Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 8, No. 5, (November 2014), pp. 729–737
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

A note on pairwise fuzzy Baire spaces

Ganesan Thangaraj, Subbiah Sethuraman

Received 14 January 2014; Revised 21 March 2014; Accepted 17 May 2014

Abstract. In this paper several characterizations of pairwise fuzzy Baire bitopological spaces are studied. The concepts of pairwise fuzzy nodec spaces, pairwise fuzzy submaximal spaces, pairwise fuzzy strongly irresolvable spaces and pairwise fuzzy almost resolvable spaces are introduced and the relation between pairwise fuzzy Baire spaces, pairwise fuzzy nodec spaces, pairwise fuzzy submaximal spaces and pairwise fuzzy almost resolvable spaces are studied.

2010 AMS Classification: 54A40, 03E72

Keywords: Fuzzy bitopological spaces, Pairwise fuzzy Baire space, Pairwise fuzzy second category space, Pairwise fuzzy nodec space, Pairwise fuzzy submaximal space, Pairwise fuzzy strongly irresolvable space.

Corresponding Author: Ganesan Thangaraj (g.thangaraj@rediffmail.com)

1. Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [14] in the year 1965. Thereafter the paper of C. L. Chang [5] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concepts of bitopological Baire spaces have been studied extensively in classical topology in [1], [2], [6] and [7]. In 1989, A. Kandil [8] introduced the concept of fuzzy bitopological space as an extension of fuzzy topological space and as a generalization of bitopological space. The concept of Baire space in fuzzy setting was introduced and studied by G. Thangaraj and S. Anjalmose in [10]. The concept of bitopological Baire space in fuzzy setting was introduced and studied by the authors in [11]. The concept of pairwise dense sets was introduced and studied by Biswanath and Bandyopadhyay in [4]. In this paper we discuss several characterizations of pairwise fuzzy Baire bitopological spaces. The concepts of pairwise fuzzy nodec spaces, pairwise fuzzy submaximal spaces and pairwise fuzzy strongly irresolvable spaces are introduced in this paper. Inter - relations between pairwise fuzzy Baire spaces, pairwise fuzzy nodec spaces, pairwise fuzzy submaximal spaces and pairwise fuzzy strongly irresolvable spaces are also investigated in this paper. While studying the properties of pairwise fuzzy nowhere dense sets, the following Questions aries when a countable union of pairwise fuzzy nowhere dense sets becomes pairwise fuzzy nowhere dense set?. The answer for this Question is obtained in this paper. Also the complement of a fuzzy dense and fuzzy open set is a fuzzy nowhere dense set in a fuzzy topological space [10]. But when extend this concept to fuzzy topological space, we find that the complement is a pairwise fuzzy dense and pairwise fuzzy open set need not be a pairwise fuzzy nowhere dense set. An example is given to illustrate the concept in this paper.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work, by a fuzzy bitopological space (Kandil, 1989), we mean an ordered triple (X, T_1, T_2) where T_1 and T_2 are fuzzy topologies on the non-empty set X. The complement λ' of a fuzzy set λ is defined by $\lambda'(x) = 1 - \lambda(x)$.

Definition 2.1 ([14])**.** Let λ and μ be any two fuzzy sets in (X, T). Then we define $\lambda \vee \mu$: X $\to$ [0,1] as follows : $(\lambda \vee \mu)(x)$ = max $\{\lambda(\mathrm{x}), \mu(\mathrm{x})\}$. Also we define $\lambda \wedge \mu$: X $\to [0, 1]$ as follows : $(\lambda \wedge \mu)(x)$ = min $\{\lambda(\mathrm{x}), \mu(\mathrm{x})\}$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the *union* $\psi = \vee_i \lambda_i$ and *intersection* $\delta = \wedge_i \lambda_i$ are defined respectively as $\psi(x) = sup_i\{\lambda_i(x) \ / \ x \in X\}$ and $\delta(x) = inf_i\{\lambda_i(x) \ / \ x \in X\}$.

