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Ideal convergence of nets in fuzzy topological spaces

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Abstract. In this paper, we have initiated the concept of convergence of nets of fuzzy points in a fuzzy topological space (X, σ) [5] via an ideal $\mathcal{I}$ (namely, $\mathcal{I}$-convergence) and investigated its several properties. Two new limits, namely fuzzy upper and lower $\mathcal{I}$-limits of nets of fuzzy sets are being introduced; several properties and their mutual relationships have been investigated. Finally, applications of the concept of the upper $\mathcal{I}$-limit of nets are given to characterize various fuzzy covering properties.

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Keywords: Fuzzy $\mathcal{I}$-convergence, Fuzzy continuous function, Fuzzy upper $\mathcal{I}$-limit, Fuzzy lower $\mathcal{I}$-limit, Weakly fuzzy compact, Strongly fuzzy compact and fuzzy compact.

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1. Introduction

In real line, the statistical convergence of sequences, introduced by Fast [13], has been developed by a good number of researchers, namely Červeňanský [4], Connor [6], Connor and Kline [7], Fridy [14, 15], Fridy et. al. [16], Kostyrko et. al. [25], Miller [30] and Šalát and Tijdeman [38]. The concept of $\mathcal{I}$-convergence of sequences of real numbers was introduced by Kostyrko et. al. [26] and $\mathcal{I}$-limit superior and inferior were initiated by Demirci [10]. Those concepts were further studied by Das and Lahiri [9] in general topological spaces. Some recent research works related to $\mathcal{I}$-convergence are found in the papers of Das et. al. [8], Komisarski [24], Kumar [27], Mursaleen and Alotaibi [31], Mursaleen et. al. [34], Mursaleen and Mohiuddine [32, 33], Şahiner et. al. [37] and Šalát et. al. [39].

After the invention of fuzzy sets by Zadeh [44], a major task for the mathematicians was to fuzzify different existing concepts of pure mathematics. In this fashion, Nuray and Savaş [36] defined the concepts of statistical convergence and statistical

Cauchy sequence of fuzzy numbers and investigated its basic properties. The works of Aytar [1], Aytar and Pehlivan [3], Esi and Özdemir [12], Nuray [35], Savaş [40] and Thillaigovindan [43] are worth to be mentioned. The statistical limit superior and limit inferior of statistically bounded sequences of fuzzy numbers were defined and studied by Aytar et al. [2]. These concepts were also studied by Aytar and Pehlivan [3] to achieve some relationships between statistical limit superior (and statistical limit inferior) and statistical cluster points of a statistically bounded sequence of fuzzy numbers. The concept of $\mathcal{I}$-convergence of sequences of real numbers was fuzzified by Kumar and Kumar [28] that was further studied by Dutta and Tripathy [11], Hazarika [18, 19, 20, 21, 22, 23] and Subramanian el. al. [42].

In this paper in section 3, we have introduced the concept of convergence of nets of fuzzy points in a fuzzy topological space (X, σ) via ideal $\mathcal{I}$ (namely, $\mathcal{I}$-convergency) and investigated its several properties. We have also found their relationships with the classical convergence of nets in fuzzy topological space. It has been shown that under a continuous function $\psi : X \to X$, where X is a fuzzy topological space, the concept of $\mathcal{I}$-convergence remains invariant. In section 4, we give our attention to introduce the concepts of two new limits, namely fuzzy upper and lower $\mathcal{I}$-limits of the nets of fuzzy sets in fuzzy topological space. Several properties and their mutual relationships have been investigated. Finally, in section 5, we have discussed some applications of the concept of upper $\mathcal{I}$-limit of the nets on some fuzzy covering properties of fuzzy topological spaces.

