic values of membership and non - membership functions of IFN. Ranking of trapezoidal IFNs based on value and ambiguity indices were given by De and Das [9], Rezvani [21] and many more approaches were subsequently developed. In 1970, Bellman and Zadeh [5] introduced the concept of decision making in fuzzy environment. The concept of optimization in intuitionistic fuzzy environment was given by Angelov [2] . This area has been extensively used in the area of linear programming. One of the important applications of linear programming is in the area of transportation of goods and services from several supply centres to several demand centres. The simplest transportation model was first presented by Hitchcock [11] in 1941. Several other extensions were successively developed.

In 1984, Chanas.et.al[6] presented a fuzzy approach to the transportation problem. Fuzzy zero point method is introduced by Pandian and Natarajan [20], which was extended to intuitionistic fuzzy zero point method by Hussain and kumar [12] to compute optimal solution of transportation problem. To the best of our knowledge, till now no one has used generalized trapezoidal intuitionistic fuzzy numbers for solving transportation problems.

In this paper, new ranking method for ordering generalized trapezoidal intuitionistic fuzzy numbers (GTRIFNs) is introduced. Intuitionistic fuzzy max - min method and generalized intuitionistic modified distribution method is introduced for computing the initial basic feasible solution (IBFS) and optimal solution respectively of transportation problem in which the costs are represented by GTRIFNs.

Rest of the paper is organized as follows. Section 2 briefly describes some basic definitions and arithmetic operations over GTRIFNs . A new ranking method for GTRIFNs and significance of the proposed ranking method over existing methods are illustrated in section 3. In section 4, mathematical model formulation of intuitionistic fuzzy transportation problem and algorithms of proposed methods to solve intuitionistic fuzzy transportation problem are illustrated. A numerical example is solved in section 5 to demonstrate the efficiency of proposed methods.

2. Preliminaries

In this section , some basic concepts related to intuitionistic fuzzy set theory are reviewed

2.1. Basic definitions.

Definition 2.1 ([4])**.** Let X be a universal set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A =< (x, \mu_A(x), \nu_A(x)) : x\epsilon X >$ where the functions $\mu_A : X \longrightarrow [0, 1]$, $\nu_A : X \longrightarrow [0, 1]$ define the degree of membership and the degree of non- membership of the element $x\epsilon X$ to the set A respectively and for every $x\epsilon X$ in A, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds.

Definition 2.2 ([4])**.** For every common intuitionistic fuzzy set A on X , intuitionistic fuzzy index of x in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A.
Obviously, for every $x\epsilon X$, $0 \leq \pi_A(x) \leq 1$.

Definition 2.3 ([17])**.** An intuitionistic fuzzy number (IFN)$\widetilde{a}^{I}$ is

(i) an intuitionistic fuzzy set of the real line.

(ii) normal, that is, there is some $x_0 \epsilon \Re \ni \mu_{\underset{a}{\sim}}(x_0) = 1, \nu_{\underset{a}{\sim}}(x_0) = 0$.

(iii) convex for the membership function $\mu_{\underset{a}{\sim}}(x)$, that is,

$$\mu_{\underset{a}{\sim}}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\underset{a}{\sim}}(x_1), \mu_{\underset{a}{\sim}}(x_2))\; \forall x_1, x_2 \epsilon \Re, \lambda \epsilon [0,1].$$

(iv) concave for the non - membership function $\nu_{\underset{a}{\sim}}(x)$, that is,

$$\nu_{\underset{a}{\sim}}(\lambda x_1 + (1-\lambda)x_2) \leq max(\nu_{\underset{a}{\sim}}(x_1), \nu_{\underset{a}{\sim}}(x_2))\; \forall x_1, x_2 \epsilon \Re, \lambda \epsilon [0,1].$$

Definition 2.4 ([17])**.** An intuitionistic fuzzy number

$$\overset{\sim I}{a} =< (a_1, a_2, a_3, a_4)(\overline{a_1}, a_2, a_3, \overline{a_4}) >$$

is said to be trapezoidal intuitionistic fuzzy number (TRIFN) if its membership and non - membership functions are given by

$$\mu_{\underset{a}{\sim}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & if\ a_1 \leq x \leq a_2 \\ 1 & if\ a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & if\ a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases} \qquad \nu_{\underset{a}{\sim}}(x) = \begin{cases} \frac{a_2-x}{a_2-\overline{a_1}} & if\ \overline{a_1} \leq x \leq a_2 \\ 0 & if\ a_2 \leq x \leq a_3 \\ \frac{a_3-x}{a_3-\overline{a_4}} & if\ a_3 \leq x \leq \overline{a_4} \\ 1 & otherwise \end{cases}$$

Definition 2.5 ([13])**.** An intuitionistic fuzzy set, defined on the universal set of real numbers $\Re$ is said to be a generalized trapezoidal intuitionistic fuzzy number (GTRIFN) denoted by $\overset{\sim I}{a} =< (a_1, a_2, a_3, a_4; \omega_a)(\overline{a_1}, a_2, a_3, \overline{a_4}; \sigma_a) >$ if its membership and non - membership functions are given by

$$\mu_{\underset{a}{\sim}}(x) = \begin{cases} \frac{(x-a_1)\omega_a}{a_2-a_1} & if\ a_1 \leq x \leq a_2 \\ \omega_a & if\ a_2 \leq x \leq a_3 \\ \frac{(a_4-x)\omega_a}{a_4-a_3} & if\ a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases}$$

$$\nu_{\underset{a}{\sim}}(x) = \begin{cases} \frac{a_2-x+\sigma_a(x-\overline{a_1})}{a_2-\overline{a_1}} & if\ \overline{a_1} \leq x \leq a_2 \\ \sigma_a & if\ a_2 \leq x \leq a_3 \\ \frac{x-a_3+\sigma_a(\overline{a_4}-x)}{\overline{a_4}-a_3} & if\ a_3 \leq x \leq \overline{a_4} \\ 1 & otherwise \end{cases}$$

where ω_a and σ_a represent the maximum degree of membership and minimum degree of non - membership respectively, satisfying $0 \leq \omega_a \leq 1, 0 \leq \sigma_a \leq 1, 0 \leq \omega_a + \sigma_a \leq 1$.