Definition 2.2 ([3])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define the interior and exterior of λ as $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.3 ([3])**.** *Let λ be any fuzzy set in a fuzzy topological space (X, T). Then* $1 - cl(\lambda) = int(1 - \lambda)$ *and* $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.4 ([9])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy dense* if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.

Definition 2.5 ([12])**.** Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1}(cl_{T_2}(\lambda)) = cl_{T_2}(cl_{T_1}(\lambda)) = 1$.

Definition 2.6 ([9])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy nowhere dense* if there exists no non - zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $intcl(\lambda) = 0$.

Definition 2.7 ([11])**.** A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called *pairwise fuzzy nowhere dense* if $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$.

Definition 2.8. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ $(i = 1, 2)$ and a pairwise fuzzy closed set if $1 - \lambda \in T_i$ $(i = 1, 2)$.

Definition 2.9 ([11]). Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy first category set if $\lambda = \bigvee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). A fuzzy set which is not of pairwise fuzzy first category, is called a *pairwise fuzzy second category set* in (X, T_1, T_2).

Definition 2.10 ([11]). If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a *pairwise fuzzy residual set* in (X, T_1, T_2).

Definition 2.11 ([10]). A fuzzy topological space (X, T) is called a fuzzy Baire space if $int(\bigvee_{k=1}^{\infty}(\lambda_k)) = 0$, where (λ_k)'s are fuzzy nowhere dense sets in (X, T).

Lemma 2.12 ([3]). *For a family $\mathscr{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case $\mathscr{A}$ is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$.*

3. Pairwise fuzzy Baire spaces

Definition 3.1 ([11]). A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $int_{T_i}(\bigvee_{k=1}^{\infty}(\lambda_k)) = 0,\ (i = 1, 2)$ where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Proposition 3.2. *If μ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set λ in (X, T_1, T_2), then λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2) .*

Proof. Let μ be a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2). Then, we have $int_{T_1}(cl_{T_2}(\mu)) = int_{T_2}(cl_{T_1}(\mu)) = 0$. Now $\lambda \leq \mu$, implies that $int_{T_1}(cl_{T_2}(\lambda)) \leq int_{T_1}(cl_{T_2}(\mu))$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq int_{T_2}(cl_{T_1}(\mu))$. Hence $int_{T_1}(cl_{T_2}(\lambda)) \leq 0$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq 0$. That is., $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Therefore λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). □

Proposition 3.3. *If μ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set λ in (X, T_1, T_2), then λ is a pairwise fuzzy first category set in (X, T_1, T_2).*

Proof. Let μ be a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2). Then, we have $\mu = \vee_{k=1}^{\infty}\mu_k$, where (μ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $\lambda \wedge \mu = \lambda \wedge \left(\vee_{k=1}^{\infty}(\mu_k)\right) = \vee_{k=1}^{\infty}(\lambda \wedge \mu_k)$. Also $\lambda \leq \mu$, implies that $\lambda \wedge \mu = \lambda$. Therefore $\lambda = \vee_{k=1}^{\infty}(\lambda \wedge \mu_k)$. Since $\lambda \wedge \mu_k \leq \mu_k$ and (μ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), by proposition 3.2, $(\lambda \wedge \mu_k)$'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence $\lambda = \vee_{k=1}^{\infty}(\lambda \wedge \mu_k)$, where $(\lambda \wedge \mu_k)$'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), implies that λ is a pairwise fuzzy first category set in (X, T_1, T_2). □

Proposition 3.4. *If λ is a pairwise fuzzy residual set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set μ in (X, T_1, T_2), then μ is a pairwise fuzzy residual set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy residual set in a fuzzy bitopological space (X, T_1, T_2). Then, $1 - \lambda$ is a pairwise fuzzy first category set in (X, T_1, T_2). Now $\lambda \leq \mu$ for a fuzzy set μ in (X, T_1, T_2), implies that $1 - \lambda \geq 1 - \mu$. Then, by proposition 3.3, $1 - \mu$ is a pairwise fuzzy first category set in (X, T_1, T_2). Hence μ is a pairwise fuzzy residual set in (X, T_1, T_2). □