2. Preliminaries

Throughout this paper, spaces (X, σ) and (Y, δ) (or simply X and Y) represent non-empty fuzzy topological spaces due to Chang [5] and the symbols I and I^X have been used for the unit closed interval $[0, 1]$ and the set of all functions with domain X and codomain I respectively. The support of a fuzzy set A is the set $\{x \in X : A(x) > 0\}$ and is denoted by $supp(A)$. A fuzzy set with only non-zero value $\lambda \in (0, 1]$ at only one element $x \in X$ is called a fuzzy point and is denoted by x_λ and the set of all fuzzy points of a fuzzy topological space is denoted by $FP(X)$. For any two fuzzy sets A, B of X, $A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy point x_λ is said to be in a fuzzy set A (denoted by $x_\lambda \in A$) if $x_\lambda \leq A$, that is, if $\lambda \leq A(x)$. The constant fuzzy sets of X with values 0 and 1 are denoted by $\underline{0}$ and $\underline{1}$ respectively. A fuzzy set A is said to be quasi-coincident with B (written as $A\hat{q}B$) [41] if $A(x) + B(x) > 1$ for some $x \in X$. A fuzzy set A is said to be not quasi-coincident with B (written as $A\bar{q}B$) [26] if $A(x) + B(x) \leq 1$, for all $x \in X$. A fuzzy open set A of X is called fuzzy quasi-neighborhood of a fuzzy point x_λ if $x_\lambda \hat{q} A$. It is well-known that a function $\psi : X \to Y$ is fuzzy continuous [5] if for every fuzzy point x_λ and every fuzzy quasi-neighborhood V of a fuzzy point $\psi(x_\lambda)$, there exists a fuzzy quasi-neighborhood U of a fuzzy point x_λ such that $\psi(U) \leq V$. In this paper, the cardinalities of an ordinary set K and of the set N of all natural numbers are denoted by $\mid K \mid$ and $\aleph_0$ respectively.

Throughout the paper, $\mathcal{N}$ stands for a directed set. An ideal on a non-empty set S is defined as a non-empty family $\mathcal{I}$ of subsets of S satisfying (i) $\varnothing \in \mathcal{I}$, (ii) $A \in \mathcal{I}$ and $B \subset A \Rightarrow B \in \mathcal{I}$ and (iii) $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$. An ideal $\mathcal{I}$ on $\mathcal{N}$ is called non-trivial if $\mathcal{N} \notin \mathcal{I}$. A non-trivial ideal $\mathcal{I}$ on $\mathcal{N}$ is called admissible [9] if

$\mathcal{N} - M_\lambda \in \mathcal{I}$, where $M_\lambda = \{n \in \mathcal{N} : n \geq \lambda\}$ for all $\lambda \in \mathcal{N}$. Throughout this paper, $\mathcal{I}$ stands for an admissible ideal on $\mathcal{N}$.

3. Fuzzy $\mathcal{I}$-convergence

Definition 3.1. A net $\{S_n : n \in \mathcal{N}\}$ of fuzzy points of a fuzzy topological space X is said to fuzzy $\mathcal{I}$-converge to a fuzzy point x_λ if for every fuzzy quasi-neighborhood U of a fuzzy point x_λ, $\{n \in \mathcal{N} : S_n \bar{q} U\} \in \mathcal{I}$.

Remark 3.2. If a net $\{S_n : n \in \mathcal{N}\}$ of fuzzy points of a fuzzy topological space fuzzy converges to a fuzzy point x_λ, then the net $\{S_n : n \in \mathcal{N}\}$ is fuzzy $\mathcal{I}$-converges to x_λ. For, if U be a fuzzy quasi-neighborhood of x_λ with $S_n \hat{q} U$ for all $n \in M_{n_0}$, then $\mathcal{N} - M_{n_0} \in \mathcal{I}$ and so $\{n \in \mathcal{N} : S_n \bar{q} U\}(\subset \mathcal{N} - M_{n_0}) \in \mathcal{I}$.

Theorem 3.3. *In Hausdorff spaces, if a net fuzzy $\mathcal{I}$-converges to two distinct fuzzy points, then their supports are same.*

Proof. If possible, let $\{S_n : n \in \mathcal{N}\}$ be a net in a fuzzy Hausdorff topological space X fuzzy $\mathcal{I}$-converging to two distinct fuzzy points x_λ and x_μ such that $supp(x_\lambda) \neq supp(x_\mu)$. Then for any two fuzzy quasi-neighborhood U and V of the fuzzy points x_λ and x_μ, $\{n \in \mathcal{N} : S_n \bar{q} U\} \in \mathcal{I}$ and $\{n \in \mathcal{N} : S_n \bar{q} V\} \in \mathcal{I}$ respectively and so $\{n \in \mathcal{N} : S_n \bar{q}(U \wedge V)\} \in \mathcal{I}$. Since $\mathcal{I}$ is non-trivial, there exists a positive integer $k \in \mathcal{N}$ such that $S_k \hat{q} U$ and $S_k \hat{q} V$. Now suppose $supp(S_k) = x$. Then $U(x) > 0$ and $V(x) > 0$, which is a contradiction. □