2.2. **Arithmetic operations.** Here, the arithmetic operations over GTRIFNs are defined in a similar way to those on TRIFNs [9] and Triangular IFNs [15]. Let

$$\overset{\sim I}{a} = < (a_1, a_2, a_3, a_4; \omega_a)(\overline{a_1}, a_2, a_3, \overline{a_4}; \sigma_a) >$$

and

$$\tilde{b}^{I} = <(b_1, b_2, b_3, b_4; \omega_b)(\overline{b_1}, b_2, b_3, \overline{b_4}; \sigma_b)>$$

be two GTRIFNs, then

(1) $\tilde{a}^{I} \oplus \tilde{b}^{I} = <(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4; min(\omega_a, \omega_b)(\overline{a_1}+\overline{b_1}, a_2+b_2, a_3+b_3, \overline{a_4}+\overline{b_4}; max(\sigma_a, \sigma_b))>$

(2) $\tilde{a}^{I} \ominus \tilde{b}^{I} = <(a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1; min(\omega_a, \omega_b)(\overline{a_1}-\overline{b_4}, a_2-b_3, a_3-b_2, \overline{a_4}-\overline{b_1}; max(\sigma_a, \sigma_b))>$

(3) $\lambda\tilde{a}^{I} = <(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \omega_a)(\lambda\overline{a_1}, \lambda a_2, \lambda a_3, \lambda\overline{a_4}; \sigma_a)> if \lambda > 0$

(4) $\lambda\tilde{a}^{I} = <(\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; \omega_a)(\lambda\overline{a_4}, \lambda a_3, \lambda a_2, \lambda\overline{a_1}; \sigma_a)> if \lambda < 0$

3. Ranking index of GTRIFN

In literature there are various algorithms for ranking IFNs, but most of the algorithms are used to rank triangular IFNs or TRIFNs with $\overline{a_1} = a_1$ and $\overline{a_4} = a_4$ [8, 9].So in order to rank GTRIFN, firstly we define a new single function ρ_a involving both membership and non - membership function of GTRIFN $\tilde{a}^{I}$ as follows:
Define $\rho_a : \Re \longrightarrow [0, \omega_a]$ such that

$$\rho_a(x) = \frac{(\mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x) + 1)\omega_a}{\omega_a - \sigma_a + 1} \;\forall x \in \Re$$

Here, $\mu_{\tilde{a}}(x)$ and $\nu_{\tilde{a}}(x)$ are membership and non - membership functions of GTRIFN $\tilde{a}^{I}$

Definition 3.1 ([22]). A fuzzy set $\widetilde{A}$ defined on the universal set of real numbers $\Re$ is said to be generalized fuzzy number if its membership function is defined as

$$\mu_A(x) = \begin{cases} A^L(x) & if\; a \le x \le b \\ \omega_a & if\; b \le x \le c \\ A^U(x) & if\; c \le x \le d \\ 0 & otherwise \end{cases}$$

where $0 < \omega_a \le 1$ is constant, $A^L : [a, b] \to [0, \omega_a]$ is monotonically increasing continous from the right and $A^U : [c, d] \to [0, \omega_a]$ is monotonically decreasing continous from the left. If the membership function $\mu_A(x)$ is piecewise linear and continuous, then $\widetilde{A}$ is referred to as generalized trapezoidal fuzzy number.

Proposition 3.2. $\overline{\rho_a} =< (x, \rho_a(x)); x\epsilon\Re >$ *is generalized trapezoidal fuzzy number.*

Proof. Let $x \epsilon \Re$ be arbitrary. Then,

$$\rho_a(x) = \begin{cases} 0 & if\ x \leq \overline{a_1}, x \geq \overline{a_4} \\ \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{-a_2 + x - \sigma_a(x - \overline{a_1})}{a_2 - \overline{a_1}} + 1\right\} & if\ \overline{a_1} \leq x \leq a_1 \\ \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{(x - a_1)\omega_a}{a_2 - a_1} - \dfrac{a_2 - x + \sigma_a(x - \overline{a_1})}{a_2 - \overline{a_1}} + 1\right\} & if a_1 \leq x \leq a_2 \\ \omega_a & if a_2 \leq x \leq a_3 \\ \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{(a_4 - x)\omega_a}{a_4 - a_3} - \dfrac{x - a_3 + \sigma_a(\overline{a_4} - x)}{\overline{a_4} - a_3} + 1\right\} & if\ a_3 \leq x \leq a_4 \\ \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{-x + a_3 - \sigma_a(\overline{a_4} - x)}{\overline{a_4} - a_3} + 1\right\} & if\ a_4 \leq x \leq \overline{a_4} \end{cases}$$

Therefore $\rho_a(x)$ can be written as

$$\rho_a(x) = \begin{cases} q(x) & if\ \overline{a_1} \leq x \leq a_2 \\ \omega_a & if\ a_2 \leq x \leq a_3 \\ r(x) & if\ a_3 \leq x \leq \overline{a_4} \\ 0 & otherwise \end{cases}$$

where $q(x)$ is defined as $q : [\overline{a_1}, a_2] \longrightarrow [0, \omega_a]$ such that

$$q(x) = \begin{cases} \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{-a_2 + x - \sigma_a(x - \overline{a_1})}{a_2 - \overline{a_1}} + 1\right\} & if\ \overline{a_1} \leq x \leq a_1 \\ \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{(x - a_1)\omega_a}{a_2 - a_1} - \dfrac{a_2 - x + \sigma_a(x - \overline{a_1})}{a_2 - \overline{a_1}} + 1\right\} & if\ a_1 \leq x \leq a_2 \end{cases}$$

and $r(x)$ is defined as $r : [a_3, \overline{a_4}] \longrightarrow [0, \omega_a]$ such that

$$r(x) = \begin{cases} \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{(a_4 - x)\omega_a}{a_4 - a_3} - \dfrac{x - a_3 + \sigma_a(\overline{a_4} - x)}{\overline{a_4} - a_3} + 1\right\} & if\ a_3 \leq x \leq a_4 \\ \dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{-x + a_3 - \sigma_a(\overline{a_4} - x)}{\overline{a_4} - a_3} + 1\right\} & if\ a_4 \leq x \leq \overline{a_4} \end{cases}$$