Theorem 3.5 ([11]). *Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:*

(i) *(X, T_1, T_2) is a pairwise fuzzy Baire space.*
(ii) *$int_{T_i}(\lambda) = 0$, (i=1,2) for every pairwise fuzzy first category set λ in (X, T_1, T_2).*
(iii) *$cl_{T_i}(\mu) = 1$, (i=1,2) for every pairwise fuzzy residual set μ in (X, T_1, T_2).*

Proposition 3.6. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then no non-zero pairwise fuzzy open set is a pairwise fuzzy first category set in (X, T_1, T_2).*

Proof. Let λ be a non-zero pairwise fuzzy open set in (X, T_1, T_2). Then, $int_{T_i}(\lambda) = \lambda$ $(i = 1, 2)$. Suppose that λ is a pairwise fuzzy first category set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 3.5, $int_{T_i}(\lambda) = 0$ $(i = 1, 2)$. This implies that $\lambda = 0$, a contradiction. Hence no non-zero pairwise fuzzy open set is a pairwise fuzzy first category set in (X, T_1, T_2). □

Proposition 3.7. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then each pairwise fuzzy residual set is a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy residual set in a pairwise fuzzy Baire bitopological space (X, T_1, T_2). Then, by theorem 3.5, $cl_{T_i}(\lambda) = 1$ $(i = 1, 2)$ in (X, T_1, T_2). Hence we have $cl_{T_1}(cl_{T_2}(\lambda)) = cl_{T_2}(cl_{T_1}(\lambda)) = 1$. Therefore λ is a pairwise fuzzy dense set in (X, T_1, T_2). □

Remark 3.8. In a fuzzy topological space (X,T), if λ is a fuzzy dense and fuzzy open set, then $1-\lambda$ is a fuzzy nowhere dense set in (X,T)[10]. But, this does not hold in a fuzzy bitopological space (X, T_1, T_2). If λ is a pairwise fuzzy dense and pairwise fuzzy open set in (X, T_1, T_2), then $1 - \lambda$ need not be a pairwise fuzzy nowhere dense set in (X, T_1, T_2) .For, consider the following example:

Example 3.9. Let $X = \{a, b, c\}$. The fuzzy sets $\lambda_k, (k = 1, 2)$ and μ_1 are defined on X as follows :

λ_1 : X $\longrightarrow$ [0,1] is defined as λ_1(a) = 0.2; λ_1(b) = 0.3; λ_1(c) = 0.4;
λ_2 : X $\longrightarrow$ [0,1] is defined as λ_2(a) = 0.8; λ_2(b) = 0.7; λ_2(c) = 0.6;
μ_1 : X $\longrightarrow$ [0,1] is defined as μ_1(a) = 0.4; μ_1(b) = 0; μ_1(c) = 0.

Clearly $T_1 = \{ 0, \lambda_1, \lambda_2, 1 \}$ and $T_2 = \{ 0, \mu_1, \lambda_2, 1 \}$ are fuzzy topologies on X. By easy computations, $cl_{T_1}(cl_{T_2}(\lambda_2)) = cl_{T_1}(1) = 1$ and $cl_{T_2}(cl_{T_1}(\lambda_2)) = cl_{T_2}(1 - \lambda_1) = 1$. Hence λ_2 is a pairwise fuzzy dense and pairwise fuzzy open set in (X, T_1, T_2). But $int_{T_1}(cl_{T_2}(1-\lambda_2)) = int_{T_1}(cl_{T_2}(\lambda_1)) = int_{T_1}(\lambda_1) = \lambda_1 \neq 0$ and $int_{T_2}(cl_{T_1}(1-\lambda_2)) = 0$. Therefore $1 - \lambda_2$ is not a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition 3.10. *If the fuzzy bitopological space* (X, T_1, T_2) *is a pairwise fuzzy Baire space and if* $\vee_{k=1}^{\infty}(\lambda_k)=1$*, where the* (λ_k)*'s are fuzzy sets defined on* X*, then there is at least one fuzzy set* λ_k *such that either* $int_{T_1}(cl_{T_2}(\lambda_k)) \neq 0$ *or* $int_{T_2}(cl_{T_1}(\lambda_k)) \neq 0$.