Theorem 3.4. *Let $\psi : X \to X$ be a fuzzy continuous function and $\{S_n : n \in \mathcal{N}\}$ be a fuzzy $\mathcal{I}$-convergent net in X. Then $\{\psi(S_n)\}$ is fuzzy $\mathcal{I}$-convergent.*

Proof. Suppose $\psi : X \to X$ be a fuzzy continuous function at x_λ and $\{S_n : n \in \mathcal{N}\}$ be a net in X fuzzy $\mathcal{I}$-converging to x_λ. Consider V be a fuzzy quasi-neighborhood of $\psi(x_\lambda)$. Then there exists a fuzzy quasi-neighborhood U of x_λ such that $\psi(U) \leq V$. Since $\{S_n : n \in \mathcal{N}\}$ fuzzy $\mathcal{I}$-converges to x_λ, we have $\{n \in \mathcal{N} : S_n \bar{q} U\} \in \mathcal{I}$. So the inclusion $\{n \in \mathcal{N} : \psi(S_n) \bar{q} V\} \subset \{n \in \mathcal{N} : S_n \bar{q} U\}$ ensures that $\{n \in \mathcal{N} : \psi(S_n) \bar{q} V\} \in \mathcal{I}$. □

Theorem 3.5. *Let A be a fuzzy subset of X. If a net $\{S_n : n \in \mathcal{N}\}$ in A $\mathcal{I}$-converges to a fuzzy point x_λ, then $x_\lambda \in cl(A)$.*

Proof. Let U be any fuzzy quasi-neighborhood of x_λ. Then $\{n \in \mathcal{N} : S_n \bar{q} U\} \in \mathcal{I}$. Since $\mathcal{I}$ is non-trivial, there exists an $m \in \mathcal{N}$ such that $S_m \hat{q} U$. Let $S_m = x_\mu$. Then $U(x) + \mu > 1$ and $A(x) \geq \mu$ and so $U(x) + A(x) > 1$. Thus $A \hat{q} U$ and so $x_\lambda \in cl(A)$. □

4. Fuzzy upper $\mathcal{I}$-limits and lower $\mathcal{I}$-limits

Definition 4.1. Let $\{A_n : n \in \mathcal{N}\}$ be a net of fuzzy sets of a fuzzy topological space X. Then the fuzzy upper $\mathcal{I}$-limit of $\{A_n : n \in \mathcal{N}\}$ is defined and denoted by $FIUL(A_n) = \vee\{x_\lambda \in FP(X)$: for every fuzzy quasi-neighborhood U of x_λ, $\{n \in \mathcal{N} : A_n \hat{q} U\} \notin \mathcal{I}\}$.

Theorem 4.2. *Let $\{A_n : n \in \mathcal{N}\}$ be a net of fuzzy sets of a fuzzy topological space X. Then the following properties hold:*
(i) $FIUL(A_n)$ is a closed set.
(ii) $FIUL(A_n) = FIUL(cl(A_n))$.
(iii) $FIUL(A_n) \leq cl(\vee_{i=1}^{\infty} A_i)$.
(iv) If for each $n \in \mathcal{N}$, $A_n = A \in \mathcal{I}^X$, then $FIUL(A_n) = cl(A)$.

Proof. (i) Let $x_\lambda \in cl(FIUL(A_n))$ and U be any fuzzy quasi-neighborhood of x_λ with $FIUL(A_n)\hat{q}U$. Then there exists an $y \in X$ such that $FIUL(A_n)(y)+U(y) > 1$. Consider $FIUL(A_n)(y) = \mu$. Then $y_\mu \hat{q} U$ and $y_\mu \in FIUL(A_n)$. Hence $\{n \in \mathcal{N} : A_n\hat{q}U\} \notin \mathcal{I}$ and so $x_\lambda \in FIUL(A_n)$.