Here, $q(x)$ is continous and monotonically increasing function and $r(x)$ is continous and monotonically decreasing function. Also, $\rho_a(x)$ is piecewise linear and continuous. Therefore, $\overline{\rho}_a = < (x, \rho_a(x)); x \epsilon \Re >$ is generalized trapezoidal fuzzy number. □

To rank GTRIFNs, firstly we will find the centroid of fuzzy number $\overline{\rho}_a$.Functions $q(x)$ and $r(x)$ defined in the proposition are both strictly monotone and continuous functions, so their inverse function exists and should be continuous and strict monotone. Let $q^I(y) : [0, \omega_a] \longrightarrow [\overline{a_1}, a_2]$ and $r^I(y) : [0, \omega_a] \longrightarrow [a_3, \overline{a_4}]$ be the inverse functions of $q(x)$ and $r(x)$ respectively. Then,

$$q^I(y) = \begin{cases} \dfrac{y(a_2 - \overline{a_1})(\omega_a - \sigma_a + 1) + \omega_a \overline{a_1}(1 - \sigma_a)}{(1 - \sigma_a)\omega_a} & if\ 0 \le y \le t \\ \dfrac{\begin{pmatrix} y(\omega_a - \sigma_a + 1)(a_2 - a_1)(a_2 - \overline{a_1}) - \omega_a(a_1\overline{a_1}\omega_a - \\ a_1 a_2 \omega_a + a_2\overline{a_1}\sigma_a - a_1\overline{a_1}\sigma_a - \overline{a_1}a_2 + a_1\overline{a_1}) \end{pmatrix}}{(a_2\omega_a - \overline{a_1}\omega_a + a_2 - a_1 - a_2\sigma_a + a_1\sigma_a)\omega_a} & if\ t \le y \le \omega_a \end{cases}$$

where $t = \dfrac{(a_1 - \overline{a_1})(1 - \sigma_a)\omega_a}{(\omega_a - \sigma_a + 1)(a_2 - \overline{a_1})}$

$$r^I(y) = \begin{cases} \dfrac{y(\overline{a_4} - a_3)(\omega_a - \sigma_a + 1) - \overline{a_4}(1 - \sigma_a)\omega_a}{(\sigma_a - 1)\omega_a} & if\ 0 \le y \le s \\ \dfrac{\begin{pmatrix} y(\omega_a - \sigma_a + 1)(a_4 - a_3)(\overline{a_4} - a_3) - (a_4\overline{a_4}\omega_a - \\ a_4 a_3 \omega_a - a_4\overline{a_4}\sigma_a + a_3\overline{a_4}\sigma_a + \overline{a_4}a_4 - a_3\overline{a_4})\omega_a \end{pmatrix}}{(-\overline{a_4}\omega_a + a_3\omega_a - a_4 + a_3 + a_4\sigma_a - a_3\sigma_a)\omega_a} & if\ s \le y \le \omega_a \end{cases}$$

where $s = \dfrac{(\overline{a_4} - a_4)(1 - \sigma_a)\omega_a}{(\omega_a - \sigma_a + 1)(\overline{a_4} - a_3)}$

Since $\overline{\rho}_a$ is generalized trapezoidal fuzzy number, so centroid point (x_0, y_0) of a fuzzy number $\overline{\rho}_a$ (based on formula of Wang .et.al. [24]) is given by

$$\begin{aligned} x_0(\widetilde{a}^I) &= \frac{\int_{-\infty}^{-\infty} x\rho_a(x)dx}{\int_{-\infty}^{-\infty} \rho_a(x)dx} \\ &= \frac{\int_{\overline{a_1}}^{a_2} xq(x)dx + \int_{a_2}^{a_3} \omega_a x dx + \int_{a_3}^{\overline{a_4}} xr(x)dx}{\int_{\overline{a_1}}^{a_2} q(x)dx + \int_{a_2}^{a_3} \omega_a dx + \int_{a_3}^{\overline{a_4}} r(x)dx} \\ &= \frac{\begin{pmatrix} (1 - \sigma_a)(-\overline{a_1}^2 - a_2^2 - \overline{a_1}a_2 + a_3^2 + a_3\overline{a_4} + \overline{a_4}^2) + \\ \omega_a(-a_1^2 - a_2^2 - a_1 a_2 + a_3^2 + a_3 a_4 + a_4^2) \end{pmatrix}}{3\left\{(1 - \sigma_a)(-\overline{a_1} - a_2 + a_3 + \overline{a_4}) + \omega_a(-a_1 - a_2 + a_3 + a_4)\right\}} \end{aligned}$$

$$y_0(\widetilde{a}^I) = \frac{\int_0^{\omega_a} y(r^I(y) - q^I(y))dy}{\int_0^{\omega_a}(r^I(y) - q^I(y))dy} \quad \text{provided } r^I(y) - q^I(y) \ne 0 \text{ and } \omega_a \ne 0$$

Remark 3.3. Let $\mu_{\widetilde{a}}(x) = 1 - \nu_{\widetilde{a}}(x)$, then $\overline{a_1} = a_1, \overline{a_4} = a_4, \omega_a = 1 - \sigma_a$. Also $\overline{\rho}_a =< (x, \mu_{\widetilde{a}}(x)); x\epsilon\Re >$.Thus, $\overline{\rho}_a$ reduces to a generalized trapezoidal fuzzy number with membership function $\mu_{\widetilde{a}}(x)$. By substituting the values in the above centroid formula, we get

$x_0(\tilde{a}^I) = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{a_4a_3 - a_1a_2}{(a_4 + a_3) - (a_1 + a_2)}\right]$
$y_0(\tilde{a}^I) = \frac{\omega_a}{3}\left[1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)}\right].$
This is exactly the same as derived by Wang.et.al [24].