Proof. Suppose $int_{T_1}(cl_{T_2}(\lambda_k)) = 0$ and $int_{T_2}(cl_{T_1}(\lambda_k)) = 0$ for all $k \in N$. Then (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Then $\vee_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy first category set in a pairwise fuzzy Baire space (X, T_1, T_2). By theorem 3.5, $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ $(i = 1, 2)$. But, this is a contradiction, since $\vee_{k=1}^{\infty}(\lambda_k) = 1$, implies that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = int_{T_i}(1) = 1$ $(i = 1, 2)$. Hence, there is at least one fuzzy set λ_k defined on X such that either $int_{T_1}(cl_{T_2}(\lambda_k)) \neq 0$ or $int_{T_2}(cl_{T_1}(\lambda_k)) \neq 0$. □

Definition 3.11 ([11]). A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy first category space* if the fuzzy set 1_X is a pairwise fuzzy first category set in (X, T_1, T_2). That is, $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Otherwise (X, T_1, T_2) will be called a *pairwise fuzzy second category space* in (X, T_1, T_2).

Theorem 3.12 ([11]). *If* λ *is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space* (X, T_1, T_2)*, then* $1 - \lambda$ *is a pairwise fuzzy dense set in* (X, T_1, T_2).

Proposition 3.13. *If* $(\wedge_{k=1}^{\infty}(\lambda_k)) \neq 0$*, where* (λ_k)*'s are pairwise fuzzy dense sets in a fuzzy bitopological space* (X, T_1, T_2)*, then* (X, T_1, T_2) *is a pairwise fuzzy second category space.*

Proof. Suppose that (X, T_1, T_2) is a pairwise fuzzy first category space. Then, we have $\vee_{k=1}^{\infty}(\mu_k) = 1$, where (μ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). This implies that $\wedge_{k=1}^{\infty}(1 - \mu_k) = 0$. Since (μ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), by theorem 3.12, $(1 - \mu_k)$'s are pairwise fuzzy dense sets in (X, T_1, T_2). Let $\lambda_k = 1 - \mu_k$. Thus, we have $(\wedge_{k=1}^{\infty}(\lambda_k)) =0$, where (λ_k)'s are pairwise fuzzy dense sets in (X, T_1, T_2). But this is a contradiction to the hypothesis. Therefore (X, T_1, T_2) is not a pairwise fuzzy first category space and hence (X, T_1, T_2) is a pairwise fuzzy second category space. □

Remark 3.14. In classical topology, a finite union of nowhere dense sets in a topological space is a nowhere dense set. But if λ and μ are pairwise fuzzy nowhere dense sets in a fuzzy bitopological space (X, T_1, T_2), then $\lambda \vee \mu$ need not be a pairwise fuzzy nowhere dense set in (X, T_1, T_2) [11] . When do a countable union of pairwise fuzzy nowhere dense sets in a fuzzy bitopological space is a pairwise fuzzy nowhere dense set?. The answer, for this Question, are given in the following propositions.

Proposition 3.15. *If the pairwise fuzzy first category set* λ*, is a pairwise fuzzy closed set, in a pairwise fuzzy Baire space* (X, T_1, T_2)*, then* λ *is a pairwise fuzzy nowhere dense set in* (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy first category set in a pairwise fuzzy Baire space (X, T_1, T_2) and $cl_{T_i}(\lambda) = \lambda ...(1)$ $(i = 1, 2)$. By theorem 3.5, $int_{T_i}(\lambda) = 0 ...(2)$ $(i = 1, 2)$, for the pairwise fuzzy first category set λ in (X, T_1, T_2). Then, from (1) and (2), we have $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Hence, λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). □

Definition 3.16. A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy submaximal space* if each pairwise fuzzy dense set in (X, T_1, T_2), is a pairwise fuzzy open set in (X, T_1, T_2). That is., if λ is a pairwise fuzzy dense set in a fuzzy bitopological space (X, T_1, T_2), then $\lambda \in T_i$ $(i = 1, 2)$.