(ii) Let U be any fuzzy quasi-neighborhood of x_λ. It is sufficient to show that $\{n \in \mathcal{N} : cl(A_n)\hat{q}U\} = \{n \in \mathcal{N} : A_n\hat{q}U\}$. Let $n \in \{n \in \mathcal{N} : A_n\hat{q}U\}$. Then there exists an $y \in X$ such that $A_n(y) + U(y) > 1$, that is, $cl(A_n)(y) + U(y) > 1$ and so $n \in \{n \in \mathcal{N} : cl(A_n)\hat{q}U\}$. Conversely, let $n \in \{n \in \mathcal{N} : cl(A_n)\hat{q}U\}$. Then there exists an $y \in X$ such that $cl(A_n)(y) + U(y) > 1$. Suppose $cl(A_n)(y) = \mu$. Then $y_\mu \in cl(A_n)$ and U is fuzzy quasi-neighborhood of y_μ. Thus $U\hat{q}A_n$ and so $n \in \{n \in \mathcal{N} : A_n\hat{q}U\}$.

(iii) Let $x_\lambda \in FIUL(A_n)$ and U be any fuzzy quasi-neighborhood of x_λ. Then $\{n \in \mathcal{N} : A_n\hat{q}U\} \notin \mathcal{I}$. Since $\mathcal{I}$ is non-trivial, there exists an $m \in \mathcal{N}$, such that $A_m\hat{q}U$ and so $\vee_{i=1}^{\infty} A_i\hat{q}U$. Thus $x_\lambda \in cl(\vee_{i=1}^{\infty} A_i)$.

(iv) (iii) implies that $FIUL(A_n) \leq cl(A)$. So let $x_\lambda \notin FIUL(A_n)$. Then there exists a fuzzy quasi-neighborhood U of x_λ such that $\{n \in \mathcal{N} : A_n\hat{q}U\} \in \mathcal{I}$. Since $\mathcal{N} \notin \mathcal{I}$, there exists an $m \in \mathcal{N}$ satisfying $A_m\bar{q}U$, that is, $A\bar{q}U$ and so $x_\lambda \notin cl(A)$. □

Theorem 4.3. *Let $\{A_n : n \in \mathcal{N}\}$ and $\{B_n : n \in \mathcal{N}\}$ be any two nets of fuzzy sets of a fuzzy topological space X. Then:*
(i) $A_n \leq B_n$ for all $n \in \mathcal{N}$ implies that $FIUL(A_n) \leq FIUL(B_n)$.
(ii) $FIUL(A_n \vee B_n) = FIUL(A_n) \vee FIUL(B_n)$.
(iii) $FIUL(A_n \wedge B_n) \leq FIUL(A_n) \wedge FIUL(B_n)$.

Proof. (i) Let $x_\lambda \in FIUL(A_n)$. Then for each fuzzy quasi-neighborhood U of a fuzzy point x_λ, $\{n \in \mathcal{N} : A_n\hat{q}U\} \notin \mathcal{I}$. Since $\{n \in \mathcal{N} : A_n\hat{q}U\} \subset \{n \in \mathcal{N} : B_n\hat{q}U\}$, $\{n \in \mathcal{N} : B_n\hat{q}U\} \notin \mathcal{I}$.

(ii) By (i), $FIUL(A_n) \vee FIUL(B_n) \leq FIUL(A_n \vee B_n)$. Now let $x_\lambda \notin FIUL(A_n)$ and $x_\lambda \notin FIUL(B_n)$. Then there exist fuzzy quasi-neighborhoods U_1 and U_2 of the fuzzy point x_λ such that $\{n \in \mathcal{N} : A_n\hat{q}U_1\} \in \mathcal{I}$ and $\{n \in \mathcal{N} : B_n\hat{q}U_2\} \in \mathcal{I}$. Consider $U = U_1 \wedge U_2$. Then $\{n \in \mathcal{N} : (A_n \vee B_n)\hat{q}U\} \subset \{n \in \mathcal{N} : A_n\hat{q}U_1\} \cup \{n \in \mathcal{N} : B_n\hat{q}U_2\}$. Thus $\{n \in \mathcal{N} : (A_n \vee B_n)\hat{q}U\} \in \mathcal{I}$ and so $x_\lambda \notin FIUL(A_n \vee B_n)$.