Remark 3.4. Let $\mu_{\tilde{a}}(x) = 1 - \nu_{\tilde{a}}(x)$ and $a_2 = a_3$,then $\omega_a = 1 - \sigma_a$, $\overline{a_1} = a_1$, $\overline{a_4} = a_4$. Also $\overline{\rho}_a$ reduces to a generalized triangular fuzzy number and by substituting the values, we get,
$x_0(\tilde{a}^I) = \frac{a_1 + a_2 + a_3}{3}$
$y_0(\tilde{a}^I) = \frac{\omega_a}{3}.$
This is the centroid formula of a triangle.

We employ Wang and Lee [23] method for the centroid of $\overline{\rho}_a$ to order GTRIFNs. Let $\tilde{a}^I$ and $\tilde{b}^I$ be two GTRIFNs. Then,

(1) $\tilde{a}^I \preceq \tilde{b}^I \; iff \quad x_0(\tilde{a}^I) < x_0(\tilde{b}^I)$ or $\left(x_0(\tilde{a}^I) = x_0(\tilde{b}^I) \quad \text{and} \quad y_0(\tilde{a}^I) < y_0(\tilde{b}^I)\right)$
(2) $\tilde{a}^I \simeq \tilde{b}^I \; iff \quad x_0(\tilde{a}^I) = x_0(\tilde{b}^I) \quad \text{and} \quad y_0(\tilde{a}^I) = y_0(\tilde{b}^I)$
(3) $\tilde{a}^I \succeq \tilde{b}^I \; iff \quad x_0(\tilde{a}^I) > x_0(\tilde{b}^I)$ or $\left(x_0(\tilde{a}^I) = x_0(\tilde{b}^I) \quad \text{and} \quad y_0(\tilde{a}^I) > y_0(\tilde{b}^I)\right)$

Definition 3.5. GTRIFN $\tilde{a}^I$ is said to be positive iff $x_0(\tilde{a}^I) > 0$

Definition 3.6. GTRIFN $\tilde{a}^I$ is said to be negative iff $x_0(\tilde{a}^I) < 0$

Definition 3.7. GTRIFN $\tilde{a}^I$ is said to be zero GTRIFN iff $x_0(\tilde{a}^I) = 0$

Significance of the proposed ranking method over existing methods

(1) Algorithms discussed in [8, 9] cannot be used to rank those GTRIFNs where $\overline{a_1} \neq a_1$ or $\overline{a_4} \neq a_4$ but the proposed method can be used to rank such GTRIFNs. *Example*: Let
$$\tilde{a}^I = (2, 4, 8, 15 : 0.6)(1, 4, 8, 18; 0.3)$$
and
$$\tilde{b}^I = (2, 5, 8, 10; 0.6)(1, 5, 8, 12; 0.2),$$
then clearly, method discussed in [8, 9] cannot be used to rank $\tilde{a}^I$ and $\tilde{b}^I$ but by proposed method $\tilde{a}^I \succ \tilde{b}^I$.
(2) Algorithm in [18] fails if membership score of $\tilde{a}^I \leq$ membership score of $\tilde{b}^I$ and non - membership score of $\tilde{a}^I \leq$ non - membership score of $\tilde{b}^I$, where $\tilde{a}^I$ and $\tilde{b}^I$ are IFNs. But in the proposed method, we overcome this situation by defining a single function $\overline{\rho}_a$ involving both membership and non - membership function of GTRIFN $\tilde{a}^I$.

Example: Let $\tilde{a}^I = (0.5, 0.5, 0.5; 1)(0.5, 0.5, 1; 0)$ and $\tilde{b}^I = (0.6, 0.6, 0.6; 1)(0.6, 0.6, 0.7; 0)$,then method discussed in [18] cannot be used to rank $\tilde{a}^I$ and $\tilde{b}^I$ but by proposed method $\tilde{a}^I \succ \tilde{b}^I$.

(3) Most of the existing methods discussed in literature [10, 16] and many more can be used only for triangular IFNs. These methods cannot be used to rank GTRIFNs. But our method can be used to rank GTRIFNs as well as triangular IFNs by taking $a_2 = a_3$.
Example: Let

$$\tilde{a}^I = (4, 8, 10, 13 : 0.4)(3, 8, 10, 15; 0.3)$$

and

$$\tilde{b}^I = (2, 7, 11, 15; 0.5)(1, 7, 11, 18; 0.3),$$

then clearly, method discussed in [10, 16] cannot be used to rank $\tilde{a}^I$ and $\tilde{b}^I$ but by proposed method $\tilde{a}^I \prec \tilde{b}^I$.