Proposition 3.17. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal pairwise fuzzy Baire space, then each pairwise fuzzy first category set in (X, T_1, T_2) is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy submaximal pairwise fuzzy Baire space and λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then, $1 - \lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by proposition 3.7, we have $1 - \lambda$ is a pairwise fuzzy dense set. Also, since (X, T_1, T_2) is a pairwise fuzzy submaximal space, for the pairwise fuzzy dense set $1 - \lambda$, we have $1 - \lambda \in T_i$ $(i = 1, 2)$. Hence the pairwise fuzzy first category set λ is a pairwise fuzzy closed set in (X, T_1, T_2). Then, by proposition 3.15, λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). □

Definition 3.18. A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy strongly irresolvable space if for each pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_1}(int_{T_2}(\lambda)) = cl_{T_2}(int_{T_1}(\lambda)) = 1$.

Proposition 3.19. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable pairwise fuzzy Baire space, then each pairwise fuzzy first category set in (X, T_1, T_2) is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable pairwise fuzzy Baire space and λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then, $1 - \lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by proposition 3.7, $1 - \lambda$ is a pairwise fuzzy dense set. Also since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense set $1 - \lambda$, we have

$$cl_{T_1}(int_{T_2}(1 - \lambda)) = cl_{T_2}(int_{T_1}(1 - \lambda)) = 1.$$

Then $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Hence the pairwise fuzzy first category set λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). □

4. Relations between pairwise fuzzy Baire spaces and other fuzzy topological spaces

The concept of nodec spaces in classical topology was introduced and studied by Eric K. Van Douwen[13]. Motivated by this we shall now define :

Definition 4.1. A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy nodec space* if every non-zero pairwise fuzzy nowhere dense set in (X, T_1, T_2), is a pairwise fuzzy closed set in (X, T_1, T_2). That is., if λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda \in T_i$ $(i = 1, 2)$.

Proposition 4.2. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy nodec space, then (X, T_1, T_2) is not a pairwise fuzzy Baire space.*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy nodec space and (λ_k)'s ($k = 1$ *to* ∞) be pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Then (λ_k)'s are pairwise fuzzy closed sets in (X, T_1, T_2) and hence $int_{T_1}(cl_{T_2}(\lambda_k)) = int_{T_1}(\lambda_k))$ and $int_{T_2}(cl_{T_1}(\lambda_k))$ $= int_{T_2}(\lambda_k)$. Since (λ_k)'s are pairwise fuzzy nowhere dense in (X, T_1, T_2),

$$int_{T_1}(cl_{T_2}(\lambda_k)) = int_{T_2}(cl_{T_1}(\lambda_k)) = 0.$$

This implies that $int_{T_1}(\lambda_k) = int_{T_2}(\lambda_k) = 0$. That is., $int_{T_i}(\lambda_k) = 0$, $(i = 1, 2)$. Now $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) > \vee_{k=1}^{\infty} int_{T_1}(\lambda_k)$ $(i = 1, 2)$, implies that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) >$ $\vee_{k=1}^{\infty}(0)$. Then, we have $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) \neq 0$. Hence (X, T_1, T_2) is not a pairwise fuzzy Baire space. □

Definition 4.3. A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy almost resolvable space, if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets λ_k's in (X, T_1, T_2) are such that $int_{T_i}(\lambda_k) = 0$, $(i = 1, 2)$. Otherwise (X, T_1, T_2) is called a pairwise fuzzy almost irresolvable space.