(iii) It follows from (i). □

Theorem 4.4. *Let $\{A_n : n \in \mathcal{N}\}$ and $\{B_n : n \in \mathcal{N}\}$ be any two nets of fuzzy sets of fuzzy topological spaces X and Y respectively. Then $FIUL(A_n \times B_n) \leq FIUL(A_n) \times FIUL(B_n)$.*

Proof. Let $(x,y)_\lambda \in FIUL(A_n \times B_n)$ and U_1 (in X) and U_2 (in Y) be fuzzy quasi-neighborhoods of the fuzzy points x_λ and y_λ respectively. Then $U_1 \times U_2$ is fuzzy quasi-neighborhoods of $(x,y)_\lambda$ in $X \times Y$. So $\{n \in \mathcal{N} : (A_n \times B_n)\hat{q}(U_1 \times U_2)\} \notin \mathcal{I}$. Clearly $\{n \in \mathcal{N} : (A_n \times B_n)\hat{q}(U_1 \times U_2)\} \subset \{n \in \mathcal{N} : A_n\hat{q}U_1\}$ and $\{n \in \mathcal{N} : (A_n \times B_n)\hat{q}(U_1 \times U_2)\} \subset \{n \in \mathcal{N} : B_n\hat{q}U_2\}$. So $x_\lambda \in FIUL(A_n)$ and $y_\lambda \in FIUL(A_n)$. □

Definition 4.5. Let $\{A_n : n \in \mathcal{N}\}$ be a net of fuzzy sets of a fuzzy topological space X. Then its lower fuzzy $\mathcal{I}$-limit is denoted and defined by $FILL(A_n) = \vee\{x_\lambda \in FP(X)$: for every fuzzy quasi-neighborhood U of x_λ, $\{n \in \mathcal{N} : A_n\bar{q}U\} \in \mathcal{I}\}$. If $FILL(A_n) = FIUL(A_n)$, then the net $\{A_n : n \in \mathcal{N}\}$ is said to converge to the limit $FL(A_n)(= FILL(A_n) = FIUL(A_n))$.

Theorem 4.6. *Let $\{A_n : n \in \mathcal{N}\}$ be a net of fuzzy sets of a fuzzy topological space X. Then the following properties hold:*
(i) $FILL(A_n)$ is a closed set.
(ii) $FIUL(A_n) = FILL(cl(A_n))$.
(iii) $FILL(A_n) \leq cl(\vee_{i=1}^{\infty} A_i)$.
(iv) If for each $n \in \mathcal{N}$, $A_n = A \in I^X$, then $FILL(A_n) = cl(A)$.
(v) $\wedge_{i=1}^{\infty} A_i \leq FILL(A_n)$.
(vi) $FILL(A_n) \leq FIUL(A_n)$.

Proof. The proofs of (i),(ii) and (iii) are parallel to the proofs of Theorem 4.2. So we prove only (iv)-(vi).
(iv) (iii) implies that $FILL(A_n) \leq cl(A)$. So let $x_\lambda \notin FILL(A_n)$. Then there exists a fuzzy quasi-neighborhood U of x_λ satisfying $\{n \in \mathcal{N} : A_n\bar{q}U\} \notin \mathcal{I}$. Since $\varnothing \in \mathcal{I}$, there exists an $m \in \mathcal{N}$ such that $A_m\bar{q}U$, that is, $A\bar{q}U$ and so $x_\lambda \notin cl(A)$.

(v) Suppose $x_\lambda \notin FILL(A_n)$. Then there exists a fuzzy quasi-neighborhood U of x_λ satisfying $\{n \in \mathcal{N} : A_n\bar{q}U\} \notin \mathcal{I}$. So there exists an $m \in \mathcal{N}$ such that $A_m\bar{q}U$. So, for every $y \in X$, $A_m(y) + U(y) \leq 1$. Again since U is a fuzzy quasi-neighborhood of x_λ, $U(x) + \lambda > 1$. Thus $A_m(x) < \lambda$, that is, $x_\lambda \notin A_m$.