4. Mathematical formulation of intuitionistic fuzzy transportation problem (IFTP)

Consider a intuitionistic fuzzy transportation problem with m origins and n destinations. Let $\tilde{c}^I_{ij}$ be the intuitionistic fuzzy (IF) cost of transporting one unit of the product from i^{th} origin to the j^{th} destination. Here, the cost $\tilde{c}^I_{ij}$ $(i = 1, 2, ..., m, j = 1, 2, ..., n)$are represented by GTRIFNs. Let a_i be the total availability of the product at the i^{th} origin. Let b_j be the total demand of the product at the j^{th} destination. Let x_{ij} be the quantity transported from i^{th} origin to the j^{th} destination so as to minimize the total IF transportation cost. Therefore, IFTP in which the DM is uncertain about the precise values of transportation cost from i^{th} origin to the j^{th} destination but sure about the supply and demand of the product can be formulated as

$$\begin{array}{lll} Minimize & \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}^I_{ij}\, x_{ij} & \\ subject\,to & \sum_{j=1}^{n} x_{ij} \leq a_i & i = 1, 2, ..., m \\ & \sum_{i=1}^{m} x_{ij} \geq b_j & j = 1, 2, ..., n \\ & x_{ij} \geq 0 & \forall i, j. \end{array}$$

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{m} b_j$, then IFTP is said to be balanced, otherwise it is said to be unbalanced IFTP.

Definition 4.1. A feasible solution $x_{ij}(i = 1, 2, ..., m; j = 1, 2, ..., n)$ of IFTP is said to be an intuitionistic fuzzy optimal solution of IFTP if and only if for all feasible solutions $y_{ij}(i = 1, 2, ...m; j = 1, 2, ..., n)$, $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}^I_{ij}\, x_{ij} \succeq \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}^I_{ij}\, y_{ij}$.

TABLE 1. Tabular form of above IFTP

	1	2	n	Supply
1	$\tilde{c}^I_{11}$	$\tilde{c}^I_{12}$	$\tilde{c}^I_{1n}$	a_1
2	$\tilde{c}^I_{21}$	$\tilde{c}^I_{22}$	$\tilde{c}^I_{2n}$	a_2
m	$\tilde{c}^I_{m1}$	$\tilde{c}^I_{m2}$	$\tilde{c}^I_{mn}$	a_m
Demand	b_1	b_2	b_n	

4.1. **Proposed IF Max - Min method for finding initial basic feasible solution (IBFS) of intuitionistic fuzzy transportation problem (IFTP).** Here, IF Max- Min method is proposed to compute initial basic feasible solution of IFTP. The steps of the proposed method are as follows:

Step 1: Set up the formulated IFTP into tabular form known as intuitionistic fuzzy transportation table (IFTT) of order $m \times n$. Represent the approximate cost by GTRIFNs.

Step 2: Examine whether $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ or $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$.

(a) If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, then go to step 3.

(b) If $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, then introduce a dummy column having all its cost as zero GTRIFNs.Assume $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ as demand at the dummy destination. Go to step 3.

(c) If $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$, then introduce a dummy row having all its cost zero GTRIFNs. Assume $\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$ as availability of the product at the dummy source. Go to step 3.

Step 3: Take the first row and choose its smallest entry (cost) and write it in the front of first row on the right. This is the intuitionistic fuzzy penalty of first row. Similarly, compute the intuitionistic fuzzy penalty of each row and write them in front of each corresponding row.
In the similar way, compute intuitionistic fuzzy penalty for each column and write them in the bottom of each corresponding column.

Step 4: Select the highest intuitionistic fuzzy penalty computed in step 3 and select the entry for which this corresponds. Let it be $\tilde{c}^I_{ij}$. Find $x_{ij} = min(a_i, b_j)$. Then the following cases arises:

(a) If $min(a_i, b_j) = a_i$, then allocate $x_{ij} = a_i$ in the $(i, j)^{th}$ cell of $m \times n$ IFTT. Ignore the i^{th} row to obtain a new IFTT of order $(m-1) \times n$. Replace b_j by $b_j - a_i$ in obtained IFTT. Go to step 5.

(b) If $min(a_i, b_j) = b_j$, then allocate $x_{ij} = b_j$ in the $(i, j)^{th}$ cell of $m \times n$ IFTT. Ignore the j^{th} row to obtain a new IFTT of order $m \times (n-1)$. Replace a_i by $a_i - b_j$ in obtained IFTT. Go to step 5.

(c) If $a_i = b_j$, then either follow case (a) or case (b) but not simultaneously. Go to step 5.

Step 5: Calculate the fresh penalties for the reduced IFTT as in step 4.
Repeat step 4 until IFTT is reduced into IFTT of order 1×1.

Step 6: Allocate all x_{ij} in the $(i,j)^{th}$ cell of the given IFTT.

Step 7: The IBFS and initial intuitionistic fuzzy transportation cost are x_{ij} and $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{I}\, x_{ij}$ respectively.

4.2. **Generalized intuitionistic modified distribution method (GIMDM) for finding optimal solution.** Here, generalized intuitionistic modified distribution method is proposed to find the intuitionistic fuzzy optimal solution of IFTP. The proposed algorithm is an extension of generalized fuzzy modified distribution method[14]. Algorithm of GIMDM is illustrated as follows:

Step 1: Find IBFS by proposed IF Max- Min method.

Step 2: Calculate GTRIFNs $\tilde{u}_i^{I}$ and $\tilde{v}_j^{I}$ for each row and column respectively, such that $\tilde{u}_i^{I} \oplus \tilde{v}_j^{I} = \tilde{c}_{ij}^{I}$ for each occupied cell. To start with, take any $\tilde{u}_i^{I}$ or $\tilde{v}_j^{I}$ as $(-\delta,0,0,\delta;1)(-\delta,0,0,\delta;0)$, where δ is any positive real number.

Step 3: For unoccupied cells, find $\tilde{d}_{ij}^{I}$ by the relation $\tilde{d}_{ij}^{I} = \tilde{c}_{ij}^{I} \ominus (\tilde{u}_i^{I} \oplus \tilde{v}_j^{I})$.

Step 4: Calculate the x_0 value of each $\tilde{d}_{ij}^{I}$.

(a) If $x_0(\tilde{d}_{ij}^{I}) \geq 0$ for all unoccupied cells, then IBFS obtained in step 1 will be an intuitionistic fuzzy optimal solution.