Proposition 4.4. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then (X, T_1, T_2) is a pairwise fuzzy almost irresolvable space.*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy Baire space. Since every pairwise fuzzy Baire space is a pairwise fuzzy second category space, (X, T_1, T_2) is not a pairwise fuzzy first category space. This implies $\bigvee_{k=1}^{\infty}(\lambda_k) \neq 1$, where (λ_k)'s ($k = 1$ *to* ∞) are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Since (λ_k)'s ($k = 1$ *to* ∞) are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), $int_{T_1}(cl_{T_2}(\lambda_k)) = int_{T_2}(cl_{T_1}(\lambda_k)) = 0$. Also, since $int_{T_1}(\lambda_k) \leq int_{T_1}(cl_{T_2}(\lambda_k))$ and $int_{T_2}(\lambda_k) \leq int_{T_2}(cl_{T_1}(\lambda_k))$, $int_{T_i}(\lambda_k) =$ 0 $(i = 1, 2)$. Hence $\bigvee_{k=1}^{\infty}(\lambda_k) \neq 1$, where $int_{T_i}(\lambda_k) = 0$, $(i = 1, 2)$. Therefore (X, T_1, T_2) is a pairwise fuzzy almost irresolvable space. □

Proposition 4.5. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal space, then (X, T_1, T_2) is a pairwise fuzzy nodec space.*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy submaximal space and λ be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then, by theorem 3.12, $1 - \lambda$ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, $1 - \lambda$ is a pairwise fuzzy open set in (X, T_1, T_2). This implies that λ is a pairwise fuzzy closed set in (X, T_1, T_2). Hence each pairwise fuzzy nowhere dense set is a pairwise fuzzy closed set in (X, T_1, T_2) and therefore (X, T_1, T_2) is a pairwise fuzzy nodec space. □

Proposition 4.6. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal space, then (X, T_1, T_2) is not a pairwise Baire space.*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy submaximal space. By proposition 4.5, (X, T_1, T_2) is a pairwise fuzzy nodec space. Then by proposition 4.2, (X, T_1, T_2) is not a pairwise fuzzy Baire space. □

Proposition 4.7. *If* $cl_{T_i}(\wedge_{k=1}^{\infty}(\lambda_k)) = 1$ $(i = 1, 2)$*, where* (λ_k)*'s* $(k = 1$ *to* $\infty)$ *are pairwise fuzzy dense sets in a pairwise fuzzy strongly irresolvable space* (X, T_1, T_2)*, then* (X, T_1, T_2) *is a pairwise Baire space.*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Suppose that (λ_k)'s $(k = 1$ *to* $\infty)$ are pairwise fuzzy dense sets in (X, T_1, T_2). Then, $cl_{T_1}(int_{T_2}(\lambda_k)) = cl_{T_2}(int_{T_1}(\lambda_k)) = 1$. Now

$$1 - cl_{T_1}(int_{T_2}(\lambda_k)) = 1 - cl_{T_2}(int_{T_1}(\lambda_k)) = 0.$$

Then, we have $int_{T_1}(cl_{T_2}(1 - \lambda_k)) = int_{T_2}(cl_{T_1}(1 - \lambda_k)) = 0$. Hence $(1 - \lambda_k)$'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $cl_{T_i}(\wedge_{k=1}^{\infty}(\lambda_k)) = 1$, implies that $1 - cl_{T_i}(\wedge_{k=1}^{\infty}(\lambda_k)) = 0$. Then, $int_{T_i}(1 - (\wedge_{k=1}^{\infty}(\lambda_k))) = 0$ and hence $int_{T_i}(\vee_{k=1}^{\infty}(1 - \lambda_k)) = 0$, $(i = 1, 2)$, where $(1 - \lambda_k)$'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) and therefore the fuzzy bitopological space (X, T_1, T_2) is a pairwise Baire space. □

Conclusions

In this paper several characterizations of fuzzy Baire space are studied. The relations between pairwise fuzzy Baire spaces, pairwise fuzzy nodec spaces, pairwise fuzzy submaximal spaces and pairwise fuzzy almost resolvable spaces are obtained.

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Ganesan Thangaraj (g.thangaraj@rediffmail.com)
Department of Mathematics, Thiruvalluvar University, Vellore - 632 115, Tamil Nadu, India

Subbiah Sethuraman (koppan60@gmail.com)
Department of Mathematics, T.K.Govt. Arts College, Vriddhachalam - 606 001, Tamil Nadu, India