(vi) Let $x_\lambda \notin FIUL(A_n)$. There exists a fuzzy quasi-neighborhood U of x_λ such that $\{n \in \mathcal{N} : A_n\hat{q}U\} \in \mathcal{I}$. since $\mathcal{N} \notin \mathcal{I}$, $\{n \in \mathcal{N} : A_n\bar{q}U\} \notin \mathcal{I}$. □

Theorem 4.7. *Let $\{A_n : n \in \mathcal{N}\}$ and $\{B_n : n \in \mathcal{N}\}$ be any two nets of fuzzy sets of a fuzzy topological space X. Then:*
(i) For all $n \in \mathcal{N}$, $A_n \leq B_n$ implies that $FILL(A_n) \leq FILL(B_n)$.
(ii) $FILL(A_n \vee B_n) \geq FILL(A_n) \vee FILL(B_n)$.
(iii) $FILL(A_n \wedge B_n) \leq FILL(A_n) \wedge FILL(B_n)$.
(iv) $FIL(A_n \wedge B_n) \leq FIL(A_n) \wedge FIL(B_n)$.
(v) $FIL(A_n \vee B_n) = FIL(A_n) \vee FIL(B_n)$.

Proof. (iv) and (v) follow from Theorem 4.3 (ii) and (iii) follow from (i). So we prove only (i).

Let $x_\lambda \in FILL(A_n)$. Then for each fuzzy quasi-neighborhood U of a fuzzy point x_λ, $\{n \in \mathcal{N} : A_n \bar{q} U\} \in \mathcal{I}$. Since $\{n \in \mathcal{N} : B_n \bar{q} U\} \subset \{n \in \mathcal{N} : A_n \bar{q} U\}$, $\{n \in \mathcal{N} : B_n \bar{q} U\} \in \mathcal{I}$. □

Theorem 4.8. *Let $\{A_n : n \in \mathcal{N}\}$ and $\{B_n : n \in \mathcal{N}\}$ be any two nets of fuzzy sets of a fuzzy topological spaces X and Y respectively. Then*
(i) $FILL(A_n \times B_n) \leq FILL(A_n) \times FILL(B_n)$.
(ii) $FIL(A_n \times B_n) \leq FIL(A_n) \times FIL(B_n)$.

Proof. The proof is parallel to the proof of Theorem 4.4. □

5. Applications

In this section, we have given some applications of the concepts studied in the earlier section.

Lowen [29] defined weakly fuzzy compactness in a fuzzy topological spaces as follows:

A fuzzy topological space X is called weakly fuzzy compact if for every fuzzy open cover $\{U_\alpha : \alpha \in \Delta\}$ of X and for each $\epsilon > 0$, there exists finite number of indices $\alpha_1, \alpha_2, ..., \alpha_p \in \Delta$ such that $\vee_{i=1}^{p} U_i \geq \overline{1-\epsilon}$.

Theorem 5.1. *A fuzzy topological space X is weakly fuzzy compact if and only if for every net $\{F_n : n \in \mathcal{N}\}$ of fuzzy closed sets, for every ideal $\mathcal{I}$ on $\mathcal{N}$ with $FIUL(F_n) = \bar{0}$ and for every $\epsilon > 0$, $\{n \in \mathcal{N} : F_n \not\leq \bar{\epsilon}\} \in \mathcal{I}$.*

Proof. Let X be weakly fuzzy compact and $\{F_n : n \in \mathcal{N}\}$ be a net of fuzzy closed sets, $\mathcal{I}$ be an ideal on $\mathcal{N}$ with $FIUL(F_n) = \bar{0}$ and $\epsilon > 0$. Then for each fuzzy point x_λ of X, there exists a fuzzy quasi-neighborhood U_{x_λ} of x_λ such that $\{n \in \mathcal{N} : F_n \hat{q} U_{x_\lambda}\} \in \mathcal{I}$. Since X is weakly fuzzy compact and $\{U_{x_\lambda} : x_\lambda \in FP(X)\}$ is a fuzzy open cover of X, there exists finite number of fuzzy points $e_1, e_2, ..e_p \in FP(X)$ such that $\vee_{i=1}^{p} U_{e_i} \geq \overline{1-\epsilon}$. Here $\{n \in \mathcal{N} : F_n \hat{q} \vee_{i=1}^{p} U_{e_i}\} = \bigcup_{i=1}^{p} \{n \in \mathcal{N} : F_n \hat{q} U_{e_i}\} \in \mathcal{I}$. Since $\{n \in \mathcal{N} : F_n \not\leq \bar{\epsilon}\} \subset \{n \in \mathcal{N} : F_n \hat{q} \vee_{i=1}^{p} U_{e_i}\}$, $\{n \in \mathcal{N} : F_n \not\leq \bar{\epsilon}\} \in \mathcal{I}$. □