(b) If at least one $x_0(\tilde{d}_{ij}^{I}) < 0$, then IBFS obtained in step 1 is not optimal. Go to step 5.

Step 5: Select the unoccupied cell corresponding to which x_0 value of $\tilde{d}_{ij}^{I}$ is most negative.

Step 6: Construct the closed loop as follows:
Start the closed loop with the selected unoccupied cell (in step 5) and move horizontally and vertically with corner cells occupied and return to selected unoccupied cell to complete the loop. Assign + and - sign alternatively at the corners of the closed loop, by assigning the + sign to the selected unoccupied cell first.

Step 7: Find the minimum allocation value from the cells having - sign.

Step 8: Allocate this value to the selected unoccupied cell and add it to the other occupied cells having + sign and subtract it to the other occupied cell having - sign.

Step 9: Allocation in step 8 will yield an improved basic feasible solution.

Step 10: Repeat steps 2 - 9 for the improved basic feasible solution obtained in step 9. The process terminates when $x_0(\tilde{d}_{ij}^{I}) \geq 0$ for all unoccupied cells.

5. Numerical example

Example: A company has 3 warehouses w_1, w_2, w_3. It is required to deliver a product from these three warehouses to three customers C_1, C_2, C_3. Amount in stock at these three warehouse w_1, w_2 and w_3 are 25, 30 and 40 units respectively and the requirement of the three customers C_1, C_2 and C_3 of the product are 35, 45 and 15 units respectively. But due to frequently variation in the rates of fuel and several other reasons, the owner of the company is uncertain about the transportation cost. Therefore, approximate cost for transporting one unit quantity of product from each warehouse to each customer is represented by GTRIFNs. Determine the optimal shipping of products such that the total intuitionistic fuzzy transportation cost is minimum.Let the data be represented in the following Table 2, in which each cell entry $w_iC_j(i = 1,2,3; j = 1,2,3)$ represent the IF transportation cost per unit of the product.

Table 2.

	C_1	C_2	C_3	Supply
w_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15; 0.3)	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
w_2	(2,5,8,10;0.6) (1,5,8,12,0.2)	(4,8,10,13;0.4) (3,8,10,15; 0.3)	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
w_3	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3)	40
Demand	35	45	15	

Sol. We apply GIMDM to compute optimal solution.
*Step 1:*Compute IBFS by IF max - min method.
Consider the 3×3 IFTT in which the costs are represented by GTRIFNs. Since $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j$, the problem is balanced.After the first iteration we get following table.

	C_1	C_2	C_3	Supply
w_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15; 0.3)	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
w_2	(2,5,8,10;0.6) (1,5,8,12,0.2)	(4,8,10,13;0.4) (3,8,10,15; 0.3)	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
w_3	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3) 15	25
Demand	35	45		

Therefore, after first iteration, IFTT reduces to the following 3×2 IFTT

	C_1	C_2	Supply
w_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15; 0.3)	25
w_2	(2,5,8,10;0.6) (1,5,8,12,0.2)	(4,8,10,13;0.4) (3,8,10,15; 0.3)	30
w_3	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2)	25
Demand	35	45	

After allocating 25 to (w_3, C_1) cell, IFTT reduces to following 2×2 IFTT

	C_1	C_2	Supply
w_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	3,5,7,12;0.5) (1,5,7,15;0.3)	25
w_2	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15;0.3)	30
Demand	10	45	

By allocating 25 to (w_1, C_2) cell, IFTT reduces to following 1×2 IFTT

	C_1	C_2	Supply
w_2	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15;0.3)	30
Demand	10	20	

By allocating 20 to (w_2, C_2) cell, IFTT reduces to following 1×1 IFTT

	C_1	Supply
w_2	(2,5,8,10;0.6) (1,5,8,12;0.2)	10
Demand	10	

Finally, all allocations of IBFS is represented in the following IFTT

	C_1	C_2	C_3	Supply
w_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15; 0.3) 25	(2,5,9,16;0.7) (1,5,9,18; 0.3)	25
w_2	(2,5,8,10;0.6) (1,5,8,12,0.2) 10	(4,8,10,13;0.4) (3,8,10,15; 0.3) 20	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
w_3	(2,7,11,15;0.5) (1,7,11,18;0.3) 25	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12; 0.3) 15	40
Demand	35	45	15	

Thus IBFS is $x_{12} = 25, x_{21} = 10, x_{22} = 20, x_{31} = 25, x_{33} = 15$ and the transportation cost is
$25(3,5,7,12;0.5)(1,5,7,15;0.3)\oplus 10(2,5,8,10;0.6)(1,5,8,12;0.2)\oplus 20(4,8,10,13;0.4)$ $(3,8,10,15;0.3)\oplus 25(2,7,11,15;0.5)(1,7,11,18;0.3)\oplus 15(4,6,8,10;0.6)(3,6,8,12;0.3)$ $=(285,600,850,1185;0.4)(165,600,850,1425;0.3)$.

Step 2: Calculate GTRIFNs $\tilde{u}_i^I$ and $\tilde{v}_j^I$ for each row and column respectively, satisfying $\tilde{u}_i^I \oplus \tilde{v}_j^I = \tilde{c}_{ij}^I$ for each occupied cell. So, for sake of simplicity, we assume that $\tilde{v}_1^I$ = (-1,0,0,1;1)(-1,0,0,1;0).

For each occupied cell,
$\tilde{u}_1^I \oplus \tilde{v}_2^I$ = (3,5,7,12;0.5)(1,5,7,15;0.3),
$\tilde{u}_2^I \oplus \tilde{v}_1^I$ = (2,5, 8, 10; 0.6) (1, 5, 8, 12; 0.2)
$\tilde{u}_2^I \oplus \tilde{v}_2^I$ = (4, 8, 10, 13; 0.4) (3, 8, 10, 15; 0.3),
$\tilde{u}_3^I \oplus \tilde{v}_1^I$ = (2, 7, 11, 15; 0.5)(1, 7, 11, 18; 0.3),
$\tilde{u}_3^I \oplus \tilde{v}_3^I$ = (4, 6, 8, 10; 0.6)(3, 6, 8, 12; 0.3).