Gantner et. al. [17] introduced the concept of strongly fuzzy compact in fuzzy topological space as follows:

A fuzzy topological space X is called strongly fuzzy compact if for each $\alpha \in [0,1)$ and each family $\mathcal{U}_\alpha$ with the property that for each $x \in X$, there exists an $U \in \mathcal{U}_\alpha$ satisfying $U(x) > \alpha$, has finite subfamily satisfying the same property.

Theorem 5.2. *A fuzzy topological space X is strongly fuzzy compact if and only if for each $\alpha \in [0,1)$, for each net $\{F_n : n \in \mathcal{N}\}$ of fuzzy closed sets and for every ideal $\mathcal{I}$ on $\mathcal{N}$ with $FIUL(F_n) \leq \overline{1-\alpha}$, $\{n \in \mathcal{N} : F_n \not\leq \overline{1-\alpha}\} \in \mathcal{I}$.*

Proof. Let X be strongly fuzzy compact and $\{F_n : n \in \mathcal{N}\}$ be a net of fuzzy closed sets, $\mathcal{I}$ be an ideal on $\mathcal{N}$ with $FIUL(F_n) \leq \overline{1-\alpha}$ and an $\alpha \in [0,1)$. Then for each fuzzy point x_λ of X satisfying $FIUL(F_n) < x_\lambda \leq \overline{1-\alpha}$, there exists a fuzzy quasi-neighborhood U_{x_λ} of x_λ such that $\{n \in \mathcal{N} : F_n \hat{q} U_{x_\lambda}\} \in \mathcal{I}$. Since X is strongly fuzzy compact and $\{U_{x_\lambda} : x_\lambda \in FP(X), FIUL(F_n) < x_\lambda \leq \overline{1-\alpha}\}$ satisfies the property that for each $x \in X$, $U_{x_\lambda}(x) > 1 - \lambda \geq \alpha$, there exist finite number of fuzzy points $e_1, e_2, ..e_p \in FP(X)$ such that for each $x \in X$, $U_{e_i}(x) > \alpha$ for some $i \in \{1, 2, ..p\}$. Here $\{n \in \mathcal{N} : F_n \hat{q} \vee_{i=1}^{p} U_{e_i}\} = \bigcup_{i=1}^{p} \{n \in \mathcal{N} : F_n \hat{q} U_{e_i}\} \in \mathcal{I}$. Since $\{n \in \mathcal{N} : F_n \not\leq \overline{1-\alpha}\} \subset \{n \in \mathcal{N} : F_n \hat{q} \vee_{i=1}^{p} U_{e_i}\}$, $\{n \in \mathcal{N} : F_n \not\leq \overline{1-\alpha}\} \in \mathcal{I}$. □

Lowen [29] defined fuzzy compactness in a fuzzy topological spaces as follows:

A fuzzy topological spaces X is called fuzzy compact if for each family $\mathcal{U}$ of fuzzy open sets of X and for each $\alpha \in [0, 1]$ such that $\vee\{U : U \in \mathcal{U}\} \geq \bar{\alpha}$ and for each $\epsilon \in (0, \alpha]$, there exists a finite subfamily $\mathcal{U}_0 \subset \mathcal{U}$ satisfying $\vee\{U : U \in \mathcal{U}_0\} \geq \overline{\alpha - \epsilon}$.

Theorem 5.3. *A fuzzy topological space X is fuzzy compact if and only if for each $\alpha \in [0, 1]$, for each net $\{F_n : n \in \mathcal{N}\}$ of fuzzy closed sets and for every ideal $\mathcal{I}$ on $\mathcal{N}$ with $FIUL(F_n) \leq \overline{1-\alpha}$ and for each $\epsilon \in (0, \alpha]$, $\{n \in \mathcal{N} : F_n \not\leq \overline{1-\alpha+\epsilon}\} \in \mathcal{I}$.*

Proof. The proof is analogous to that of Theorem 5.2. □

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