Thus we get,
$\tilde{u}_3^I$ = (1, 7, 11, 16; 0.5) (0, 7, 11, 19; 0.3),
$\tilde{u}_2^I$ = (1, 5, 8, 11; 0.6) (0, 5, 8, 13; 0.2),
$\tilde{v}_3^I$ = (-12,-5, 1, 9; 0.5) (-16,-5, 1, 12; 0.3),
$\tilde{v}_2^I$ = (-7, 0, 5, 12; 0.4) (-10, 0, 5, 15; 0.3),
$\tilde{u}_1^I$ = (-9, 0, 7, 19; 0.4) (-14, 0, 7, 25; 0.3)

Step 3: Calculate $\tilde{d}_{ij}^I$ for each unoccupied cell.
Therefore,after calculating, we get,

$\tilde{d}_{11}^I$ = (-18, -3, 8, 25; 0.4) (-25, -3, 8, 33; 0.3),
$\tilde{d}_{13}^I$ = (-26,-3, 14, 37; 0.4) (-36, -3, 14, 48; 0.3),
$\tilde{d}_{23}^I$ = (-17, -3, 10, 26; 0.5) (-23, -3, 10, 34; 0.3),
$\tilde{d}_{32}^I$ = (-23, -7, 5, 22; 0.4) (-31, -7, 5, 29; 0.3)

Step 4: Calculate the x_0 value of each $\tilde{d}_{ij}^I$.
$x_0(\tilde{d}_{11}^I) = 3.32$
$x_0(\tilde{d}_{13}^I) = 5.71$
$x_0(\tilde{d}_{23}^I) = 4.48$
$x_0(\tilde{d}_{32}^I) = -0.907$
Since the $x_o(\tilde{d}_{32}^I)$ is negative, so IBFS is not intuitionistic fuzzy optimal.

*Step 5:*Since the $x_o(\tilde{d}_{32}^{\,I})$ is most negative, so cell (3,2) is selected.
Step 6: Construct the loop as follows:

	C_1	C_2	C_3	Supply
w_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15;0.3) **25**	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
w_2	(2,5,8,10;0.6) (1,5,8,12;0.2) **10** **(+)**	(4,8,10,13;0.4) (3,8,10,15;0.3) **20** **(-)**	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
w_3	(2,7,11,15;0.5) (1,7,11,18;0.3) **25** **(-)**	(5,9,12,16;0.7) (3,9,12,19;0.2) **(+)**	(4,6,8,10;0.6) (3,6,8,12;0.3) **15**	40
Dema-nd	35	45	15	

Step 7: Minimum allocation in the cell marked with (-) sign is 20
*Step 8:*Allocate 20 to the unoccupied cell (3,2)and add 20 to the cell with (+) sign, and subtract 20 from the cell with (-) sign
*Step 9:*Improved basic feasible solution is represented in the following IFTT

	C_1	C_2	C_3	Supply
w_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15; 0.3) 25	(2,5,9,16;0.7) (1,5,9,18; 0.3)	25
w_2	(2,5,8,10;0.6) (1,5,8,12,0.2) 30	(4,8,10,13;0.4) (3,8,10,15; 0.3)	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
w_3	(2,7,11,15;0.5) (1,7,11,18;0.3) 5	(5,9,12,16;0.7) (3,9,12,19;0.2) 20	(4,6,8,10;0.6) (3,6,8,12; 0.3) 15	40
Demand	35	45	15	

Step 10: Compute $\tilde{u}_i^{\,I}$ and $\tilde{v}_j^{\,I}$ satisfying $\tilde{u}_i^{\,I} \oplus \tilde{v}_j^{\,I} = \tilde{c}_{ij}^{\,I}$ for each occupied cell in the improved basic feasible solution obtaind in *step 9* . Let

$$\tilde{u}_3^{\,I} = (-1, 0, 0, 1; 1)(-1, 0, 0, 1; 0).$$

For each occupied cell, we get,
$\tilde{u}_1^{\,I} \oplus \tilde{v}_2^{\,I}$ = (3,5,7,12;0.5)(1,5,7,15;0.3),
$\tilde{u}_2^{\,I} \oplus \tilde{v}_1^{\,I}$ = (2,5, 8, 10; 0.6) (1, 5, 8, 12; 0.2)

$\tilde{u}_3^{\,I} \oplus \tilde{v}_1^{\,I} = (2,\ 7,\ 11,\ 15;\ 0.5)\ (1,\ 7,\ 11,\ 18;\ 0.3),$
$\tilde{u}_3^{\,I} \oplus \tilde{v}_2^{\,I} = (5,\ 9,\ 12,\ 16;\ 0.7)(3,\ 9,\ 12,\ 19;\ 0.2),$
$\tilde{u}_3^{\,I} \oplus \tilde{v}_3^{\,I} = (4,\ 6,\ 8,\ 10;\ 0.6)(3,\ 6,\ 8,\ 12;\ 0.3).$

After solving above equations, we get,
$\tilde{v}_3^{\,I} = (3, 6, 8, 11; 0.6)(2, 6, 8, 13; 0.3)$
$\tilde{v}_2^{\,I} = (4, 9, 12, 17; 0.7)(2, 9, 12, 20; 0.2)$
$\tilde{v}_1^{\,I} = (1, 7, 11, 16; 0.5)(0, 7, 11, 19; 0.3)$
$\tilde{u}_2^{\,I} = (-14, -6, 1, 9; 0.5)(-18, -6, 1, 12; 0.3)$
$\tilde{u}_1^{\,I} = (-14, -7, -2, 8; 0.5)(-19, -7, -2, 13; 0.3)$
Thus, for each unoccupied cell,
$\tilde{d}_{11}^{\,I} = (-22, -5, 8, 28; 0.5)(-31, -5, 8, 37; 0.3)$
$\tilde{d}_{13}^{\,I} = (-17, -1, 10, 27; 0.5)(-25, -1, 10, 35; 0.3)$
$\tilde{d}_{22}^{\,I} = (-22, -5, 7, 23; 0.4)(-29, -5, 7, 31; 0.3)$
$\tilde{d}_{23}^{\,I} = (-17, -3, 10, 26; 0.5)(-23, -3, 10, 34; 0.3)$
Since, $x_0(\tilde{d}_{ij}^{\,I}) \geq 0$ for all unoccupied cells, so optimal solution is
$x_{12} = 25, x_{21} = 30, x_{31} = 5, x_{32} = 20, x_{33} = 15$, and the minimum transportation intuitionistic fuzzy cost is
$25(3, 5, 7, 12; 0.5)(1, 5, 7, 15; 0.3) \oplus 30(2, 5, 8, 10; 0.6)(1, 5, 8, 12; 0.2) \oplus 5(2, 7, 11, 15; 0.5)$
$(1, 7, 11, 18; 0.3) \oplus 20(5, 9, 12, 16; 0.7)(3, 9, 12, 19; 0.2) \oplus 15(4, 6, 8, 10; 0.6)(3, 6, 8, 12; 0.3)$
$= (305, 580, 830, 1145; 0.5)(165, 580, 830, 1385; 0.3).$

Conclusions

In this paper, new ranking technique is defined and applied to solve IFTP in which the costs are represented by GTRIFNs. Also, new methods are proposed to compute IBFS and optimal solution of IFTP, which are very simple and easy to understand and can be easily applied by decision maker to solve real life transportation problem.

Acknowledgements. Corresponding author would like to thank "Council of Scientific and Industrial Research" New Delhi, India, for its financial support.

References

[1] S. Abbasbandy and T. Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, Comput. Math. Appl. 57(3) (2009) 413–419.
[2] P. P. Angelov, Optimization in an intuitionistic fuzzy environment, Fuzzy Sets and Systems 86 (1997) 299–306.
[3] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[4] K. T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Physica- Verlag, Heidelberg, New York, 1999
[5] R. E. Bellman and L. A. Zadeh, Decision making in a fuzzy environment, Management Sci. 17 (1970/71) B141–B164.
[6] S. Chanas, W. Kolodziejckzy and A. Machaj, A fuzzy approach to the transportation problem, Fuzzy Sets and Systems 13(3) (1984) 211–221.

[7] S. M Chen and J. H. Chen, Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads, Expert Systems with Applications 36 (2009) 6833–6842.
[8] S. Das and D. Guha, Ranking of intuitionistic fuzzy number by centroid point, Journal of Industrial and Intelligent Information 1(2) (2013) 107–110.
[9] P. K. De and Debaroti Das, Ranking of trapezoidal intuitionistic fuzzy numbers, 12th International Conference on Intelligent Systems Design and Applications (ISDA) (2012) 184–188.
[10] D. Dubey and A. Mehra, Linear programming with triangular intuitionistic fuzzy number, EUSFLAT-LFA (2011) 563–569.
[11] F. L. Hitchcock, The distribution of a product from several sources to numerous localities, J. Math. Phys. Mass. Inst. Tech. 20 (1941) 224–230.
[12] R. J. Hussain and P. Senthil Kumar, Algorithm approach for solving intuitionistic fuzzy transportation problem, Appl. Math. Sci. 6(80) (2012) 3981–3989.
[13] W. Jianqiang and Z. Zhong, Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems, Journal of Systems Engineering and Electronics 20(2) (2009) 321–326 .
[14] A. Kaur and A. Kumar, A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers, Applied Soft Computing 12 (2012) 1201–1213.
[15] D. F. Li, A note on "using intuitionistic fuzzy sets for fault tree analysis on printed circuit board assembly", Microelectronics Reliability 48 (2008) 1741.
[16] D. F. Li, A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to madm problems. Comput. Math. Appl. 60 (2010) 1557–1570.
[17] G. S. Mahapatra and B. S. Mahapatra, Intuitionistic fuzzy fault tree analysis using intuitionistic fuzzy numbers, Int. Math. Forum 5(21) (2010) 1015–1024.
[18] V. L. G. Nayagam, G. Venkateshwari and G. Sivaraman, Ranking of intuitionistic fuzzy numbers, Proceedings of IEEE International Conference on Fuzzy Systems, HongKong, (2008), 1971 - 1974.
[19] H. M. Nehi, A new ranking method for intuitionistic fuzzy numbers, Int. J. Fuzzy Syst. 12(1) (2010) 80–86.
[20] P. Pandian and G. Natarajan, A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems, Appl. Math. Sci. 4(2) (2010) 79–90.
[21] S. Rezvani, Ranking method of trapezoidal intuitionistic fuzzy numbers, Ann. Fuzzy Math. Inform. 5(3) (2013) 515–523.
[22] B. S. Shieh, An approach to centroids of fuzzy numbers, Int. J. Fuzzy Syst. 9(1) (2007) 51–54.
[23] Y. J. Wang and H. S. Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points, Comput. Math. Appl. 55(9) (2008) 2033–2042.
[24] Y. M. Wang, J. B. Yang, D. L. Xu and K. S. Chin, On the centroids of fuzzy numbers, Fuzzy Sets and Systems 157 (2006) 919–926.
[25] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Shashi Aggarwal (shashi.aggarwal@mirandahouse.ac.in)
Department of Mathematics, Miranda House, University of Delhi, Delhi - 110007, India

Chavi Gupta (chavi.gupta85@gmail.com)
Department of Mathematics, University of Delhi, Delhi - 110007, India